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Coexistence and extinction of a periodic stochastic predator–prey model with general functional response

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Abstract

This note deals with a stochastic predator–prey system with periodic coefficients and general functional response, and provides the threshold between coexistence and extinction. The result refines and evolves some prior investigations.

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Keywords: Predator–prey model; Stochasticity; Periodic; Coexistence-and-extinction threshold

1 Introduction

This note dissects the following predator–prey model with random perturbations:

$$\begin{cases} \mathrm{d}N_1 = \{N_1[r_1(t) - a_1(t)N_1] - f(N_1, N_2, t)N_2\}\,\mathrm{d}t + \xi_1(t)N_1\,\mathrm{d}B_1(t), \\ \mathrm{d}N_2 = N_2[-r_2(t) + \eta(t)f(N_1, N_2, t) - a_2(t)N_2]\,\mathrm{d}t + \xi_2(t)N_2\,\mathrm{d}B_2(t), \end{cases} \tag{1}$$

where $N_1 = N_1(t)$ and $N_2 = N_2(t)$ indicate the prey and predator population sizes, respectively. The growth rate $r_1(t)$, the death rate $r_2(t)$, the intra-specific competition rate $a_i(t)$, and the intensity of the white noise $\xi_i(t)$ are continuous, positive, and T-periodic functions on $\mathbb{R}_+ = [0, +\infty)$; $B_1(t)$ and $B_2(t)$ are two independent Brownian motions defined on a complete probability space $(\Omega, \mathcal{F}, \mathrm{P})$ which obeys the usual conditions; $\eta(t) > 0$, a T-periodic function, means the food conversion, the T-periodic function $f(N_1, N_2, t) \in \mathrm{C}(\mathbb{R}_+^3, \mathbb{R}_+)$ denotes the functional response which complies with the follow conditions:

(C1) $f(e^{N_1}, e^{N_2}, t)$ and $g(e^{N_1}, e^{N_2}, t)$ are locally Lipschitz (see, e.g., [16]), where $g(N_1, N_2, t) = f(N_1, N_2, t)N_2/N_1$;

(C2) $f(0, N_2, t) \equiv 0$ for all $N_2 \geq 0$ and $t \geq 0$, which indicates that without the prey, the predator will go to extinction; $g(N_1, 0, t) \equiv 0$ for all $N_1 \geq 0$ and $t \geq 0$, which indicates that without the predator, the growth of the prey will follow the logistic rule;

(C3) $\frac{\partial f(N_1,N_2,t)}{\partial N_1} \geq 0$ and $-M \leq \frac{\partial f(N_1,N_2,t)}{\partial N_2} \leq 0$ for all $(N_1, N_2, t) \in \mathbb{R}_+^3$, where M is a positive constant. The former indicates that the more the prey, the more food each predator will obtain. The later indicates that the more the predator, the less food each predator will obtain;

(C4) For arbitrary $(N_1,N_2,t)\in\mathbb{R}_+^3$ and arbitrary $(x_1,x_2,t)\in\mathbb{R}_+^3$, $f(N_1,N_2,t)\le M_1N_1$, $g(N_1,N_2,t)\le M_1N_2$, $|f(x_1,0,t)-f(x_2,0,t)|\le M_2|x_1-x_2|$, where M_1 and M_2 are positive constants.

Establishing theoretical coexistence-and-extinction threshold is an interesting issue in the exploration of stochastic ecological models [18] and has attracted many scholars' attention (see, e.g., [4, 7, 11–15, 19]). However, most prior threshold explorations have concentrated on autonomous models (see, e.g., [12]) or single-species nonautonomous models (see, e.g., [14, 19]), little research has been conducted to provide the coexistence-and-extinction threshold for the multi-species nonautonomous model (1). For this reason, this note dissects model (1).

Many functional response functions satisfy (C1)–(C4), for example:

- (Lotka–Volterra) $f(N_1,N_2,t)=m(t)N_1$;
- (Holling II [17]) $f(N_1,N_2,t)=\frac{m_1(t)N_1}{1+m_2(t)N_1}$;
- (Beddington–DeAngelis [1, 3]) $f(N_1,N_2,t)=\frac{m_1(t)N_1}{m_2(t)N_1+m_3(t)N_2+m_4(t)}$,

where $m(t)$ and $m_i(t)$ are continuous, positive, and T-periodic functions on $\mathbb{R}_+$.

For model (1), we are going to prove

Theorem 1 *Define*

$$b_1(t)=r_1(t)-\xi_1^2(t)/2,\qquad \varLambda_1=\frac{1}{T}\int_0^T b_1(s)\,\mathrm{d}s,$$
$$b_2(t)=r_2(t)+\xi_2^2(t)/2,\qquad \varLambda_2=\frac{1}{T}\int_0^T\left[\eta(s)\int_0^{+\infty}f(\tau,0,s)\rho_s(\mathrm{d}\tau)-b_2(s)\right]\mathrm{d}s,$$

where $\rho_t(\cdot)$ is a periodic measure which is given in Lemma 1 below. Under (C1)–(C4),

(i) *If $\varLambda_1<0$, then both species 1 and 2 die out almost surely (a.s.), i.e., $\lim_{t\to+\infty}N_1(t)=0$ and $\lim_{t\to+\infty}N_2(t)=0$ a.s.;*

(ii) *If $\varLambda_1>0$ and $\varLambda_2<0$, then species 2 dies out a.s. and the distribution of $N_1(t)$ converges weakly to $\rho_t(\cdot)$;*

(iii) *If $\varLambda_1>0$ and $\varLambda_2>0$, then species 1 and 2 are uniformly weakly persistent in the mean (UWPIM), i.e., there is a pair of positive constants β_1 and β_2 such that*

$$\beta_1<\limsup_{t\to+\infty}t^{-1}\int_0^t N_i(s)\,\mathrm{d}s<\beta_2,\quad i=1,2.$$

Remark 1 Zu et al. [20] investigated model (1) with $f(N_1,N_2,t)=m(t)N_1$, and testified that

– If $\varLambda_1>0$ and $T^{-1}\int_0^T[\eta^u m^u b_1(s)-a_1^l b_2(s)]\,\mathrm{d}s<0$, then species 2 dies out, where $c^u=\max_{0\le t\le T}\{c(t)\}$, $c^l=\min_{0\le t\le T}\{c(t)\}$;

– If $\varLambda_1>0$ and $T^{-1}\int_0^T[\eta^l m^l b_1(s)-a_1^u b_2(s)]\,\mathrm{d}s>0$, then both species 1 and 2 are weakly persistent in the mean (WPIM), i.e., $\limsup_{t\to+\infty}t^{-1}\int_0^t N_i(s)\,\mathrm{d}s>0$, $i=1,2$.

Theorem 1 refines and evolves the work of [20]: firstly, the model in [20] is a special case of model (1); secondly, there is a gap in [20] while we provide the coexistence-and-extinction threshold; thirdly, we provide the conditions for UWPIM which indicates WPIM, however, WPIM does not indicate UWPIM.

Remark 2 Li and Zhang [8] delved into model (1) with

$$f(N_1,N_2,t)=\frac{m_1(t)N_1}{m_2(t)N_1+m_3(t)N_2+m_4(t)}$$

in thee nonautonomous case. When their results are restricted to the periodic case, Li and Zhang [8] testified that

- If $\Lambda_1>0$ and $T^{-1}\int_0^T[\frac{\eta^u m_1^u}{m_4^l}b_1(s)-a_1^l b_2(s)]\,\mathrm{d}s<0$, then species 2 dies out;
- If $\Lambda_1>0$ and $T^{-1}\int_0^T[\frac{\eta(s)m_1(s)\psi^*(s)}{m_2(s)\psi^*(s)+m_3(s)\phi^*(s)+m_4(s)}-b_2(s)]\,\mathrm{d}s>0$, then both species 1 and are WPIM, where $(\psi^*(t),\phi^*(t))$ is the periodic solution of the following equations:

$$\begin{aligned}\mathrm{d}\psi&=\psi\bigl[r_1(t)-a_1(t)\psi\bigr]\,\mathrm{d}t+\xi_1(t)\psi\,\mathrm{d}B_1(t),\\ \mathrm{d}\phi&=\phi\bigl[-r_2(t)+\eta(t)m_1(t)/m_2(t)-a_2(t)\phi\bigr]\,\mathrm{d}t+\xi_2(t)\phi\,\mathrm{d}B_2(t).\end{aligned}\tag{2}$$

Theorem 1 refines and evolves the above results in the periodic case: firstly, model (1) is more general; secondly, we provide the coexistence-and-extinction threshold while there is a gap in [8]; thirdly, we provide the conditions for UWPIM, the authors [8] provided conditions for WPIM.

2 Proof

Lemma 1 *If $\Lambda_1>0$, then Eq. (2) possesses a positive T-periodic solution $\psi^*(t)$ which complies with*

$$\lim_{t\to+\infty}\bigl|\psi(t)-\psi^*(t)\bigr|=0,\tag{3}$$

where $\psi(t)$ is an arbitrary solution of Eq. (2) with $\psi(0)>0$, and there is a continuous T-periodic function $\rho_s(\cdot)$ such that the transition function $p(s,\psi(s),s+t,\cdot)$ converges weakly to $\rho_s(\cdot)$ as $t\to+\infty$.

Proof The existence of $\psi^*(t)$ follows from Theorem 3.1 in [20]. In accordance with Theorem 6.2 in [9], one can obtain (3). Analogous to the proof of Lemma 2.6 in [12], one can obtain the last assertion. □

Lemma 2 *For any $(N_1(0),N_2(0))\in\mathbb{R}_+^{2,0}=\{x\in\mathbb{R}^2:x_1>0,x_2>0\}$, under* (C1), *model* (1) *possesses a unique solution $(N_1(t),N_2(t))$ which is global and positive a.s. Additionally,*

$$\limsup_{t\to+\infty}\ln N_1(t)/\ln t\le 1,\qquad \limsup_{t\to+\infty}\ln N_2(t)/\ln t\le 1,\quad a.s.\tag{4}$$

Proof On the basis of (C1), model (1) possesses a unique solution which is local and positive. Analogous to the proof of Theorem 2.1 in [9], namely, using Itô's formula to $V(N_1,N_2)=\sum_{i=1,2}(N_i-1-\ln N_i)$, one can illustrate that this local solution is global. On the other hand, analogous to the proof of Lemma 3.4 in [9], one obtains (4). □

Proof of Theorem 1 In the light of Itô's formula,

$$\begin{aligned}t^{-1}\ln\bigl(N_1(t)/N_1(0)\bigr)={}&t^{-1}\int_0^t b_1(s)\,\mathrm{d}s-t^{-1}\int_0^t a_1(s)N_1(s)\,\mathrm{d}s\\ &-t^{-1}\int_0^t g\bigl(N_1(s),N_2(s),s\bigr)\,\mathrm{d}s+t^{-1}\Psi_1(t),\end{aligned}\tag{5}$$

$$t^{-1}\ln\bigl(N_2(s)/N_2(0)\bigr) = -t^{-1}\int_0^t b_2(s)\,\mathrm{d}s - t^{-1}\int_0^t a_2(s)N_2(s)\,\mathrm{d}s + t^{-1}\int_0^t \eta(s)f\bigl(N_1(s),N_2(s),s\bigr)\,\mathrm{d}s + t^{-1}\Psi_2(t), \tag{6}$$

where $\Psi_i(t) = \int_0^t \xi_i(s)\,\mathrm{d}B_i(s)$, $i = 1,2$. Notice that (see [10])

$$\lim_{t\to+\infty} \Psi_i(t)/t = 0 \quad \text{a.s.} \tag{7}$$

(I) By (5) and (7),

$$\limsup_{t\to+\infty} t^{-1}\ln N_1(t) \le \limsup_{t\to+\infty} t^{-1}\int_0^t b_1(s)\,\mathrm{d}s = \Lambda_1 < 0.$$

As a result, $\lim_{t\to+\infty} N_1(t) = 0$. Then (6), (7), and (C2) suggest that

$$\limsup_{t\to+\infty} t^{-1}\ln N_2(t) \le -\limsup_{t\to+\infty} t^{-1}\int_0^t b_2(s)\,\mathrm{d}s < 0.$$

Accordingly, $\lim_{t\to+\infty} N_2(t) = 0$.

(II) We first show

$$\lim_{t\to+\infty} t^{-1}\int_0^t \eta(s)f\bigl(\psi(s),0,s\bigr)\,\mathrm{d}s = T^{-1}\int_0^T \eta(s)\int_0^{+\infty} f(\tau,0,s)\rho_s(\mathrm{d}\tau)\,\mathrm{d}s \quad \text{a.s.} \tag{8}$$

As a matter of fact, in accordance with (C4) and (3),

$$\lim_{t\to+\infty} t^{-1}\int_0^t \eta(s)\bigl|f\bigl(\psi(s),0,s\bigr) - f\bigl(\psi^*(s),0,s\bigr)\bigr|\,\mathrm{d}s = 0 \quad \text{a.s.} \tag{9}$$

Define $N^*(t) = \eta(t)f(\psi^*(t),0,t)$, then $N^*(t)$ is a T-periodic stochastic process which possesses a unique T-periodic measure $\lambda_t(\cdot)$. For this reason, $\bar{\lambda}(\cdot) := T^{-1}\int_0^T \lambda_s(\cdot)\,\mathrm{d}s$ is the unique invariant measure of $N^*(t)$ (see [5]). In accordance with Theorem 3.2.6 and Theorem 3.3.1 in [2], $\bar{\lambda}(\cdot)$ is ergodic. Accordingly,

$$\lim_{t\to+\infty} t^{-1}\int_0^t N^*(s)\,\mathrm{d}s = \int_A \mu\bar{\lambda}(\mathrm{d}\mu) \quad \text{a.s.,}$$

where $A = \{\mu | \mu = \eta(t)f(x,0,t), 0 \le t \le T, x > 0\}$. Hence

$$\begin{aligned}
&\lim_{t\to+\infty} t^{-1}\int_0^t \eta(s)f\bigl(\psi^*(s),0,s\bigr)\,\mathrm{d}s \\
&\quad= \lim_{t\to+\infty} t^{-1}\int_0^t N^*(s)\,\mathrm{d}s = T^{-1}\int_0^T\int_A \mu\lambda_s(\mathrm{d}\mu)\,\mathrm{d}s \\
&\quad= T^{-1}\int_0^T \eta(s)\int_0^{+\infty} f(\tau,0,s)\rho_s(\mathrm{d}\tau)\,\mathrm{d}s \quad \text{a.s.}
\end{aligned}$$

Then (8) follows from (9). Now we prove $\lim_{t\to+\infty} N_2(t) = 0$. Let $\bar{\psi}(t)$ be a solution of Eq. (2) with $\bar{\psi}(0) = N_1(0)$. In accordance with the comparison theorem (see [6]) and (C3), one can see that

$$\eta(t)f\big(N_1(t),N_2(t),t\big) \le \eta(t)f\big(N_1(t),0,t\big) \le \eta(t)f\big(\bar{\psi}(t),0,t\big).$$

Then by (6), we get

$$\begin{aligned} t^{-1}\ln\big(N_2(t)/N_2(0)\big) &= -t^{-1}\int_0^t b_2(s)\,\mathrm{d}s - t^{-1}\int_0^t a_2(s)N_2(s)\,\mathrm{d}s \\ &\quad + t^{-1}\int_0^t \eta(s)f\big(N_1(s),N_2(s),s\big)\,\mathrm{d}s + t^{-1}\Psi_2(t) \\ &\le -t^{-1}\int_0^t b_2(s)\,\mathrm{d}s - t^{-1}\int_0^t a_2(s)N_2(s)\,\mathrm{d}s \\ &\quad + t^{-1}\int_0^t \eta(s)f\big(\bar{\psi}(s),0,s\big)\,\mathrm{d}s + t^{-1}\Psi_2(t). \end{aligned} \tag{10}$$

Applying (7) and (8), one gets

$$\begin{aligned} \limsup_{t\to+\infty} t^{-1}\ln N_2(t) &\le -T^{-1}\int_0^T b_2(s)\,\mathrm{d}s + T^{-1}\int_0^T \eta(s)\int_0^{+\infty} f(\tau,0,s)\rho_s(\mathrm{d}\tau)\,\mathrm{d}s \\ &= \Lambda_2 < 0. \end{aligned}$$

Accordingly, $\lim_{t\to+\infty} N_2(t) = 0$ a.s. This suggests that the distribution of $N_1(t)$ converges weakly to $\rho_t(\cdot)$.

(III) We first show that

$$\limsup_{t\to+\infty} t^{-1}\int_0^t N_2(s)\,\mathrm{d}s \ge \beta_1 \quad \text{a.s.} \tag{11}$$

Otherwise, for any $\varepsilon > 0$, Eq. (1) possesses a solution $(\tilde{N}_1(t), \tilde{N}_2(t))$ with $\tilde{N}_1(0) > 0$ and $\tilde{N}_2(0) > 0$ such that $\mathrm{P}\{\limsup_{t\to+\infty} t^{-1}\int_0^t \tilde{N}_2(s)\,\mathrm{d}s < \varepsilon\} > 0$. Choose a sufficiently small ε such that

$$\Lambda_2 - \big(a_2^u + M\eta^u\big)\varepsilon - 2M_1M_2\eta^u\varepsilon > 0.$$

In the light of (6),

$$\begin{aligned} &t^{-1}\ln\big(\tilde{N}_2(t)/\tilde{N}_2(0)\big) \\ &\quad = -t^{-1}\int_0^t b_2(s)\,\mathrm{d}s + t^{-1}\int_0^t \eta(s)f\big(\tilde{N}_1(s),\tilde{N}_2(s),s\big)\,\mathrm{d}s - t^{-1}\int_0^t a_2(s)\tilde{N}_2(s)\,\mathrm{d}s + t^{-1}\Psi_2(t) \\ &\quad = -t^{-1}\int_0^t b_2(s)\,\mathrm{d}s + t^{-1}\int_0^t \eta(s)f\big(\tilde{\psi}(s),0,s\big)\,\mathrm{d}s - t^{-1}\int_0^t a_2(s)\tilde{N}_2(s)\,\mathrm{d}s + t^{-1}\Psi_2(t) \\ &\qquad + t^{-1}\int_0^t \eta(s)f\big(\tilde{N}_1(s),0,s\big)\,\mathrm{d}s - t^{-1}\int_0^t \eta(s)f\big(\tilde{\psi}(s),0,s\big)\,\mathrm{d}s \\ &\qquad + t^{-1}\int_0^t \eta(s)f\big(\tilde{N}_1(s),\tilde{N}_2(s),s\big)\,\mathrm{d}s - t^{-1}\int_0^t \eta(s)f\big(\tilde{N}_1(s),0,s\big)\,\mathrm{d}s, \end{aligned} \tag{12}$$

where $\tilde{\psi}(t)$ is the solution of Eq. (2) with $\tilde{\psi}(0) = \tilde{N}_1(0)$. In accordance with the comparison theorem, one has $\tilde{N}_1(t) \le \tilde{\psi}(t)$. Then (C4) suggests that

$$f(\tilde{N}_1(t), 0, t) - f(\tilde{\psi}(t), 0, t) \ge -M_2(\tilde{\psi}(t) - \tilde{N}_1(t)). \tag{13}$$

On the basis of (C3),

$$f(\tilde{N}_1(t), \tilde{N}_2(t), t) - f(\tilde{N}_1(t), 0, t) \ge -M\tilde{N}_2(t). \tag{14}$$

Substituting (13) and (14) into (12) yields

$$\begin{aligned}
& t^{-1}\ln(\tilde{N}_2(t)/\tilde{N}_2(0)) \\
&\quad \ge -t^{-1}\int_0^t b_2(s)\,\mathrm{d}s + t^{-1}\int_0^t \eta(s)f(\tilde{\psi}(s), 0, s)\,\mathrm{d}s - t^{-1}\int_0^t (a_2(s) + M\eta(s))\tilde{N}_2(s)\,\mathrm{d}s \\
&\quad\quad + t^{-1}\Psi_2(t) - M_2 t^{-1}\int_0^t \eta(s)(\tilde{\psi}(s) - \tilde{N}_1(s))\,\mathrm{d}s \\
&\quad \ge -t^{-1}\int_0^t b_2(s)\,\mathrm{d}s + t^{-1}\int_0^t \eta(s)f(\tilde{\psi}(s), 0, s)\,\mathrm{d}s - (a_2^u + M\eta^u)t^{-1}\int_0^t \tilde{N}_2(s)\,\mathrm{d}s \\
&\quad\quad + t^{-1}\Psi_2(t) - M_2\eta^u t^{-1}\int_0^t (\tilde{\psi}(s) - \tilde{N}_1(s))\,\mathrm{d}s.
\end{aligned} \tag{15}$$

Define $V(t) = |\ln\tilde{\psi}(t) - \ln\tilde{N}_1(t)|$, then

$$\mathrm{d}^+V(t) \le [g(\tilde{N}_1(t), \tilde{N}_2(t), t) - a_1^l|\tilde{\psi}(t) - \tilde{N}_1(t)|]\,\mathrm{d}t. \tag{16}$$

For any $\omega \in \{\limsup_{t\to+\infty} t^{-1}\int_0^t \tilde{N}_2(s)\,\mathrm{d}s < \varepsilon\}$, (16) and (C4) suggest that

$$\begin{aligned}
a_1^l t^{-1}\int_0^t |\tilde{\psi}(s,\omega) - \tilde{N}_1(s,\omega)|\,\mathrm{d}s &\le t^{-1}\int_0^t g(\tilde{N}_1(s,\omega), \tilde{N}_2(s,\omega), s)\,\mathrm{d}s + t^{-1}V(0) \\
&\le t^{-1}\int_0^t M_1\tilde{N}_2(s,\omega)\,\mathrm{d}s + t^{-1}V(0) \\
&\le M_1\varepsilon + t^{-1}V(0).
\end{aligned}$$

Let t be sufficiently large such that $t^{-1}V(0) \le M_1\varepsilon$. Accordingly, for sufficiently large t,

$$t^{-1}\int_0^t |\tilde{\psi}(s,\omega) - \tilde{N}_1(s,\omega)|\,\mathrm{d}s \le 2M_1\varepsilon/a_1^l.$$

When this inequality is utilized in (15), one can obtain that

$$\limsup_{t\to+\infty} t^{-1}\ln\tilde{N}_2(t,\omega) \ge \Lambda_2 - (a_2^u + M\eta^u)\varepsilon - 2M_1M_2\eta^u\varepsilon > 0.$$

This is a contradiction to (4). It follows that (11) holds.

Next we show that

$$\limsup_{t\to+\infty} t^{-1}\int_0^t N_1(s)\,\mathrm{d}s \ge \beta_1 \quad \text{a.s.}$$

Hypothesize, on the contrary, that for arbitrary $\varepsilon > 0$, there is a solution $(\bar{N}_1(t), \bar{N}_2(t))$ with $\bar{N}_i(0) > 0$ $(i = 1, 2)$ such that $\mathrm{P}\{\limsup_{t\to+\infty} t^{-1}\int_0^t \bar{N}_1(s)\,\mathrm{d}s < \varepsilon\} > 0$. Let ε be sufficiently small such that

$$\eta^u M_1\varepsilon - T^{-1}\int_0^T b_2(s)\,\mathrm{d}s < 0. \tag{17}$$

For any $\omega \in \{\limsup_{t\to+\infty} t^{-1}\int_0^t \bar{N}_1(s)\,\mathrm{d}s < \varepsilon\}$, on the basis of (C4), (6), (7), and (17),

$$\limsup_{t\to+\infty} t^{-1}\ln\bar{N}_2(t,\omega) \le -T^{-1}\int_0^T b_2(s)\,\mathrm{d}s + \eta^u M_1\varepsilon < 0.$$

Accordingly $\lim_{t\to+\infty}\bar{N}_2(t,\omega) = 0$. This is a contradiction to (11).

Finally, we establish

$$\limsup_{t\to+\infty} t^{-1}\int_0^t N_1(s)\,\mathrm{d}s \le \beta_2, \qquad \limsup_{t\to+\infty} t^{-1}\int_0^t N_2(s)\,\mathrm{d}s \le \beta_2 \quad \text{a.s.}$$

Applying Lemma 4(I) in [15] to (5) and (10), one has

$$\limsup_{t\to+\infty} t^{-1}\int_0^t N_1(s)\,\mathrm{d}s \le \Lambda_1/a_1^l$$

and

$$\limsup_{t\to+\infty} t^{-1}\int_0^t N_2(s)\,\mathrm{d}s \le \Lambda_2/a_2^l,$$

respectively. This completes the proof of (III). □

Remark 3 This note imposes technical assumptions (i.e., (C1) and (C4)) in Theorem 1. How to weaken them is an interesting topic. In addition, this note assumes that the coefficients in model (1) are T-periodic. For nonperiodic model (1), this note fails to provide the coexistence-and-extinction threshold. We leave these problems for further research.

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Authors' contributions
WMJ mainly wrote the whole content of the paper. WMJ and MLD mainly established the model and completed its development. Both authors read and approved the final manuscript.

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