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Dynamics in a ratio-dependent Lotka–Volterra competitive-competitive-cooperative system with feedback controls and delays

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Abstract
This study investigates the dynamical behavior of a ratio-dependent Lotka–Volterra competitive-competitive-cooperative system with feedback controls and delays. Compared with previous studies, both ratio-dependent functional responses and time delays are considered. By employing the comparison method, the Lyapunov function method, and useful inequality techniques, some sufficient conditions on the permanence, periodic solution, and global attractivity for the considered system are derived. Finally, a numerical example is also presented to validate the practicability and feasibility of our proposed results.

Keywords: Ratio-dependent competitive-competitive-cooperative system; Time delay; Permanence; Feedback control; Global attractivity

1 Introduction

As is well known, competition and mutualism(cooperation) are two important interactions among species. Competition occurs when two species use the same resources or harm each other when seeking resources, whereas mutualism is defined as the living of two species in close association with one another with the benefit of both [1]. Notably, the Lotka–Volterra models were proposed by Lotka [2] and Volterra [3] for the first time, and now they have become the most important means to explain this type of ecological phenomenon. In particular, the Lotka–Volterra competitive model, mutualism (cooperative) model, and predator-prey model characterize competitive, cooperative, and predator-prey interactions between species that are of great interest in the study of dynamical behaviors of systems [4–17]. However, pure competition as described by the Lotka–Volterra model often results in species exclusion or coexistence with reduced carrying capacity of both species and does not help the coexistence of multiple species, although it is a driving force for natural selection [1]. Hence, when modeling we should consider more interactions between species such as competition, cooperation, and predator-prey [18–26].

For example, in [18], the authors considered the following delayed competitive-cooperative systems:

$$
\begin{aligned}
\dot{z}_1(t) &= z_1(t)\big(r_1(t) - a_{11}^1(t)z_1(t-\tau) - a_{11}^2(t)z_1(t-2\tau) \\
&\quad - a_{12}(t)z_2(t-2\tau) + a_{13}(t)z_3(t-\tau)\big), \\
\dot{z}_2(t) &= z_2(t)\big(r_2(t) - a_{21}(t)z_1(t-2\tau) - a_{22}^1(t)z_2(t-\tau) \\
&\quad - a_{22}^2(t)z_2(t-2\tau) + a_{23}(t)z_3(t-\tau)\big), \\
\dot{z}_3(t) &= z_3(t)\big(r_3(t) + a_{31}(t)z_1(t-\tau) + a_{32}(t)z_2(t-\tau) \\
&\quad - a_{33}^1(t)z_3(t) - a_{33}^2(t)z_3(t-\tau)\big).
\end{aligned}
\tag{1.1}
$$

They established some sufficient conditions that ensured the system is permanent and globally attractive. In [22], the authors proposed the following Lotka–Volterra predator-prey-competition model with feedback controls:

$$
\begin{aligned}
\dot{z}_1(t) &= z_1(t)\left[\gamma_1(t) - b_{11}(t)z_1(t) - \frac{b_{12}(t)z_2(t)}{a_{12}(t)z_2(t)+z_1(t)} - \frac{b_{13}(t)z_3(t)}{a_{13}(t)z_3(t)+z_1(t)} - c_1(t)v_1(t)\right], \\
\dot{z}_2(t) &= z_2(t)\left[-\gamma_2(t) + \frac{b_{21}(t)z_1(t)}{a_{12}(t)z_2(t)+z_1(t)} - b_{23}(t)z_3(t) + c_2(t)v_2(t)\right], \\
\dot{z}_3(t) &= z_3(t)\left[-\gamma_3(t) + \frac{b_{31}(t)z_1(t)}{a_{13}(t)z_3(t)+z_1(t)} - b_{32}(t)z_2(t) + c_3(t)v_3(t)\right], \\
\dot{v}_1(t) &= q_1(t) - e_1(t)v_1(t) + f_1(t)z_1(t), \\
\dot{v}_2(t) &= q_2(t) - e_2(t)v_2(t) - f_2(t)z_2(t), \\
\dot{v}_3(t) &= q_3(t) - e_3(t)v_3(t) - f_3(t)z_3(t).
\end{aligned}
\tag{1.2}
$$

They have obtained some sufficient conditions for the permanence, global attractivity, existence, and stability of the positive periodic solution for system (1.2) by using a comparison theorem, constructing a suitable Lyapunov function, the fixed-point theory, and a new analysis method. In [22] the authors presented an open problem adding a delay term to the proposed model (1.2) and studying the dynamical properties of system (1.2). In [25], the authors further analyzed systems (1.1) and (1.2) and subsequently proposed the following delayed Lotka–Volterra competitive-competitive-cooperative model with feedback controls:

$$
\begin{aligned}
\dot{z}_1(t) &= z_1(t)\big[r_1(t) - a_{11}^1(t)z_1(t-\tau) - a_{11}^2(t)z_1(t-2\tau) \\
&\quad - a_{12}(t)z_2(t-2\tau) + a_{13}(t)z_3(t-\tau) - c_1(t)v_1(t)\big], \\
\dot{z}_2(t) &= z_2(t)\big[r_2(t) - a_{21}(t)z_1(t-2\tau) - a_{22}^1(t)z_2(t-\tau) \\
&\quad - a_{22}^2(t)z_2(t-2\tau) + a_{23}(t)z_3(t-\tau) + c_2(t)v_2(t)\big], \\
\dot{z}_3(t) &= z_3(t)\big[r_3(t) + a_{31}(t)z_1(t-\tau) + a_{32}(t)z_2(t-\tau) \\
&\quad - a_{33}^1(t)z_3(t) - a_{33}^2(t)z_3(t-\tau) + c_3(t)v_3(t)\big], \\
\dot{v}_1(t) &= q_1(t) - e_1(t)v_1(t) + f_1(t)x_1(t),
\end{aligned}
\tag{1.3}
$$

$$\dot{v}_2(t) = q_2(t) - e_2(t)v_2(t) - f_2(t)x_2(t),$$
$$\dot{v}_3(t) = q_3(t) - e_3(t)v_3(t) - f_3(t)x_3(t).$$

They further obtained sufficient conditions for the permanence and global attractivity for system (1.3) by developing a new analysis technique and constructing a new and suitable Lyapunov function.

However, to the best of our knowledge, no study has been conducted to date for dynamics on the three-species ratio-dependent Lotka–Volterra competitive-competitive-cooperative system with feedback controls and delays. Therefore, based on the above models, analysis, and reasons, in this study we extend systems (1.2) and (1.3) to the following system:

$$\begin{aligned}
\dot{x}_1(t) &= x_1(t)\Bigg[r_1(t) - a_1(t)x_1(t) - \frac{b_1(t)x_2(t-\tau_1)}{c_1(t)x_2(t-\tau_1) + x_1(t-\tau_1)} \\
&\quad + \frac{d_1(t)x_3(t-\tau_1)}{e_1(t)x_3(t-\tau_1) + x_1(t-\tau_1)} - f_1(t)u_1(t)\Bigg], \\
\dot{x}_2(t) &= x_2(t)\Bigg[r_2(t) - a_2(t)x_2(t) - \frac{b_2(t)x_1(t-\tau_2)}{c_2(t)x_1(t-\tau_2) + x_2(t-\tau_2)} \\
&\quad + \frac{d_2(t)x_3(t-\tau_2)}{e_2(t)x_3(t-\tau_2) + x_2(t-\tau_2)} + f_2(t)u_2(t)\Bigg], \\
\dot{x}_3(t) &= x_3(t)\Bigg[r_3(t) - a_3(t)x_3(t) + \frac{g_1(t)x_1(t-\tau_3)}{e_1(t)x_3(t-\tau_3) + x_1(t-\tau_3)} \\
&\quad + \frac{g_2(t)x_2(t-\tau_3)}{e_2(t)x_3(t-\tau_3) + x_2(t-\tau_3)} + f_3(t)u_3(t)\Bigg], \\
\dot{u}_1(t) &= q_1(t) - p_1(t)u_1(t) + h_1(t)x_1(t), \\
\dot{u}_2(t) &= q_2(t) - p_2(t)u_2(t) - h_2(t)x_2(t), \\
\dot{u}_3(t) &= q_3(t) - p_3(t)u_3(t) - h_3(t)x_3(t).
\end{aligned} \tag{1.4}$$

The aim of this study is to use the inequality techniques, comparison method and to construct suitable Lyapunov functionals to establish some new and sufficient conditions on the permanence, periodic solution, and global attractivity for system (1.4).

2 Preliminaries

In system (1.4), $x_i(t)$ $(i = 1,2,3)$ represents the density of three species x_i $(i = 1,2,3)$ at time t. $r_i(t)$ $(i = 1,2,3)$ represents the intrinsic growth rate of three species x_i $(i = 1,2,3)$ at time t. $a_i(t)$ $(i = 1,2,3)$ represents the intrapatch restriction density of three species x_i $(i = 1,2,3)$ at time t. $d_i(t), g_i(t)$ $(i = 1,2)$ represent the cooperative coefficients between species x_1, x_2, and x_3 at time t. $b_i(t)$ $(i = 1,2)$ represents the competitive coefficients between species x_1 and x_2 at time t. $u_i(t)$ $(i = 1,2,3)$ represents the indirect control variables at time t. $q_i(t), p_i(t), h_i(t), f_i(t)$ $(i = 1,2,3)$ are the feedback control coefficients at time t. τ_i $(i = 1,2,3)$ is a positive constant.

In this paper, the initial conditions for system (1.4) take the following form:

$$\begin{aligned} &x_i(t) = \varphi_i(t) \quad \text{for all } t \in [-\tau, 0], i = 1,2,3, \\ &u_i(t) = \phi_i(t) \quad \text{for all } t \in [0, +\infty), i = 1,2,3, \end{aligned} \tag{2.1}$$

where $\varphi_i(0) > 0, \phi_i(0) > 0$ $(i = 1,2,3)$, and $\tau = \max\{\tau_i \ (i = 1,2,3)\}$.

For system (1.4) we introduce the following assumptions:

(H_1) $r_i(t), a_i(t)q_i(t), p_i(t), h_i(t), f_i(t)$ $(i = 1,2,3)$ and $b_i(t), c_i(t), d_i(t), e_i(t), g_i(t)$ $(i = 1,2)$ are continuous, bounded, and positive functions on $[0, +\infty)$.

(H_2) $r_i(t), a_i(t)q_i(t), p_i(t), h_i(t), f_i(t)$ $(i = 1,2,3)$ and $b_i(t), c_i(t), d_i(t), e_i(t), g_i(t)$ $(i = 1,2)$ are all continuously positive ω-periodic functions on $[0, \omega]$.

For a continuous and bounded function $f(t)$ defined on $[0, +\infty)$, we define $f^L = \inf_{t\in[0,+\infty)}\{f(t)\}$ and $f^M = \sup_{t\in[0,+\infty)}\{f(t)\}$.

In this paper, we need the following definition and lemmas.

Definition 2.1 ([17]) System (1.4) is called permanent if there exist positive constants M_i, N_i, m_i, n_i $(i = 1,2,3)$ and $T > 0$ such that $m_i \le x_i(t) \le M_i, n_i \le u_i(t) \le N_i$ $(i = 1,2,3)$ for any positive solution $Z(t) = (x_1(t), x_2(t), x_3(t), u_1(t), u_2(t), u_3(t))$ of system (1.4) with the initial values (2.1) as $t > T$.

Lemma 2.1 ([6]) *Consider the following equation*:

$$\dot{u}(t) = u(t)\big(d_1 - d_2 u(t)\big),$$

where $d_2 > 0$, *we have*

(1) *If* $d_1 > 0$, *then* $\lim_{t\to+\infty} u(t) = d_1/d_2$.

(2) *If* $d_1 < 0$, *then* $\lim_{t\to+\infty} u(t) = 0$.

Lemma 2.2 ([25]) *If* $a > 0, b > 0$, *and* $\dot{x}(t) \ge (\le) b - ax(t)$,*when* $t \ge 0$ *and* $x(0) > 0$, *we have*

$$x(t) \ge (\le) \frac{b}{a}\left[1 + \left(\frac{ax(0)}{b} - 1\right)e^{-at}\right].$$

Consider the following periodic differential equation with solution $x(t, 0, \Phi)$:

$$\frac{dx}{dt} = F(t, x_t), \tag{2.2}$$

where $F(t, x_t)$ is an n-dimensional continuous functional and $x(t) \in R^n$, $x(t, 0, \Phi) = (x_1(t, 0, \Phi), x_2(t, 0, \Phi), \dots, x_n(t, 0, \Phi))$ is a solution of the functional differential equation with the initial condition $x_0 = \Phi$.

Lemma 2.3 ([26]) *If there exist positive constants* m *and* M *for any* $\Phi \in C_+^n[-\tau, 0]$ *such that*

$$m < \liminf_{t\to\infty} x_i(t, 0, \Phi) \le \limsup_{t\to\infty} x_i(t, 0, \Phi) < M, \quad i = 1,2,\dots,n,$$

then system (2.2) *admits at least one positive* ω*-periodic solution*.

3 Permanence and periodic solution

In this section, we obtain some new and sufficient conditions for the permanence and periodic solution of system (1.4).

Theorem 3.1 *Let $x(t) = (x_1(t), x_2(t), x_3(t), u_1(t), u_2(t), u_3(t))$ be any positive solution of system (1.4) with the initial conditions (2.1). Assume that (H_1) holds, then there exist positive constants $M_1, M_2, M_3, N_1, N_2, N_3, T$ and m_3, n_1 such that $x_i(t) \le M_i$ $(i = 1, 2, 3)$, $u_i(t) \le N_i$ $(i = 1, 2, 3)$ and $x_3(t) \ge m_3, u_1(t) \ge n_1$ as $t > T$.*

Proof From the first equation of system (1.4), for $t \ge \tau$, we have

$$\dot{x}_1(t) \le x_1(t)\left[r_1^M + \frac{d_1^M}{e_1^L} - a_1^L x_1(t)\right].$$

Then, by Lemma 2.1 and the comparison theorem, there exists a constant $T_0 > 0$ such that

$$x_1(t) \le \frac{r_1^M + \frac{d_1^M}{e_1^L}}{a_1^L} = M_1 \quad \text{as } t > T_0.$$

By the fourth equation of system (1.4), for $t > T_0$, we have

$$\dot{u}_1(t) \le q_1^M + h_1^M M_1 - p_1^L u_1(t).$$

Then, by Lemma 2.2 and the comparison theorem, there exists a constant $T_1 > 0$ such that

$$u_1(t) \le \frac{q_1^M + h_1^M M_1}{p_1^L} = N_1 \quad \text{as } t > T_1.$$

From the fifth and sixth equation of system (1.4), for $t \ge \tau$ and $i = 2, 3$, we have

$$\dot{u}_i(t) \le q_i^M - p_i^L u_i(t).$$

Then, by Lemma 2.2 and the comparison theorem, there exists a constant $T_2 > 0$ such that

$$u_i(t) \le \frac{q_i^M}{p_i^L} = N_i, \quad i = 2, 3, \text{ as } t > T_2.$$

Next, from the second equation of system (1.4), for $t > T_2$, we have

$$\dot{x}_2(t) \le x_2(t)\left[r_2^M + \frac{d_2^M}{e_2^L} + f_2^M N_2 - a_2^L x_2(t)\right].$$

Then, by Lemma 2.1 and the comparison theorem, there exists a constant $T_3 > 0$ such that

$$x_2(t) \le \frac{r_2^M + \frac{d_2^M}{e_2^L} + f_2^M N_2}{a_2^L} = M_2 \quad \text{as } t > T_3.$$

Similar to the above discussion, from the third equation of system (1.4), there exists a constant $T_4 > 0$ such that

$$x_3(t) \le \frac{r_3^M + g_1^M + g_2^M + f_3^M N_3}{a_3^L} = M_3 \quad \text{as } t > T_4.$$

On the other hand, from the third equation of system (1.4), for $t \ge \tau$, we have

$$\dot{x}_3(t) \ge x_3(t)\big[r_3^L - a_3^M x_3(t)\big].$$

Then, by Lemma 2.1 and the comparison theorem, there exists a constant $T_5 > 0$ such that

$$x_3(t) \ge \frac{r_3^L}{a_3^M} = m_3 \quad \text{as } t > T_5.$$

Finally, from the fourth equation of system (1.4), for $t \ge \tau$, we have

$$\dot{u}_1(t) \ge q_1^L - p_1^M u_1(t).$$

Then, by Lemma 2.2 and the comparison theorem, there exists a constant $T_6 > 0$ such that

$$u_1(t) \ge \frac{q_1^L}{p_1^M} = n_1 \quad \text{as } t > T_6.$$

This completes the proof of Theorem 3.1. □

Theorem 3.2 *Assume that* (H_1) *holds and*

$$A_1 = r_1^L + \frac{d_1^L m_3}{e_1^M M_3 + M_1} - \frac{b_1^M}{c_1^L} - f_1^M N_1 > 0, \qquad B_2 = q_2^L - h_2^M M_2 > 0,$$
$$A_2 = r_2^L + \frac{d_2^L m_3}{e_2^M M_3 + M_2} - \frac{b_2^M}{c_2^L} > 0, \qquad B_3 = q_3^L - h_3^M M_3 > 0,$$

then system (1.4) *is permanent, where* M_1, M_2, M_3, N_1, *and* m_3 *are defined in Theorem* 3.1.

Proof Firstly, from the first and second equation of system (1.4), we can obtain a sufficiently large positive constant T_1^M such that

$$\dot{x}_i(t) \ge x_i(t)\big[A_i - a_i^M x_i(t)\big] \quad \text{as } t > T_1^M, i = 1, 2.$$

Then, by Lemma 2.2 and the comparison theorem, there exists a constant $T_2^M > T_1^M$ such that

$$x_i(t) \ge \frac{A_i}{a_i^M} = m_i \quad \text{as } t > T_2^M, i = 1, 2.$$

Next, from the fifth and sixth equation of system (1.4), we can obtain a sufficiently large positive constant T_1^N such that

$$\dot{u}_i(t) \ge B_i - p_i^M u_i(t) \quad \text{as } t > T_1^N, i = 2, 3.$$

Then, by Lemma 2.2 and the comparison theorem, there exists a constant $T_2^N > T_1^N$ such that

$$u_i(t) \geq \frac{B_i}{p_i^M} = n_i \quad \text{as } t > T_2^N, i = 2, 3.$$

This completes the proof of Theorem 3.2. □

As a direct result of Lemma 2.3, from Theorem 3.1 and Theorem 3.2 we have the following.

Corollary 3.1 *Assume that* (H_2) *holds and* $A_1 > 0, A_2 > 0, B_2 > 0, B_3 > 0$, *then system* (1.4) *is permanent and has at least one positive* ω*-periodic solution.*

4 Global attractivity

In this section, we obtain the sufficient conditions for the global attractivity of system (1.4). Firstly, for convenience we denote

$$\begin{aligned}
&C_1(t) = \frac{b_1(t)y_2(t-\tau_1)}{\gamma_1(t)} \leq \frac{b_1^M M_2}{\gamma_1^L} = C_1^M, \qquad C_2(t) = \frac{b_1(t)y_1(t-\tau_1)}{\gamma_1(t)} \leq \frac{b_1^M M_1}{\gamma_1^L} = C_2^M,\\
&C_3(t) = \frac{d_1(t)y_3(t-\tau_1)}{\gamma_2(t)} \leq \frac{d_1^M M_3}{\gamma_2^L} = C_3^M, \qquad C_4(t) = \frac{d_1(t)y_1(t-\tau_1)}{\gamma_2(t)} \leq \frac{d_1^M M_1}{\gamma_2^L} = C_4^M,\\
&C_5(t) = \frac{b_2(t)y_2(t-\tau_2)}{\gamma_3(t)} \leq \frac{b_2^M M_2}{\gamma_3^L} = C_5^M, \qquad C_6(t) = \frac{b_2(t)y_1(t-\tau_2)}{\gamma_3(t)} \leq \frac{b_2^M M_1}{\gamma_3^L} = C_6^M,\\
&C_7(t) = \frac{d_2(t)y_3(t-\tau_2)}{\gamma_4(t)} \leq \frac{d_2^M M_3}{\gamma_4^L} = C_7^M, \qquad C_8(t) = \frac{d_2(t)y_2(t-\tau_2)}{\gamma_4(t)} \leq \frac{d_2^M M_2}{\gamma_4^L} = C_8^M,\\
&C_9(t) = \frac{g_1(t)e_1(t)y_3(t-\tau_3)}{\gamma_5(t)} \leq \frac{g_1^M e_1^M M_3}{\gamma_2^L} = C_9^M,\\
&C_{10}(t) = \frac{g_1(t)e_1(t)y_1(t-\tau_3)}{\gamma_5(t)} \leq \frac{g_1^M e_1^M M_1}{\gamma_2^L} = C_{10}^M,\\
&C_{11}(t) = \frac{g_2(t)e_2(t)y_3(t-\tau_3)}{\gamma_6(t)} \leq \frac{g_2^M e_2^M M_3}{\gamma_4^L} = C_{11}^M,\\
&C_{12}(t) = \frac{g_2(t)e_2(t)y_2(t-\tau_3)}{\gamma_6(t)} \leq \frac{g_2^M e_2^M M_2}{\gamma_4^L} = C_{12}^M,\\
&D_1 = a_1^L - C_1^M - C_3^M - C_5^M - C_9^M - h_1^M, \qquad D_2 = a_2^L - C_2^M - C_6^M - C_7^M - C_{11}^M - h_2^M,\\
&D_3 = a_3^L - C_4^M - C_8^M - C_{10}^M - C_{12}^M - h_3^M, \qquad E_i = p_i^L - f_i^M \quad (i = 1, 2, 3),\\
&G = \min\bigl\{D_i, E_i (i = 1, 2, 3)\bigr\},
\end{aligned}$$

where

$$\begin{aligned}
&\gamma_1(t) = \bigl(c_1(t)x_2(t-\tau_1) + x_1(t-\tau_1)\bigr)\bigl(c_1(t)y_2(t-\tau_1) + y_1(t-\tau_1)\bigr),\\
&\gamma_1^L = \bigl(c_1^L m_2 + m_1\bigr)^2,\\
&\gamma_2(t) = \bigl(e_1(t)x_3(t-\tau_1) + x_1(t-\tau_1)\bigr)\bigl(e_1(t)y_3(t-\tau_1) + y_1(t-\tau_1)\bigr),
\end{aligned}$$

$$
\begin{aligned}
&\gamma_2^L = \left(e_1^L m_3 + m_1\right)^2, \\
&\gamma_3(t) = \left(c_2(t)x_1(t-\tau_2) + x_2(t-\tau_2)\right)\left(c_2(t)y_1(t-\tau_2) + y_2(t-\tau_2)\right), \\
&\gamma_3^L = \left(c_2^L m_1 + m_2\right)^2, \\
&\gamma_4(t) = \left(e_2(t)x_3(t-\tau_2) + x_2(t-\tau_2)\right)\left(e_2(t)y_3(t-\tau_2) + y_2(t-\tau_2)\right), \\
&\gamma_4^L = \left(e_2^L m_3 + m_2\right)^2, \\
&\gamma_5(t) = \left(e_1(t)x_3(t-\tau_3) + x_1(t-\tau_3)\right)\left(e_1(t)y_3(t-\tau_3) + y_1(t-\tau_3)\right), \\
&\gamma_5^L = \gamma_2^L, \\
&\gamma_6(t) = \left(e_2(t)x_3(t-\tau_3) + x_2(t-\tau_3)\right)\left(e_2(t)y_3(t-\tau_3) + y_2(t-\tau_3)\right), \\
&\gamma_6^L = \gamma_4^L.
\end{aligned}
$$

Theorem 4.1 *Suppose that the conditions of Theorem 3.2 hold and $G > 0$. Then system (1.4) is globally attractive.*

Proof Let $(x_1(t), x_2(t), x_3(t))$ and $(y_1(t), y_2(t), y_3(t))$ be any two positive solutions of system (1.4). Choose positive constants M_i, m_i $(i = 1,2,3)$, and T such that $m_i \le y_i(t), x_i(t) \le M_i$ $(i = 1,2,3)$ for all $t \ge T$. Firstly, let

$$
V_1(t) = \sum_{i=1}^{3} \left|\ln x_i(t) - \ln y_i(t)\right|.
$$

Calculating the upper right derivative of $V_1(t)$ along system (1.4), we have

$$
\begin{aligned}
D^+V_1(t) = {} & \operatorname{sign}\left(x_1(t) - y_1(t)\right)\bigg[-a_1(t)\left(x_1(t) - y_1(t)\right) - f_1(t)\left(u_1(t) - v_1(t)\right) \\
& - b_1(t)\left(\frac{x_2(t-\tau_1)}{c_1(t)x_2(t-\tau_1) + x_1(t-\tau_1)} - \frac{y_2(t-\tau_1)}{c_1(t)y_2(t-\tau_1) + y_1(t-\tau_1)}\right) \\
& + d_1(t)\left(\frac{x_3(t-\tau_1)}{e_1(t)x_3(t-\tau_1) + x_1(t-\tau_1)} - \frac{y_3(t-\tau_1)}{e_1(t)y_3(t-\tau_1) + y_1(t-\tau_1)}\right)\bigg] \\
& + \operatorname{sign}\left(x_2(t) - y_2(t)\right)\bigg[-a_2(t)\left(x_2(t) - y_2(t)\right) + f_2(t)\left(u_2(t) - v_2(t)\right) \\
& - b_2(t)\left(\frac{x_1(t-\tau_2)}{c_2(t)x_1(t-\tau_2) + x_2(t-\tau_2)} - \frac{y_1(t-\tau_2)}{c_2(t)y_1(t-\tau_2) + y_2(t-\tau_2)}\right) \\
& + d_2(t)\left(\frac{x_3(t-\tau_2)}{e_2(t)x_3(t-\tau_2) + x_2(t-\tau_2)} - \frac{y_3(t-\tau_2)}{e_2(t)y_3(t-\tau_2) + y_2(t-\tau_2)}\right)\bigg] \\
& + \operatorname{sign}\left(x_3(t) - y_3(t)\right)\bigg[-a_3(t)\left(x_3(t) - y_3(t)\right) + f_3(t)\left(u_3(t) - v_3(t)\right) \\
& + g_1(t)\left(\frac{x_1(t-\tau_3)}{e_1(t)x_3(t-\tau_3) + x_1(t-\tau_3)} - \frac{y_1(t-\tau_3)}{e_1(t)y_3(t-\tau_3) + y_1(t-\tau_3)}\right) \\
& + g_2(t)(\frac{x_2(t-\tau_3)}{e_2(t)x_3(t-\tau_3) + x_2(t-\tau_3)} - \frac{y_2(t-\tau_3)}{e_2(t)y_3(t-\tau_3) + y_2(t-\tau_3)}\bigg] \\
= {} & \operatorname{sign}\left(x_1(t) - y_1(t)\right)\left[-a_1(t)\left(x_1(t) - y_1(t)\right) - f_1(t)\left(u_1(t) - v_1(t)\right)\right.
\end{aligned}
$$

$$
\begin{aligned}
&- C_2(t)\big(x_2(t-\tau_1) - y_2(t-\tau_1)\big) + C_4(t)\big(x_3(t-\tau_1) - y_3(t-\tau_1)\big) \\
&+ \big(C_1(t) - C_3(t)\big)\big(x_1(t-\tau_1) - y_1(t-\tau_1)\big)\big] + \operatorname{sign}\big(x_2(t) - y_2(t)\big) \\
&\times \big[-a_2(t)\big(x_2(t) - y_2(t)\big) + f_2(t)\big(u_2(t) - v_2(t)\big) - C_5(t)\big(x_1(t-\tau_2) \\
&- y_1(t-\tau_2)\big) + \big(C_6(t) - C_7(t)\big)\big(x_2(t-\tau_2) - y_2(t-\tau_2)\big) + C_8(t) \\
&\times \big(x_3(t-\tau_2) - y_3(t-\tau_2)\big)\big] + \operatorname{sign}\big(x_3(t) - y_3(t)\big)\big[-a_2(t)\big(x_2(t) - y_2(t)\big) \\
&+ f_3(t)\big(u_3(t) - v_3(t)\big) - \big(C_{10}(t) + C_{12}(t)\big)\big(x_3(t-\tau_3) - y_3(t-\tau_3)\big) \\
&+ C_9(t)\big(x_1(t-\tau_3) - y_1(t-\tau_3)\big) + C_{11}(t)\big(x_2(t-\tau_3) - y_2(t-\tau_3)\big)\big] \\
\leq{}& -\sum_{i=3}^{3} a_i^L\big|x_i(t) - y_i(t)\big| + \sum_{i=3}^{3} f_i^M\big|u_i(t) - v_i(t)\big| + C_2^M\big|x_2(t-\tau_1) - y_2(t-\tau_1)\big| \\
&+ \big(C_1^M + C_3^M\big)\big|x_1(t-\tau_1) - y_1(t-\tau_1)\big| + C_4^M\big|x_3(t-\tau_1) - y_3(t-\tau_1)\big| \\
&+ \big(C_6^M + C_7^M\big)\big|x_2(t-\tau_2) - y_2(t-\tau_2)\big| + C_5^M\big|x_1(t-\tau_2) - y_1(t-\tau_2)\big| \\
&+ C_8^M\big|x_3(t-\tau_2) - y_3(t-\tau_2)\big| + \big(C_{10}^M + C_{12}^M\big)\big|x_3(t-\tau_3) - y_3(t-\tau_3)\big| \\
&+ C_9^M\big|x_1(t-\tau_3) - y_1(t-\tau_3)\big| + C_{11}^M\big|x_2(t-\tau_3) - y_2(t-\tau_3)\big|. \qquad (4.1)
\end{aligned}
$$

Next, we let

$$
\begin{aligned}
V_2(t) ={}& \big(C_1^M + C_3^M\big)\int_{t-\tau_1}^{t}\big|x_1(s) - y_1(s)\big|\,ds + C_2^M\int_{t-\tau_1}^{t}\big|x_2(s) - y_2(s)\big|\,ds \\
&+ C_4^M\int_{t-\tau_1}^{t}\big|x_3(s) - y_3(s)\big|\,ds + \big(C_6^M + C_7^M\big)\int_{t-\tau_2}^{t}\big|x_2(s) - y_2(s)\big|\,ds \\
&+ C_5^M\int_{t-\tau_2}^{t}\big|x_1(s) - y_1(s)\big|\,ds + C_8^M\int_{t-\tau_2}^{t}\big|x_3(s) - y_3(s)\big|\,ds \\
&+ \big(C_{10}^M + C_{12}^M\big)\int_{t-\tau_3}^{t}\big|x_3(s) - y_3(s)\big|\,ds + C_9^M\int_{t-\tau_3}^{t}\big|x_1(s) - y_1(s)\big|\,ds \\
&+ C_{11}^M\int_{t-\tau_3}^{t}\big|x_2(s) - y_2(s)\big|\,ds. \qquad (4.2)
\end{aligned}
$$

Calculating the upper right derivative of $V_2(t)$ and from (4.1), we have

$$
\begin{aligned}
D^+V_1(t) + D^+V_2(t) \leq{}& -\big(a_1^L - C_1^M - C_3^M - C_5^M - C_9^M\big)\big|x_1(t) - y_1(t)\big| \\
&- \big(a_2^L - C_2^M - C_6^M - C_7^M - C_{11}^M\big)\big|x_2(t) - y_2(t)\big| \\
&- \big(a_3^L - C_4^M - C_8^M - C_{10}^M - C_{12}^M\big)\big|x_3(t) - y_3(t)\big| \\
&+ f_1^M\big|u_1(t) - v_1(t)\big| + f_2^M\big|u_2(t) - v_2(t)\big| + f_3^M\big|u_3(t) - v_3(t)\big|. \qquad (4.3)
\end{aligned}
$$

Moreover, we let

$$
V_3(t) = \sum_{i=1}^{3}\big|u_i(t) - v_i(t)\big|.
$$

Calculating the upper right derivative of $V_3(t)$ and from (4.3), we have

$$\begin{aligned}\sum_{i=1}^{3} D^+ V_i(t) \leq &-\left(a_1^L - C_1^M - C_3^M - C_5^M - C_9^M - h_1^M\right)\left|x_1(t) - y_1(t)\right| \\ &- \left(a_2^L - C_2^M - C_6^M - C_7^M - C_{11}^M - h_2^M\right)\left|x_2(t) - y_2(t)\right| \\ &- \left(a_3^L - C_4^M - C_8^M - C_{10}^M - C_{12}^M - h_3^M\right)\left|x_3(t) - y_3(t)\right| \\ &- \sum_{i=1}^{3}\left(p_i^L - f_i^M\right)\left|u_i(t) - v_i(t)\right|. \end{aligned} \tag{4.4}$$

Finally, we let a Lyapunov function be as follows:

$$V(t) = V_1(t) + V_2(t) + V_3(t).$$

Calculating the upper right derivation of $V(t)$, from (4.4) we finally can obtain, for all $t \geq T$,

$$D^+ V(t) \leq -\sum_{i=1}^{3}\left(D_i\left|x_i(t) - y_i(t)\right| + E_i\left|u_i(t) - v_i(t)\right|\right). \tag{4.5}$$

Integrating from T to t on both sides of (4.5) produces

$$V(t) + G\int_{T_4}^{t}\left(\sum_{i=1}^{3}\left[\left|x_i(s) - y_i(s)\right| + \left|u_i(s) - v_i(s)\right|\right]\right) ds \leq V(T) < +\infty. \tag{4.6}$$

Hence, $V(t)$ is bounded on $[T, \infty)$, and we have

$$\int_{T_4}^{t}\left(\sum_{i=1}^{3}\left[\left|x_i(s) - y_i(s)\right| + \left|u_i(s) - v_i(s)\right|\right]\right) ds \leq \frac{V(T)}{G} < +\infty. \tag{4.7}$$

Then we have

$$\sum_{i=1}^{3}\left(\left|x_i(s) - y_i(s)\right| + \left|u_i(s) - v_i(s)\right|\right) \in L^1(T, +\infty). \tag{4.8}$$

From the permanence of system (1.4), we can obtain that $\sum_{i=1}^{3}(|x_i(s) - y_i(s)| + |u_i(s) - v_i(s)|)$ is uniformly continuous on $[T, +\infty)$. By Barbalat's lemma it follows that

$$\lim_{t\to\infty}\left|x_i(t) - y_i(t)\right| = 0, \qquad \lim_{t\to\infty}\left|u_i(t) - v_i(t)\right| = 0, \quad (i = 1, 2, 3).$$

This completes the proof of Theorem 4.1. □

Corollary 4.1 *Suppose that the conditions of Corollary 3.1 hold and $G > 0$, then system (1.4) has a positive ω- periodic solution which is globally attractive.*

5 One example

In this section one example is given to illustrate the effectiveness of our results obtained in this paper.

Example We consider the following system:

$$\begin{aligned}
\dot{x}_1(t) &= x_1(t)\bigg[4.45 + 0.35\cos(t) - \big(4.15 + 0.35\cos(t)\big)x_1(t) \\
&\quad - \frac{(0.75 + 0.2\cos(t))x_2(t-0.75)}{(3 + 0.25\cos(t))x_2(t-0.75) + x_1(t-0.75)} \\
&\quad + \frac{(2.6 + 0.25\cos(t))x_3(t-0.75)}{(2.25 + 0.25\cos(t))x_3(t-0.75) + x_1(t-0.75)} \\
&\quad - \big(0.25 + 0.15\cos(t)\big)u_1(t)\bigg], \\
\dot{x}_2(t) &= x_2(t)\bigg[4.5 + 0.45\cos(t) - \big(4.2 + 0.25\cos(t)\big)x_2(t) \\
&\quad - \frac{(0.85 + 0.2\cos(t))x_1(t-0.5)}{(3 + 0.35\cos(t))x_1(t-0.5) + x_2(t-0.5)} \\
&\quad + \frac{(2.75 + 0.2\cos(t))x_3(t-0.5)}{(2.35 + 0.35\cos(t))x_3(t-0.5) + x_2(t-0.5)} + \big(0.2 + 0.15\cos(t)\big)u_2(t)\bigg], \qquad (5.1) \\
\dot{x}_3(t) &= x_3(t)\bigg[3.75 + 0.35\cos(t) - \big(4.3 + 0.25\cos(t)\big)x_3(t) \\
&\quad + \frac{(0.4 + 0.2\cos(t))x_1(t-0.25)}{(3 + 0.15\cos(t))x_3(t-0.25) + x_1(t-0.25)} \\
&\quad + \frac{(0.35 + 0.1\cos(t))x_2(t-0.25)}{(2.15 + 0.15\cos(t))x_3(t-0.25) + x_2(t-0.25)} + \big(0.2 + 0.1\cos(t)\big)u_3(t)\bigg], \\
\dot{u}_1(t) &= 2.25 + 0.35\cos(t) - \big(1.95 + 0.25\cos(t)\big)u_1(t) + \big(0.2 + 0.1\cos(t)\big)x_1(t), \\
\dot{u}_2(t) &= 2.65 + 0.25\cos(t) - \big(1.85 + 0.35\cos(t)\big)u_2(t) - \big(0.2 + 0.15\cos(t)\big)x_2(t), \\
\dot{u}_2(t) &= 1.65 + 0.15\cos(t) - \big(1.75 + 0.45\cos(t)\big)u_3(t) - \big(0.15 + 0.12\cos(t)\big)x_3(t).
\end{aligned}$$

By direct calculation, we can get

$$\begin{aligned}
&M_1 \approx 1.6382, \quad M_2 \approx 1.7979, \quad M_3 \approx 1.3742, \quad m_1 \approx 0.7323, \\
&m_2 \approx 0.8959, \quad m_3 \approx 0.7473, \quad N_1 \approx 1.8185, \quad N_2 \approx 1.9333, \\
&N_3 \approx 1.3846, \quad n_1 \approx 0.8636, \quad n_2 \approx 0.8049, \quad n_3 \approx 0.5132, \\
&\gamma_1^L \approx 10.2143, \quad \gamma_2^L \approx 4.9587, \quad \gamma_3^L \approx 8.0458, \quad \gamma_4^L \approx 5.7139, \\
&C_1^M \approx 0.1672, \quad C_2^M \approx 0.1524, \quad C_3^M \approx 0.7898, \quad C_4^M \approx 0.9415, \\
&C_5^M \approx 0.2346, \quad C_6^M \approx 0.2138, \quad C_7^M \approx 0.7095, \quad C_8^M \approx 0.9282, \\
&C_9^M \approx 0.4157, \quad C_{10}^M \approx 0.4955, \quad C_{11}^M \approx 0.2922, \quad C_{12}^M \approx 0.3823, \\
&D_1 \approx 1.8927 \quad D_2 \approx 2.2322, \quad D_3 \approx 1.0324, \quad E_1 \approx 1.3, \quad E_2 \approx 1.15, \\
&E_3 \approx 1, \quad G = 1.
\end{aligned}$$

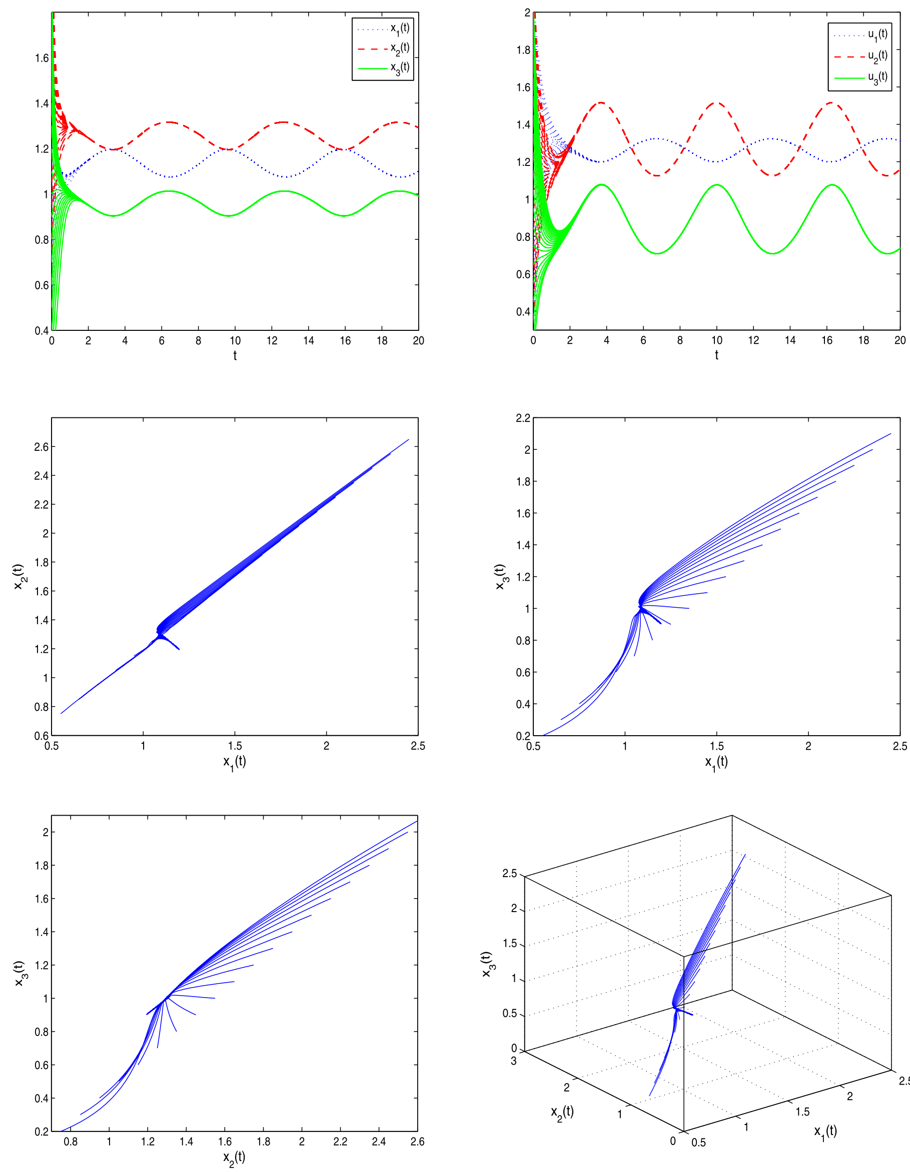

Figure 1 The numerical solutions of system (5.1) with different initial conditions

It is easy to show that system (5.1) satisfies the conditions of Theorem 3.2, Theorem 4.1, Corollary 3.1, and Corollary 4.1. Hence, system (5.1) is permanent, globally attractive and has a globally attractive positive periodic solution.

From Fig. 1 we can see that system (5.1) is permanent and has a globally attractive positive periodic solution.

6 Conclusion

In this study, we are concerned with system (1.4). First, using the inequality techniques and the comparison method, we obtained a set of conditions that ensure that the system is permanent and at least has a positive periodic solution. Second, using the Lyapunov function method, we derived sufficient conditions on the global attractivity of the system.

Finally, we provided a suitable example to illustrate the feasibility of our main results. Because we extended systems (1.2) and (1.3) to system (1.4), we also obtained some sufficient conditions for the permanence, periodic solution, and global attractivity of system (1.4). Hence, system (1.4) and the results obtained in this study can be seen as the supplements and extensions of previously known studies [13, 17, 20].

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