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New findings on exponential convergence of a Nicholson's blowflies model with proportional delay

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Abstract

We deal with Nicholson's blowflies model with proportional delays. Employing the differential inequality theory, we give a new sufficient condition that guarantees the exponential convergence of all solutions of Nicholson's blowflies model with proportional delays. Numerical simulations are put into effect to examine our theoretical findings. The derived results of this manuscript are innovative and complement some known investigations.

MSC: 34C27; 34D23

Keywords: Nicholson; s blowflies model; Exponential convergence; Proportional delay

1 Introduction

To describe the periodic oscillation in Nicholson's classic experiments [1] with the Australian sheep blowfly, Lucilia cuprina, Gurney et al. [2] put up the following Nicholson's blowflies model:

$$\dot{w}(t) = -dw(t) + bw(t-\vartheta)e^{-\gamma x(t-\vartheta)}, \tag{1.1}$$

where b is the maximum per capita daily egg production rate, $\frac{1}{\gamma}$ is the size at which the blowfly population reproduces at its maximum rate, d is the per capita daily adult death rate, and ϑ is the generation time. Due to the immense application of Nicholson's blowflies model in biology, model (1.1) and its modifications have been extensively discussed by lots of authors (see, e.g., [3–6] and the references therein). Noticing the periodic change of real environment, many scholars [7–9] generalized model (1.1) into the following Nicholson's blowflies model:

$$\dot{w}(t) = -d(t)w(t) + \sum_{j=1}^{m} b_j(t)w\bigl(t-\vartheta_j(t)\bigr)e^{-\gamma_j(t)w(t-\vartheta_j(t))}, \tag{1.2}$$

where m is a positive integer, $d: R \to R$ and $b_j, \gamma_j, \vartheta_j : R \to [0,+\infty)$, $j = 1,2,\ldots,m$, are bounded continuous functions, and $w(t)$ is the size of the population at time t. Noting that the exponential convergent rate can be unveiled [10–29], Long [30] investigated the exponential convergence of model (1.2).

Some researchers think that time delays appearing in many biological models are proportional; in other words, the proportional delay function takes the form $\vartheta(t) = t - at$ ($0 < a < 1$ is a constant). In objective world, proportional delay plays a key role in numerous areas such as web quality, current collection [31], biological systems and many nonlinear models [32, 33], electrodynamics [34], and probability principle [35]. So it is valuable to study the global exponential convergence of Nicholson's blowflies model with proportional delays. But so far there are no manuscripts about the global exponential convergence of Nicholson's blowflies model with proportional delays.

Stimulated by the above analysis, it is important for us to analyze the global exponential convergence on Nicholson's blowflies model with proportional delays. In this paper, we focus on the following Nicholson's blowflies model with proportional delays:

$$\dot{w}(t) = -d(t)w(t) + \sum_{j=1}^{m} b_j(t)w(a_j t)e^{-\gamma_j(t)w(a_j t)}, \tag{1.3}$$

where m is a positive integer, $d : R \to R$ and $b_j, \gamma_j, \vartheta_j : R \to [0, +\infty)$, $j = 1, 2, \dots, m$, are bounded continuous functions, a_j is the proportional delay factor such that $0 < a_j < 1$, $a_j t = t - (1 - a_j)t$, and $(1 - a_j)t \to +\infty$ as $t \to +\infty$.

The initial condition of model (1.3) takes the form

$$w(s) = \psi(s), \quad s \in [a_0 t_0, t_0], t_0 > 0, \tag{1.4}$$

where $a_0 = \min_{i=1,2,\dots,m}\{a_i\}$, and ψ is a real-valued continuous function on $[a_0 t_0, t_0]$.

For convenience, we denote $l^+ = \sup_{t\in[t_0,+\infty)} |l(t)|$ and $l^- = \inf_{t\in[t_0,+\infty)} |l(t)|$ for a bounded continuous function l on $[t_0, +\infty)$.

Throughout this paper, we also make the following assumptions:

- (K1) There exist a bounded continuous function: $d^* : [t_0, +\infty) \to (0, +\infty)$ and a positive constant μ such that $e^{-\int_s^t d(\theta)\,d\theta} \le \mu e^{-\int_s^t d^*(\theta)\,d\theta}$ for all $t, s \in R$ and $t - s \ge 0$.
- (K2) $\sup_{t\ge t_0}\{-d^*(t) + \mu \sum_{j=1}^{m} |b_j(t)|\} < 0$.
- (K3) $ma > 1$, where $a = \max_{1\le i\le m}\{a_i\}$.

The key task of this paper is finding a sufficient condition that ensures the global exponential convergence of all solutions of (1.3). The key contributions of this paper are the following: (i) For the first time, the new Nicholson's blowflies model with proportional delays is presented; (ii) A new sufficient condition that guarantees the global exponential convergence of Nicholson's blowflies model with proportional delays is established; (iii) Until now, the global exponential convergence for Nicholson's blowflies model with proportional delays has not been studied.

2 Main findings

Now we will discuss the global exponential convergence of model (1.3)

Lemma 2.1 *Let $d^{*-} > 0$ and $\sigma \ge 0$ be constants such that*

$$\int_s^t \big(d^*(v) - d(v)\big)\,dv \le \sigma \quad \text{for all } t, s \in R \text{ and } t - s \ge 0.$$

Then for any $t_0 \in R$, the solution $w(t;t_0,\psi)$ of system (1.3) with the initial value (1.4) satisfies $w(t;t_0,\psi) > 0$ for all $t \in [t_0,\eta(\psi))$ and $\eta(\psi) = +\infty$, where $[t_0,\eta(\psi))$ is the maximal right interval of the existence of a solution $w(t;t_0,\psi)$.

In view of the proof of Lemma 2.1 in Long [30], we can easily prove Lemma 2.1.

Theorem 2.1 *For system (1.3), under the assumptions of Lemma 2.1, if* (K1)–(K3) *hold, then there exists a constant $\xi > 0$ such that $w(t) = O(e^{-\xi t})$ as $t \to +\infty$.*

Proof Assume that $w(t)$ is an arbitrary solution of model (1.3). By (1.3) we have

$$\dot{w}(t) + d(t)w(t) = \sum_{j=1}^{m} b_j(t)w(a_j t)e^{-\gamma_j(t)w(a_j t)}. \tag{2.1}$$

Define the continuous function

$$\Phi(\omega) = \sup_{t\geq t_0}\left\{\omega - d^*(t) + \mu\left[\sum_{j=1}^{m}\left|b_j(t)\right| + \omega\right]\right\}, \quad \omega \in [0,+\infty). \tag{2.2}$$

It follows from (K2) that

$$\Phi(0) = \sup_{t\geq t_0}\left\{-d^*(t) + \mu\left[\sum_{j=1}^{m}\left|b_j(t)\right|\right]\right\} < 0. \tag{2.3}$$

In view of the continuity of $\Phi(\omega)$, we can choose a constant $\xi \in (0,\inf_{t\geq t_0} d^*(t))$ such that

$$\Phi(\xi) = \sup_{t\geq t_0}\left\{\xi - d^*(t) + \mu\left[\sum_{j=1}^{m}\left|b_j(t)\right| + \xi\right]\right\} < 0. \tag{2.4}$$

Let

$$\|\psi\|_{\varrho} = \max_{t\in[a_0 t_0,t_0]}\left|\psi(t)\right|. \tag{2.5}$$

For all $\epsilon > 0$, we get

$$\left|w(t)\right| < \left(\|\psi\|_{\varrho} + \epsilon\right)e^{-\xi(t-t_0)} < P\left(\|\psi\|_{\varrho} + \epsilon\right)e^{-\xi(t-t_0)} \tag{2.6}$$

for $t \in [a_0 t_0, t_0]$, where $P > \mu + 1$. We will further prove that

$$\left|w(t)\right| < P\left(\|\psi\|_{\varrho} + \epsilon\right)e^{-\xi(t-t_0)} \tag{2.7}$$

fir $t \geq t_0$. Otherwise, there exists $t^* > t_0$ such that

$$\left|w(t^*)\right| = P\left(\|\psi\|_{\varrho} + \epsilon\right)e^{-\xi(t^*-t_0)} \tag{2.8}$$

and

$$\left|w(t)\right| < P\left(\|\psi\|_{\varrho} + \epsilon\right)e^{-\xi(t-t_0)} \tag{2.9}$$

for $t \in [a_0 t_0, t^*]$. Note that

$$\dot{w}(s) + d(t)w(s) = \sum_{j=1}^{m} b_j(s) w(a_j s) e^{-\gamma_j(s) w(a_j s)} \tag{2.10}$$

for $s \in [t_0, t]$ and $t \in [t_0, t^*]$. By (2.10) we get

$$w(t) = w(t_0) e^{-\int_{t_0}^{t} d(v)\,dv} + \int_{t_0}^{t} e^{-\int_{s}^{t} d(v)\,dv} \left[\sum_{j=1}^{m} b_j(s) w(a_j s) e^{-\gamma_j(s) w(a_j s)} \right] ds, t \in \left[t_0, t^*\right]. \tag{2.11}$$

By (2.4) and (K3) we have

$$\begin{aligned}
\left|w(t)\right| &= \left| w(t_0) e^{-\int_{t_0}^{t} d(v)\,dv} + \int_{t_0}^{t} e^{-\int_{s}^{t} d(v)\,dv} \left[\sum_{j=1}^{m} b_j(s) w(a_j s) e^{-\gamma_j(s) w(a_j s)} \right] ds \right| \\
&\leq \left(\|\psi\|_{\varrho} + \epsilon\right) \mu e^{-\int_{t_0}^{t^*} d^*(v)\,dv} + \int_{t_0}^{t^*} e^{-\int_{s}^{t^*} d^*(v)\,dv} \mu \left[\sum_{j=1}^{m} \left|b_j(s)\right| \left|w(a_j s)\right| \right] ds \\
&\leq \left(\|\psi\|_{\varrho} + \epsilon\right) \mu e^{-\int_{t_0}^{t^*} d^*(v)\,dv} \\
&\quad + \int_{t_0}^{t^*} e^{-\int_{s}^{t^*} d^*(v)\,dv} \mu \left[\sum_{j=1}^{m} \left|b_j(s)\right| P\left(\|\psi\|_{\varrho} + \epsilon\right) e^{-\xi(a_j s - t_0)} \right] ds \\
&\leq \left(\|\psi\|_{\varrho} + \epsilon\right) \mu e^{-\int_{t_0}^{t^*} d^*(v)\,dv} + \int_{t_0}^{t^*} e^{-\int_{s}^{t^*} d^*(v)\,dv} e^{-\xi(s-t_0)} \left(d^*(s) - \xi\right) ds \\
&\quad \times e^{-\xi(t^*-t_0)} P\left(\|\psi\|_{\varrho} + \epsilon\right) \\
&= \left(\|\psi\|_{\varrho} + \epsilon\right) e^{-\xi(t^*-t_0)} \mu e^{-\int_{t_0}^{t^*} (d^*(v)-\xi)\,dv} + \int_{t_0}^{t^*} e^{-\int_{s}^{t^*} (d^*(v)-\xi)\,dv} \left(d^*(s) - \xi\right) ds \\
&\quad \times e^{-\xi(t^*-t_0)} P\left(\|\varphi\|_{\varrho} + \epsilon\right) \\
&= P\left(\|\psi\|_{\varrho} + \epsilon\right) e^{-\xi(t^*-t_0)} \left[\frac{e^{-\int_{t_0}^{t^*} (d^*(v)-\xi)\,dv} \mu}{P} + 1 - e^{-\int_{t_0}^{t^*} (d^*(v)-\xi)\,dv} \right] \\
&= P\left(\|\psi\|_{\varrho} + \epsilon\right) e^{-\xi(t^*-t_0)} \left[\left(\frac{\kappa}{P} - 1\right) e^{-\int_{t_0}^{t^*} (d^*(v)-\xi)\,dv} + 1 \right] \\
&< P\left(\|\psi\|_{\varrho} + \epsilon\right) e^{-\xi(t^*-t_0)},
\end{aligned} \tag{2.12}$$

which contradicts (2.8). Then (2.7) is true. Thus $w(t) = O(e^{-\xi t})$ as $t \to +\infty$. The theorem is proved. □

Remark 2.1 In [36, 37] the authors dealt with neural networks with proportional delays, but they did not consider the global exponential convergence of involved models. In [10, 38] the authors studied the exponential convergence of neural networks with proportional delays, but they did not investigate Nicholson's blowflies models. In this paper, we study the global exponential convergence of Nicholson's blowflies model with proportional delays. All the derived results in [10, 36–38] cannot be applied to model (1.3) to obtain the global exponential convergence of system (1.3). So far, no results about the global exponential convergence of Nicholson's blowflies model with proportional delays are reported.

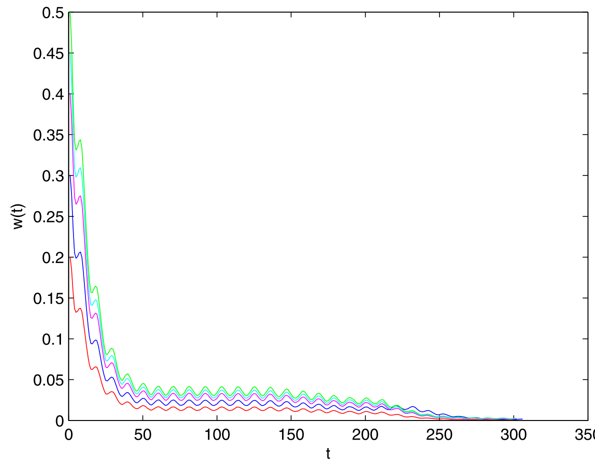

Figure 1 The relation of t and $w(t)$. The initial values are $w_0 = 0.2, 0.3, 0.5, 0.4, 0.45$

Therefore our findings on the global exponential convergence of Nicholson's blowflies model with proportional delays are essentially innovative and supplement earlier publications to a certain extent.

3 Example

Consider the model

$$\dot{w}(t) = -d(t)w(t) + \sum_{j=1}^{2} b_j(t)w(a_j t)e^{-\gamma_j(t)w(a_j t)}, \tag{3.1}$$

where $d(t) = 0.2(1 + 0.5\sin t)$, $b_1(t) = 0.07 + 0.07|\cos\sqrt{5t}|$, $b_2(t) = 0.05 + 0.05|\sin\sqrt{5t}|$, $\gamma_1(t) = 1 + 0.1|\cos\sqrt{3t}|$, $\gamma_2(t) = 1 + 0.1|\sin\sqrt{3t}|$, $a_1 = 0.1$, $a_2 = 0.6$ Then $d^*(t) = 0.2$ and $\mu = e^{\frac{1}{5}}$, Let $\sigma = \frac{1}{200}$. Then $e^{-\int_s^t d(\theta)\,d\theta} \leq e^{\frac{1}{5}}e^{-(t-s)}$, $t \geq s$, and $\int_s^t (d^*(v) - d(v))\,dv \leq \sigma$, $\sup_{t\geq t_0}\{-d^*(t) + \mu\sum_{j=1}^{2}|b_j(t)|\} \approx -0.6052 < 0$. Thus all the conditions in Theorem 2.1 are satisfied, and all solutions of model (3.1) converge exponentially to $(0,0)^T$. This fact is shown in Fig. 1.

4 Conclusions

Exponential convergence is an important dynamical behavior of differential dynamical systems. During the past decades, many researchers payed much attention to it. In this paper, we have discussed Nicholson's blowflies model with proportional delays. By means of the differential inequality knowledge, we derived a sufficient criterion ensuring the exponential convergence of all solutions for Nicholson's blowflies model with proportional delays. The sufficiency criterion can be easily checked by simple computation. Up to now, there are no papers that focus on the exponential convergence of Nicholson's blowflies model with proportional delays, which shows that the results derived in this paper are new and extend earlier publications to some extent.

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Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors have read and approved the final manuscript.

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