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Survival analysis of a stochastic delay single-species system in polluted environment with psychological effect and pulse toxicant input

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Abstract

We propose and study a stochastic delay single-species population system in polluted environment with psychological effect and pulse toxicant input. We establish sufficient conditions for the extinction, nonpersistence in the mean, weak persistence, and strong persistence of the single-species population and obtain the threshold value between extinction and weak persistence. Finally, we confirm the efficiency of the main results by numerical simulations.

Keywords: Psychological effect; Persistence; Extinction; Polluted environment

1 Introduction

Along with fast development of agriculture and modern industry, a large number of toxic pollutants are discharged into the ecosystem, and therefore it is an undeniable fact that environmental pollution becomes increasingly serious, such as pollution of pollutants from burning agricultural plant straw, heavy metal pollution, water pollution caused by crop fertilization and pesticide application. As it is well known, the existence of various poisons are becoming a threat to the survival of unprotected populations, which has prompted many scholars to investigate the impact of toxins on the population and assess the risk of the population. An important tool to analyze the effects of toxins on population is establishing a mathematical model [1–7].

Wei and Chen [8] proposed a mathematical model for the first time to study the physiological effects of vertebrates on population in a polluted environment. The so-called "psychological" effect refers to: in the heavy pollution environment, because the organism with spine has a good sensory nervous system, which can transmit the information of the polluted environment to the area where the brain can explain the information, the effective contact between the organism and the live environment will be reduced, which plays the role of self-protection; for instance, fish can identify the information in the polluted environment through their own neurosensory system and make decisions to either escape from the polluted area or bear the environmental toxicant [8]. Afterwards, Lan and Wei

[9] considered that the exogenous input of toxins is regular in some practical situations. They proposed the following single-species population model with psychological effect and impulsive toxicant in a pollution environment:

$$\begin{cases} \dot{x}(t) = x(t)(r - r_0 c_0(t) - a x(t) - \frac{\lambda c_e(t)}{1+\alpha c_e^2(t)}), \\ \dot{c}_0(t) = (k c_e(t) - (g + m + b) c_0(t)), \\ \dot{c}_e(t) = -h c_e(t), \quad t \neq n\gamma, n \in z^+, \\ \Delta x(t) = 0, \qquad \Delta c_0(t) = 0, \qquad \Delta c_e(t) = \mu, \quad t = n\gamma, n \in z^+, \end{cases} \tag{1}$$

where $x(t)$ represents the density of the population, $c_0(t)$ and $c_e(t)$ represent the concentration of toxins in the organism and the concentration of toxins in the environment, respectively, r and r_0 stand for the net growth rate in nonpolluted environment and response intensity of biological growth to toxins, $kc_e(t)$ stands for the uptake of toxins in the environment, $gc_0(t)$ represents the emission rate of toxins, $mc_0(t)$ represents the purification rate of toxins because of metabolic process of organisms, $bc_0(t)$ is the loss due to giving birth at time t, $-hc_e(t)$ denotes the amount of reduction in the purification of toxins by the environment itself, μ and γ denote the toxicant input amount and the period of pulse input toxin, respectively, and $\Delta\phi(t) = \phi(t^+) - \phi(t)$.

In reality, population growth is more or less disturbed by environmental factors, such as temperature, humidity, and seasonal climate change, and almost all the observed data show that there are obvious random fluctuations in the growth process of organisms. Therefore, in some practical cases, ignoring the randomness of the system and using deterministic models to describe and predict the system behavior are not always satisfactory; especially, it is not suitable to use deterministic population model to study how to protect endangered species [10]. May [11] pointed out that due to the impact of environmental noises, the birth rate, death rate, carrying capacity, competition coefficients, and other parameters involved with the system exhibit random fluctuation to a greater or lesser extent [12–17]. On the other hand, the influence of delay has played an important role in the population dynamic [18–22].

In this paper, we suppose that the environmental noises affect all parameters of model (1) (see e.g. [23, 24]). Considering the work of population psychological effect, we propose the following stochastic impulsive single-species population model with delay and psychological effect in polluted environment:

$$\begin{cases} dx(t) = x(t)(r - r_0 c_0(t) - a x(t) - c x(t-\tau) - \frac{\lambda c_e(t)}{1+\alpha c_e^2(t)})\,dt \\ \qquad\quad + \sigma_1 x(t)\,dB_1(t) + \sigma_2 c_0(t) x(t)\,dB_2(t) + \sigma_3 x^2(t)\,dB_3(t) \\ \qquad\quad + \sigma_4 x(t) x(t-\tau)\,dB_4(t) + \frac{\sigma_5 x(t) c_e(t)}{1+\alpha c_e^2(t)}\,dB_5(t), \\ dc_0(t) = (k c_e(t) - (g + m + b) c_0(t))\,dt, \\ dc_e(t) = -h c_e(t)\,dt, \quad t \neq n\gamma, n \in z^+, \\ \Delta x(t) = 0, \qquad \Delta c_0(t) = 0, \qquad \Delta c_e(t) = \mu, \quad t = n\gamma, n \in z^+, \end{cases} \tag{2}$$

where τ represents the time delay, $B_i(t)$ stands for a standard Brownian motion defined on a complete probability space (Ω, F, P) with filtration $\{F_t\}_{t \in R_+}$, and σ_i^2 $(i = 1, 2, 3, 4, 5)$ represents the intensity of noise. Let $\varphi(\theta)$ be a continuous function on $[-\tau, 0]$, and let the solution $x(t)$ of system (2) satisfy the initial condition

$$x(\theta) = \varphi(\theta) > 0, \quad \theta \in [-\tau, 0].$$

Remark 1.1 (see [7]) In model (2), since $c_0(t)$ and $c_e(t)$ denote the concentrations of toxicant, we must have $0 \le c_0(t) \le 1$ and $0 \le c_e(t) \le 1$ for all $t \ge 0$. To this end, we need the following constraints: $k \le g + m$ and $b \le 1 - e^{-h\gamma}$.

2 Preliminaries

Let $R_+ = \{a \in R : a > 0\}$, and let $C(\Omega \times [0, +\infty); R_+)$ denote the family of continuous functions from $\Omega \times [0, +\infty)$ to R_+. In addition, for convenience, we also introduce some notations:

$$\begin{aligned}
&\langle x(t)\rangle = t^{-1}\int_0^t x(s)\,ds, \qquad x_* = \liminf_{t\to+\infty} x(t), \qquad x^* = \limsup_{t\to+\infty} x(t),\\
&\eta = \frac{-1}{h\gamma\sqrt{\alpha}}\left(\arctan\left(\frac{\mu\sqrt{\alpha}e^{-k\gamma}}{1-e^{-k\gamma}}\right) - \arctan\left(\frac{\mu\sqrt{\alpha}}{1-e^{-k\gamma}}\right)\right),\\
&\xi = \frac{\mu^2(1-e^{-h\gamma})^3(1+e^{-h\gamma})}{2h\gamma((1-e^{-h\gamma})^2+\alpha\mu^2e^{-2h\gamma})((1-e^{-h\gamma})^2+\alpha\mu^2)},\\
&\nu = \left(\frac{k\mu}{h-g-m-b}\right)^2\left(\frac{1+e^{-(g+m+b)\gamma}}{2(g+m+b)\gamma(1-e^{-(g+m+b)\gamma})} + \frac{1+e^{-h\gamma}}{2h\gamma(1-e^{-h\gamma})}\right.\\
&\qquad \left. - \frac{2(1-e^{-(g+m+b+h)\gamma})}{(1-e^{-h\gamma})(1-e^{-(g+m+b)\gamma})(g+m+b+h)\gamma}\right),\\
&A = r - \frac{r_0k\mu}{h(g+m+b)\gamma} - \lambda\eta - 0.5\sigma_1^2 - 0.5\sigma_2^2\nu - 0.5\sigma_5^2\xi.
\end{aligned}$$

Definition 2.1

(1) population $x(t)$ goes to extinction if $\lim_{t\to+\infty} x(t) = 0$ a.s.;
(2) population $x(t)$ is nonpersistent in the mean if $\langle x(t)\rangle^* = 0$;
(3) population $x(t)$ is weakly persistent if $x^* > 0$;
(4) population $x(t)$ is strongly persistent in the mean if $\langle x(t)\rangle_* > 0$.

Lemma 2.1 (see [25]) *Consider the following subsystem of* (2):

$$\begin{cases}
dc_0(t) = (kc_e(t) - (g+m+b)c_0(t))\,dt,\\
dc_e(t) = -hc_e(t)\,dt, \quad t \ne n\gamma, n \in Z^+,\\
\Delta c_0(t) = 0, \qquad \Delta c_e(t) = \mu, \quad t = n\gamma, n \in Z^+.
\end{cases} \tag{3}$$

Model (3) *has a unique positive* γ*-periodic solution* $(\widetilde{c}_0(t), \widetilde{c}_e(t))$, *and for each solution* $(c_0(t), c_e(t))$ *of model* (2), $c_0(t) \to \widetilde{c}_0(t)$ *and* $c_e(t) \to \widetilde{c}_e(t)$ *as* $t \to +\infty$. *Moreover*, $c_0(t) > \widetilde{c}_0(t)$ *and* $c_e(t) > \widetilde{c}_e(t)$, *where*

$$\begin{cases}
\widetilde{c}_0(t) = \widetilde{c}_0(0)e^{-(g+m+b)(t-n\gamma)} + \frac{k\mu(e^{-(g+m+b)(t-n\gamma)}-e^{-h(t-n\gamma)})}{(h-g-m-b)(1-e^{-h\gamma})},\\
\widetilde{c}_e(t) = \frac{\mu e^{-h(t-n\gamma)}}{1-e^{-h\gamma}},\\
\widetilde{c}_0(0) = \frac{k\mu(e^{-(g+m+b)\gamma}-e^{-h\gamma})}{(h-g-m-b)(1-e^{-(g+m+b)\gamma})(1-e^{-h\gamma})},\\
\widetilde{c}_e(0) = \frac{\mu}{1-e^{-h\gamma}}
\end{cases}$$

for $t \in (n\gamma, (n+1)\gamma]$ *and* $n \in z^+$.

Lemma 2.2 *The positive γ-periodic solution $(\tilde{c}_o(t), \tilde{c}_e(t))$ has the following properties*:

$$\lim_{t\to+\infty} t^{-1}\int_0^t \widetilde{c}_0(s)\,ds = \frac{k\mu}{h(g+m+b)\gamma},$$
$$\lim_{t\to+\infty} t^{-1}\int_0^t \widetilde{c}_e(s)\,ds = \frac{\mu}{h\gamma}, \qquad \lim_{t\to+\infty} t^{-1}\int_0^t \frac{\widetilde{c}_e(s)}{1+\alpha\widetilde{c}_e^2(s)}\,ds = \eta,$$
$$\lim_{t\to+\infty} t^{-1}\int_0^t \widetilde{c}_0^2(s)\,ds = \omega, \qquad \lim_{t\to+\infty} t^{-1}\int_0^t \left(\frac{\widetilde{c}_e(s)}{1+\alpha\widetilde{c}_e^2(s)}\right)^2 ds = \xi.$$

Proof By the lemma of Yang [26] and Lemma 2.1 of Lan [9] we get

$$\lim_{t\to+\infty} t^{-1}\int_0^t \widetilde{c}_0(s)\,ds = \frac{k\mu}{h(g+m+b)\gamma},$$
$$\lim_{t\to+\infty} t^{-1}\int_0^t \widetilde{c}_e(s)\,ds = \frac{\mu}{h\gamma},$$
$$\lim_{t\to+\infty} t^{-1}\int_0^t \frac{\widetilde{c}_e(s)}{1+\alpha\widetilde{c}_e^2(s)}\,ds = \eta.$$

Next, we will prove the last two limits. From

$$\widetilde{c}_0(t) = \widetilde{c}_0(0)e^{-(g+m+b)t} + \frac{k\mu(e^{-(g+m+b)t} - e^{-h(t-n\gamma)})}{(h-g-m-b)(1-e^{-h\gamma})},$$
$$\widetilde{c}_e(t) = \frac{\mu e^{-ht}}{1-e^{-h\gamma}}$$

for all $t \in (0,\gamma]$ we have

$$\begin{aligned}
&\gamma^{-1}\int_0^\gamma \tilde{c}_0^2(t)\,dt \\
&= \gamma^{-1}\int_0^\gamma \left(\tilde{c}_0(0)e^{-(g+m+b)t} + \frac{k\mu(e^{-(g+m+b)t} - e^{-ht})}{(h-g-m-b)(1-e^{-h\gamma})}\right)^2 dt \\
&= \left(\frac{k\mu}{(h-g-m-b)(1-e^{-(g+m+b)\gamma})}\right)^2 \gamma^{-1}\int_0^\gamma e^{-2(g+m+b)t}\,dt \\
&\quad + \left(\frac{k\mu}{(h-g-m-b)(1-e^{-h\gamma})}\right)^2 \gamma^{-1}\int_0^\gamma e^{-2h\gamma}\,dt \\
&\quad - 2\left(\frac{(k\mu)^2}{(h-g-m-b)^2(1-e^{-(g+m+b)\gamma})(1-e^{-h\gamma})}\right)\gamma^{-1}\int_0^\gamma e^{-(g+m+b+h)t}\,dt \\
&= \left(\frac{k\mu}{(h-g-m-b)(1-e^{-(g+m+b)\gamma})}\right)^2 \frac{1-e^{-2(g+m+b)\gamma}}{2(g+m+b)\gamma} \\
&\quad - \left(\frac{k\mu}{(h-g-m-b)(1-e^{-h\gamma})}\right)^2 \frac{1-e^{-2h\gamma}}{2h\gamma} \\
&\quad - 2\left(\frac{(k\mu)^2}{(h-g-m-b)^2(1-e^{-(g+m+b)\gamma})(1-e^{-h\gamma})}\right) \frac{1-e^{-(g+m+b+h)\gamma}}{(g+m+b+h)\gamma} \\
&= \nu.
\end{aligned}$$

Similarly,

$$\begin{aligned}\gamma^{-1}\int_0^\gamma \left(\frac{\tilde{c}_e(s)}{1+\alpha\tilde{c}_e^2(s)}\right)^2 ds &= \frac{-1}{h\gamma}\int_0^\gamma \frac{\tilde{c}_e(s)(-h\tilde{c}_e(s))}{(1+\alpha\tilde{c}_e^2(s))^2}\,ds \\ &= \frac{-1}{2h\gamma\alpha}\int_0^\gamma \frac{1}{(1+\alpha\tilde{c}_e{}^2(s))^2}\,d\bigl(1+\alpha\tilde{c}_e{}^2(s)\bigr) \\ &= \frac{1}{2h\gamma\alpha}\left(\frac{1}{1+\alpha\tilde{c}_e{}^2(\gamma)}-\frac{1}{1+\alpha\tilde{c}_e{}^2(0)}\right) \\ &= \xi.\end{aligned}$$

In view of the periodicity of $\tilde{c}_0(t)$ and $\tilde{c}_e(t)$ we can observe that

$$\begin{aligned}&\lim_{t\to+\infty} t^{-1}\int_0^t \tilde{c}_0^2(s)\,ds = \gamma^{-1}\int_0^\gamma \tilde{c}_0^2(s)\,ds = \nu, \\ &\lim_{t\to+\infty} t^{-1}\int_0^t \left(\frac{\tilde{c}_e(s)}{1+\alpha\tilde{c}_e^2(s)}\right)^2 ds = \gamma^{-1}\int_0^\gamma \left(\frac{\tilde{c}_e(s)}{1+\alpha\tilde{c}_e^2(s)}\right)^2 ds = \xi.\end{aligned}$$

□

Lemma 2.3 (see [27]) *Suppose that $x(t)\in C(\Omega\times[0,+\infty);R_+)$.*

(1) *If there are λ and positive constants λ_0, T such that*

$$\ln x(t) \le \lambda t - \lambda_0\int_0^t x(s)\,ds + \sum_{i=1}^n \beta_i\,dB_i(t), \quad t\ge T,$$

where β_i $(1\le i\le n)$ are constants, then

$$\begin{cases}\limsup_{t\to+\infty} t^{-1}\int_0^t x(s)\,ds \le \frac{\lambda}{\lambda_0} & a.s., \lambda\ge 0, \\ \lim_{t\to+\infty} x(t) = 0 & a.s., \lambda<0.\end{cases}$$

(2) *If there are positive constants λ, λ_0, and T such that*

$$\ln x(t) \ge \lambda t - \lambda_0\int_0^t x(s)\,ds + \sum_{i=1}^n \beta_i\,dB_i(t), \quad t\ge T,$$

then $\liminf_{t\to+\infty} t^{-1}\int_0^t x(s)\,ds \le \frac{\lambda}{\lambda_0}$ *a.s.*

Lemma 2.4 (see[14]) *Consider the stochastic differential equation*

$$dx(t) = x(t)\bigl(r-ax(t)\bigr)\,dt + \sigma x(t)\,dB(t),$$

where r, a, and σ are positive constants. If $r-0.5\sigma^2>0$, then

$$\lim_{t\to+\infty}\langle x(t)\rangle = \frac{r-0.5\sigma^2}{a}, \qquad \lim_{t\to+\infty}\frac{\ln x(t)}{t} = 0 \quad a.s.$$

3 Main results

Lemma 3.1 *For any given initial value $x(\theta)=\varphi(\theta)\in C([-\tau,0];R_+)$, the first equation of model (2) has a unique global positive solution $x(t)$. Moreover,* $\limsup_{t\to+\infty}\frac{\ln x(t)}{t}\le 0$ *a.s.*

Proof Considering the stochastic differential equation

$$
\begin{aligned}
dx(t) &= x(t)\left(r - r_0c_0(t) - ax(t) - cx(t-\tau) - \frac{\lambda c_e(t)}{1+\alpha c_e^2(t)}\right)dt \\
&\quad + \sigma_1 x(t)\,dB_1(t) + \sigma_2 c_0(t)x(t)\,dB_2(t) + \sigma_3 x^2(t)\,dB_3(t) \\
&\quad + \sigma_4 x(t)x(t-\tau)\,dB_4(t) + \frac{\sigma_5 x(t)c_e(t)}{1+\alpha c_e^2(t)}\,dB_5(t).
\end{aligned}
\tag{4}
$$

The proof of the existence and uniqueness of the global positive solution of the equation is similar to that of Dai et al. [24] by defining the nonnegative function

$$
V_1(x) = \sqrt{x} - 1 - 0.5\ln x + \frac{1+\sigma_4^2}{4}\int_{t-\tau}^{t} x^2(s)\,ds, \quad x > 0.
$$

Hence we omit it.

Next, we will focus on proving that $\lim_{t\to+\infty}\frac{\ln x(t)}{t} \le 0$ a.s. Applying Itô's formula to $\ln x(t)$, we obtain

$$
\begin{aligned}
d\ln x(t) &= \left(r - r_0c_0(t) - ax(t) - cx(t-\tau) - \frac{\lambda c_e(t)}{1+\alpha c_e^2(t)}\right)dt \\
&\quad - \frac{1}{2}\left(\sigma_1^2 + \sigma_2^2 C_0^2(t) + \sigma_3^2 x^2(t) + \sigma_4^2 x^2(t-\tau) + \sigma_5^2\left(\frac{c_e(t)}{1+\alpha c_e^2(t)}\right)^2\right)dt \\
&\quad + \sigma_1\,dB_1(t) + \sigma_2 c_0(t)\,dB_2(t) + \sigma_3 x(t)\,dB_3(t) \\
&\quad + \sigma_4 x(t-\tau)\,dB_4(t) + \frac{\sigma_5 c_e(t)}{1+\alpha c_e^2(t)}\,dB_5(t).
\end{aligned}
\tag{5}
$$

Using Itô's formula again, we get

$$
\begin{aligned}
de^t\ln x(t) &= e^t\ln x(t)\,dt + e^t\,d\bigl(\ln x(t)\bigr) \\
&= e^t\ln x(t) + e^t\left(r - r_0c_0(t) - ax(t) - cx(t-\tau) - \frac{\lambda c_e(t)}{1+\alpha c_e^2(t)}\right)dt \\
&\quad - 0.5e^t\left(\sigma_1^2 + \sigma_2^2 C_0^2(t) + \sigma_3^2 x^2(t) + \sigma_4^2 x^2(t-\tau) + \sigma_5^2\left(\frac{c_e(t)}{1+\alpha c_e^2(t)}\right)^2\right)dt \\
&\quad + e^t\left(\sigma_1\,dB_1(t) + \sigma_2 c_0(t)\,dB_2(t) + \sigma_3 x(t)\,dB_3(t) + \sigma_4 x(t-\tau)\,dB_4(t)\right. \\
&\quad \left. + \frac{\sigma_5 c_e(t)}{1+\alpha c_e^2(t)}\,dB_5(t)\right).
\end{aligned}
\tag{6}
$$

Integrating both sides of equality (6) from 0 to t, we get

$$
\begin{aligned}
e^t\ln x(t) &= \int_0^t e^s\left(r - r_0c_0(s) - ax(s) - cx(s-\tau) - \frac{\lambda c_e(s)}{1+\alpha c_e^2(s)}\right)ds \\
&\quad - \int_0^t \frac{e^s}{2}\left(\sigma_1^2 + \sigma_2^2 C_0^2(s) + \sigma_3^2 x^2(s) + \sigma_4^2 x^2(s-\tau) + \sigma_5^2\left(\frac{c_e(s)}{1+\alpha c_e^2(s)}\right)^2\right)ds \\
&\quad + \ln x(0) + M_1(t) + M_2(t) + M_3(t) + M_4(t) + M_5(t),
\end{aligned}
\tag{7}
$$

where

$$M_1(t) = \int_0^t \sigma_1 e^s \, dB_1(s), \qquad M_2(t) = \int_0^t \sigma_2 e^s c_0(s) \, dB_2(s),$$
$$M_3(t) = \int_0^t \sigma_3 e^s x(s) \, dB_3(s), \qquad M_4(t) = \int_0^t \sigma_4 e^s x(s-\tau) \, dB_4(s),$$
$$M_5(t) = \int_0^t \frac{\sigma_5 e^s c_e(s)}{1+\alpha c_e^2(s)} \, dB_5(s).$$

Note that $M_i(t)$ $(i = 1,2,3,4,5)$ is a local martingale. Therefore the quadratic variation of $M_i(t)$ is

$$\langle M_1(t), M_1(t)\rangle = \int_0^t \sigma_1^2 e^{2s} \, ds, \qquad \langle M_2(t), M_2(t)\rangle = \int_0^t \sigma_2^2 e^{2s} c_0^2(s) \, ds,$$
$$\langle M_3(t), M_3(t)\rangle = \int_0^t \sigma_3^2 e^{2s} x^2(s) \, ds, \qquad \langle M_4(t), M_4(t)\rangle = \int_0^t \sigma_4^2 e^{2s} x^2(s-\tau) \, ds,$$
$$\langle M_5(t), M_5(t)\rangle = \int_0^t \sigma_5^2 e^{2s} \left(\frac{c_e(s)}{1+\alpha c_e^2(s)}\right)^2 ds.$$

Using the exponential martingale inequality, for all $\delta > 0$, $\beta > 0$, $T > 0$, and $\theta > 1$, we have

$$P\left\{\sup_{0\le t\le T}\left[M_1(t) - \frac{\delta}{2}\langle M_1(t), M_1(t)\rangle\right] > \beta\right\} \le e^{-\alpha\beta}.$$

Taking $\delta = e^{-k}$, $\beta = \theta e^k \ln k$, and $T = k$, we have

$$P\left\{\sup_{0\le t\le k}\left[M_1(t) - \frac{e^{-k}}{2}\langle M_1(t), M_1(t)\rangle\right] > \theta e^k \ln k\right\} \le k^{-\theta}.$$

By the Borel–Cantelli lemma there are an event Ω and positive integers $k_1 = k_1(\omega)$ such that $P(\Omega) = 1$ and for all $\omega \in \Omega$ and $k > k_1$, we have

$$M_1(t) \le \frac{e^{-k}}{2}\langle M_1(t), M_1(t)\rangle + \theta e^k \ln k, \quad 0 \le t \le k.$$

Similarly,

$$M_i(t) \le \frac{e^{-k}}{2}\langle M_i(t), M_i(t)\rangle + \theta e^k \ln k, \quad 0 \le t \le k, i = 2,3,4,5.$$

By (7) we get

$$\begin{aligned}
e^t \ln x(t) \le{} & \int_0^t e^s \left(r - r_0 c_0(s) - a x(s) - c x(s-\tau) - \frac{\lambda c_e(s)}{1+\alpha c_e^2(s)}\right) ds \\
& - \int_0^t \frac{e^s}{2}\left(\sigma_1^2 + \sigma_2^2 C_0^2(s) + \sigma_3^2 x^2(s) + \sigma_4^2 x^2(s-\tau) + \sigma_5^2 \left(\frac{c_e(s)}{1+\alpha c_e^2(s)}\right)^2\right) ds \\
& + \int_0^t \frac{e^{2s-k}}{2}\left(\sigma_1^2 + \sigma_2^2 C_0^2(s) + \sigma_3^2 x^2(s) + \sigma_4^2 x^2(s-\tau) + \sigma_5^2 \left(\frac{c_e(s)}{1+\alpha c_e^2(s)}\right)^2\right) ds \\
& + \ln x(0) + 5\theta e^k \ln k, \quad 0 \le t \le k.
\end{aligned} \tag{8}$$

Because $a > 0$, we can easily obtain that there is a positive constant $K > 0$ such that $\ln x + r - ax \le K$, and from (8) it follows that

$$e^t \ln x(t) \le \ln x(0) + K\left(e^t - 1\right) + 5\theta e^k \ln k, \quad 0 \le t \le k.$$

For $k - 1 \le t \le k$,

$$\frac{\ln x(t)}{t} \le \frac{\ln x(0)}{te^t} + \frac{K(e^t - 1)}{te^t} + \frac{5\theta e^t \ln k}{(k-1)e^{k-1}},$$

and as $t \to +\infty$ ($k \to +\infty$), we have $\limsup_{t\to+\infty} \frac{\ln x(t)}{t} \le 0$ a.s. □

Theorem 3.2 *Let $x(t)$ be the solution of system* (2).

(i) *If $A < 0$, then the population $x(t)$ will die out almost surely.*

(ii) *If $A = 0$, then the population $x(t)$ is nonpersistent in the mean almost surely.*

(iii) *If $A > 0$, then the population $x(t)$ is weakly persistent almost surely.*

Proof Integrating from 0 to t both sides of equation (5), we get

$$\begin{aligned}\ln x(t)/x(0) &= \int_0^t \left(r - r_0 c_0(s) - ax(s) - cx(s-\tau) - \frac{\lambda c_e(s)}{1 + \alpha c_e^2(s)}\right) ds \\ &\quad - \int_0^t \frac{1}{2}\left(\sigma_1^2 + \sigma_2^2 C_0^2(s) + \sigma_3^2 x^2(s) + \sigma_4^2 x^2(s-\tau) + \sigma_5^2 \left(\frac{c_e(s)}{1 + \alpha c_e^2(s)}\right)^2\right) dt \\ &\quad + N_1(t) + N_2(t) + N_3(t) + N_4(t) + N_5(t),\end{aligned} \tag{9}$$

where

$$\begin{aligned} &N_1(t) = \int_0^t \sigma_1 \, dB_1(s), \qquad N_2(t) = \int_0^t \sigma_2 c_0(s) \, dB_2(s), \\ &N_3(t) = \int_0^t \sigma_3 x(s) \, dB_3(s), \qquad N_4(t) = \int_0^t \sigma_4 x(s-\tau) \, dB_4(s), \\ &N_5(t) = \int_0^t \frac{\sigma_5 c_e(s)}{1 + \alpha c_e^2(s)} \, dB_5(s). \end{aligned}$$

The quadratic variations of $N_2(t)$, $N_3(t)$, $N_4(t)$, and $N_5(t)$ are

$$\begin{aligned} &\langle N_2(t), N_2(t)\rangle = \int_0^t \sigma_2^2 c_0^2(s) \, ds, \qquad \langle N_3(t), N_3(t)\rangle = \int_0^t \sigma_3^2 x^2(s) \, ds, \\ &\langle N_4(t), N_4(t)\rangle = \int_0^t \sigma_4^2 x^2(s-\tau) \, ds, \qquad \langle N_5(t), N_5(t)\rangle = \int_0^t \left(\frac{\sigma_5 c_e(s)}{1 + \alpha c_e^2(s)}\right)^2 ds. \end{aligned}$$

For all $t \in (n\gamma, (n+1)\gamma]$, $n \in N_+$, we have

$$\frac{1}{(n+1)\gamma} \int_0^{n\gamma} c_0^2(s) \, ds \le \frac{1}{t} \int_0^t c_0^2(s) \, ds \le \frac{1}{n\gamma} \int_0^{(n+1)\gamma} c_0^2(s) \, ds.$$

Noting that $0 \le c_0(t) \le 1$ and $0 \le c_e(t) \le 1$, by Lemmas 2.1 and 2.2 we get

$$\begin{aligned}\lim_{t\to+\infty} \frac{\langle N_2(t), N_2(t)\rangle}{t} &= \lim_{t\to+\infty} \sigma_2^2 t^{-1} \int_0^t c_0^2(s)\,ds \\ &= \sigma_2^2\gamma^{-1}\int_0^\gamma c_0^2(s)\,ds = \nu\sigma_2^2 < +\infty. \end{aligned} \tag{10}$$

Similarly,

$$\lim_{t\to+\infty} \frac{\langle N_5(t), N_5(t)\rangle}{t} = \sigma_5^2\gamma^{-1}\int_0^\gamma \left(\frac{c_e(s)}{1+\alpha c_e^2(s)}\right)^2 ds = \sigma_5^2\xi < +\infty. \tag{11}$$

By the strong law of large numbers it follows that

$$\lim_{t\to+\infty}\frac{N_1(t)}{t} = 0, \qquad \lim_{t\to+\infty}\frac{N_2(t)}{t} = 0, \qquad \lim_{t\to+\infty}\frac{N_5(t)}{t} = 0 \quad \text{a.s.} \tag{12}$$

By the exponential martingale inequality, choosing $\delta = 1$, $\beta = 2\ln k$, and $T = k$, have

$$P\left\{\sup_{0\le t\le k}\left[N_i(t) - \frac{1}{2}\langle N_i(t), N_i(t)\rangle\right] > 2\ln k\right\} \le k^{-2}, \quad i = 3,4.$$

It follows from Borel–Cantelli lemma that there exists a positive constant k_1 such that, for $k > k_1$, we have

$$N_i(t) \le 0.5\langle N_i(t), N_i(t)\rangle + 2\ln k, \quad i = 3,4, 0 \le t \le k.$$

Substituting this inequality into (9), we obtain that

$$\begin{aligned}\frac{\ln x(t)/x(0)}{t} &\le r - 0.5\sigma_1^2 - r_0\langle c_0(t)\rangle - a\langle x(t)\rangle - c\langle x(t-\tau)\rangle \\ &\quad - \lambda\left\langle\frac{c_e(t)}{1+\alpha c_e^2(t)}\right\rangle - 0.5\sigma_2^2\langle c_0^2(t)\rangle - 0.5\sigma_5^2\left\langle\left(\frac{c_e(t)}{1+\alpha c_e^2(t)}\right)^2\right\rangle \\ &\quad + \frac{N_1(t)+N_2(t)+N_5(t)}{t} + \frac{4\ln k}{t}. \end{aligned} \tag{13}$$

For all $t \in (n\gamma, (n+1)\gamma]$, we have

$$\begin{aligned}&\frac{1}{(n+1)\gamma}\int_0^{n\gamma} c_0(s)\,ds \le \frac{1}{t}\int_0^t c_0(s)\,ds \le \frac{1}{n\gamma}\int_0^{(n+1)\gamma} c_0(s)\,ds, \\ &\frac{1}{(n+1)\gamma}\int_0^{n\gamma}\frac{c_e(s)}{1+\alpha c_e(s)}\,ds \le \frac{1}{t}\int_0^t \frac{c_e(s)}{1+\alpha c_e(s)}\,ds \le \frac{1}{n\gamma}\int_0^{(n+1)\gamma}\frac{c_e(s)}{1+\alpha c_e(s)}\,ds.\end{aligned}$$

It follows from Lemmas 2.1 and 2.2 that

$$\lim_{t\to+\infty} t^{-1}\int_0^t c_0(s)\,ds = \gamma^{-1}\int_0^\gamma \tilde{c}_0(s)\,ds = \frac{ku}{h(g+m+b)\gamma}, \tag{14}$$

$$\lim_{t\to+\infty} t^{-1}\int_0^t \frac{c_e(s)}{1+\alpha c_e^2(s)}\,ds = \gamma^{-1}\int_0^\gamma \frac{\tilde{c}_e(s)}{1+\alpha \tilde{c}_e^2(s)}\,ds = \eta. \tag{15}$$

Let $k-1\leq t\leq k$, $k\in N_+$, $A=r-0.5\sigma_1^2-\frac{r_0ku}{h(g+m+b)\gamma}-\lambda\eta-0.5\sigma_2^2\nu-0.5\sigma_5^2\xi$. By virtue of (10), (11), (12), (14), and (15), for all $\epsilon>0$, there exists $T>0$ such that for all $t>T$ (i.e., $k>T$), we have

$$\begin{aligned}
&-\frac{\epsilon}{4}\leq\frac{\ln x(0)}{t}\leq\frac{\epsilon}{4},\qquad -\frac{\epsilon}{4}\leq\frac{2\ln k}{t}\leq\frac{\epsilon}{4},\qquad -\frac{\epsilon}{4}\leq\frac{N_1(t)+N_2(t)+N_5(t)}{t}\leq\frac{\epsilon}{4},\\
&A-\frac{\epsilon}{4}\leq r-\frac{\sigma_1^2}{2}-r_0\langle c_0(t)\rangle-\lambda\left\langle\frac{c_e(t)}{1+\alpha c_e^2(t)}\right\rangle-\frac{\sigma_2^2}{2}\langle c_0^2(t)\rangle\\
&\qquad-\frac{\sigma_5^2}{2}\left\langle\left(\frac{c_e(t)}{1+\alpha c_e^2(t)}\right)^2\right\rangle\leq A+\frac{\epsilon}{4}.
\end{aligned}\tag{16}$$

(i) If $A<0$, then for ϵ small enough such that $A+\epsilon<0$, by (13) and (16) we get

$$\frac{\ln x(t)}{t}\leq A+\epsilon-a\langle x(t)\rangle+\frac{B_1(t)}{t},\quad t\geq T.\tag{17}$$

Then by Lemma 2.3 we get $\lim_{t\to+\infty}x(t)=0$ a.s.

(ii) If $A=0$, then by (17) and Lemma 2.3 we have $\langle x(t)\rangle^*\leq\frac{\epsilon}{a}$ a.s. By the arbitrariness of ϵ we have $\langle x(t)\rangle^*=0$ a.s.

(iii) If $A>0$, then suppose the conclusion is not true, that is, $P(\{\omega|x^*(t,\omega)=0\})>0$. Then for all $\omega\in\{\omega|x^*(t,\omega)=0\}$, we have $\lim_{t\to+\infty}x(t,\omega)=0$ a.s.

Because

$$t^{-1}\int_0^t x(s-\tau)\,ds=t^{-1}\left[\int_{-\tau}^0 x(s)\,ds+\int_t^{t-\tau}x(s)\,ds\right]+t^{-1}\int_0^t x(s)\,ds,$$

it follows from $\lim_{t\to+\infty}x(\omega,t)=0$ a.s. that

$$\begin{aligned}
&\lim_{t\to+\infty}t^{-1}\left[\int_{-\tau}^0 x(s)\,ds+\int_t^{t-\tau}x(s)\,ds\right]=0\quad\text{a.s.},\\
&\lim_{t\to+\infty}\langle x(t)\rangle=\lim_{t\to+\infty}\langle x^2(t)\rangle=\lim_{t\to+\infty}\langle x(t-\tau)\rangle=\lim_{t\to+\infty}\langle x^2(t-\tau)\rangle=0.
\end{aligned}\tag{18}$$

It follows from (9) that

$$\begin{aligned}
\frac{\ln x(t)}{t}&=\frac{\ln x(0)}{t}+r-0.5\sigma_1^2-r_0\langle c_0(t)\rangle-\lambda\left\langle\frac{c_e(t)}{1+\alpha c_e^2(t)}\right\rangle-0.5\sigma_2^2\langle c_0^2(t)\rangle\\
&\quad-0.5\sigma_5^2\left\langle\left(\frac{c_e(t)}{1+\alpha c_e^2(t)}\right)^2\right\rangle-a\langle x(t)\rangle-c\langle x(t-\tau)\rangle-0.5\sigma_3^2\langle x^2(t)\rangle\\
&\quad-0.5\langle x^2(t-\tau)\rangle+\frac{N_1(t)+N_2(t)+N_3(t)+N_4(t)+N_5(t)}{t}.
\end{aligned}\tag{19}$$

For all $\omega\in\{\omega|x^*(t,\omega)=0\}$, by (18) we get that $\lim_{t\to+\infty}\frac{N_i(t)}{t}=0$ a.s. ($i=1,2,3,4,5$). Taking the limit superior of both sides of (19), we obtain that

$$\limsup_{t\to+\infty}\frac{\ln x(t)}{t}=A>0\quad\text{a.s.},$$

which is a contradiction to the result of Lemma 3.1. □

Theorem 3.3 *Suppose that $\sigma_3 = \sigma_4 = 0$, and let $x(t)$ be the positive solution of model (2) with initial value $x(\theta) = \varphi(\theta) \in C([-\tau, 0], R_+)$. Then*:

(I) *If $A < 0$, then the population x goes to die out a.s., that is, $\lim_{t\to+\infty} x(t) = 0$ a.s.*

(II) *If $A > 0$, then the population x is strongly persistent in the mean a.s.; moreover, $\lim_{t\to+\infty} t^{-1}\int_0^t x(s)\,ds = \frac{A}{a+c}$ a.s.*

Proof If $\sigma_3 = \sigma_4 = 0$, then by (19) we have

$$\begin{aligned}\frac{\ln x(t)}{t} &= \frac{\ln x(0)}{t} + r - 0.5\sigma_1^2 - r_0\langle c_0(t)\rangle - \left\langle \frac{\lambda c_e(t)}{1+\alpha c_e^2(t)}\right\rangle - 0.5\sigma_2^2\langle c_0^2(t)\rangle \\ &\quad - 0.5\sigma_5^2\left\langle\left(\frac{c_e(t)}{1+\alpha c_e^2(t)}\right)^2\right\rangle - a\langle x(t)\rangle - c\langle x(t-\tau)\rangle + \frac{N_1(t)+N_2(t)+N_5(t)}{t}. \end{aligned}\tag{20}$$

It follows from (12) and (16) that for all $\epsilon > 0$, there exists a positive constant T_1 such that for $t > T_1$, we have

$$\frac{\ln x(t)}{t} \le A + \epsilon - a\langle x(t)\rangle - c\langle x(t-\tau)\rangle + \frac{B_1(t)}{t}, \tag{21}$$

$$\frac{\ln x(t)}{t} \ge A - \epsilon - a\langle x(t)\rangle - c\langle x(t-\tau)\rangle + \frac{B_1(t)}{t}. \tag{22}$$

(I) If $A < 0$, then by Lemma 2.3 and (21) we have $\lim_{t\to+\infty} x(t) = 0$ a.s.

Now we will prove (II). Let us consider the following auxiliary equation:

$$\begin{aligned} dy(t) &= y(t)\left(r - r_0c_0(t) - ay(t) - \frac{\lambda c_e(t)}{1+\alpha c_e^2(t)}\right)dt + \sigma_1 y(t)\,dB_1(t) \\ &\quad + \sigma_2 c_0(t)y(t)\,dB_2(t) + \frac{\sigma_5 y(t)c_e(t)}{1+\alpha c_e^2(t)}\,dB_5(t), \end{aligned}\tag{23}$$

where

$$y(\theta) = \varphi(\theta) > 0, \quad \theta \in [-\tau, 0].$$

It follows from the stochastic comparison theorem [14] that $x(t) \le y(t)$. Hence, for all $\epsilon > 0$, there is a positive constant T_2 such that for all $t > T_2$, we have

$$\frac{\ln y(t)}{t} \le A + \epsilon - a\langle y(t)\rangle + \frac{B_1(t)}{t}, \tag{24}$$

$$\frac{\ln y(t)}{t} \ge A - \epsilon - a\langle y(t)\rangle + \frac{B_1(t)}{t}. \tag{25}$$

By (24), (25), and Lemma 2.3 this implies that

$$\frac{A-\epsilon}{a} \le \langle y(t)\rangle_* \le \langle y(t)\rangle^* \le \frac{A+\epsilon}{a}. \tag{26}$$

Due to the arbitrariness of ϵ, from (26) we get

$$\lim_{t\to+\infty} t^{-1}\int_0^t y(s)\,ds = \frac{A}{a} \quad \text{a.s.}$$

Consequently,

$$\lim_{t\to+\infty} t^{-1}\left[\int_{-\tau}^{0} y(s)\,ds + \int_{0}^{t-\tau} y(s)\,ds - \int_{0}^{t} y(s)\,ds\right] = 0 \quad \text{a.s.} \tag{27}$$

Since $x(t) \le y(t)$, by (27) we easily see that

$$\lim_{t\to+\infty} t^{-1}\left[\int_{-\tau}^{0} x(s)\,ds + \int_{0}^{t-\tau} x(s)\,ds - \int_{0}^{t} x(s)\,ds\right] = 0 \quad \text{a.s.} \tag{28}$$

It follows from (28) that, for all $\epsilon > 0$, there exists a positive constant T_3 such that for $t > T_3$, have

$$-\epsilon \le t^{-1}\left[\int_{-\tau}^{0} x(s)\,ds + \int_{0}^{t-\tau} x(s)\,ds - \int_{0}^{t} x(s)\,ds\right] \le \epsilon.$$

For all $\epsilon > 0$, there exists $T = \max\{T_1, T_3\}$ such that, for $t > T$, have

$$\frac{\ln x(t)}{t} \le A + 2\epsilon - (a+c)\langle x(t)\rangle + \frac{B_1(t)}{t}, \tag{29}$$

$$\frac{\ln x(t)}{t} \ge A - 2\epsilon - (a+c)\langle x(t)\rangle + \frac{B_1(t)}{t}. \tag{30}$$

By (29) and (30) we derive that

$$\lim_{t\to+\infty} t^{-1}\int_{0}^{t} x(s)\,ds = \frac{A}{a+c} \quad \text{a.s.}$$

□

Remark 3.4 From Theorems 3.2 and 3.3 we easily see that A is the threshold of system (2) for the single-species population extinction and weak persistence. If $A < 0$, then the population will be extinct, and if $A > 0$, then the population is weakly persistent. Particularly, when $\sigma_3 = \sigma_4 = 0$, A is also the threshold of system (2) for the single-species population extinction and strong persistence; moreover, if $A > 0$, then the single-species population is stable in the mean.

Remark 3.5 Note that from the expression for $A = r - \frac{r_0 k\mu}{h(g+m+b)\gamma} - \lambda\eta - 0.5\sigma_1^2 - 0.5\sigma_2^2\nu - 0.5\sigma_5^2\xi$ we can find that the parameters μ and γ obviously affect the persistence and extinction of system (2), that is, we can control the persistence and extinction of population x by controlling the toxicant input amount μ and the period of pulse input toxicant γ.

Remark 3.6 Theorems 3.2 and 3.3 show that the persistence population x of deterministic system (1) may be extinct when σ_1, σ_2, and σ_5 are large enough; however, τ, σ_3, and σ_4 have no effect on the persistence and extinction for population x.

4 Numerical simulations and discussion

Next, we show the numerical simulation results to illustrate the accuracy of analytical results in the previous section by using the famous Milstein method [28]. We choose the parameters of system (2) as follows:

$$r = 0.4, \qquad r_0 = 0.3, \qquad a = 0.4, \qquad c = 0.4, \qquad \lambda = 0.8, \qquad k = 0.6,$$

$$g = 0.3, \qquad m = 0.3, \qquad b = 0.1, \qquad h = 0.4, \qquad \alpha = 0.4, \qquad \tau = 1.$$

To understand the effects of white noise and pulse toxicant input on population dynamics, we change the values of μ, γ, and σ_i ($i = 1,2,3,4,5$).

(1) We firstly adopt $u = 0.3$, $\sigma_1 = 0.3$, $\sigma_2 = 0.2$, $\sigma_3 = 0.2$, $\sigma_4 = 0.4$, $\sigma_5 = 0.4$. If $\gamma = 5$, then simple calculation shows that $A = 0.1955 > 0$. In view of the Theorem 3.2, we obtain that the single-species population x is weakly persistent; see Fig. 1. If $\gamma = 2$, then by computing we have $A = -0.0439 < 0$, so condition (i) of Theorem 3.2 holds, that is, the population x of system (2) will extinct (see Fig. 2).

(2) Next, to analyze the effect of the toxicant input amount each time μ on the persistence of the single species, we adopt $u = 0.8$, $\sigma_1 = 0.3$, $\sigma_2 = 0.2$, $\sigma_3 = 0.2$, $\sigma_4 = 0.4$, $\sigma_5 = 0.4$, and $\gamma = 5$. Simple calculation shows that $A = -0.0555 < 0$, and from Theorem 3.2 it follows that the population x will die out a.s. (see Fig. 3). From Figs. 1 and 3 we can see that the population x will die out when the environmental toxicant amount of each time μ is large enough.

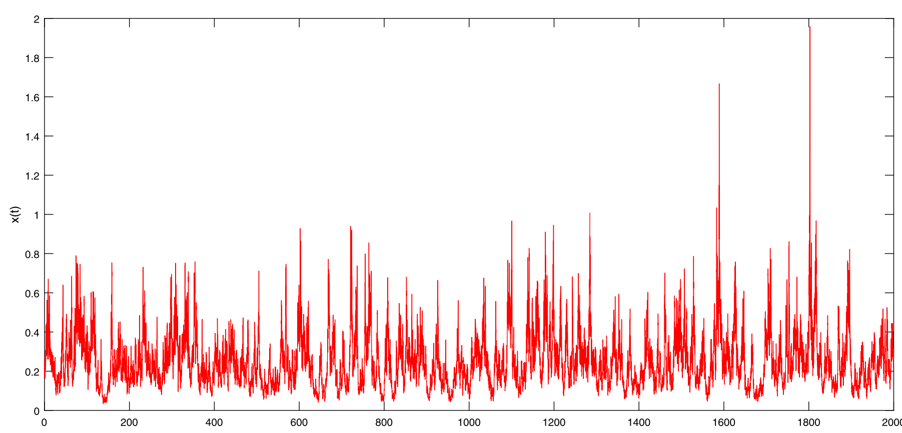

Figure 1 Solution of system (2) with $\sigma_1 = 0.3$, $\sigma_2 = 0.2$, $\sigma_3 = 0.2$, $\sigma_4 = 0.4$, $\sigma_5 = 0.4$, $u = 0.3$, $\gamma = 5$

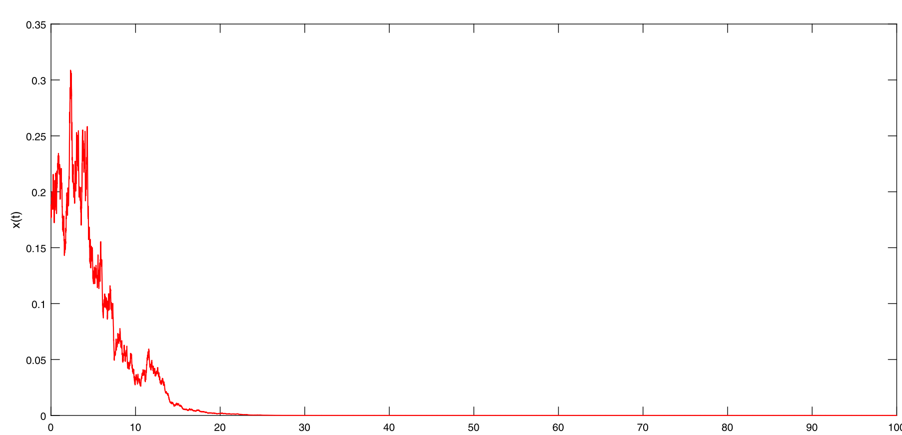

Figure 2 Solution of system (2) with $\sigma_1 = 0.3$, $\sigma_2 = 0.2$, $\sigma_3 = 0.2$, $\sigma_4 = 0.4$, $\sigma_5 = 0.4$, $u = 0.3$, $\gamma = 2$

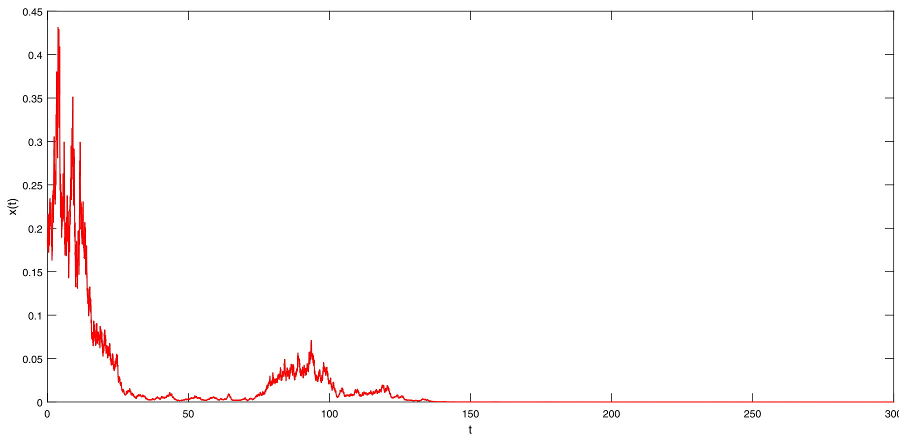

Figure 3 Solution of system (2) with $\sigma_1 = 0.3$, $\sigma_2 = 0.2$, $\sigma_3 = 0.2$, $\sigma_4 = 0.4$, $\sigma_5 = 0.4$, $u = 0.8$, $\gamma = 5$

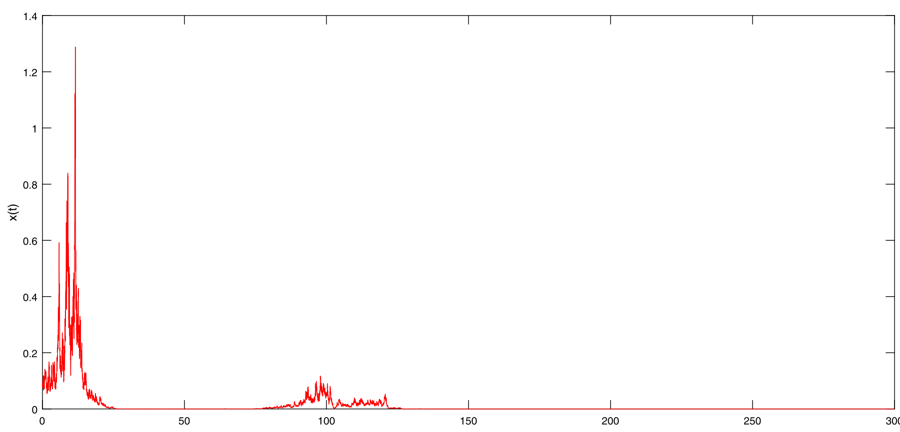

Figure 4 Solution of system (2) with $\sigma_1 = 0.8$, $\sigma_2 = 0.2$, $\sigma_3 = 0.2$, $\sigma_4 = 0.6$, $\sigma_5 = 0.6$, $u = 0.3$, $\gamma = 5$

(3) On the other hand, we will focus on the influence of the intensity of white noises on the survival for the population x. We adopt $u = 0.3$ and $\gamma = 5$ and suppose that $\sigma_1 = 0.8$, $\sigma_2 = 0.2$, $\sigma_3 = 0.2$, $\sigma_4 = 0.6$, and $\sigma_5 = 0.6$. Simple calculation shows that $A = -0.0823 < 0$, so that the population x will die out (see Fig. 4). Suppose $\sigma_1 = 0.3$, $\sigma_2 = 0.2$, $\sigma_3 = 0$, $\sigma_4 = 0$, and $\sigma_5 = 0.4$. Then $A = 0.1955 > 0$, and by Theorem 3.3 we obtain that the population x is strongly persistent in the mean; moreover, $\lim_{t\to+\infty} t^{-1}\int_0^t x(s)\,ds = 0.2443$, that is, the population is stable in the mean (see Fig. 5). From Figs. 1 and 5 we can see that the population x will die out when $\sigma_1, \sigma_2, \sigma_5$ are large enough, but σ_3 and σ_4 have no effect on the survival of the population x.

5 Conclusions

We studied a stochastic impulsive single-species population model with psychological effect and delay in pollution environment. We obtained the threshold A between weak persistence and extinction. Particularly, A is also a threshold of system (2) for the strong persistence and extinction when $\sigma_3 = \sigma_4 = 0$. These results have revealed that the environmental noise, the impulsive period, and the amount of toxicant input for each time have

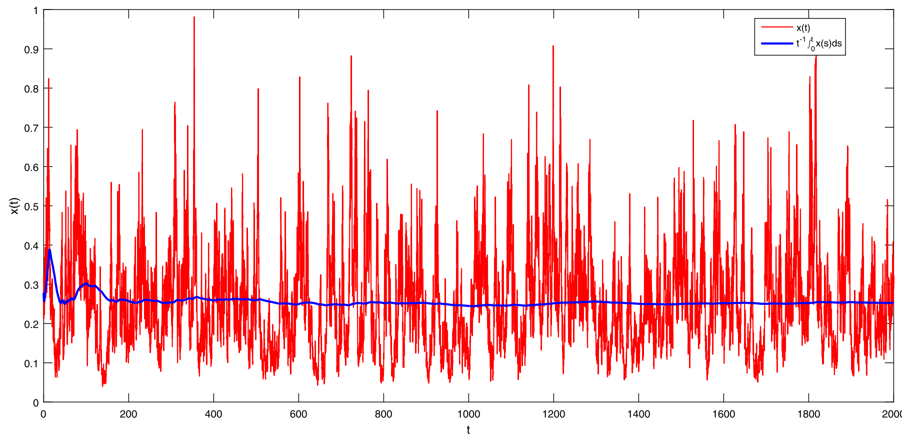

Figure 5 Solution of system (2) with $\sigma_1 = 0.3$, $\sigma_2 = 0.2$, $\sigma_3 = 0$, $\sigma_4 = 0$, $\sigma_5 = 0.4$, $u = 0.3$, $\gamma = 5$

an influence on the persistence and extinction for the single-species population and show that the delay τ and the intensities of the white noises σ_3^2 and σ_4^2 have no effect on the persistence and extinction for the population x. Therefore Theorems 3.2 and 3.3 extend the corresponding results in [8, 9].

However, there are still many interesting questions to be further studied. On the one hand, we can propose more realistic and complex models, and the method used in this paper can be also applied to study other interesting models, such as Gompertz models, Gilpin–Ayala models, and so on. On the other hand, in recent years, optimal harvesting problems have received a lot of attention [29], and hence we could add harvest items to model (2) and discuss the optimal harvesting problem. We leave these investigations for future work.

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Availability of data and materials
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Competing interests
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Authors' contributions
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