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Results on fixed circles and discs for $L_{(\omega,C)}$-contractions and related applications

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Abstract

In this paper, we study the behavior of $L_{(\omega,C)}$-contraction mappings and establish some results on common fixed circles and discs. We explain the significance of our main theorems through examples and applications.

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1 Introduction

The Banach contraction principle [1] and its generalizations have been applied in various disciplines of mathematics, economics, and engineering. One of the interesting applications of Banach contraction principle is to study the graph neural network model [2] (see also [3, 4]). In some cases when we do not have uniqueness of the fixed point, such a map fixes a circle which we call a fixed circle, the fixed-circle problem arises naturally in practice. There exist a lot of examples of self-mappings that map a circle onto themselves and fix all the points of the circle, whereas the circle is not fixed by the self-mapping.

Take $x_0 \in \Im$ and given $\ell \geq 0$ an arbitrary real. A circle and a disc with center x_0 and radius ℓ are defined on a metric space $(\Im, d)$ as follows, respectively:

$$C_{(x_0,\ell)} = \{x \in \Im : d(x_0,x) = \ell\},$$

and

$$D_{(x_0,\ell)} = \{x \in \Im : d(x_0,x) \leq \ell\}.$$

A mapping $\phi : \Im \to \Im$ fixes the circle $C_{(x_0,\ell)}$ (resp. the disc $D_{(x_0,\ell)}$) if $\phi(x) = x$ for all $x \in C_{(x_0,\ell)}$(resp. $x \in D_{(x_0,\ell)}$).

Denote by $\mathbb{C}$ the set of all complex numbers with the usual metric $d(r_1,r_2) = |r_1 - r_2|$ for all $r_1, r_2 \in \mathbb{C}$, and let $\phi : \mathbb{C} \to \mathbb{C}$ be defined by

$$\phi(r) = \begin{cases} \frac{1}{r} & \text{if } r \neq 0, \\ 0 & \text{if } r = 0, \end{cases}$$

where $\overline{r}$ is the conjugate of r. Then ϕ fixes all points of the circle $\mathbb{C}_{0,1} = \{r \in \mathbb{C} : |r| \leq 1\}$. It is worth pointing out that there exist some mappings which map the circle $\mathbb{C}_{r_0,\iota}$ to themselves, but do not fix all points of the circle $\mathbb{C}_{r_0,\iota}$. For instance, let $\psi : \mathbb{C} \to \mathbb{C}$ be defined by

$$\psi(r) = \begin{cases} \frac{1}{r} & \text{if } r \neq 0, \\ 0 & \text{if } r = 0. \end{cases}$$

Then $\psi(\mathbb{C}_{0,1}) = \mathbb{C}_{0,1}$, but ψ does not fix all points of $\mathbb{C}_{0,1}$. In fact, the mapping ψ fixes only two points of the unit circle.

The results on fixed circles and fixed discs have been studied in metric and generalized metric spaces via different contractive conditions (see [5–14]).

The concept of an F-contraction given by Wardowski [15] proved to be another milestone in fixed point theory. Numerous research papers on F -contractions have been published, see for instance [16–32].

Jleli and Samet [33, 34] introduced another generalization of BCP, known as a θ-contraction, and obtained some fixed point theorems for θ-contraction type mappings. Liu [35] introduced the concept of (Υ, Λ)-contraction and established fixed point results for such mappings in metric spaces. Recently, Ameer [36] introduced common fixed point results for generalized multivalued $(\alpha_K^*, \Upsilon, \Lambda)$-contractions in α_K-complete partial b-metric spaces. Moreover, Ameer [37, 38] introduced common fixed point results for generalized multivalued (Υ, Λ)-contractions in complete metric and b-metric spaces. Ameer et al. [39] initiated the notion of rational $(\Lambda, \Upsilon, \Re)$-contractive pair of mappings (where $\Re$is a binary relation) and established new common fixed point results for these mappings in complete metric spaces, see also ([40, 41]).

In this paper, we study mappings that not only fix one element, but also fix a well-defined set of "fixed points" which is either a circle, or a disc. We present some results on fixed circles and discs for some types of contraction self-mappings, namely $L_{(\omega,C)}$-contractions [42], Ćirić [43] type $L_{(\omega,C)}$-contractions, and Hardy–Rogers [44] type $L_{(\omega,C)}$-contractions in the setting of metric spaces. We discuss some results on fixed circles (discs) of integral type contractive single-valued maps. Moreover, we give an application of our obtained results to a discontinuous self-mapping that has a fixed circle.

2 Preliminaries

First, let $F : (0,\infty) \to (-\infty,\infty)$ be such that

(F_1) F is strictly increasing;

(F_2) for each sequence $\{\gamma_n\}_{n=1}^{\infty} \subset (0,\infty)$,

$$\lim_{n\to\infty} F(\gamma_n) = -\infty \quad \Leftrightarrow \quad \lim_{n\to\infty} \gamma_n = 0;$$

(F_3) there is $\kappa \in (0,1)$ such that $\lim_{t\to 0^+} t^\kappa F(t) = 0$.

Define $\mathcal{F}_1 = \{F : (0,\infty) \to (-\infty,\infty), F \text{ satisfies } (F_1)\}$ and $\mathcal{F}_2 = \{F : (0,\infty) \to (-\infty,\infty), F \text{ satisfies } (F_1) - (F_3)\}$.

Definition 2.1 ([15]) Let $T:\chi\to\chi$ be defined on a metric space (χ,d). Such T is named an F-contraction if there exist $\tau>0$ and $F\in\mathcal{F}_2$ such that

$$\ell,\jmath\in\chi,\quad d\bigl(T(\ell),T(\jmath)\bigr)>0\quad\Rightarrow\quad\tau+F\bigl(d\bigl(T(\ell),T(\jmath)\bigr)\bigr)\le F\bigl(d(\ell,\jmath)\bigr).\tag{2.1}$$

The related fixed point result is as follows:

Theorem 2.2 ([15]) *Each F- contraction self-mapping on a complete metric space admits a unique fixed point.*

Piri *and Kumam* [45] replaced assumption (F_3) of F by:
$(\acute{F}_3)$ F is continuous.
Take $\mathcal{F}^*=\{F:(0,\infty)\to(-\infty,\infty), F \text{ verifies } (F_1),(F_2), \text{ and } (\acute{F}_3)\}$.

Definition 2.3 ([45]) $T:\chi\to\chi$ defined on a metric space (χ,d) is named an F-contraction (with $F\in\mathcal{F}^*$) if there exist $\tau>0$ and $F\in\mathcal{F}^*$ such that (2.1) holds.

The main result of Piri *and Kumam* [45] is as follows.

Theorem 2.4 ([45]) *Each F-contraction self-mapping* (*with* $F\in\mathcal{F}^*$) *on a complete metric space admits a unique fixed point.*

Remark 2.5 Mention that the function F given as $F(\varsigma)=-\frac{1}{\varsigma^q}$ (with $q\ge1$) is in the set $\mathcal{F}^*$, but it does not belong to $\mathcal{F}_2$. Also, the function F defined by

$$F(\varsigma)=-\frac{1}{(\varsigma+[\varsigma])^k},\quad k\in\left(0,\frac{1}{l}\right), l>1,$$

is in $\mathcal{F}_2$, but does not belong to $\mathcal{F}^*$. However, there is also at least one function F defined by $F(\varsigma)=\ln(\varsigma)$ which belongs to both $\mathcal{F}^*$ and $\mathcal{F}_2$. Hence, the families $\mathcal{F}^*$ and $\mathcal{F}_2$ are overlapping.

In 2014, the concept of θ-contractions was initiated by Jleli and Samet [33].

The self-mapping T defined on the metric space (χ,d) is named a θ-contraction, whenever there exist $\theta\in\Theta$ and $k\in(0,1)$ such that

$$\ell,\jmath\in\chi,\quad d\bigl(T(\ell),T(\jmath)\bigr)\ne0\quad\Longrightarrow\quad\theta\bigl(d\bigl(T(\ell),T(\jmath)\bigr)\bigr)\le\bigl[\theta(d(\ell,\jmath)\bigr]^k,$$

where $\Theta=\{\theta:(0,\infty)\to(1,\infty)|\theta \text{ verifies } (\Theta_1)-(\Theta_3)\}$
(Θ_1) θ is nondecreasing;
(Θ_2) for each sequence $\{t_n\}$ in $(0,\infty)$,

$$\lim_{n\to\infty}\theta(t_n)=1\quad\text{iff}\quad\lim_{n\to\infty}t_n=0^+;$$

(Θ_3) there are $\rho\in(0,1)$ and $\vartheta\in(0,\infty]$ such that $\lim_{t\longrightarrow0^+}\frac{\theta(t)-1}{t^\rho}=\vartheta$.
Define $\Psi=\{\Theta:\Theta \text{ satisfies } (\Theta_1)\}$.

Theorem 2.6 ([33]) *Each* θ*-contraction mapping on a complete metric space admits a unique fixed point.*

3 Results on fixed circles and discs

Throughout this paper, we denote by Ξ be the set of functions $L:(0,\infty)\to(0,\infty)$ such that L is nondecreasing. We consider $\Omega^* = \{\omega:(0,\infty)\to(0,\infty)|\omega$ satisfies (ω_1) and $(\omega_2)\}$:

(ω_1) ω is monotone increasing, that is, $x<\varsigma$ implies $\omega(x)\le\omega(\varsigma)$;

(ω_2) $\omega(t)<t$ for every $t>0$.

In this section, we initiate the notion of $L_{(\omega,C)}$-contractions, Ćirić type $L_{(\omega,C)}$-contractions, and $L_{(\omega,C)}$-weak contractions and establish some related results on fixed circles (discs).

Definition 3.1 The mapping $\phi:\chi\to\chi$ is called an $L_{(\omega,C)}$-contraction if there are $\omega\in\Omega^*$, $L\in\Xi$, and $x_0\in\chi$ such that, for all $x\in\chi$,

$$d\bigl(x,\phi(x)\bigr)>0 \quad\Rightarrow\quad L\bigl(d\bigl(x,\phi(x)\bigr)\bigr)\le\omega\bigl(L\bigl(d(x,x_0)\bigr)\bigr).$$

Definition 3.2 The mapping $\phi:\chi\to\chi$ is said to be a Ćirić type $L_{(\omega,C)}$-contraction if there are $\omega\in\Omega^*$, $L\in\Xi$, and $x_0\in\chi$ such that, for all $x\in\chi$,

$$d\bigl(x,\phi(x)\bigr)>0 \quad\Rightarrow\quad L\bigl(d\bigl(x,\phi(x)\bigr)\bigr)\le\omega\bigl(L\bigl(M_c(x,x_0)\bigr)\bigr),$$

where

$$M_c(x,x_0)=\max\left\{\begin{array}{c} d(x,x_0), d(x,\phi(x)),\\ d(x_0,\phi(x_0)), \frac{d(x,\phi(x_0))+d(x_0,\phi(x))}{2}\end{array}\right\}.$$

Definition 3.3 The mapping $\phi:\chi\to\chi$ is said to be an $L_{(\omega,C)}$-weak contraction if there exist $\omega\in\Omega^*$, $L\in\Xi$, and $x_0\in\chi$ such that, for all $x\in\chi$,

$$d\bigl(x,\phi(x)\bigr)>0 \quad\Rightarrow\quad L\bigl(d\bigl(x,\phi(x)\bigr)\bigr)\le\omega\bigl(L\bigl(M_c(x,x_0)\bigr)\bigr),$$

where

$$M_c(x,x_0)=\max\left\{\begin{array}{c} d(x,x_0), \nu d(x,\phi(x))+(1-\nu)\,d(x_0,\phi(x_0)),\\ \nu d(x_0,\phi(x_0))+(1-\nu)\,d(x,\phi(x)), \frac{d(x,\phi(x_0))+d(x_0,\phi(x))}{2}\end{array}\right\}, \tag{3.1}$$

and $\nu\in[0,1)$.

Remark 3.4 Take $\nu=0$ in Definition 3.3, then Definition 3.3 reduces to Definition 3.2.

Proposition 3.5 *Let (χ,d) be a metric space and $\phi:\chi\to\chi$ be an $L_{(\omega,C)}$-weak contraction with $x_0\in\chi$, then $x_0=\phi(x_0)$.*

Proof Suppose that $x_0\ne\phi(x_0)$. By Definition 3.2, one gets

$$L\bigl(d\bigl(x_0,\phi(x_0)\bigr)\bigr)\le\omega\bigl(L\bigl(M_c(x_0,x_0)\bigr)\bigr),$$

where

$$M_c(x_0,x_0)=\max\left\{\begin{array}{c} d(x_0,x_0), d(x_0,\phi(x_0)),\\ d(x_0,\phi(x_0)), \frac{d(x_0,\phi(x_0))+d(x_0,\phi(x_0))}{2}\end{array}\right\}.$$

This implies that

$$L\bigl(d\bigl(x_0,\phi(x_0)\bigr)\bigr) \le \omega\bigl(L\bigl(d\bigl(x_0,\phi(x_0)\bigr)\bigr)\bigr) < L\bigl(d\bigl(x_0,\phi(x_0)\bigr)\bigr).$$

It is a contradiction. Therefore, $x_0 = \phi(x_0)$. □

Now, we introduce our result on fixed circles.

Theorem 3.6 *Let (χ,d) be a metric space, $\phi : \chi \to \chi$, and $\ell = \inf\{d(x,\phi(x)) : x \neq \phi(x)\}$. If ϕ is an $L_{(\omega,C)}$-weak contraction, $x_0 \in \chi$, and $d(x_0,\phi(x)) = \ell$ for all $x \in C_{(x_0,\ell)}$, then ϕ fixes the circle $C_{(x_0,\ell)}$.*

Proof Let $x \in C_{(x_0,\ell)}$. Suppose that $x \in C_{(x_0,\ell)}$, $x \neq \phi(x)$. By definition of ℓ, we have $d(x,\phi(x)) \geq \ell$. By Proposition 3.5 and (3.1), we get

$$L\bigl(d\bigl(x,\phi(x)\bigr)\bigr) \le \omega\bigl(L\bigl(M_c(x,x_0)\bigr)\bigr) < L\bigl(M_c(x,x_0)\bigr), \tag{3.2}$$

where

$$\begin{aligned} M_c(x,x_0) &= \max\left\{\begin{matrix} d(x,x_0), \nu d(x,\phi(x)) + (1-\nu)\, d(x_0,\phi(x_0)), \\ \nu d(x_0,\phi(x_0)) + (1-\nu)\, d(x,\phi(x)), \frac{d(x,\phi(x_0))+d(x_0,\phi(x))}{2} \end{matrix}\right\} \\ &= \max\bigl\{\ell, \nu d\bigl(x,\phi(x)\bigr), (1-\nu)\, d\bigl(x,\phi(x)\bigr)\bigr\}. \end{aligned}$$

Now, we have three cases:

(1) $\max\{\ell, \nu d(x,\phi(x)), (1-\nu)\, d(x,\phi(x))\} = \ell$, then from (3.2), the definition of ℓ, and the monotony of L, we get

$$L(\ell) \le \omega\bigl(L(\ell)\bigr) < L(\ell).$$

It is a contradiction.

(2) $\max\{\ell, \nu d(x,\phi(x)), (1-\nu)\, d(x,\phi(x))\} = \nu d(x,\phi(x))$, then we have two possibilities, $\nu = 0$ or $\nu \in (0,1)$. If $\nu = 0$, then from (3.2) we have

$$L\bigl(d\bigl(x,\phi(x)\bigr)\bigr) \le \omega\bigl(L(0)\bigr) < L(0),$$

which contradicts the definition of L. Suppose that $\nu \in (0,1)$. Hence, from (3.2) and the monotony of L, we get

$$\begin{aligned} L\bigl(d\bigl(x,\phi(x)\bigr)\bigr) &\le \omega\bigl(L\bigl(\nu d\bigl(x,\phi(x)\bigr)\bigr)\bigr) < L\bigl(\nu d\bigl(x,\phi(x)\bigr)\bigr) \\ &\le L\bigl(d\bigl(x,\phi(x)\bigr)\bigr), \end{aligned}$$

which is a contradiction.

(3) $\max\{\ell, \nu d(x,\phi(x)), (1-\nu)\, d(x,\phi(x))\} = (1-\nu)\, d(x,\phi(x))$, then from (3.2) and the monotony of L, we get

$$L\bigl(d\bigl(x,\phi(x)\bigr)\bigr) \le \omega\bigl(L\bigl((1-\nu)\, d\bigl(x,\phi(x)\bigr)\bigr)\bigr)$$

$$\leq \omega\big(L\big(d\big(x,\phi(x)\big)\big)\big) < L\big(d\big(x,\phi(x)\big)\big).$$

It is a contradiction. Therefore, $\phi(x) = x$ for all $x \in C_{(x_0,\ell)}$. Thus, $C_{(x_0,\ell)}$ is a fixed circle of ϕ. □

The following is a result on fixed discs.

Theorem 3.7 *Let (χ, d) be a metric space, $\phi : \chi \to \chi$, and $\ell = \inf\{d(x,\phi(x)) : x \neq \phi(x)\}$. If ϕ is an $L_{(\omega,C)}$-weak contraction, $x_0 \in \chi$, and $d(x_0,\phi(x)) = \ell$ for all $x \in D_{(x_0,\ell)}$, then ϕ fixes the disc $D_{(x_0,\ell)}$.*

Proof The mapping ϕ fixes the disc $C_{(x_0,\ell)}$ (from Theorem 3.6). Now, to show that ϕ fixes the disc $D_{(x_0,\ell)}$, it is sufficient to prove that ϕ fixes any circle $C_{(x_0,\varrho)}$ with $\varrho < \ell$. Let $x \in C_{(x_0,\varrho)}$. Suppose that $x \in C_{(x_0,\ell)}$ with $x \neq \phi(x)$. From Proposition 3.5 and (3.1), we get

$$L\big(d\big(x,\phi(x)\big)\big) \leq \omega\big(L\big(M_c(x,x_0)\big)\big) < L\big(M_c(x,x_0)\big), \tag{3.3}$$

where

$$\begin{aligned} M_c(x,x_0) &= \max\left\{\begin{matrix} d(x,x_0), \nu d(x,\phi(x)) + (1-\nu)d(x_0,\phi(x_0)), \\ \nu d(x_0,\phi(x_0)) + (1-\nu)d(x,\phi(x)), \frac{d(x,\phi(x_0))+d(x_0,\phi(x))}{2} \end{matrix}\right\} \\ &= \max\left\{\varrho, \nu d\big(x,\phi(x)\big), (1-\nu)d\big(x,\phi(x)\big), \frac{\varrho+\ell}{2}\right\}. \end{aligned}$$

Now, we have three cases:

(1) $\max\{\varrho, \nu d(x,\phi(x)), (1-\nu)d(x,\phi(x)), \frac{\varrho+\ell}{2}\} = \nu d(x,\phi(x))$, then we have two possibilities: $\nu = 0$ or $\nu \in (0,1)$. If $\nu = 0$, then from (3.3) we have

$$L\big(d\big(x,\phi(x)\big)\big) \leq \omega\big(L(0)\big) < L(0),$$

which contradicts the definition of L. Suppose that $\nu \in (0,1)$. Hence, from (3.3) and the monotony of L, we get

$$\begin{aligned} L\big(d\big(x,\phi(x)\big)\big) &\leq \omega\big(L\big(\nu d\big(x,\phi(x)\big)\big)\big) < L\big(\nu d\big(x,\phi(x)\big)\big) \\ &\leq L\big(d\big(x,\phi(x)\big)\big). \end{aligned}$$

It is a contradiction.

(2) $\max\{\varrho, \nu d(x,\phi(x)), (1-\nu)d(x,\phi(x)), \frac{\varrho+\ell}{2}\} = (1-\nu)d(x,\phi(x))$, then from (3.3) and the monotony of L, we get

$$\begin{aligned} L\big(d\big(x,\phi(x)\big)\big) &\leq \omega\big(L\big((1-\nu)d\big(x,\phi(x)\big)\big)\big) \\ &\leq \omega\big(L\big(d\big(x,\phi(x)\big)\big)\big) < L\big(d\big(x,\phi(x)\big)\big), \end{aligned}$$

which is a contradiction.

(3) $\max\{\varrho, \nu d(x,\phi(x)), (1-\nu)d(x,\phi(x)), \frac{\varrho+\ell}{2}\} = \frac{\varrho+\ell}{2}$, then from (3.3), the definition of ℓ, and the monotony of L, we get

$$\begin{aligned} L(\ell) \leq L\big(d\big(x,\phi(x)\big)\big) &\leq \omega\left(L\left(\frac{\varrho+\ell}{2}\right)\right) \\ &\leq \omega\big(L(\ell)\big) < L(\ell). \end{aligned}$$

It is a contradiction. Therefore, $\phi(x) = x$ for all $x \in D_{(x_0,\ell)}$. Thus, ϕ fixes the disc $D_{(x_0,\ell)}$. □

Corollary 3.8 *Let (χ,d) be a metric space, $\phi:\chi \to \chi$, and $\ell = \inf\{d(x,\phi(x)) : x \neq \phi(x)\}$. If ϕ is an $L_{(\omega,C)}$-contraction, $x_0 \in \chi$, and $d(x_0,\phi(x)) = \ell$ for all $x \in C_{(x_0,\ell)}$ (resp. $D_{(x_0,\ell)}$), then ϕ fixes the circle $C_{(x_0,\ell)}$ (resp. the disc $D_{(x_0,\ell)}$).*

Corollary 3.9 *Let (χ,d) be a metric space, $\phi:\chi \to \chi$, and $\ell = \inf\{d(x,\phi(x)) : x \neq \phi(x)\}$. If ϕ is a Ćirić type $L_{(\omega,C)}$-contraction, $x_0 \in \chi$, and $d(x_0,\phi(x)) = \ell$ for all $x \in C_{(x_0,\ell)}$(resp. the disc $D_{(x_0,\ell)}$), then ϕ fixes the circle $C_{(x_0,\ell)}$ (resp. the disc $D_{(x_0,\ell)}$).*

Example 3.10 Let $\chi = [-3,\infty)$ be a metric space endowed with the usual metric d. Define ϕ: $\chi \to \chi$ by

$$\phi(x) = \begin{cases} x & \text{if } -3 \leq x < 3, \\ x+2 & x \geq 3. \end{cases}$$

Then ϕ is an $L_{(\omega,C)}$-contraction. In fact, let $L(t) = te^t$, $x_0 = 0$, and $\omega(t) = \frac{t}{2}$. For all $x \in \chi$ such that $x \geq 3$, we have $d(x,\phi(x)) = 2 > 0$ and

$$L\big(d\big(x,\phi(x)\big)\big) = 2e^2 \leq \frac{1}{2}d(x,x_0)e^{d(x,x_0)} = \omega\big(L\big(d(x,x_0)\big)\big).$$

The mapping ϕ is an $L_{(\omega,C)}$-weak contraction. To show this, let $L(t) = te^t$, $x_0 = 0$, $\nu = \frac{1}{2}$, and $\omega = \frac{t}{2}$. For all $x \in \chi$ such that $x \geq 3$, we have $d(x,\phi(x)) = 2 > 0$ and

$$\begin{aligned} M_c(x,x_0) &= \max\left\{ \begin{matrix} d(x,x_0), \nu d(x,\phi(x)) + (1-\nu)d(x_0,\phi(x_0)), \\ \nu d(x_0,\phi(x_0)) + (1-\nu)d(x,\phi(x)), \frac{d(x,\phi(x_0))+d(x_0,\phi(x))}{2} \end{matrix} \right\} \\ &= \max\left\{ \begin{matrix} d(x,x_0), \frac{1}{2}d(x,\phi(x)), \\ \frac{1}{2}d(x,\phi(x)), \frac{d(x,\phi(x_0))+d(x_0,\phi(x))}{2} \end{matrix} \right\} \\ &= \max\left\{ d(x,x_0), \frac{1}{2}d\big(x,\phi(x)\big), \frac{d(x,\phi(x_0)) + d(x_0,\phi(x))}{2} \right\} \\ &= \max\left\{ |x|, 1, \frac{|x| + |x+2|}{2} \right\} = \frac{|x| + |x+2|}{2}. \end{aligned}$$

Also,

$$\begin{aligned} L\big(d\big(x,\phi(x)\big)\big) = 2e^2 &\leq \frac{1}{4}\big(|x| + |x+2|\big)e^{\frac{|x|+|x+2|}{2}} \\ &= \frac{1}{2}M_c(x,x_0)e^{M_c(x,x_0)} = \omega\big(L\big(M_c(x,x_0)\big)\big). \end{aligned}$$

We also have $\ell = \inf\{d(x,\phi(x)) : x \neq \phi(x)\} = 2$. Therefore, all the conditions of Theorem 3.6 and Theorem 3.7 (and also Corollary 3.8) are satisfied. Observe that ϕ fixes the circle $C_{0,2} = \{-2,2\}$ and the disc $D_{0,2} = [-2,2]$.

Next, we introduce the concepts of Reich type $L_{(\omega,C)}$-contractions, Chatterjea type $L_{(\omega,C)}$-contractions, and Hardy–Rogers type $L_{(\omega,C)}$-contractions.

Definition 3.11 The mapping $\phi : \chi \to \chi$ is said to be a Reich type $L_{(\omega,C)}$-contraction if there exist $\omega \in \Omega^*$, $L \in \Xi$, and $x_0 \in \chi$ so that, for all $x \in \chi$,

$$d\bigl(x,\phi(x)\bigr) > 0 \quad \Rightarrow \quad L\bigl(d\bigl(x,\phi(x)\bigr)\bigr) \leq \omega\left(L\begin{pmatrix} a_1 d(x,x_0), a_2 d(x,\phi(x)) \\ + a_3 d(x_0,\phi(x_0)) \end{pmatrix}\right),$$

where $a_1, a_2, a_3 \geq 0$ and $a_1 + a_2 + a_3 < 1$.

Definition 3.12 The mapping $\phi : \chi \to \chi$ is said to be a Chatterjea type $L_{(\omega,C)}$-contraction if there exist $\omega \in \Omega^*$, $L \in \Xi$, and $x_0 \in \chi$ such that, for all $x \in \chi$,

$$d\bigl(x,\phi(x)\bigr) > 0 \quad \Rightarrow \quad L\bigl(d\bigl(x,\phi(x)\bigr)\bigr) \leq \omega\bigl(L\bigl(b\bigl[d\bigl(x,\phi(x_0)\bigr) + d\bigl(x_0,\phi(x)\bigr)\bigr]\bigr)\bigr),$$

where b$\in (0, \frac{1}{2})$.

Definition 3.13 The mapping $\phi : \chi \to \chi$ is said to be a Hardy–Rogers type $L_{(\omega,C)}$-contraction if there exist $\omega \in \Omega^*$, $L \in \Xi$, and $x_0 \in \chi$ such that, for all $x \in \chi$,

$$\begin{aligned} &d\bigl(x,\phi(x)\bigr) > 0 \quad \Rightarrow \\ &L\bigl(d\bigl(x,\phi(x)\bigr)\bigr) \leq \omega\left(L\begin{pmatrix} a_1 d(x,x_0), a_2 d(x,\phi(x)) \\ + a_3 d(x_0,\phi(x_0)) + a_4 d(x,\phi(x_0)) + a_5 d(x_0,\phi(x)) \end{pmatrix}\right), \end{aligned} \tag{3.4}$$

where $a_1, a_2, a_3, a_4, a_3 \geq 0$ and $a_1 + a_2 + a_3 + a_4 + a_5 < 1$.

Proposition 3.14 *Let (χ,d) be a metric space and $\phi : \chi \to \chi$ be a Hardy–Rogers type $\Lambda_{(\Upsilon,C)}$- contraction with $x_0 \in \chi$, then $x_0 = \phi(x_0)$.*

Proof Suppose that $x_0 \neq \phi(x_0)$. By Definition 3.13, one gets

$$L\bigl(d\bigl(x_0,\phi(x_0)\bigr)\bigr) \leq \omega\left(L\begin{pmatrix} a_1 d(x_0,x_0) + a_1 d(x_0,\phi(x_0)), \\ a_3 d(x_0,\phi(x_0)) + a_4 d(x_0,\phi(x_0)) + a_5 d(x_0,\phi(x_0)) \end{pmatrix}\right).$$

It implies that

$$\begin{aligned} L\bigl(d\bigl(x_0,\phi(x_0)\bigr)\bigr) &\leq \omega\bigl(L\bigl((a_1 + a_2 + a_3 + a_4 + a_5)d\bigl(x_0,\phi(x_0)\bigr)\bigr)\bigr) \\ &< L\bigl(d\bigl(x_0,\phi(x_0)\bigr)\bigr). \end{aligned}$$

It is a contradiction. Therefore, $x_0 = \phi(x_0)$. □

Now, we introduce our result on fixed circles.

Theorem 3.15 *Let (χ, d) be a metric space, $\phi : \chi \to \chi$, and $\ell = \inf\{d(x,\phi(x)) : x \neq \phi(x)\}$. If ϕ is a Hardy–Rogers type $L_{(\omega,C)}$-contraction, $x_0 \in \chi$, and $d(x_0,\phi(x)) = \ell$ for all $x \in C_{(x_0,\ell)}$, then ϕ fixes the circle $C_{(x_0,\ell)}$.*

Proof Let $x \in C_{(x_0,\ell)}$. Suppose that $x \in C_{(x_0,\ell)}$, $x \neq \phi(x)$. By definition of ℓ, we have $d(x,\phi(x)) \geq \ell$. From Proposition 3.14 and (3.4), we get

$$\begin{aligned} L\bigl(d\bigl(x,\phi(x)\bigr)\bigr) &\leq \omega\left(L\left(\begin{array}{c} a_1 d(x,x_0) + a_2 d(x,\phi(x)) \\ + a_3 d(x_0,\phi(x_0)) + a_4 d(x,\phi(x_0)) + a_5 d(x_0,\phi(x)) \end{array}\right)\right) \\ &= \omega\left(L\left(\begin{array}{l} a_1\ell + a_2 d(x,\phi(x)) \\ + a_3\ell + a_4\ell + a_5\ell \end{array}\right)\right) \\ &\leq \omega\bigl(L\bigl((a_1+a_2+a_3+a_4+a_5)d\bigl(x,\phi(x)\bigr)\bigr)\bigr)\omega\bigl(L\bigl(d\bigl(x,\phi(x)\bigr)\bigr)\bigr) \\ &< L\bigl(d\bigl(x,\phi(x)\bigr)\bigr). \end{aligned}$$

It is a contradiction. Therefore, $\phi(x) = x$ for all $x \in C_{(x_0,\ell)}$. Thus, ϕ fixes the circle $C_{(x_0,\ell)}$. □

Theorem 3.16 *Let (χ, d) be a metric space, $\phi : \chi \to \chi$, and $\ell = \inf\{d(x,\phi(x)) : x \neq \phi(x)\}$. If ϕ is a Hardy–Rogers type $L_{(\omega,C)}$-contraction, $x_0 \in \chi$, and $d(x_0,\phi(x)) = \ell$ for all $x \in D_{(x_0,\ell)}$, then ϕ fixes the disc $D_{(x_0,\ell)}$.*

Proof The mapping ϕ fixes the circle $C_{(x_0,\ell)}$ (from Theorem 3.15). Now, to show that $D_{(x_0,\ell)}$ is a fixed disc of ϕ, it is sufficient to prove that ϕ fixes any circle $C_{(x_0,\varrho)}$ with $\varrho < \ell$. Let $x \in C_{(x_0,\varrho)}$. Suppose that $x \in C_{(x_0,\ell)}$, $x \neq \phi(x)$. From Proposition 3.14 and (3.4), we get

$$\begin{aligned} L\bigl(d\bigl(x,\phi(x)\bigr)\bigr) &\leq \omega\left(L\left(\begin{array}{c} a_1 d(x,x_0) + a_2 d(x,\phi(x)) \\ + a_3 d(x_0,\phi(x_0)) + a_4 d(x,\phi(x_0)) + a_5 d(x_0,\phi(x)) \end{array}\right)\right) \\ &= \omega\left(L\left(\begin{array}{l} a_1\varrho + a_2 d(x,\phi(x)) \\ + a_3\varrho + a_4\varrho + a_5\varrho \end{array}\right)\right) \\ &\leq \omega\bigl(L\bigl((a_1+a_2+a_3+a_4+a_5)d\bigl(x,\phi(x)\bigr)\bigr)\bigr)\omega\bigl(L\bigl(d\bigl(x,\phi(x)\bigr)\bigr)\bigr) \\ &< L\bigl(d\bigl(x,\phi(x)\bigr)\bigr). \end{aligned}$$

It is a contradiction. Therefore, $\phi(x) = x$. Thus, ϕ fixes the circle $D_{(x_0,\varrho)}$. □

Example 3.17 Let $\chi = \{2, 3, \ln(2e), \ln(2), \ln(\frac{2}{e})\}$ be endowed with the usual metric d. Define $\phi : \chi \to \chi$ as

$$\phi(x) = \begin{cases} 3, & \text{if } x = 2, \\ x, & \text{otherwise.} \end{cases}$$

Let $L(\mathrm{t}) = te^t$, $x_0 = \ln(2)$, $\omega(t) = \frac{9t}{10}$, $a_1 = a_2 = a_3 = a_4 = \frac{1}{5}$, and $a_5 = 0$. Then ϕ is a Hardy–Rogers type $L_{(\omega,C)}$-contraction. Indeed, for $x = 2$,

$$d\bigl(x,\phi(x)\bigr) = d\bigl(2,\phi(2)\bigr) = 1 > 0,$$

$$L\big(d\big(x,\phi(x)\big)\big)=L\big(d(2,3)\big)=e^{1},$$

and

$$\begin{aligned}
&\omega\left(L\begin{pmatrix}a_1d(x,x_0)+a_2d(x,\phi(x))+a_3d(x_0,\phi(x_0))\\ +a_4d(x,\phi(x_0))+a_5d(x_0,\phi(x))\end{pmatrix}\right)\\
&\quad=\omega\left(L\begin{pmatrix}\frac{1}{5}d(x,x_0)+\frac{1}{5}d(x,\phi(x))+\\ +\frac{1}{5}d(x,\phi(x_0))\end{pmatrix}\right)\\
&\quad=\omega\left(L\left(\frac{2}{5}+\frac{1}{5}+\frac{1}{5}|2-\ln 2|\right)\right)\\
&\quad=\omega\left(L\left(\frac{1}{5}\big(3+|2-\ln 2|\big)\right)\right)\frac{9}{50}\big(3+|2-\ln 2|\big)e^{\frac{1}{5}(3+|2-\ln 2|)}.
\end{aligned}$$

Thus,

$$L\big(d\big(x,\phi(x)\big)\big)\le\omega\left(L\begin{pmatrix}a_1d(x,x_0)+a_2d(x,\phi(x))+a_3d(x_0,\phi(x_0))\\ +a_4d(x,\phi(x_0))+a_5d(x_0,\phi(x))\end{pmatrix}\right).$$

We also have

$$\ell=\inf\big\{d\big(x,\phi(x)\big)\big):x\neq\phi(x)\big\}=\big\{d(2,3)\big)\big\}=1.$$

Hence, all the conditions of Theorem 3.15 and Theorem 3.16 are satisfied. Observe that ϕ fixes the circle $C_{\ln(2),1}=\{\ln(2e),\ln(\frac{2}{e})\}$ and the disc $D_{\ln(2),1}=\{\ln(2e),\ln 2,\ln(\frac{2}{e})\}$.

Corollary 3.18 *Let (χ,d) be a metric space, $\phi:\chi\to\chi$, and $\ell=\inf\{d(x,\phi(x)):x\neq\phi(x)\}$. If ϕ is a Reich type $L_{(\omega,C)}$-contraction, $x_0\in\chi$, and $d(x_0,\phi(x))=\ell$ for all $x\in C_{(x_0,\ell)}$(resp. $x\in D_{(x_0,\ell)}$), then ϕ fixes the circle $C_{(x_0,\ell)}$ (resp. the disc $D_{(x_0,\ell)}$).*

Corollary 3.19 *Let (χ,d) be a metric space, $\phi:\chi\to\chi$, and $\ell=\inf\{d(x,\phi(x)):x\neq\phi(x)\}$. If ϕ is a Chatterjea type $L_{(\omega,C)}$-contraction, $x_0\in\chi$, and $d(x_0,\phi(x))=\ell$ for all $x\in C_{(x_0,\ell)}$(resp. $x\in D_{(x_0,\ell)}$), then ϕ fixes the circle $C_{(x_0,\ell)}$ (resp. the disc $D_{(x_0,\ell)}$).*

4 Some consequences

Next corollaries are generalizations of fixed point results of Jleli and Samet [33] and Wardowski [15].

Corollary 4.1 *Let (χ,d) be a metric space, $\phi:\chi\to\chi$, and $\ell=\inf\{d(x,\phi(x)):x\neq\phi(x)\}$. If there exist $F\in\mathcal{F}_1$, $\tau>0$, and $x_0\in\chi$ such that, for all $x\in\chi$,*

$$d\big(x,\phi(x)\big)>0\quad\Rightarrow\quad\tau+F\big(d\big(x,\phi(x)\big)\big)\le F\big(M_c(x,x_0)\big),$$

where

$$M_c(x,x_0)=\max\left\{\begin{matrix}d(x,x_0),d(x,\phi(x)),\\ d(x_0,\phi(x_0)),\frac{d(x,\phi(x_0))+d(x_0,\phi(x))}{2}\end{matrix}\right\}$$

and $d(x_0,\phi(x))=\ell$ for all $x\in C_{(x_0,\ell)}$, then ϕ fixes the circle $C_{(x_0,\ell)}$.

Proof Take $\omega(t) = e^{-\tau}t$, $L(t) = e^{F(t)}$, and $\nu = 0$ in Theorem 3.6. □

Corollary 4.2 *Let (χ, d) be a metric space, $\phi : \chi \to \chi$, and $\ell = \inf\{d(x,\phi(x)) : x \neq \phi(x)\}$. If there exist $\theta \in \Psi$, $k \in (0,1)$, and $x_0 \in \chi$ so that, for all $x \in \chi$,*

$$d\big(x,\phi(x)\big) > 0 \quad \Rightarrow \quad \theta\big(d\big(x,\phi(x)\big)\big) \leq \big[\boldsymbol{\theta}\big(M_c(x,x_0)\big)\big]^k,$$

where

$$M_c(x,x_0) = \max\left\{\begin{array}{c} d(x,x_0), d(x,\phi(x)), \\ d(x_0,\phi(x_0)), \frac{d(x,\phi(x_0))+d(x_0,\phi(x))}{2} \end{array}\right\}$$

and $d(x_0,\phi(x)) = \ell$ for all $x \in C_{(x_0,\ell)}$, then ϕ fixes the circle $C_{(x_0,\ell)}$.

Proof Set $\omega(t) := kt$ and $L(t) = \ln(\theta(t))$ in Theorem 3.6. □

Denote by Λ the family of functions $\beta : [0,\infty) \to [0,\infty)$ so that $\lim_{r\to t^+} \beta(r) < 1$ for every $t \in (0,\infty)$

Corollary 4.3 *Let (χ, d) be a metric space, $\phi : \chi \to \chi$, and $\ell = \inf\{d(x,\phi(x)) : x \neq \phi(x)\}$. Let there exist a function $\beta \in \Lambda$ and $x_0 \in \chi$ so that, for all $x \in \chi$,*

$$d\big(x,\phi(x)\big) \leq \beta\big(M_c(x,x_0)\big).M_c(x,x_0),$$

where

$$M_c(x,x_0) = \max\left\{\begin{array}{c} d(x,x_0), d(x,\phi(x)), \\ d(x_0,\phi(x_0)), \frac{d(x,\phi(x_0))+d(x_0,\phi(x))}{2} \end{array}\right\}.$$

and $d(x_0,\phi(x)) = \ell$ for all $x \in C_{(x_0,\ell)}$. Then ϕ fixes the circle $C_{(x_0,\ell)}$.

Proof Take $\psi(t) := \beta(t)t$ and $\phi(t) = t$ in Theorem 3.6. □

5 Results on fixed circles and discs for integral type $L_{(\omega,C)}$-contractions

In this section, we discuss some results on fixed circles and discs for integral type $L_{(\omega,C)}$-contractions. Let $\Omega : [0,\infty) \to [0,\infty)$ be a locally integrable function such that, for every $t > 0$,

$$\int_0^t \Omega(u)\,du > 0.$$

Definition 5.1 The mapping $\phi : \chi \to \chi$ is said to be an integral type $L_{(\omega,C)}$-contraction if there exist $\omega \in \Omega^*$, $L \in \Xi$, and $x_0 \in \chi$ so that, for all $x \in \chi$,

$$d\big(x,\phi(x)\big) > 0 \quad \Rightarrow \quad \int_0^{L(d(x,\phi(x)))} \Omega(t)\,dt \leq \int_0^{\omega(L(d(x,x_0)))} \Omega(t)\,dt. \tag{5.1}$$

.

Proposition 5.2 *Let (χ, d) be a metric space and $\phi : \chi \to \chi$ be an integral type $\Lambda_{(\Upsilon,C)}$-contraction with $x_0 \in \chi$, then $x_0 = \phi(x_0)$.*

Proof Suppose that $x_0 \neq \phi(x_0)$. By Definition 5.1, one gets

$$\int_0^{L(d(x_0,\phi(x_0)))} \Omega(t)\,dt \leq \int_0^{\omega(L(d(x_0,x_0)))} \Omega(t)\,dt.$$

It contradicts $d(x_0, x_0) = 0$ and the definition of ω and L. Thus, $x_0 = \phi(x_0)$. □

Now, we introduce our result on fixed circles.

Theorem 5.3 *Let (χ, d) be a metric space, $\phi : \chi \to \chi$, and $\ell = \inf\{d(x,\phi(x)) : x \neq \phi(x)\}$. If ϕ is an integral type $L_{(\omega,C)}$-contraction, $x_0 \in \chi$, and $d(x_0, \phi(x)) = \ell$ for all $x \in C_{(x_0,\ell)}$, then ϕ fixes the circle $C_{(x_0,\ell)}$.*

Proof Let $x \in C_{(x_0,\ell)}$. Suppose that $x \in C_{(x_0,\ell)}$, $x \neq \phi(x)$. By definition of ℓ, we have $d(x,\phi(x)) \geq \ell$. Using the monotony of L, we get

$$L(\ell) \leq L\big(d\big(x,\phi(x)\big)\big), \tag{5.1}$$

hence

$$\int_0^{L(\ell)} \Omega(t)\,dt \leq \int_0^{L(d(x,\phi(x)))} \Omega(t)\,dt.$$

From (5.1), one gets

$$\begin{aligned}\int_0^{L(\ell)} \Omega(t)\,dt &\leq \int_0^{L(d(x,\phi(x)))} \Omega(t)\,dt \leq \int_0^{\omega(L(d(x,x_0)))} \Omega(t)\,dt \\ &< \int_0^{L(d(x,x_0))} \Omega(t)\,dt = \int_0^{L(\ell)} \Omega(t)\,dt.\end{aligned}$$

It is a contradiction. Therefore, $\phi(x) = x$ for all $x \in C_{(x_0,\ell)}$. Thus, ϕ fixed the circle $C_{(x_0,\ell)}$. □

The following theorem describes a result on fixed discs.

Theorem 5.4 *Let (χ, d) be a metric space, $\phi : \chi \to \chi$, and $\ell = \inf\{d(x,\phi(x)) : x \neq \phi(x)\}$. Let ϕ be an integral type $L_{(\omega,C)}$- contraction, $x_0 \in \chi$, and $d(x_0, \phi(x)) = \ell$ for all $x \in D_{(x_0,\ell)}$, then ϕ fixes the disc $D_{(x_0,\ell)}$.*

Proof ϕ fixes the disc $C_{(x_0,\ell)}$ (from Theorem 5.3). Now, we show that $D_{(x_0,\ell)}$ is a fixed disc of ϕ, it is sufficient to prove that ϕ fixes any circle $C_{(x_0,\varrho)}$ with $\varrho < \ell$. Let $x \in C_{(x_0,\varrho)}$. Suppose that $x \in C_{(x_0,\ell)}$, $x \neq \phi(x)$. From the definition of ℓ and the monotony of L, we get

$$L(\varrho) < L(\ell) \leq L\big(d\big(x,\phi(x)\big)\big) \tag{5.2}$$

and

$$\int_0^{L(\varrho)} \Omega(t)\,dt \leq \int_0^{L(d(x,\phi(x)))} \Omega(t)\,dt. \tag{5.3}$$

From (5.3), one gets

$$\int_0^{L(\varrho)} \Omega(t)\,dt \leq \int_0^{L(d(x,\phi(x)))} \Omega(t)\,dt \leq \int_0^{\omega(L(d(x,x_0)))} \Omega(t)\,dt$$
$$< \int_0^{L(d(x,x_0))} \Omega(t)\,dt = \int_0^{L(\varrho)} \Omega(t)\,dt.$$

It is a contradiction. Therefore, $\phi(x) = x$ for all $x \in D_{(x_0,\ell)}$. Thus, ϕ fixes the disc $D_{(x_0,\ell)}$. □

6 Applications to discontinuous functions

In this section, we give some examples of discontinuous functions and obtain a discontinuity result related to fixed circles.

Example 6.1 Let $\Im = \{1, 2, e^4 - 1, e^4, e^4 + 1\}$be the metric space with the usual metric. Let us define the self-mapping $\phi : \Im \to \Im$ by

$$\phi(u) = \begin{cases} 3, & \text{if } u < e^4 - 1, \\ u, & \text{if } u \geq e^4 - 1 \end{cases}$$

for all $u \in \Im$. As in Example 3.10, it is easily verified that the self-mapping ϕ is a Ćirić $L_{(\omega,C)}$-contractive self-mapping and $C_{e^4,1} = \{e^4 - 1, e^4 + 1\}$is a fixed circle of ϕ. We note that the self-mapping ϕ is continuous at the point $e^4 + 1$, while the self-mapping ϕ is discontinuous at the point $e^4 - 1$.

Example 6.2 Let $\Im = \{1, 2, e^4 - 1, e^4, e^4 + 1\}$be the metric space with the usual metric. Let us define the self-mapping $\phi : \Im \to \Im$ by

$$\phi(u) = \begin{cases} 3, & \text{if } u < e^4 - 1, \\ e^3 - 1 & \text{if } e^4 - 1 \leq u < e^4, \\ u & \text{if } e^4 \leq u \leq e^4 + 1, \end{cases}$$

for all $u \in \Im$. It is easily verified that the self-mapping ϕ is an $L_{(\omega,C)}$-weak contractive self-mapping and $C_{e^4,1} = \{e^4 - 1, e^4 + 1\}$ is a fixed circle of ϕ. We note that the self-mapping $\phi$is discontinuous at the center e^4 and on the circle $C_{e^4,1}$. We note that the self-mapping ϕ is continuous at the point $e^4 + 1$, while the self-mapping ϕ is discontinuous at the point $e^4 - 1$ and on the circle $C_{e^4,1}$

From the above examples, we give the following theorem.

Theorem 6.3 *Let ϕ be an $L_{(\omega,C)}$-weak contraction with $u_0 \in \Im$and ι be defined as in Theorem 3.7. If $d(u_0, \phi(u)) = \iota$ for all $u \in C_{u_0,\iota}$, then $C_{u_0,\iota}$ is a fixed circle of ϕ. Also, ϕ is discontinuous at $u \in C_{u_0,\iota}$ if and only if $\lim\limits_{\upsilon \to u} M_c(u, v) \neq 0$.*

Proof From Theorem 3.7, we see that $C_{u_0,\iota}$ is a fixed circle of ϕ. Using the idea given in Theorem 2.1 in [46] (see also [47]), we see that ϕ is discontinuous at $u \in C_{u_0,\iota}$ if and only if $\lim\limits_{\upsilon \to u} M_c(u, v) \neq 0$. □

7 Conclusion

We have introduced new generalized results on fixed circles and discs for $L_(\omega, C)$-contractive mappings, Ćirić type $L_{(\omega,C)}$-contractive mappings, and $L_{(\omega,C)}$-weak contractive mappings on metric spaces. We provided some results on fixed circles (discs) of integral type contractive single-valued maps. Furthermore, we applied our main results to discontinuous self-mappings that have a fixed circle. The obtained results are generalizations of variant corresponding results in literature and are applicable to be used in other research areas.

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