Advances in Difference Equations
a SpringerOpen Journal

RESEARCH Open Access

New results on Caputo fractional-order neutral differential inclusions without compactness

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Abstract

This article deals with existence results of Caputo fractional neutral inclusions without compactness in Banach space using weak topology. In fact, for weakly sequentially closed maps we apply fixed point theorems to obtain the existence of the solution. Furthermore, the results are manifested for fractional neutral system held by nonlocal conditions. To justify the application of the reported results an illustration is presented.

MSC: 34A08; 93B05; 34G20; 34G25; 47H10; 47H20

Keywords: Existence; Fractional differential inclusions; Neutral equations; Semigroup theory; Multivalued map

1 Introduction

The dynamical behavior of real life phenomenon are summarized by essential tools such as fractional differential equations (FDEs) in a precise manner. This aspect is the main convenience of derivatives with fractional-order versus integer-order models. FDEs and inclusions have obtained many interest for their applications in different fields, such as engineering, physics, mechanics, and mathematical modelling, because they are more practical and realistic to describe many natural phenomena. Compared with ordinary and partial differential systems, fractional differential systems have the strong prospect to modulate the real time issues with high efficiency. The goal of analyzing fractional differential systems for the above, major analysis [2–4, 6, 7, 12, 23, 25–28, 30–32, 38, 40, 42, 47, 48, 56–58] had been carried out. El-Sayed and Ibrahim in [24]. were the first who considered fractional differential inclusions.

Furthermore, differential inclusions are used to model many realistic problems, arising from optimal control, economics, and so on. Recently, by using various techniques, the mild solutions together with other issues for different types of nonlinear fractional evolution inclusions have been studied in [1, 5, 13–17, 21, 22, 33, 43–45, 49–52, 54, 55, 59].

We recall that several techniques and noncompact measures are used to achieve the outcome of the differential systems. Most of these papers, assumed the compactness of the semi group or alternatively a compactness condition on the nonlinear part (generally a measure of noncompactness). In [46], Ravichandran et al. analyzed the controllability of impulsive fractional integro-differential systems utilizing a contraction principle. Li [33] studied the controllability results for neutral impulsive inclusion systems using the Dhage fixed point theorem. The controllability for evolution inclusions without compactness was studied by Benedetti et al. [15]. To the best of our knowledge, the existence of mild solutions of Caputo fractional neutral differential inclusions without compactness has not been studied and this is the main motivation of this work, is to prove the existence of Caputo fractional neutral differential inclusions with weak topology, and without compactness. We investigate the following Caputo fractional neutral inclusion in Banach space:

$$ {}^{C}D_{u}^{q}\big[z(u)-h\big(u,z(u)\big)\big]\in \mathscr{A}z(u)+\mathscr{H}\big(u,z(u)\big),\quad \text{a.e. } u\in[0,b], 0<q\leq 1, \tag{1.1}$$

$$ z(0)=z_0, \tag{1.2}$$

where b is positive in nature, $\mathscr{A}$ represents the infinitesimal generator of a C_0-strongly continuous semigroup $T(u), u\geq 0$, defined from $\mathscr{A}:D(\mathscr{A})\to\mathbb{Y}$. Besides, $z(\cdot)$ assumes values in the Banach space $\mathbb{Y}$, z_0 is for an element of $\mathbb{Y}$, $\mathscr{H}:[0,b]\times\mathbb{Y}\multimap\mathbb{Y}$ denotes a multivalued map, $h:[0,b]\times\mathbb{Y}\to\mathbb{Y}$ is equicontinuous and bounded.

The aim of this paper is to derive some sufficient conditions for the existence of neutral differential inclusions in Banach space. Furthermore, we expand the result to get the conditions for neutral differential inclusions with nonlocal conditions. In this paper another procedure is considered, it utilize the weak topology of the state space.

Considering the importance of modeling of crisis phenomena, one may extend the analysis to the existence of solutions for a three step crisis integro-differential equation. Also in order to rise the applicability of the fractional calculus, many researchers assumed a new type of fractional derivatives with different kernels. By exploiting it, one can examine the existence of solutions for high-order fractional differential equations using the Caputo–Fabrizio derivative [8–11, 36, 37].

The layout of this artical is as follows: The preliminaries and notations are listed in Sect. 2. The existence results are discussed in Sect. 3, and in Sect. 4 we investigate the same fractional system supported by nonlocal conditions. An illustration is offered to enhance the abstract technique.

2 Basic tools

Here, we present a few fundamental facts on fractional theory and theorems in order to use them in the manuscript.

Let $\mathbb{Y}$ possessing $\|\cdot\|$. For some constant $\mathscr{M}_1>0$ provided $\sup_{u\in[0,b]}\|T(u)\|\leq\mathscr{M}_1$. Let $(\mathbb{Y},\|\cdot\|)$ be a reflexive Banach space and $\mathbb{Y}_w$ denotes $\mathbb{Y}$ equipped with the weak topology. $\mathfrak{C}([0,b];\mathbb{Y})$ refers the Banach space of all continuous functions from $[0,b]$ into $\mathbb{Y}$. For $\mathscr{D}\subset\mathbb{Y}$, $\overline{\mathscr{D}}^{w}$ signifies the weak closure of $\mathscr{D}$. Moreover, the bounded subset $\mathscr{D}$ of a reflexive Banach space $\mathbb{Y}$ is weakly relatively compact. Let us express by $\|\cdot\|_p$ both the $L^p([0,b];\mathbb{Y})$-norm and the $L^p([0,b];\mathbb{R})$-norm and by $\|\cdot\|_0$ the $\mathfrak{C}([0,b];\mathbb{Y})$-norm. We evoke (see [18], Theorem 4.3) that the sequence $\{\chi_n\}\subset\mathfrak{C}([0,b];\mathbb{Y})$ converges weakly to $\chi\in\mathfrak{C}([0,b];\mathbb{Y})$ if and only if

(i) there exists $M>0$ such that, for every $m\in\mathbb{N}$ and $u\in[0,b]$, $\|\chi_m(u)\|\leq M$;

(ii) $\chi_m(u) \rightharpoonup \chi(u)$, for every $u \in [0, b]$.

In addition to that, we state some results that will be utilized further as a part of this manuscript.

Theorem 2.1 (Donal O'Regan fixed point theorem [39]) *Let F be a metrizable locally convex linear topological space and U be a weakly compact, convex subset of F and $\mathbb{C}(U)$ the family of nonempty closed, convex subsets of U. If $G : U \to \mathbb{C}(U)$ possesses weakly sequentially closed graph then G admits a fixed point.*

Theorem 2.2 ([29]) *Let Ψ be a subset of the Banach space $\mathbb{Y}$ then the subsequent affirmations are equivalent, namely*:

(i) *Ψ is relatively weakly compact*;

(ii) *Ψ is relatively weakly sequentially compact.*

Remark 2.1 ([29]) Let Ψ be a subset of the Banach space $\mathbb{Y}$, then the subsequent affirmations are equivalent:

(a) Ψ is weakly compact;

(b) Ψ is weakly sequentially compact.

Theorem 2.3 (Krein–Smulian theorem [23, p. 434]) *The convex hull of a weakly compact set in a Banach space $\mathbb{Y}$ is weakly compact.*

Theorem 2.4 (Pettis measurability theorem [41]) *Let (E, Σ) be a measurable space, $\mathbb{Y}$ be a separable Banach space. Then $f : E \longrightarrow \mathbb{Y}$ is measurable if and only if, for every $e \in E'$, the function $e \circ f : E \longrightarrow \mathbb{R}$ is measurable with respect to $\sum$ and the Borel σ-algebra in $\mathbb{R}$.*

Now we summarize the subsequent interpretations [7, 31, 32, 38, 42].

Definition 2.1 The form of the fractional integral for f is

$$I^{\alpha} g(p) = \frac{1}{\Gamma(\alpha)} \int_0^p \frac{g(w)}{(p-w)^{1-\alpha}}\, dw, \quad p > 0, \alpha > 0,$$

as the right hand-side is point-wise on $[0, \infty)$ and $\Gamma(\alpha) = \int_0^{\infty} p^{\alpha-1} e^{-p}\, dp$.

Definition 2.2 ([31]) The form of the R-L fractional derivative for $g : [0, \infty) \to \mathbb{R}$ is characterized by

$$^{(\mathrm{R-L})}D^{\alpha}_{0+} g(p) = \frac{1}{\Gamma(m-\alpha)} \left(\frac{d}{dp}\right)^m \int_0^p (p-w)^{m-\alpha-1} g(w)\, dw, \quad p > 0, m-1 < \alpha < m,$$

such that the function $g(p)$ posses absolutely continuous derivative up to $(m-1)$.

Definition 2.3 ([31]) The expression of the Caputo derivative for $g : [0, \infty) \to \mathbb{R}$ is

$$^{\mathrm{C}}D^{\alpha} g(p) = {}^{(\mathrm{R-L})}D^{\alpha} \left(g(p) - \sum_{k=0}^{m-1} \frac{p^k}{k!} g^{(k)}(0) \right), \quad p > 0, m-1 < \alpha < m.$$

Remark 2.2 (i) If $g(p) \in C^m[0,\infty)$, then

$$^{C}D^{\alpha}g(p) = \frac{1}{\Gamma(m-\alpha)}\int_0^p \frac{g^{(m)}(w)}{(p-w)^{\alpha+1-m}}\,dw = I^{m-\alpha}g^{(m)}(p), \quad p>0, m-1<\alpha<m.$$

3 Existence results

Below, we demonstrate some sufficient conditions for the existence of (1.1)–(1.2) coupled with weak topology.

(H₁) For $\{T(u)\}_{u\geq 0}$ in $\mathbb{Y}$, there is a constant $\mathscr{M}_1 \geq 1$ fulfilling $\sup_{u\in[0,b]}\|T(u)\| \leq \mathscr{M}_1$. Additionally, we require that the multivalued nonlinearity function $\mathscr{H}:[0,b]\times\mathbb{Y}\multimap\mathbb{Y}$ possess nonempty convex and weakly compact values.

(H₂) For all $z\in\mathbb{Y}$, the multivalued function $\mathscr{H}(\cdot,z):[0,b]\multimap\mathbb{Y}$ has a measurable selection.

(H₃) $\mathscr{H}(u,\cdot):\mathbb{Y}\multimap\mathbb{Y}$ is weakly sequentially closed for almost everywhere u in $[0,b]$.

(H₄) For a real valued function $h:[0,b]\times\mathbb{Y}\to\mathbb{Y}$, for all $u>0$ and some constant $\mathfrak{M}_h>0$ we have $\|h(u,\cdot)\|\leq\mathfrak{M}_h$.

(H₅) For $\kappa_1\in(0,q)$, for every $r>0$ and a function $\delta_r\in L^{\frac{1}{\kappa_1}}([0,b];\mathbb{R}_+)$ as for each $d\in\mathbb{Y}$, $\|d\|\leq r$:

$$\|h(u,d)\| = \sup\{\|z\|: z\in h(u,d)\}\leq\delta_r(u),$$

for almost everywhere $u\in[0,b]$.

(H₆) For $\kappa_1\in(0,q)$, for every $r>0$ and a function $\mu_r\in L^{\frac{1}{\kappa_1}}([0,b];\mathbb{R}_+)$ as for each $d\in\mathbb{Y}$, $\|d\|\leq r$:

$$\|\mathscr{H}(u,d)\| = \sup\{\|z\|: z\in\mathscr{H}(u,d)\}\leq\mu_r(u),$$

for almost everywhere $u\in[0,b]$.

In connection with the above consideration, we determine the solution of (1.1)–(1.2).

Definition 3.1 ([56]) $z:[0,b]\to\mathbb{Y}$ is a mild solution of (1.1)–(1.2) if the accompanying recognize $z(0)=z_0$ and there is $\chi\in L^{\frac{1}{\kappa_1}}([0,b];\mathbb{Y})$ provided $\chi(u)\in\mathscr{H}(u,z(u))$ on $u\in[0,b]$ and z fulfills

$$\begin{aligned} z(u) &= \mathbb{T}(u)\big[z_0-h(0,z_0)\big]+h\big(u,z(u)\big)+\int_0^u(u-w)^{q-1}\mathscr{A}\mathbb{S}(u-w)h\big(w,z(w)\big)\,dw \\ &\quad+\int_0^u(u-w)^{q-1}\mathbb{S}(u-w)\chi(w)\,dw, \quad u\in[0,b], \end{aligned}$$

where

$$\mathbb{T}(u)=\int_0^\infty\xi_q(\theta)T\big(u^q\theta\big)\,d\theta, \qquad \mathbb{S}(u)=q\int_0^\infty\theta\xi_q(\theta)T\big(u^q\theta\big)\,d\theta,$$

and, for $\theta\in(0,\infty)$,

$$\xi_q(\theta)=\frac{1}{q}\theta^{-1-\frac{1}{q}}\overline{w}_q\big(\theta^{-\frac{1}{q}}\big)\geq 0,$$

$$\overline{w}_q(\theta) = \frac{1}{\pi}\sum_{n=1}^{\infty}(-1)^{n-1}\theta^{-nq-1}\frac{\Gamma(nq+1)}{n!}\sin(n\pi q),$$

$$\int_0^{\infty}\xi_q(\theta)\,d\theta = 1.$$

Remark 3.1 Obviously, for $\nu \in [0,1]$,

$$\int_0^{\infty}\theta^{\nu}\xi_q(\theta)\,d\theta = \int_0^{\infty}\theta^{-q\nu}\overline{w}_q(\theta)\,d\theta = \frac{\Gamma(1+\nu)}{\Gamma(1+q\nu)}.$$

Lemma 3.1 (See [56]) *$\mathbb{T}$ and $\mathbb{S}$ obey the subsequent assertions*:

(i) *For a constant $\mathscr{M}_2 \geq 1$, for any $z \in \mathbb{Y}$, fixed $u \geq 0$ and for the bounded linear operators $\mathbb{T}$ and $\mathbb{S}$ we have*

$$\|\mathbb{T}(u)z\| \leq \mathscr{M}_1\|z\| \quad \textit{and} \quad \|\mathbb{S}(u)z\| \leq \frac{q\mathscr{M}_1}{\Gamma(1+q)}\|z\|,$$

$$\|\mathscr{A}\mathbb{S}(u)z\| \leq \frac{q\mathscr{M}_1\mathscr{M}_2}{\Gamma(1+q)}\|z\|.$$

(ii) *The operators $\{\mathbb{T}(u), u \geq 0\}$ and $\{\mathbb{S}(u), u \geq 0\}$ are strongly continuous.*

Construct the set $\Upsilon_{\wp}$, for given $\wp \in \mathfrak{C}([0,b];\mathbb{Y})$ as $\Upsilon_{\wp} = \{\chi \in L^{\frac{1}{\kappa_1}}([0,b];\mathbb{Y}) : \chi(u) \in \mathscr{H}(u,\wp(u))$ for almost everywhere $u \in [0,b]\}$. $\Upsilon_{\wp}$ is nonempty as the next Proposition 3.1 mentions.

Proposition 3.1 (See [15, 59]) *Let us assume that a multivalued map $\mathscr{H} : [0,b] \times \mathbb{Y} \multimap \mathbb{Y}$ obeys* $(\mathbf{H_2})$–$(\mathbf{H_6})$, *the set $\Upsilon_{\wp}$ is nonempty for any $\wp \in \mathfrak{C}([0,b];\mathbb{Y})$.*

We define the operator $\Lambda : \mathfrak{C}([0,b];\mathbb{Y}) \multimap \mathfrak{C}([0,b];\mathbb{Y})$ by

$$\Lambda(\wp) = \bigl\{z \in \mathfrak{C}\bigl([0,b];\mathbb{Y}\bigr) : z(u) = \mathbb{T}(u)\bigl(z_0 - h(0,z_0)\bigr) + h\bigl(u,\wp(u)\bigr) \\ + S_1(z)(u) + S_2\chi(u), \chi \in \Upsilon_{\wp}\bigr\},$$

where

$$S_1 : \mathfrak{C}\bigl([0,b];\mathbb{Y}\bigr) \to \mathfrak{C}\bigl([0,b];\mathbb{Y}\bigr),$$

$$S_1(z) = \int_0^u (u-w)^{q-1}\mathscr{A}\mathbb{S}(u-w)h\bigl(w,\wp(w)\bigr)\,dw,$$

and

$$S_2 : L^{\frac{1}{\kappa_1}}\bigl([0,b];\mathbb{Y}\bigr) \to \mathfrak{C}\bigl([0,b];\mathbb{Y}\bigr),$$

$$S_2(\chi) = \int_0^u (u-w)^{q-1}\mathbb{S}(u-w)\chi(w)\,dw.$$

At first, we show that S_1 and S_2 are continuous.

For any $z_n, z \in \mathfrak{C}([0,b];\mathbb{Y})$ and $z_n \to z$ $(n \to \infty)$, using $(\mathbf{H_5})$, for every $u \in [0,b]$, we get

$$(u-w)^{q-1}\|z_n(w) - z(w)\| \leq 2(u-w)^{q-1}\mu_r(w), \quad \text{almost everywhere } w \in [0,u).$$

Also for any $\chi_n, \chi \in L^{\frac{1}{\kappa_1}}([0,b];\mathbb{Y})$ and $\chi_n \to \chi$ ($n \to \infty$), using (**H₆**), we can have, for every $u \in [0,b]$,

$$(u-w)^{q-1}\|\chi_n(w)-\chi(w)\| \leq 2(u-w)^{q-1}\delta_r(w), \quad \text{almost everywhere } w \in [0,u).$$

Moreover, the functions

$$\int_0^u (u-w)^{q-1}\mu_r(w)\,dw = \left[\left(\frac{1-\kappa_1}{q-\kappa_1}\right)b^{\frac{q-\kappa_1}{1-\kappa_1}}\right]^{1-\kappa_1}\|\mu_r\|_{\frac{1}{\kappa_1}}$$

and

$$\int_0^u (u-w)^{q-1}\delta_r(w)\,dw = \left[\left(\frac{1-\kappa_1}{q-\kappa_1}\right)b^{\frac{q-\kappa_1}{1-\kappa_1}}\right]^{1-\kappa_1}\|\delta_r\|_{\frac{1}{\kappa_1}}$$

becomes integrable for $u \in [0,b]$. Taking into account the Lebesgue theorem, we conclude, as $n \to \infty$,

$$\int_0^u (u-w)^{q-1}\|z_n(w)-z(w)\|\,dw \to 0 \quad \text{and} \quad \int_0^u (u-w)^{q-1}\|\chi_n(w)-\chi(w)\|\,dw \to 0.$$

Therefore

$$\begin{aligned}\|S_1(z_n)-S_1(z)\| &\leq \left\|\int_0^u (u-w)^{q-1}\mathscr{A}\mathbb{S}(u-w)\big(z_n(w)-z(w)\big)\,dw\right\| \\ &\leq \frac{q\mathscr{M}_1\mathscr{M}_2}{\Gamma(1+q)}\int_0^u (u-w)^{q-1}\|z_n(w)-z(w)\|\,dw \to 0, \quad \text{as } n \to \infty, \\ \|S_2(\chi_n)-S_2(\chi)\| &\leq \left\|\int_0^u (u-w)^{q-1}\mathbb{S}(u-w)\big(\chi_n(w)-\chi(w)\big)\,dw\right\| \\ &\leq \frac{q\mathscr{M}_1}{\Gamma(1+q)}\int_0^u (u-w)^{q-1}\|\chi_n(w)-\chi(w)\|\,dw \to 0, \quad \text{as } n \to \infty.\end{aligned}$$

It indicates that the operators S_1 and S_2 are continuous.

For $n \in \mathbb{N}$, Φ_n, the closed ball of radius n in $\mathfrak{C}([0,b];\mathbb{Y})$ described by $\Lambda_n = \Lambda \mid \Phi_n : \Phi_n \multimap \mathfrak{C}([0,b];\mathbb{Y})$, the limitation of Λ on Φ_n. Next we illustrate the qualities of Λ_n.

Proposition 3.2 *Λ_n possess a weakly sequentially closed graph.*

Proof Let a sequence $\{\wp_m\} \subset \Phi_n$, $\{z_m\} \subset \mathfrak{C}([0,b];\mathbb{Y})$ obeying $z_m \subset \Lambda_n(\wp_m)$, for all m and $\wp_m \rightharpoonup \wp$, $z_m \rightharpoonup z$ in $\mathfrak{C}([0,b];\mathbb{Y})$; we claim $z \in \Lambda_n(\wp)$.

Since $\wp_m \in \Phi_n$, for each m and $\wp_m(u) \rightharpoonup \wp(u)$, for every $u \in [0,b]$, we conclude $\|\wp(u)\| \leq \lim_{\inf_{m\to\infty}}\|\wp_m(u)\| \leq n$, for all u (see [19]). By $z_m \in \Lambda_n(p_m)$, there is a sequence $\{\chi_m\}$, $\chi_m \in \Upsilon_{\wp m,}$ provided for all $u \in [0,b]$, we get

$$\begin{aligned}z_m(u) = {}&\mathbb{T}(u)\big(z_0-h(0,z_0)\big)+h\big(u,\wp_m(u)\big)+\int_0^u (u-w)^{q-1}\mathscr{A}\mathbb{S}(u-w)h\big(w,\wp_m(w)\big)\,dw \\ &+\int_0^u (u-w)^{q-1}\mathbb{S}(u-w)\chi_m(w)\,dw.\end{aligned}$$

By reference to $(\mathbf{H_6})$, $\|\chi_m(u)\| \le \mu_n(u)$, for almost everywhere u and every m. It means that $\{\chi_m\}$ is bounded, uniformly integrable and $\{\chi_m(u)\}$ is bounded in $\mathbb{Y}$ for almost everywhere $u \in [0,b]$. By the Dunford-Pettis theorem [13], we can conclude that there exist a subsequence, represented as the sequence, and functions g_1, g_2 provided $z_m \rightharpoonup g_1$ in $\mathfrak{C}([0,b];\mathbb{Y})$ and $\chi_m \rightharpoonup g_2$ in $L^{\frac{1}{\kappa_1}}([0,b];\mathbb{Y})$.

Therefore, we have $S_1 z_m \rightharpoonup S_1 g_1$ and $S_2\chi_m \rightharpoonup S_2 g_2$. In this connection, let the linear continuous operator $e' : \mathbb{Y} \to \mathbb{R}$. The operators S_1 and S_2 are linear and continuous, therefore we have

$$g_1 \to e'(S_1 g_1)(u), \qquad g_1 \in \mathfrak{C}\bigl([0,b];\mathbb{Y}\bigr),$$

is linear continuous operator on $\mathfrak{C}([0,b];\mathbb{Y})$ to $\mathbb{R}$ for every $u \in [0,b]$. Also,

$$g_2 \to e'(S_2 g_2)(u), \qquad g_2 \in L^{\frac{1}{\kappa_1}}\bigl([0,b];\mathbb{Y}\bigr),$$

is linear continuous operator on $L^{\frac{1}{\kappa_1}}([0,b];\mathbb{Y})$ to $\mathbb{R}$ for every $u \in [0,b]$. By weak convergence, we get

$$\begin{aligned} &e'\biggl(\int_0^u (u-w)^{q-1}\mathscr{A}\mathbb{S}(u-w)h\bigl(w,\wp_m(w)\bigr)\,dw + \int_0^u (u-w)^{q-1}\mathbb{S}(u-w)\chi_m(w)\,dw\biggr) \\ &\quad\to e'\biggl(\int_0^u (u-w)^{q-1}\mathscr{A}\mathbb{S}(u-w)g_1(w)\,dw + \int_0^u (u-w)^{q-1}\mathbb{S}(u-w)g_2(w)\,dw\biggr). \end{aligned}$$

Therefore

$$\begin{aligned} z_m(u) &\rightharpoonup \mathbb{T}(u)\bigl(z_0 - h(0,z_0)\bigr) + h\bigl(u,\wp_m(u)\bigr) + \int_0^u (u-w)^{q-1}\mathscr{A}\mathbb{S}(u-w)g_1(w)\,dw \\ &\quad + \int_0^u (u-w)^{q-1}\mathbb{S}(u-w)g_2(w)\,dw \\ &= z_0(u), \end{aligned}$$

for every $u \in [0,b]$. This indicates that $z_0(u) = z(u)$, for all $u \in [0,b]$. Hence by Proposition 3.1, $g_2(u) \in \mathscr{H}(u,\wp(u))$, for almost everywhere $u \in [0,b]$. □

Proposition 3.3 *Λ_n is weakly compact.*

Proof At first, we show that $\Lambda_n(\Phi_n)$ is relatively weakly sequentially compact.

Let us consider $\wp_m \in \Phi_n$ and $z_m \subset \mathfrak{C}([0,b];\mathbb{Y})$ such that $z_m \in \Lambda_n(\wp_m)$ for all m. For Λ_n, there exists a sequence $\{\chi_m\}$, $\chi_m \in \Upsilon_{\wp m}$, provided that

$$\begin{aligned} z_m(u) &= \mathbb{T}(u)\bigl(z_0 - h(0,z_0)\bigr) + h\bigl(u,\wp_m(u)\bigr) + \int_0^u (u-w)^{q-1}\mathscr{A}\mathbb{S}(u-w)h\bigl(w,\wp_m(w)\bigr)\,dw \\ &\quad + \int_0^u (u-w)^{q-1}\mathbb{S}(u-w)\chi_m(w)\,dw, \end{aligned}$$

for every $u \in [0,b]$. Therefore, by Proposition 3.2, there exist a subsequence, represented by the sequence, and functions g_1, g_2 provided $z_m \rightharpoonup g_1$ in $\mathfrak{C}([0,b];\mathbb{Y})$ and $\chi_m \rightharpoonup g_2$ in

$L^{\frac{1}{\kappa_1}}([0,b];\mathbb{Y})$. Hence

$$\begin{aligned} z_m(u) &\rightharpoonup l(u) \\ &= \mathbb{T}\big(z_0 - h(0,z_0)\big) + h\big(u,\wp_m(u)\big) + \int_0^u (u-w)^{q-1}\mathscr{A}\mathbb{S}(u-w)g_1(w)\,dw \\ &\quad + \int_0^u (u-w)^{q-1}\mathbb{S}(u-w)g_2(w)\,dw. \end{aligned}$$

Furthermore, by the nature of weak convergence of χ_m, by $(\mathbf{H_1})$, we have

$$\begin{aligned} \|z_m(u)\| &\le \mathscr{M}_1\|z_0\| + \delta_r(0) + \mathfrak{M}_h + \frac{\mathscr{M}_1\mathscr{M}_2 q}{\Gamma(1+q)}\left[\left(\frac{1-\kappa_1}{q-\kappa_1}\right)b^{\frac{q-\kappa_1}{1-\kappa_1}}\right]^{1-\kappa_1}\|\delta_r\|_{\frac{1}{\kappa_1}} \\ &\quad + \frac{\mathscr{M}_1 q}{\Gamma(1+q)}\left[\left(\frac{1-\kappa_1}{q-\kappa_1}\right)b^{\frac{q-\kappa_1}{1-\kappa_1}}\right]^{1-\kappa_1}\|\mu_r\|_{\frac{1}{\kappa_1}}, \end{aligned}$$

for all $m \in \mathbb{N}$ and $u \in [0,b]$. By utilizing the Proposition 3.2, we ensure that $z_m \rightharpoonup l$ in $\mathfrak{C}([0,b];\mathbb{Y})$. Thus, $\Lambda_n(\Phi_n)$ is relatively weakly compact by Theorem 2.2. □

Proposition 3.4 *Λ_n has convex and weakly compact values.*

Proof Fixing $\wp \in \Phi_n$, taking into account that $\mathscr{H}$ is convex valued and the characteristics of $\mathbb{T}(u)$ and $\mathbb{S}(u)$, it implies that $\Lambda_n(\wp)$ is convex. By reference to Proposition 3.2 and Proposition 3.3, $\Lambda_n(\wp)$ has weakly compact values. □

Next we list out the essential outcomes of this part.

Theorem 3.1 *Assuming* $(\mathbf{H_1})$–$(\mathbf{H_6})$ *hold. Suppose* $(\mathbf{H_7})$ *for a sequence of functions* $\{u_n\} \in L^{\frac{1}{\kappa_1}}([0,b];\mathbb{R}_+)$ *provided*

$$\sup_{\|d\|\le n}\|\mathscr{H}(u,d)\| \le u_n(u),$$

for almost everywhere $u \in [0,b]$, $n \in \mathbb{N}$ *with*

$$\lim_{n\to\infty}\inf\frac{\|u_n\|_{\frac{1}{\kappa_1}}}{n} = 0. \tag{3.1}$$

Then (1.1)–(1.2) *recognizes at least a mild solution.*

Proof We have to confirm that Λ maps Φ_n into itself for $n \in \mathbb{N}$.

Assume by way of contradiction that there exist $\{z_n\}$, $\{x_n\}$ such that $z_n \in \Phi_n$, $x_n \in \Lambda_n(z_n)$ and $x_n \notin \Phi_n$, for every $n \in \mathbb{N}$. Therefore for a sequence $\{\chi_n\} \subset L^{\frac{1}{\kappa_1}}([0,b];\mathbb{Y})$, $\chi_n(w) \in \mathscr{H}(w,z_n(w))$, we can have

$$\begin{aligned} x_n(u) &= \mathbb{T}(u)\big(z_0 - h(0,z_0)\big) + h\big(u,z_n(u)\big) + \int_0^u (u-w)^{q-1}\mathscr{A}\mathbb{S}(u-w)h\big(w,z_n(w)\big)\,dw \\ &\quad + \int_0^u (u-w)^{q-1}\mathbb{S}(u-w)\chi_n(w)\,dw, \end{aligned}$$

for every $u \in [0,b]$. By Proposition 3.3, we have

$$\begin{aligned} n &\leq \|x_n\|_0 \\ &\leq \mathscr{M}_1\|z_0\| + \delta_r(0) + \mathfrak{M}_h + \frac{\mathscr{M}_1\mathfrak{M}_h\mathscr{M}_2 q}{\Gamma(1+q)}\left[\left(\frac{1-\kappa_1}{q-\kappa_1}\right)b^{\frac{q-\kappa_1}{1-\kappa_1}}\right]^{1-\kappa_1} \\ &\quad + \frac{\mathscr{M}_1 q}{\Gamma(1+q)}\left[\left(\frac{1-\kappa_1}{q-\kappa_1}\right)b^{\frac{q-\kappa_1}{1-\kappa_1}}\right]^{1-\kappa_1}\|u_n\|_{\frac{1}{\kappa_1}}. \end{aligned}$$

Then, for $n \in \mathbb{N}$,

$$\begin{aligned} 1 &\leq \frac{\|x_n\|_0}{n} \\ &\leq \frac{1}{n}\left[\mathscr{M}_1\|z_0\| + \delta_r(0) + \mathfrak{M}_h + \frac{\mathscr{M}_1\mathfrak{M}_h\mathscr{M}_2 q}{\Gamma(1+q)}\left[\left(\frac{1-\kappa_1}{q-\kappa_1}\right)b^{\frac{q-\kappa_1}{1-\kappa_1}}\right]^{1-\kappa_1}\right. \\ &\quad \left. + \frac{\mathscr{M}_1 q}{\Gamma(1+q)}\left[\left(\frac{1-\kappa_1}{q-\kappa_1}\right)b^{\frac{q-\kappa_1}{1-\kappa_1}}\right]^{1-\kappa_1}\|u_n\|_{\frac{1}{\kappa_1}}\right], \end{aligned}$$

which leads to a contradiction. Therefore $x_n \in \Phi_n$.

Now, fix $n \in \mathbb{N}$ such that $\Lambda_n(\Phi_n) \subset \Phi_n$. By Proposition 3.3, the set $\mathscr{V}_n = \overline{\Lambda_n(\Phi_n)}^w$ is weakly compact. Let $\xi_n = \overline{\text{co}}(\mathscr{V}_n)$, be the closed convex hull of $\mathscr{V}_n$. According to Theorem 2.3, ξ_n denotes a weakly compact set. In addition to that $\Lambda_n(\Phi_n) \subset \Phi_n$ and Φ_n is a closed convex set. Furthermore we have $\xi_n \subset \Phi_n$, and we have

$$\Lambda_n(\xi_n) = \Lambda_n\big(\overline{\text{co}}\big(\Lambda_n(\Phi_n)\big)\big) \subseteq \Lambda_n(\Phi_n) \subseteq \overline{\Lambda_n(\Phi_n)}^w = \mathscr{V}_n \subset \xi_n.$$

This shows that Λ_n possesses a weakly sequentially closed graph. As a result by utilizing Theorem 2.1, we conclude that the system (1.1)–(1.2) recognizes a solution. □

Remark 3.2 There exist $\alpha \in L^{\frac{1}{\kappa_1}}([0,b];\mathbb{R}_+)$ and a nondecreasing function $\phi : [0,\infty) \to [0,\infty)$ such that $\|\mathscr{H}(u,d)\| \leq \alpha(u)\phi(\|d\|)$, for almost everywhere $u \in [0,b]$ and every $d \in \mathbb{Y}$. Then the restriction (3.1) is related to

$$\lim_{n\to\infty}\inf\frac{\phi(n)}{n} = 0.$$

Theorem 3.2 *Assume that* $(\mathbf{H_1})$–$(\mathbf{H_5})$ *holds.*

$(\mathbf{H_8})$ *There exists* $\rho \in L^{\frac{1}{\kappa_1}}([0,b];\mathbb{R}_+)$, *for almost everywhere* $u \in [0,b]$, *for every* $d \in \mathbb{Y}$ *provided*

$$\big\|\mathscr{H}(u,d)\big\| \leq \rho(u)\big(1 + \|d\|\big)$$

and

$$\frac{\mathscr{M}_1 q}{\Gamma(1+q)}\left[\left(\frac{1-\kappa_1}{q-\kappa_1}\right)b^{\frac{q-\kappa_1}{1-\kappa_1}}\right]^{1-\kappa_1}\big(\mathscr{M}_2\mathfrak{M}_h + \|\rho\|_{\frac{1}{\kappa_1}}\big) < 1, \tag{3.2}$$

then (1.1)–(1.2) *possess at least a mild solution.*

Proof By reference to Theorem 3.1, assuming that there exist $\{z_n\}$, $\{x_n\}$ provided $z_n \in \Phi_n$, $x_n \in \Lambda_n(z_n)$ and $x_n \notin \Phi_n$, for every $n \in \mathbb{N}$, we get

$$\begin{aligned} n &< \|x_n\|_0 \\ &\leq \mathscr{M}_1\|z_0\| + \delta_r(0) + \mathfrak{M}_h + \frac{\mathscr{M}_1\mathscr{M}_2\mathfrak{M}_h q}{\Gamma(1+q)}\left[\left(\frac{1-\kappa_1}{q-\kappa_1}\right)b^{\frac{q-\kappa_1}{1-\kappa_1}}\right]^{1-\kappa_1} \\ &\quad + \frac{\mathscr{M}_1 q}{\Gamma(1+q)}\left[\left(\frac{1-\kappa_1}{q-\kappa_1}\right)b^{\frac{q-\kappa_1}{1-\kappa_1}}\right]^{1-\kappa_1}\int_0^b \left|\rho(\xi)\right|^{\frac{1}{\kappa_1}}\left(1 + \left\|z_n(\xi)\right\|^{\frac{1}{\kappa_1}} d\xi\right)^{\kappa_1} \\ &\leq \mathscr{M}_1\|z_0\| + \delta_r(0) + \mathfrak{M}_h \\ &\quad + \frac{\mathscr{M}_1 q}{\Gamma(1+q)}\left[\left(\frac{1-\kappa_1}{q-\kappa_1}\right)b^{\frac{q-\kappa_1}{1-\kappa_1}}\right]^{1-\kappa_1}\left(\mathscr{M}_2\mathfrak{M}_h + (1+n)\|\rho\|_{\frac{1}{\kappa_1}}\right), \quad n \in \mathbb{N}, \end{aligned}$$

which contradicts (3.2). The conclusion refers to Theorem 2.1, like Theorem 3.1. □

Theorem 3.3 *Assuming that* $(\mathbf{H_1})$–$(\mathbf{H_5})$ *holds.* $(\mathbf{H_9})$ *there exist* $\beta \in L^{\frac{1}{\kappa_1}}([0,b];\mathbb{R}_+)$ *and a nondecreasing function* $\varrho : [0,\infty) \to [0,\infty)$ *fulfilling*

$$\left\|\mathscr{H}(u,d)\right\| \leq \beta(u)\varrho\left(\|d\|\right),$$

for almost everywhere $u \in [0,b]$, $d \in \mathbb{Y}$, *and* $\mathscr{L} > 0$ *provided*

$$\frac{\mathscr{L}}{\mathscr{M}_1\|z_0\| + \delta_r(0) + \mathfrak{M}_h + \frac{\mathscr{M}_1 q}{\Gamma(1+q)}[(\frac{1-\kappa_1}{q-\kappa_1})b^{\frac{q-\kappa_1}{1-\kappa_1}}]^{1-\kappa_1}(\mathscr{M}_2\mathfrak{M}_h + \|\rho\|_{\frac{1}{\kappa_1}})\varrho(\mathscr{L})} > 1, \tag{3.3}$$

then (1.1)–(1.2) *possess at least a mild solution.*

Proof We have to ensure that Λ maps the ball $\Phi_{\mathscr{L}}$ into itself. For any $z \in \Phi_{\mathscr{L}}$, $x \in \Gamma(z)$, we conclude

$$\begin{aligned} \|x_n\|_0 &\leq \mathscr{M}_1\|z_0\| + \delta_r(0) + \mathfrak{M}_h + \frac{\mathscr{M}_1\mathscr{M}_2\mathfrak{M}_h q}{\Gamma(1+q)}\left[\left(\frac{1-\kappa_1}{q-\kappa_1}\right)b^{\frac{q-\kappa_1}{1-\kappa_1}}\right]^{1-\kappa_1} \\ &\quad + \frac{\mathscr{M}_1 q}{\Gamma(1+q)}\left[\left(\frac{1-\kappa_1}{q-\kappa_1}\right)b^{\frac{q-\kappa_1}{1-\kappa_1}}\right]^{1-\kappa_1}\int_0^b \left|\rho(\xi)\right|^{\frac{1}{\kappa_1}}\left(\varrho\left\|z(\xi)\right\|^{\frac{1}{\kappa_1}} d\xi\right)^{\kappa_1} \\ &\leq \mathscr{M}_1\|z_0\| + \delta_r(0) + \mathfrak{M}_h \\ &\quad + \frac{\mathscr{M}_1 q}{\Gamma(1+q)}\left[\left(\frac{1-\kappa_1}{q-\kappa_1}\right)b^{\frac{q-\kappa_1}{1-\kappa_1}}\right]^{1-\kappa_1}\left[\mathscr{M}_2\mathfrak{M}_h + \|\beta\|_{\frac{1}{\kappa_1}}\right]\varrho(\mathscr{L}) < \mathscr{L}. \end{aligned}$$

This indicates that (1.1)–(1.2) possess at least a mild solution. □

4 Nonlocal conditions

The active desire for analyzing fractional systems with nonlocal problems comes mainly from theoretical physics. The outcomes regarding the existence of Cauchy problems using nonlocal conditions were firstly investigated by Byszewski [20]. Many papers [13, 21, 34, 35, 53] have acknowledged the facts of the existence, controllability and uniqueness for varied nonlinear fractional systems and abstract differential systems. Motivated by the above discussions, this part deals with the existence of (1.1)–(1.2) as

$$ {}^{C}D_t^q\big[z(u)-h\big(u,z(u)\big)\big] \in \mathscr{A}z(u)+\mathscr{H}\big(u,z(u)\big), \quad u\in[0,b], \tag{4.1} $$

$$ z(0)+\varphi(z)=z_0, \tag{4.2} $$

where $\varphi:\mathfrak{C}([0,b];\mathbb{Y})\to\mathbb{Y}$ fulfills the following conditions:

($\mathbf{H_{10}}$) For some constant $\mathscr{N}>0$ provided $\|\varphi(z)\|\le\mathscr{N}$, $z\in\mathfrak{C}([0,b];\mathbb{Y})$.

($\mathbf{H_{11}}$) There is a constant $\mathscr{L}>0$ and

$$ \frac{\mathscr{L}}{\mathscr{M}_1\|z_0\|+\mathscr{N}+\delta_r(0)+\mathfrak{M}_h+\frac{\mathscr{M}_1 q}{\Gamma(1+q)}[(\frac{1-\kappa_1}{q-\kappa_1})b^{\frac{q-\kappa_1}{1-\kappa_1}}]^{1-\kappa_1}(\mathscr{M}_2\mathfrak{M}_h+\|\rho\|_{\frac{1}{\kappa_1}})\varrho(\mathscr{L})} > 1. \tag{4.3} $$

In order to show the high accuracy, we always refer the nonlocal condition in the place of initial condition $z(0)=z_0$. Particularly, $\varphi(z)$ can be formulated as

$$ \varphi(z)=\sum_{i=1}^{n}K_i z(t_i), $$

where K_i $(i=1,2,3,\ldots,n)$ are constants and $0<t_1<t_2<\cdots<t_n\le b$.

Definition 4.1 $z:[0,b]\to\mathbb{Y}$ is called the mild solution of the neutral fractional differential model (4.1)–(4.2) if the accompanying recognize $z(0)+\varphi(z)=z_0$ and there exists $\chi\in L^{\frac{1}{\kappa_1}}([0,b];\mathbb{Y})$ provided $\chi(u)\in\mathscr{H}(u,z(u))$ on $u\in[0,b]$ and z satisfies

$$ \begin{aligned} z(u) &= \mathbb{T}(u)\big[z_0-\varphi(z)-h(0,z_0)\big]+h\big(u,z(u)\big)+\int_0^u(u-w)^{q-1}\mathscr{A}\mathbb{S}(u-w)h\big(w,z(w)\big)\,dw \\ &\quad+\int_0^u(u-w)^{q-1}\mathbb{S}(u-w)\chi(w)\,dw, \quad u\in[0,b], \end{aligned} $$

such that $\mathbb{T}(u)$ and $\mathbb{S}(u)$ are defined as in Definition 3.1.

Theorem 4.1 *If Theorem 3.1, Theorem 3.2 and Theorem 3.3 hold, and in addition hypotheses* ($\mathbf{H_{10}}$) *and* ($\mathbf{H_{11}}$) *hold, then the neutral fractional system with inclusion* (4.1)–(4.2) *has at least a mild solution.*

Proof We introduce the solution operator $\Lambda:\mathfrak{C}([0,b];\mathbb{Y})\multimap\mathfrak{C}([0,b];\mathbb{Y})$ as

$$ \begin{aligned} \Lambda(\wp) = \Big\{&z\in\mathfrak{C}\big([0,b];\mathbb{Y}\big): z(u)=\mathbb{T}\big(z_0-\varphi(z)-h(0,z_0)\big)+h\big(u,\wp(u)\big) \\ &+S_1(z)(u)+S_2(\chi)(u)\Big\}. \end{aligned} $$

It should be noted that we recognize that Λ possesses a fixed point by employing the techniques utilized in Theorems corresponding to initial conditions. The proof is similar, therefore we omitted it. □

5 An example

Let us consider the model:

$$
\begin{aligned}
&{}^{C}D_{u}^{q}\left[v(u,y)+\int_0^{\pi}k(\theta,y)v(u,\theta)\,d\theta\right]\in\frac{\partial^2}{\partial y^2}v(u,y)+s\bigl(u,v(u,y)\bigr),\quad u\in I=[0,\pi],\\
&v(u,0)=v(u,\pi)=0, \qquad (5.1)\\
&v(0,y)=0,\quad 0<y<\pi, \qquad (5.2)
\end{aligned}
$$

where $0<q\leq 1$, construct $\mathbb{Y}=L^2(0,\pi)$ and $\mathscr{A}:D(\mathscr{A})\subseteq\mathbb{Y}\to\mathbb{Y}$ by $\mathscr{A}z=z''$, together $D(\mathscr{A})=\{z\in\mathbb{Y}:y''\in\mathbb{Y}\}$, are absolutely continuous. Obviously $\mathscr{A}$ is the infinitesimal generator of $\{T(u),u\geq 0\}$ in $\mathbb{Y}$ and generates the strongly continuous semi group $T(u)$. Additionally, $\mathscr{A}$ has eigenvalues in the form of $-n^2, n\in\mathbb{N}$, and it can be denoted as

$$
\mathscr{A}z=\sum_{n=1}^{\infty}n^2\langle z,z_n\rangle,\quad z\in D(\mathscr{A}),
$$

and $z_n(x)=\sqrt{\frac{2}{\pi}}\sin(nx)$, $n=1,2,3,\dots$, represents the set of eigenvectors of $\mathscr{A}$ which are orthonormal. Also for any $z\in\mathbb{Y}$,

$$
T(u)z=\sum_{n=1}^{\infty}e^{-n^2}u\langle z,z_n\rangle z_n.
$$

Clearly, $T(u)$ fulfills $(\mathbf{H_1})$. Define $h:[0,\pi]\times\mathbb{Y}\to\mathbb{Y}$ by

$$
h(v)(z)=\int_0^{\pi}k(\theta,z)v(u,\theta)\,d\theta,
$$

where the continuous function $k:[0,\pi]\times[0,\pi]\longrightarrow\mathbb{R}$ provided $\|k(\cdot,z)\|\leq 1$, for each $z\in[0,\pi]$ and $v(u)(z)=v(u,z)$, $\mathscr{H}(u,z(u))z=s(u,v(u,z))$. With a suitable choice of $\mathscr{A}$, $\mathscr{H}$, h, the above said system can be equivalent to (1.1)–(1.2), that is,

$$
\begin{aligned}
&{}^{C}D_{t}^{q}\bigl[z(u)-h\bigl(u,z(u)\bigr)\bigr]\in\mathscr{A}z(u)+\mathscr{H}\bigl(u,z(u)\bigr),\quad u\in[0,b], \qquad (5.3)\\
&z(0)+\varphi(z)=z_0. \qquad (5.4)
\end{aligned}
$$

Besides assuming $\mathscr{H}$, h satisfies the concerned hypotheses. As a result, (5.1)–(5.2) has at least a mild solution on $[0,b]$.

6 Conclusion

This manuscript addresses the existence of Caputo fractional neutral inclusions without compactness in a Banach space by using weak topology. Further, the results are derived for fractional neutral system where nonlocal conditions hold. Our theorem guarantees the effectiveness of the existence, which is the result of the system concerned.

Acknowledgements
This research was funded by the Deanship of Scientific Research at Princess Nourah bint Abdulrahman University through the Fast-track Research Funding Program.

Availability of data and materials
Not applicable.

Competing interests
There does not exist any competing interest regarding this article.

Authors' contributions
All authors have equally made contributions to this article. All authors read and approved the final manuscript.

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Publisher's Note

Received: 9 October 2019 Accepted: 9 December 2019 Published online: 19 December 2019

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