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Dissipative criteria for Takagi–Sugeno fuzzy Markovian jumping neural networks with impulsive perturbations using delay partitioning approach

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Abstract

In this work, we investigate the result of dissipative analysis for Takagi–Sugeno fuzzy Markovian jumping neural networks with impulsive perturbations via delay partition approach. By using the Lyapunov–Krasovskii functional and delay partition approach, we derive a set of delay-dependent sufficient criteria for obtaining the required results. Furthermore, we restate the obtained sufficient conditions in the form of linear matrix inequalities (LMIs), which can be checked by the standard MATLAB LMI tool box. The main advantage of this work is reduced conservatism, which is mainly based on the delay partition approach. Finally, we provide numerical examples with simulations to demonstrate the applicability of the proposed method.

Keywords: Dissipativity; Markovian jumping neural network; Impulses; Lyapunov–Krasovskii functional; Linear matrix inequality

1 Introduction

In the last twenty years, neural networks have received increasing consideration because of their applications in various fields such as signal processing, pattern recognition, optimization problems, associative memories, and so on [1, 9, 30, 31, 42, 43]. In particular, the stability theory of neural networks has become an important topic in both theory and practice, since stability is one of the major problems related to neural network dynamic behaviors. Besides, time-delays are frequently encountered in hardware implementation of neural networks, since time-delays are the source of generation of instability and poor performance. Hence the stability analysis of neural networks with time delay have obtained remarkable consideration in recent years [12, 35, 42, 43].

In practice, most of the neural networks are represented by nonlinear models, so it is important and necessary to design an appropriate neural network approach for nonlinear systems. In this relation, the Takagi–Sugeno (T–S) fuzzy model can provide an effective approach for complex nonlinear systems in terms of fuzzy sets and linear input–output variables [21, 24]. The main advantage of this T–S fuzzy model is easy to analyze and design linear systems to nonlinear systems. Thus, many authors extended the T–S fuzzy models to describe different types of neural networks with time delays to establish the

stability of the concerned network models [5, 25]. Very recently, Shen et al. [27] obtained an asynchronous state estimation for fuzzy Markovian jumping neural networks with uncertain measurements in a finite time interval. Based on the Lyapunov stability theory and Wirtinger-based integral inequality, sufficient conditions were constructed in [4] to ensure that the state estimation error system is robustly stable along with a guaranteed dissipative performance of T–S fuzzy neural networks.

On the other side, Markovian jump systems (MJSs), as a certain class of switched systems consisting of an indexed family of subsystems and a set of Markovian chains, have received increasing attention [18, 19]. Due to the presence of sudden variation, random component failures, and abrupt environment changes in dynamic systems, by MJSs we can model aircraft control, solar receiver control, manufacturing systems, networked control systems, power systems, and other practical systems [15, 26, 32]. During the past few years, MJSs have been used to various disciplines of science and engineering fields, and a great number of results have been obtained in [2, 3, 10, 12, 13, 29]. Based on a generalized double integral inequality, dissipativity conditions were proposed under the consideration of free-matrix-based integral inequality and Finsler's lemma approach in [12]. In [13] the authors studied the problem of passivity and dissipativity analysis of Markovian jump neural networks including two types of additive time-varying delays. In [29] the results on dissipativity-based Markovian jump neural networks are established using some generalized integral inequalities. Stability analysis of neural models with Markovian jump limitations and delay constants are derived using reciprocally convex approach in [3].

On the other hand, dissipativity is an important concept of dynamical systems, which is closely related with the intuitive phenomenon of loss or dissipation of energy. Moreover, dissipativity theory gives a framework for the control design and stability analysis of practical control systems under an input–output energy-related consideration. Based on the framework of dissipativity, many problems were investigated for continuous-time neural networks [12, 13, 23, 35, 37, 41] and discrete-time neural networks [17], but there appeared a few works based on the dissipativity concept of T–S fuzzy neural networks [11, 14]. Also, it is well known that impulsive effects are used to express the dynamical models in many areas such as medicine, biology, economics, and telecommunications. Roughly speaking, the states of neural networks often undergo rapid disruption and sudden changes at certain moments of time, which leads to impulsive effects. Thus impulses should be taken into account while studying the dissipativity of neural networks and the corresponding issues studied in the literature [20, 22, 44]. However, up to now, the results of dissipativity analysis of Markovian jumping T–S fuzzy neural networks together with impulsive effects have not yet been reported, which is still an open challenge. All this motivated us to consider a new set of dissipative conditions for fuzzy Markovian jumping neural networks with impulsive control via delay partitioning approach.

The main contributions of this paper are summarized as follows:

- (i) Uncertain parameters, Markovian jumping, nonlinearities, time-delay, dissipative conditions and impulsive perturbations are considered in the framework of stability analysis and designing Takagi–Sugeno fuzzy neural networks.
- (ii) By employing a proper Lyapunov–Krasovskii functional the asymptotic stability of addressed neural networks is checked via some less conservative stability conditions.

- (iii) Some novel uncertain parameters are initially handled in the Lyapunov–Krasovskii functional, which ensures sufficient conditions for asymptotic stability of designed neural networks.
- (iv) The importance of the proposed algorithm is illustrated by numerical examples.

2 Problem formulation

Let $\{r(t), t \geq 0\}$ be a right-continuous Markovian process taking values in the finite space $S = \{1, 2, \dots, s\}$ with generator $\Gamma = (\pi_{ij})$ ($i, j \in S$) given by

$$\Pr\{r(t + \Delta t) = j \mid r(t) = i\} = \begin{cases} \pi_{ij}\Delta t + o(\Delta t), & i \neq j, \\ 1 + \pi_{ii}\Delta t + o(\Delta t), & i = j, \end{cases}$$

where $\Delta t \geq 0, \lim_{\Delta t \rightarrow 0} (o(\Delta t)/\Delta t) = 0, \pi_{ij} \geq 0$ for $j \neq i$ is the transition rate from mode i to mode j at time $t + \Delta t$, and $\pi_{ii} = -\sum_{j=1, j \neq i}^N \pi_{ij}$.

Consider the neural networks of Markovian jumping parameters with mixed interval time-varying delays of the following form:

$$\begin{aligned} \dot{v}_i(t) &= -a_i(r(t))v_i(t) + \sum_{i=1}^n w_{1ij}(r(t))f_i(v_i(t)) + \sum_{i=1}^n w_{2ij}(r(t))f_i(v_i(t - \tau(t))) \\ &\quad + \sum_{i=1}^n w_{3ij}(r(t)) \int_{t-d(t)}^t f_i(v(s)) ds + u(t), \quad t > 0, t \neq t_k, \end{aligned} \tag{1}$$

$$y(t) = f(v_i(t)),$$

$$v(t_k) = I_k(r(t))(v(t_k^-)), \quad t = t_k, k \in Z_+,$$

where $i = 1, 2, \dots, n, v_i(t)$ is the state of the i th neuron, $a_i(r(t)) > 0$ denotes the rate with which the cell i resets its potential to the resting states when isolated from the other cells and inputs, $w_{1ij}(r(t)), w_{2ij}(r(t))$ and $w_{3ij}(r(t))$ are the connection weights at the time $t, u(t) = [u_1(t), u_2(t), \dots, u_n(t)]^T$ is the external input, $y(t) = [y_1(t), y_2(t), \dots, y_n(t)]$ is the output, $I_k(r(t))(\cdot)$ is a constant real matrix at the time moments t_k , and $f_i(\cdot)$ stands for the signal function of the i th neuron. In addition, we suppose that the discrete delay $\tau(t)$ and distributed delay $d(t)$ satisfy

$$0 \leq \tau_1 \leq \tau(t) < \tau_2, \quad \tau(\dot{t}) \leq \mu_1, \quad 0 \leq d(t) \leq d, \quad d(\dot{t}) \leq \mu_2, \tag{2}$$

where $\tau_1, \tau_2, d, \mu_1, \mu_2$ are constants. We consider system (1) together with the initial condition $v(s) = \psi(s), s \in [-\max\{\tau_2, d\}, 0]$.

Then we rewrite model (1) as

$$\begin{aligned} \dot{v}(t) &= -A(r(t))v(t) + W_1(r(t))f(v(t)) + W_2(r(t))f(v(t - \tau(t))) \\ &\quad + W_3(r(t)) \int_{t-d(t)}^t f(v(s)) ds + u(t), \quad t > 0, t \neq t_k, \end{aligned} \tag{3}$$

$$y(t) = f(v(t)),$$

$$v(t_k) = I_k(r(t))(v(t_k^-)), \quad t = t_k, k \in Z_+,$$

where $v(t) = [v_1(t), v_2(t), \dots, v_n(t)]^T$, $A = \text{diag}(a_1(r(t)), \dots, a_n(r(t)))$, $W_1 = (w_{1ij}(r(t)))_{m \times n}$, $W_2 = (w_{2ij}(r(t)))_{m \times n}$, $W_3 = (w_{3ij}(r(t)))_{m \times n}$, and $f(\cdot) = (f_1(\cdot), f_2(\cdot), \dots, f_n(\cdot))^T$.

Let $v^* = (v_1^*, v_2^*, \dots, v_n^*)$ be the equilibrium points of system (3). We can obtain from (3) that in terms of the transformations $x(\cdot) = v(\cdot) - v^*$ and $f(x(t)) = f(x(t) + v^*) - f(v^*)$, system (3) can be written as

$$\begin{aligned} \dot{x}(t) &= -A(r(t))x(t) + W_1(r(t))f(x(t)) + W_2(r(t))f(x(t - \tau(t))) \\ &\quad + W_3(r(t)) \int_{t-d(t)}^t f(x(s)) ds + u(t), \quad t > 0, t \neq t_k, \\ y(t) &= f(x(t)), \\ x(t_k) &= I_k(r(t)) (x(t_k^-)), \quad t = t_k, k \in Z_+, \end{aligned} \tag{4}$$

where $x(t) = [x_1(t), x_2(t), \dots, x_n(t)]^T$, $f(\cdot) = (f_1(\cdot), f_2(\cdot), \dots, f_n(\cdot))^T$.

Further, we express the Markovian jumping neural network with impulsive control by a T-S fuzzy model. We represent the i th rule of this T-S fuzzy model in the following form:

IF $v_1(t)$ is M_{i1} , $v_2(t)$ is M_{i2} , \dots , $v_j(t)$ is M_{ij} **THEN**

$$\begin{aligned} \dot{x}(t) &= -A_i(r(t))x(t) + W_{1i}(r(t))f(x(t)) + W_{2i}(r(t))f(x(t - \tau(t))) \\ &\quad + W_{3i}(r(t)) \int_{t-d(t)}^t f(x(s)) ds + u(t), \quad t > 0, t \neq t_k, \\ y(t) &= f(x(t)), \\ x(t_k) &= I_k(r(t)) (x(t_k^-)), \quad t = t_k, k \in Z_+, i = 1, 2, \dots, r, \end{aligned} \tag{5}$$

where M_{ij} are fuzzy sets, $(v_1(t), v_2(t), \dots, v_j(t))^T$ represents the premise variable vector, $x(t)$ denotes the state variable, and r is the number of IF-THEN rules. It is known that system (5) has a unique global solution on $t \geq 0$ with initial values $\psi_x \in C([- \max\{\tau_2, d\}, 0]; \mathbb{R}^n)$. For convenience, $r(t) = i$, where $i \in s$, and in the upcoming discussion, we represent the system matrices associated together with the i th mode by $A_i(r(t)) = A_i$, $W_{1i}(r(t)) = W_{1i}$, $W_{2i}(r(t)) = W_{2i}$, $W_{3i}(r(t)) = W_{3i}$.

Then the state equation is as follows:

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^r \lambda_i(v(t)) \left\{ -A_i x(t) + W_{1i} f(x(t)) + W_{2i} f(x(t - \tau(t))) \right. \\ &\quad \left. + W_{3i} \int_{t-d(t)}^t f(x(s)) ds + u(t) \right\}, \quad t > 0, t \neq t_k, \\ y(t) &= f(x(t)), \\ x(t_k) &= I_k(x(t_k^-)), \quad t = t_k, k \in Z_+, i = 1, 2, \dots, r, \end{aligned} \tag{6}$$

where $\lambda_i(v(t)) = \frac{\beta_i(v(t))}{\sum_{i=1}^r \beta_i(v(t))}$, $\beta_i(v(t)) = \prod_{j=1}^p M_{ij}(v_j(t))$, and $M_{ij}(\cdot)$ is the degree of the membership function of M_{ij} . Further, we assume that $\beta_i(v(t)) \geq 0$, $i = 1, 2, \dots, r$, and $\sum_{i=1}^r \beta_i(v(t)) > 0$ for all $v(t)$. Therefore $\lambda_i(v(t))$ satisfy $\lambda_i(v(t)) \geq 0$, $i = 1, \dots, r$, and $\sum_{i=1}^r \lambda_i(v(t)) = 1$ for any $v(t)$.

Based on the previous simple transformation, we can equivalently rewrite model (6) as follows:

$$\begin{aligned} \dot{x}(t) &= -A_i x(t) + W_{1i} f(x(t)) + W_{2i} f(x(t - \tau(t))) \\ &\quad + W_{3i} \int_{t-d(t)}^t f(x(s)) ds + u(t), \quad t > 0, t \neq t_k, \\ y(t) &= f(x(t)), \\ x(t_k) &= I_k(x(t_k^-)), \quad t = t_k, k \in \mathbb{Z}_+, i = 1, 2, \dots, r. \end{aligned} \tag{7}$$

The following assumptions are needed to prove the required result.

Assumption (H1) ([9]) For all $j \in 1, 2, \dots, n, f_j(0) = 0$, and there exist constants F_j^- and F_j^+ such that

$$F_j^- \leq \frac{f_j(\alpha_1) - f_j(\alpha_2)}{\alpha_1 - \alpha_2} \leq F_j^+ \tag{8}$$

for all $\alpha_1, \alpha_2 \in \mathbb{R}$ and $\alpha_1 \neq \alpha_2$.

Assumption (H2) The impulsive times t_k satisfy $0 = t_0 < t_1 < \dots < t_k \rightarrow \infty$ and $\inf_{k \in \mathbb{Z}_+} \{t_k - t_{k-1}\} > 0$.

The energy function E associated with system (7) is represented by

$$E(u, y, T) = \langle y, Qy \rangle_T + 2\langle y, Su \rangle_T + \langle u, Ru \rangle_T, \tag{9}$$

where

$$\langle y, u \rangle_T = \int_0^T y^T u dt, \quad T \geq 0.$$

The following definitions and lemmas are needed to prove our results.

Definition 1 Given some value $\vartheta > 0$, real constant matrices $Q = Q^T$ and $R = R^T$, and a matrix S , the considered model (7) is (Q, S, R) - ϑ -dissipative for any $T \geq 0$. Under the zero initial condition, the following condition is holds:

$$E(u, y, T) \geq \vartheta \langle u, u \rangle_T. \tag{10}$$

Definition 2 The proposed neural network model (7) is called passive if there exists a scalar $\gamma \geq 0$ such that the following inequality holds for all $t_f \geq 0$ under the zero initial condition:

$$2 \left[\int_0^{t_f} y^T(s) u(s) ds \right] \geq -\gamma \left[\int_0^{t_f} u^T(s) u(s) ds \right]. \tag{11}$$

Lemma 2.1 ([7]) For any vectors $\tau(t) \geq 0$ and positive-definite matrix $Q \in R^{n \times n}$, we have the following inequalities:

$$-\int_{t-\tau(t)}^t \dot{x}^T(s) Q \dot{x}(s) ds \leq \tau(t) \zeta^T(t) M Q^{-1} M^T \zeta(t) + 2\zeta^T(t) M [x(t) - x(t - \tau(t))],$$

$$\begin{aligned}
 - \int_{t-\tau}^{t-\tau(t)} \dot{x}^T(s) Q \dot{x}(s) ds &\leq (\tau - \tau(t)) \zeta^T(t) N Q^{-1} N^T \zeta(t) \\
 &+ 2\zeta^T(t) N [x(t - \tau(t)) - x(t - \tau)],
 \end{aligned}$$

where M and N are free weighting matrices of appropriate dimensions, and

$$\begin{aligned}
 \zeta(t) = &\left[x^T(t) \quad x^T(t - \tau_1) \quad x^T\left(t - \frac{\tau_a}{N}\right) \quad x^T\left(t - 2\frac{\tau_a}{N}\right) \quad \dots \quad x^T\left(t - (N - 1)\frac{\tau_a}{N}\right) \right. \\
 &x^T(t - \tau_a) \quad x^T(t - \tau(t)) \quad x^T\left(t - \frac{\tau_2}{N}\right) \quad x^T\left(t - 2\frac{\tau_2}{N}\right) \quad \dots \\
 &x^T\left(t - (N - 1)\frac{\tau_2}{N}\right) \quad x^T(t - \tau_2) \quad \dot{x}^T(t) \quad f^T(x(t)) \quad f^T(x(t - \tau(t))) \\
 &\int_{t-\tau_2}^t x^T(s) ds \quad \int_{t-\tau_2}^{t-\tau_1} x^T(s) ds \quad \int_{t-\tau_2}^t f^T(x(s)) ds \quad \int_{t-\tau_2}^{t-\tau_1} f^T(x(s)) ds \\
 &\left. \int_{t-d(t)}^t f^T(x(s)) ds \quad x^T(t - d(t)) \quad u^T(t) \right]^T.
 \end{aligned}$$

3 Main results

In this section, we establish a dissipativity condition for fuzzy impulsive Markovian jumping neural networks (7) with both discrete and distributed time delays. Under a Lyapunov functional and delay fractionizing approach, in the following theorem, we provide a new set of novel delay-dependent dissipative conditions with impulsive perturbations. For presentation convenience, we denote

$$\begin{aligned}
 F_1 &= \text{diag}(F_1^- F_1^+, F_2^- F_2^+, \dots, F_n^- F_n^+), \quad F_2 = \text{diag}\left(\frac{F_1^- + F_1^+}{2}, \frac{F_2^- + F_2^+}{2}, \dots, \frac{F_n^- + F_n^+}{2}\right), \\
 F_3 &= \text{diag}(F_1^-, F_2^-, \dots, F_n^-), \quad F_4 = \text{diag}(F_1^+, F_2^+, \dots, F_n^+).
 \end{aligned}$$

Theorem 3.1 *Under Assumptions (H1) and (H2), for given scalars τ_1, τ_2, d, μ_1 , and μ_2 , the neural network described by (7) is strictly $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$ - ϑ -dissipative if there exist positive definite matrices P_{1i}, P_i ($i = 2, \dots, 4$), Q, R, S_i ($i = 1, 2, \dots, 7$), T_i ($i = 1, 2$), positive diagonal matrices U_1, U_2 , and matrices O, L_i, M_i, V_i ($i = 1, 2$) of appropriate dimensions such that the following LMIs hold:*

$$I_{ik}^T P_{1j} I_{ik} - P_{1i} < 0, \tag{12}$$

$$\begin{bmatrix} \Phi & \sqrt{\frac{\tau_a}{N}} L & \sqrt{\tau_\delta} M \\ * & -S_1 & 0 \\ * & * & -S_2 \end{bmatrix} < 0, \tag{13}$$

$$\begin{bmatrix} \Phi & \sqrt{\frac{\tau_a}{N}} L & \sqrt{\tau_\delta} V \\ * & -S_1 & 0 \\ * & * & -S_2 \end{bmatrix} < 0, \tag{14}$$

and

$$Q = \begin{bmatrix} Q_{11} & Q_{12} & \cdots & Q_{1N} \\ * & Q_{22} & \cdots & Q_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ * & * & Q_{N-1,N-1} & Q_{N-1,N} \\ * & * & * & Q_{NN} \end{bmatrix} \geq 0,$$

$$R = \begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1N} \\ * & R_{22} & \cdots & R_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ * & * & R_{N-1,N-1} & R_{N-1,N} \\ * & * & * & R_{NN} \end{bmatrix} \geq 0,$$

where

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} & \Phi_{15} & 0 & \Phi_{17} & \Phi_{18} & 0 & 0 & 0 & 0 & 0 \\ * & \Phi_{22} & \Phi_{23} & 0 & 0 & F_2U_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \Phi_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Phi_{44} & OW_{1ij} & OW_{2ij} & 0 & 0 & 0 & 0 & OW_{3ij} & 0 & 0 \\ * & * & * & * & \Phi_{55} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -S \\ * & * & * & * & * & -U_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -T_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -T_2 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -S_5 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -S_6 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & -S_7 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & * & -(1-\mu_2)P_4 & 0 \\ * & * & * & * & * & * & * & * & * & * & * & * & \gamma I - \mathcal{R} \end{bmatrix},$$

$$\Phi_{11} = \begin{bmatrix} \Phi_1 & L_2^T & Q_{12} - L_1 & Q_{13} & \cdots & Q_{1N} & V_1 \\ * & P_3 - P_2 - S_4 & -L_2 & 0 & \cdots & 0 & V_2 \\ * & * & Q_{22} - Q_{11} & Q_{23} - Q_{12} & \cdots & Q_{2N} - Q_{1,N-1} & -Q_{1N} \\ * & * & * & Q_{33} - Q_{22} & \cdots & Q_{3N} - Q_{2,N-1} & -Q_{2N} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & * & \cdots & Q_{NN} - Q_{N-1,N-1} & -Q_{N-1,N} \\ * & * & * & * & \cdots & * & -Q_{NN} \end{bmatrix},$$

$$\Phi_{33} = \begin{bmatrix} R_{22} - R_{11} & R_{23} - R_{12} & \cdots & R_{2N} - R_{1,N-1} & -R_{1N} \\ * & R_{33} - R_{22} & \cdots & R_{3N} - R_{2,N-1} & -R_{2N} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & \cdots & R_{NN} - R_{N-1,N-1} & -R_{N-1,N} \\ * & * & \cdots & * & -R_{NN} - S_3 - S_4 \end{bmatrix},$$

$$\Phi_{13} = \begin{bmatrix} R_{12} & R_{13} & \cdots & R_{1N} & -M_1 + S_3 \\ 0 & 0 & \cdots & 0 & -M_2 \\ 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix},$$

$$\begin{aligned} \Phi_1 &= P_2 + P_4 + Q_{11} + L_1 + L_1^T + R_{11} - S_3 - \tau_2^2 T_1 + \tau_{12}^2 T_2 - F_1 U_1 + \sum_{j=1}^N \pi_{ij} P_{1j}, \\ \Phi_{12} &= \begin{bmatrix} (-V_1 + M_1)^T & (-V_2 + M_2 + S_4)^T & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}^T, \\ \Phi_{22} &= -(1 - \mu_1)P_3 - 2S_4 - F_1 U_2, \\ \Phi_{23} &= \begin{bmatrix} 0 & 0 & \cdots & 0 & S_4 \end{bmatrix}, \\ \Phi_{14} &= \begin{bmatrix} (P_{1i} - A_{ij}^T O^T)^T & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}^T, \\ \Phi_{44} &= \frac{\tau_a}{N} S_1 + \tau_\delta S_2 + \tau_2^2 S_3 + \tau_{12}^2 S_4 + \frac{\tau_2^4}{4} T_1 + \tau_s^2 T_2 - O - O^T, \\ \Phi_{15} &= \begin{bmatrix} (F_2 U_1 & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}^T, \\ \Phi_{55} &= \tau_2^2 S_5 + \tau_{12}^2 S_6 + d^2 S_7 - U_1 - Q, \\ \Phi_{17} &= \begin{bmatrix} \tau_2 T_1^T & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}^T, \\ \Phi_{18} &= \begin{bmatrix} \tau_{12} T_2^T & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}^T, \\ L &= \begin{bmatrix} L_1^T & L_2^T & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \end{bmatrix}^T, \\ M &= \begin{bmatrix} M_1^T & M_2^T & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \end{bmatrix}^T, \\ V &= \begin{bmatrix} V_1^T & V_2^T & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \end{bmatrix}^T, \\ \tau_a &= \frac{(\tau_1 + \tau_2)}{2}, \quad \tau_\delta = \frac{(\tau_2 - \tau_1)}{2}, \quad \tau_{12} = \tau_2 - \tau_1, \quad \tau_s = \frac{1}{2}(\tau_2^2 - \tau_1^2). \end{aligned}$$

Proof To obtain dissipativity criteria for the fuzzy Markovian jumping impulsive neural networks (7), we examine the Lyapunov–Krasovskii functional

$$\begin{aligned} V(t, x(t), i) &= V_1(t, x(t), i) + V_2(t, x(t), i) + V_3(t, x(t), i) + V_4(t, x(t), i) \\ &\quad + V_5(t, x(t), i) + V_6(t, x(t), i), \end{aligned} \tag{15}$$

where

$$\begin{aligned} V_1(t, x(t), i) &= x^T(t)P_{1i}x(t) + \int_{t-\tau_1}^t x^T(s)P_2x(s) ds + \int_{t-\tau(t)}^{t-\tau_1} x^T(s)P_3x(s) ds \\ &\quad + \int_{t-d(t)}^t x^T(s)P_4x(s) ds, \\ V_2(t, x(t), i) &= \int_{t-\frac{\tau_a}{N}}^t \xi_1^T(s)Q\xi_1(s) ds + \int_{t-\frac{\tau_2}{N}}^t \xi_2^T(s)R\xi_2(s) ds, \\ V_3(t, x(t), i) &= \int_{-\frac{\tau_a}{N}}^0 \int_{t+\theta}^t \dot{x}^T(s)S_1\dot{x}(s) ds d\theta + \int_{-\tau_2}^{-\tau_1} \int_{t+\theta}^t \dot{x}^T(s)S_2\dot{x}(s) ds d\theta, \\ V_4(t, x(t), i) &= \tau_2 \int_{-\tau_2}^0 \int_{t+\theta}^t \dot{x}^T(s)S_3\dot{x}(s) ds d\theta + \tau_{12} \int_{-\tau_2}^{-\tau_1} \int_{t+\theta}^t \dot{x}^T(s)S_4\dot{x}(s) ds d\theta, \end{aligned}$$

$$\begin{aligned}
 V_5(t, x(t), i) &= \tau_2 \int_{-\tau_2}^0 \int_{t+\theta}^t f^T(x(s)) S_5 f(x(s)) ds d\theta \\
 &\quad + \tau_{12} \int_{-\tau_2}^{-\tau_1} \int_{t+\theta}^t f^T(x(s)) S_6 f(x(s)) ds d\theta \\
 &\quad + d \int_{-d}^0 \int_{t+\theta}^t f^T(x(s)) S_7 f(x(s)) ds d\theta, \\
 V_6(t, x(t), i) &= \frac{\tau_2^2}{2} \int_{-\tau_2}^0 \int_{\theta}^0 \int_{t+\lambda}^t \dot{x}^T(s) T_1 \dot{x}(s) ds d\lambda d\theta \\
 &\quad + \tau_s \int_{-\tau_2}^{-\tau_1} \int_{\theta}^0 \int_{t+\lambda}^t \dot{x}^T(s) T_2 \dot{x}(s) ds d\lambda d\theta
 \end{aligned}$$

with

$$\begin{aligned}
 \xi_1(t) &= \left[x^T(t) \quad x^T\left(t - \frac{\tau_a}{N}\right) \quad \dots \quad x^T\left(t - (N-1)\frac{\tau_a}{N}\right) \right]^T, \\
 \xi_2(t) &= \left[x^T(t) \quad x^T\left(t - \frac{\tau_b}{N}\right) \quad \dots \quad x^T\left(t - (N-1)\frac{\tau_b}{N}\right) \right]^T.
 \end{aligned}$$

For $t = t_k$, we have

$$\begin{aligned}
 V_1(t_k, x(t), j) - V_1(t_k^-, x(t), i) &= x^T(t_k) P_{1j} x(t_k) - x^T(t_k^-) P_{1i} x(t_k^-) \\
 &= x^T(t_k^-) I_{ik}^T P_{1j} I_{ik} x(t_k^-) - x^T(t_k^-) P_{1i} x(t_k^-) \\
 &= x^T(t_k^-) [I_{ik}^T P_{1j} I_{ik} - P_{1i}] x(t_k^-). \tag{16}
 \end{aligned}$$

Based on the assumptions and conditions, we know that I_{ik} is a constant matrix at the moment t_k and in the mode i for $i \in S, k \in N$. So

$$V_1(t_k, x(t), j) - V_1(t_k^-, x(t), i) < 0. \tag{17}$$

For $t \in [t_{k-1}, t_k]$, by (17) we obtain that the weak infinitesimal generator $\mathcal{L}V(t, x(t), i)$ satisfies

$$\begin{aligned}
 \mathcal{L}V_1(t, x(t), i) &= 2x^T(t) P_{1i} \dot{x}(t) + x^T(t) [P_2 + P_4] x(t) + x^T(t - \tau_1) (P_3 - P_2) x(t - \tau_1) \\
 &\quad - (1 - \dot{\tau}(t)) x^T(t - \tau(t)) P_3 x(t - \tau(t)) - (1 - \dot{d}(t)) x^T(t - d(t)) P_4 x(t - d(t)) \\
 &\quad + \sum_{j=1}^N \pi_{ij} x^T(t) P_{1j} x(t), \\
 &\leq 2x^T(t) P_{1i} \dot{x}(t) + x^T(t) [P_2 + P_4] x(t) + x^T(t - \tau_1) (P_3 - P_2) x(t - \tau_1) \\
 &\quad - (1 - \mu_1) x^T(t - \tau(t)) P_3 x(t - \tau(t)) - (1 - \mu_2) x^T(t - d(t)) P_4 x(t - d(t)) \\
 &\quad + \sum_{j=1}^N \pi_{ij} x^T(t) P_{1j} x(t), \tag{18}
 \end{aligned}$$

$$\begin{aligned} \mathcal{L}V_2(t, x(t), i) &= \xi_1^T(t)Q\xi_1(t) - \xi_1^T\left(t - \frac{\tau_a}{N}\right)Q\xi_1\left(t - \frac{\tau_a}{N}\right) \\ &\quad + \xi_2^T(t)R\xi_2(t) - \xi_2^T\left(t - \frac{\tau_2}{N}\right)R\xi_2\left(t - \frac{\tau_2}{N}\right), \end{aligned} \tag{19}$$

$$\begin{aligned} \mathcal{L}V_3(t, x(t), i) &= \frac{\tau_a}{N}\dot{x}^T(t)S_1\dot{x}(t) + (\tau_2 - \tau_a)\dot{x}^T(t)S_2\dot{x}(t) \\ &\quad - \int_{t-\frac{\tau_a}{N}}^t \dot{x}^T(s)S_1\dot{x}(s) ds - \int_{t-\tau_2}^{t-\tau_a} \dot{x}^T(s)S_2\dot{x}(s) ds, \end{aligned} \tag{20}$$

$$\begin{aligned} \mathcal{L}V_4(t, x(t), i) &= \tau_2^2\dot{x}^T(t)S_3\dot{x}(t) - \tau_2 \int_{t-\tau_2}^t \dot{x}^T(s)S_3\dot{x}(s) ds + \tau_{12}^2\dot{x}^T(t)S_4\dot{x}(t) \\ &\quad - \tau_{12} \int_{t-\tau_2}^{t-\tau_1} \dot{x}^T(s)S_4\dot{x}(s) ds, \end{aligned} \tag{21}$$

$$\begin{aligned} \mathcal{L}V_5(t, x(t), i) &= f^T(x(t))[\tau_2^2S_5 + \tau_{12}^2S_6 + d^2S_7]f(x(t)) - \tau_2 \int_{t-\tau_2}^t f^T(x(s))S_5f(x(s)) ds \\ &\quad - \tau_{12} \int_{t-\tau_2}^{t-\tau_1} f^T(x(s))S_6f(x(s)) ds - d \int_{t-d(t)}^t f^T(x(s))S_7f(x(s)) ds, \end{aligned} \tag{22}$$

$$\begin{aligned} \mathcal{L}V_6(t, x(t), i) &= \frac{\tau_2^4}{4}\dot{x}^T(t)T_1\dot{x}(t) - \frac{\tau_2^2}{2} \int_{-\tau_2}^0 \int_{t+\theta}^t \dot{x}^T(s)T_1\dot{x}(s) ds d\theta \\ &\quad + \tau_s^2\dot{x}^T(t)T_2\dot{x}(t) - \tau_s \int_{-\tau_2}^{-\tau_1} \int_{t+\theta}^t \dot{x}^T(s)T_2\dot{x}(s) ds d\theta. \end{aligned} \tag{23}$$

Note that

$$- \int_{t-\tau_2}^{t-\tau_a} \dot{x}^T(s)S_2\dot{x}(s) ds = - \int_{t-\tau_2}^{t-\tau(t)} \dot{x}^T(s)S_2\dot{x}(s) ds - \int_{t-\tau(t)}^{t-\tau_a} \dot{x}^T(s)S_2\dot{x}(s) ds.$$

Using Lemma 2.1, we obtain

$$- \int_{t-\frac{\tau_a}{N}}^t \dot{x}^T(s)S_1\dot{x}(s) ds \leq \frac{\tau_a}{N}\zeta^T(t)LS_1^{-1}L^T\zeta(t) + 2\zeta^T(t)L\left[x(t) - x\left(t - \frac{\tau_a}{N}\right)\right], \tag{24}$$

$$\begin{aligned} - \int_{t-\tau(t)}^{t-\tau_a} \dot{x}^T(s)S_2\dot{x}(s) ds &\leq (\tau(t) - \tau_a)\zeta^T(t)VS_2^{-1}V^T\zeta(t) \\ &\quad + 2\zeta^T(t)V[x(t - \tau_a) - x(t - \tau(t))], \end{aligned} \tag{25}$$

$$\begin{aligned} - \int_{t-\tau_2}^{t-\tau(t)} \dot{x}^T(s)S_2\dot{x}(s) ds &\leq (\tau_2 - \tau(t))\zeta^T(t)MS_2^{-1}M^T\zeta(t) \\ &\quad + 2\zeta^T(t)M[x(t - \tau(t)) - x(t - \tau_2)]. \end{aligned} \tag{26}$$

Applying the lemma in [8] and the Newton–Leibniz formula

$$\int_{t-\tau_2}^t \dot{x}(s) ds = x(t) - x(t - \tau_2),$$

we have

$$\begin{aligned}
 -\tau_2 \int_{t-\tau_2}^t \dot{x}^T(s) S_3 \dot{x}(s) ds &\leq -\left[\int_{t-\tau_2}^t \dot{x}(s) ds \right]^T S_3 \left[\int_{t-\tau_2}^t \dot{x}(s) ds \right] \\
 &\leq -[x(t) - x(t - \tau_2)]^T S_3 [x(t) - x(t - \tau_2)].
 \end{aligned}
 \tag{27}$$

Note that

$$\int_{t-\tau_2}^{t-\tau_1} \dot{x}^T(s) S_4 \dot{x}(s) ds = \int_{t-\tau_2}^{t-\tau(t)} \dot{x}^T(s) S_4 \dot{x}(s) ds + \int_{t-\tau(t)}^{t-\tau_1} \dot{x}^T(s) S_4 \dot{x}(s) ds.$$

The lemma in [6] gives

$$\begin{aligned}
 [\tau_2 - \tau(t)] \int_{t-\tau_2}^{t-\tau(t)} \dot{x}^T(s) S_4 \dot{x}(s) ds &\geq \left[\int_{t-\tau_2}^{t-\tau(t)} \dot{x}(s) ds \right]^T S_4 \left[\int_{t-\tau_2}^{t-\tau(t)} \dot{x}(s) ds \right] \\
 &\geq [x(t - \tau(t)) - x(t - \tau_2)]^T S_4 [x(t - \tau(t)) - x(t - \tau_2)].
 \end{aligned}$$

Since $\tau_2 - \tau(t) \leq \tau_2 - \tau_1$, we have

$$[\tau_2 - \tau_1] \int_{t-\tau_2}^{t-\tau(t)} \dot{x}^T(s) S_4 \dot{x}(s) ds \geq [x(t - \tau(t)) - x(t - \tau_2)]^T S_4 [x(t - \tau(t)) - x(t - \tau_2)],$$

and thus

$$\begin{aligned}
 -[\tau_2 - \tau_1] \int_{t-\tau_2}^{t-\tau(t)} \dot{x}^T(s) S_4 \dot{x}(s) ds \\
 \leq -[x(t - \tau(t)) - x(t - \tau_2)]^T S_4 [x(t - \tau(t)) - x(t - \tau_2)].
 \end{aligned}
 \tag{28}$$

Similarly, we have

$$\begin{aligned}
 -[\tau_2 - \tau_1] \int_{t-\tau(t)}^{t-\tau_1} \dot{x}^T(s) S_4 \dot{x}(s) ds \\
 \leq -[x(t - \tau_1) - x(t - \tau(t))]^T S_4 [x(t - \tau_1) - x(t - \tau(t))]
 \end{aligned}
 \tag{29}$$

and

$$\begin{aligned}
 -\tau_2 \int_{t-\tau_2}^t f^T(x(s)) S_5 f(x(s)) ds &\leq -\left(\int_{t-\tau_2}^t f(x(s)) ds \right)^T \\
 &\quad \times S_5 \left(\int_{t-\tau_2}^t f(x(s)) ds \right),
 \end{aligned}
 \tag{30}$$

$$\begin{aligned}
 -\tau_{12} \int_{t-\tau_2}^{t-\tau_1} f^T(x(s)) S_6 f(x(s)) ds &\leq -\left(\int_{t-\tau_2}^{t-\tau_1} f(x(s)) ds \right)^T \\
 &\quad \times S_6 \left(\int_{t-\tau_2}^{t-\tau_1} f(x(s)) ds \right),
 \end{aligned}
 \tag{31}$$

$$\begin{aligned}
 -d \int_{t-d(t)}^t f^T(x(s)) S_7 f(x(s)) ds &\leq -\left(\int_{t-d(t)}^t f(x(s)) ds \right)^T \\
 &\quad \times S_7 \left(\int_{t-d(t)}^t f(x(s)) ds \right), \tag{32}
 \end{aligned}$$

$$\begin{aligned}
 -\frac{\tau_2^2}{2} \int_{-\tau_2}^0 \int_{t+\theta}^t \dot{x}^T(s) T_1 \dot{x}(s) ds d\theta &\leq -\left(\int_{-\tau_2}^0 \int_{t+\theta}^t \dot{x}(s) ds d\theta \right)^T \\
 &\quad \times T_1 \left(\int_{-\tau_2}^0 \int_{t+\theta}^t \dot{x}(s) ds d\theta \right) \\
 &\leq -\left(\tau_2 x(t) - \int_{t-\tau_2}^t x(s) ds \right)^T \\
 &\quad \times T_1 \left(\tau_2 x(t) - \int_{t-\tau_2}^t x(s) ds \right), \tag{33}
 \end{aligned}$$

$$\begin{aligned}
 -\tau_s \int_{-\tau_2}^{-\tau_1} \int_{t+\theta}^t \dot{x}^T(s) T_2 \dot{x}(s) ds d\theta &\leq -\left(\int_{-\tau_2}^{-\tau_1} \int_{t+\theta}^t \dot{x}(s) ds d\theta \right)^T \\
 &\quad \times T_2 \left(\int_{-\tau_2}^{-\tau_1} \int_{t+\theta}^t \dot{x}(s) ds d\theta \right) \\
 &\leq -\left(\tau_{12} x(t) - \int_{t-\tau_2}^{t-\tau_1} x(s) ds \right)^T \\
 &\quad \times T_2 \left(\tau_{12} x(t) - \int_{t-\tau_2}^{t-\tau_1} x(s) ds \right). \tag{34}
 \end{aligned}$$

For positive diagonal matrices U_1 and U_2 , it follows from Assumption (H1) that

$$0 \leq \begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix}^T \begin{bmatrix} -F_1 U_1 & F_2 U_1 \\ F_2 U_1 & -U_1 \end{bmatrix} \begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix}, \tag{35}$$

$$0 \leq \begin{bmatrix} x(t - \tau(t)) \\ f(x(t - \tau(t))) \end{bmatrix}^T \begin{bmatrix} -F_1 U_2 & F_2 U_2 \\ F_2 U_2 & -U_2 \end{bmatrix} \begin{bmatrix} x(t - \tau(t)) \\ f(x(t - \tau(t))) \end{bmatrix}. \tag{36}$$

On the other side, for any matrix O of appropriate dimensions, from system (7) we have

$$\begin{aligned}
 2\dot{x}^T(t) O \times &\left[-A_{ij} x(t) + W_{1ij} f(x(t)) + W_{2ij} f(x(t - \tau(t))) \right. \\
 &\left. + W_{3ij} \int_{t-d(t)}^t f(x(s)) ds + u(t) - \dot{x}(t) \right] = 0. \tag{37}
 \end{aligned}$$

Combining (18)–(37), we can obtain

$$\begin{aligned}
 \mathcal{L}V(t, x(t), i) &+ \vartheta u^T(t) u(t) - y^T(t) \mathcal{Q} y(t) - 2y^T(t) S u(t) - u^T(t) \mathcal{R} u(t) \\
 &\leq \zeta^T(t) \left\{ \Phi + \frac{\tau_a}{N} L S_1^{-1} L^T + (\tau_2 - \tau(t)) M S_2^{-1} M^T + (\tau(t) - \tau_a) V S_2^{-1} V^T \right\} \zeta(t). \tag{38}
 \end{aligned}$$

By the conditions of Theorem 3.1, if $\zeta(t) \neq 0$, then we have

$$\mathcal{L}V(t, x(t), i) < 0. \tag{39}$$

For $t \in [t_{k-1}, t_k]$, in view of (16) and (39), we have

$$V(t_k, x(t), j) < V(t_k^-, x(t), i) < V(t_{k-1}, x(t), i). \tag{40}$$

By a similar proof and mathematical induction we can ensure that (40) is true for all i, j , $r(0) = i_0 \in \mathcal{S}$, $k \in \mathbb{N}$:

$$V(t_k, x(t), j) < V(t_k^-, x(t), i) < V(t_k, x(t), i) < \dots < V(t_0, x(t), i_0). \tag{41}$$

It follows from (38) that

$$\begin{aligned} & \mathcal{L}V(t, x(t), i) + \vartheta u^T(t)u(t) - y^T(t)Qy(t) - 2y^T(t)Su(t) - u^T(t)Ru(t) \\ & \leq \zeta^T(t) \left\{ \Phi + \frac{\tau_a}{N}LS_1^{-1}L^T + (\tau_2 - \tau(t))MS_2^{-1}M^T + (\tau(t) - \tau_a)VS_2^{-1}V^T \right\} \zeta(t). \end{aligned} \tag{42}$$

Let

$$\Pi = \Phi + \frac{\tau_a}{N}LS_1^{-1}L^T + (\tau_2 - \tau(t))MS_2^{-1}M^T + (\tau(t) - \tau_a)VS_2^{-1}V^T. \tag{43}$$

Then, applying the lemma in [8] to (43), we obtain the following inequalities:

$$\Phi + \frac{\tau_a}{N}LS_1^{-1}L^T + (\tau_2 - \tau_a)MS_2^{-1}M^T < 0, \tag{44}$$

$$\Phi + \frac{\tau_a}{N}LS_1^{-1}L^T + (\tau_2 - \tau_a)VS_2^{-1}V^T < 0. \tag{45}$$

Using Schur complements on (44)–(45), we obtain the LMIs of Theorem 3.1. Since $\Phi_i < 0$, we easily get

$$y^T(t)Qy(t) + 2y^T(t)Su(t) + u^T(t)Ru(t) > \mathcal{L}V(t, x(t), i) + \vartheta u^T(t)u(t). \tag{46}$$

Integrating this inequality from 0 to T and using the zero initial conditions, we get

$$E(y, u, T) \geq \vartheta \langle u, u \rangle_T + V(T) - V(0) \geq \vartheta \langle u, u \rangle_T \tag{47}$$

for all $T \geq 0$. Hence, if condition (11) holds, then the proposed model (7) is $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$ - ϑ -dissipative in the sense of Definition 1. \square

Remark 1 The LKF $V_3(t, x(t), i)$ plays a important role in reducing the conservativity of time-varying delay system, whereas in the derivative of $\dot{V}_3(t, x(t), i)$, the cross terms $-\int_{t-\frac{\tau_a}{N}}^t \dot{x}^T(s)S_1\dot{x}(s) ds$, $-\int_{t-\tau(t)}^{t-\tau_a} \dot{x}^T(s)S_2\dot{x}(s) ds$, and $-\int_{t-\tau_2}^{t-\tau(t)} \dot{x}^T(s)S_2\dot{x}(s) ds$ are defined as follows:

$$\begin{aligned} & -\int_{t-\frac{\tau_a}{N}}^t \dot{x}^T(s)S_1\dot{x}(s) ds \leq \frac{\tau_a}{N}\zeta^T(t)LS_1^{-1}L^T\zeta(t) + 2\zeta^T(t)L \left[x(t) - x\left(t - \frac{\tau_a}{N}\right) \right], \\ & -\int_{t-\tau(t)}^{t-\tau_a} \dot{x}^T(s)S_2\dot{x}(s) ds \\ & \leq (\tau(t) - \tau_a)\zeta^T(t)VS_2^{-1}V^T\zeta(t) + 2\zeta^T(t)V[x(t - \tau_a) - x(t - \tau(t))], \end{aligned}$$

$$\begin{aligned}
 & - \int_{t-\tau_2}^{t-\tau(t)} \dot{x}^T(s) S_2 \dot{x}(s) ds \\
 & \leq (\tau_2 - \tau(t)) \zeta^T(t) M S_2^{-1} M^T \zeta(t) + 2 \zeta^T(t) M [x(t - \tau(t)) - x(t - \tau_2)].
 \end{aligned}$$

Finally, to reduce the conservatism of the constructed dissipativity conditions, the convexity of the matrix function for cross term is applied. This treatment involved in our paper is different from the approaches used in [12, 35, 37, 41] and may ensure a better feasible region for dissipativity conditions. Thus, using a tighter bounding of the time derivative of LKF and a low number of slack variables, the considered dissipativity condition is less conservative than that in [12, 35, 37, 41].

Remark 2 Very recently, many researchers endeavor to focus on how to reduce conservatism of dissipativity condition for neural network delay systems. A free-matrix-based integral inequality technique is constructed by using a set of slack variables, which can be solved via convex optimization algorithms [37]. Therefore, some improved dissipativity criteria for delayed neural networks are investigated in [35, 41] using the LKF approach. In [12] the authors developed the Wirtinger double integral inequality, which was used to analyze the dissipativity behavior of continuous-time neural networks involving Markovian jumping parameters under Finsler’s lemma approach. Using a delay fractioning approach, the designed dissipativity condition is much less conservative than those in the existing works, and the derived results can ensure the dissipativity of the proposed delayed neural networks. Hence the delay-partitioning method is widely applied and exposes the potential of reducing the conservatism. However, to the best of authors’ knowledge, dissipativity analysis of fuzzy Markovian jumping neural network with discrete and distributed time varying delays and impulses has not been investigated yet, and it shows the effectiveness of our developed methods.

Remark 3 Consider the Markovian jumping neural network without fuzzy and impulsive effects of the following form:

$$\begin{aligned}
 \dot{x}(t) &= -A_i x(t) + W_{1i} f(x(t)) + W_{2i} f(x(t - \tau(t))) \\
 & \quad + W_{3i} \int_{t-d(t)}^t f(x(s)) ds + u(t), \quad t > 0, t \neq t_k, \\
 y(t) &= f(x(t)).
 \end{aligned} \tag{48}$$

Due to Theorem 3.1, we obtain a corollary for the dissipativity analysis of Markovian jumping neural networks (48).

Corollary 3.2 *Under Assumption (H1) and given scalars τ_1, τ_2, d, μ_1 , and μ_2 , the neural network model (48) is strictly $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$ - ϑ -dissipative if there exist positive definite matrices P_{1i}, P_i ($i = 2, \dots, 4$), Q, R, S_i ($i = 1, 2, \dots, 7$), T_i ($i = 1, 2$), positive diagonal matrices U_1, U_2 , and matrices O, L_i, M_i, V_i ($i = 1, 2$) of appropriate dimensions such that the following LMIs hold:*

$$\begin{bmatrix} \Phi & \sqrt{\frac{\tau_a}{N}} L & \sqrt{\tau_\delta} M \\ * & -S_1 & 0 \\ * & * & -S_2 \end{bmatrix} < 0, \tag{49}$$

$$\begin{bmatrix} \Phi & \sqrt{\frac{\tau_a}{N}}L & \sqrt{\tau_\delta}V \\ * & -S_1 & 0 \\ * & * & -S_2 \end{bmatrix} < 0. \tag{50}$$

Proof The proof is similar to that of Theorem 3.1 and therefore is omitted. □

Remark 4 When Markovian jumping parameters are not taken, that is, the Markov chain $\{r(t), t \geq 0\}$ only takes a unique value 1 (i.e., $S = 1$), then system (48) becomes the following neural network model:

$$\begin{aligned} \dot{x}(t) &= -Ax(t) + W_1f(x(t)) + W_2f(x(t - \tau(t))) \\ &+ W_3 \int_{t-d(t)}^t f(x(s)) ds + u(t), \quad t > 0, t \neq t_k, \\ y(t) &= f(x(t)). \end{aligned} \tag{51}$$

For system (51), we obtain the following corollary by Theorem 3.1 and Corollary 3.2.

Corollary 3.3 *Based on Assumption (H1) and given scalars τ_1, τ_2, d, μ_1 , and μ_2 , the neural network (51) is strictly $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$ - ϑ -dissipative if there exist positive definite matrices P_1, P_i ($i = 2, \dots, 4$), Q, R, S_i ($i = 1, 2, \dots, 7$), T_i ($i = 1, 2$), positive diagonal matrices U_1, U_2 , and matrices O, L_i, M_i, V_i ($i = 1, 2$) of appropriate dimensions such that the following LMIs hold:*

$$\begin{bmatrix} \Phi & \sqrt{\frac{\tau_a}{N}}L & \sqrt{\tau_\delta}M \\ * & -S_1 & 0 \\ * & * & -S_2 \end{bmatrix} < 0, \tag{52}$$

$$\begin{bmatrix} \Phi & \sqrt{\frac{\tau_a}{N}}L & \sqrt{\tau_\delta}V \\ * & -S_1 & 0 \\ * & * & -S_2 \end{bmatrix} < 0, \tag{53}$$

and

$$Q = \begin{bmatrix} Q_{11} & Q_{12} & \cdots & Q_{1N} \\ * & Q_{22} & \cdots & Q_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ * & * & Q_{N-1,N-1} & Q_{N-1,N} \\ * & * & * & Q_{NN} \end{bmatrix} \geq 0,$$

$$R = \begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1N} \\ * & R_{22} & \cdots & R_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ * & * & R_{N-1,N-1} & R_{N-1,N} \\ * & * & * & R_{NN} \end{bmatrix} \geq 0,$$

where

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} & \Phi_{15} & 0 & \Phi_{17} & \Phi_{18} & 0 & 0 & 0 & 0 & 0 \\ * & \Phi_{22} & \Phi_{23} & 0 & 0 & F_2U_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \Phi_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Phi_{44} & OW_1 & OW_2 & 0 & 0 & 0 & 0 & OW_3 & 0 & 0 \\ * & * & * & * & \Phi_{55} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -S \\ * & * & * & * & * & -U_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -T_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -T_2 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -S_5 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -S_6 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & -S_7 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & * & -(1-\mu_2)P_4 & 0 \\ * & * & * & * & * & * & * & * & * & * & * & * & \gamma I - \mathcal{R} \end{bmatrix},$$

$$\Phi_{11} = \begin{bmatrix} \Phi_1 & L_2^T & Q_{12} - L_1 & Q_{13} & \dots & Q_{1N} & V_1 \\ * & P_3 - P_2 - S_4 & -L_2 & 0 & \dots & 0 & V_2 \\ * & * & Q_{22} - Q_{11} & Q_{23} - Q_{12} & \dots & Q_{2N} - Q_{1,N-1} & -Q_{1N} \\ * & * & * & Q_{33} - Q_{22} & \dots & Q_{3N} - Q_{2,N-1} & -Q_{2N} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & * & \dots & Q_{NN} - Q_{N-1,N-1} & -Q_{N-1,N} \\ * & * & * & * & \dots & * & -Q_{NN} \end{bmatrix},$$

$$\Phi_{33} = \begin{bmatrix} R_{22} - R_{11} & R_{23} - R_{12} & \dots & R_{2N} - R_{1,N-1} & -R_{1N} \\ * & R_{33} - R_{22} & \dots & R_{3N} - R_{2,N-1} & -R_{2N} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & \dots & R_{NN} - R_{N-1,N-1} & -R_{N-1,N} \\ * & * & \dots & * & -R_{NN} - S_3 - S_4 \end{bmatrix},$$

$$\Phi_{13} = \begin{bmatrix} R_{12} & R_{13} & \dots & R_{1N} & -M_1 + S_3 \\ 0 & 0 & \dots & 0 & -M_2 \\ 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 \end{bmatrix},$$

$$\Phi_1 = P_2 + P_4 + Q_{11} + L_1 + L_1^T + R_{11} - S_3 - \tau_2^2 T_1 + \tau_{12}^2 T_2 - F_1 U_1,$$

$$\Phi_{12} = \begin{bmatrix} (-V_1 + M_1)^T & (-V_2 + M_2 + S_4)^T & 0 & 0 & \dots & 0 & 0 \end{bmatrix}^T,$$

$$\Phi_{22} = -(1 - \mu_1)P_3 - 2S_4 - F_1 U_2,$$

$$\Phi_{23} = \begin{bmatrix} 0 & 0 & \dots & 0 & S_4 \end{bmatrix},$$

$$\Phi_{14} = \begin{bmatrix} (P_1 - A^T O^T)^T & 0 & 0 & 0 & \dots & 0 & 0 \end{bmatrix}^T,$$

$$\Phi_{44} = \frac{\tau_a}{N} S_1 + \tau_8 S_2 + \tau_2^2 S_3 + \tau_{12}^2 S_4 + \frac{\tau_2^4}{4} T_1 + \tau_s^2 T_2 - O - O^T,$$

$$\Phi_{15} = \begin{bmatrix} F_2 U_1 & 0 & 0 & 0 & \dots & 0 & 0 \end{bmatrix}^T,$$

$$\Phi_{55} = \tau_2^2 S_5 + \tau_{12}^2 S_6 + d^2 S_7 - U_1 - Q,$$

$$\Phi_{17} = \begin{bmatrix} \tau_2 T_1^T & 0 & 0 & 0 & \dots & 0 & 0 \end{bmatrix}^T,$$

$$\begin{aligned} \Phi_{18} &= \left[\tau_{12} T_2^T \quad 0 \quad 0 \quad 0 \quad \dots \quad 0 \quad 0 \right]^T, \\ L &= \left[L_1^T \quad L_2^T \quad 0 \quad \dots \quad 0 \quad 0 \quad \dots \quad 0 \quad 0 \quad 0 \quad \dots \quad 0 \quad 0 \quad 0 \right]^T, \\ M &= \left[M_1^T \quad M_2^T \quad 0 \quad \dots \quad 0 \quad 0 \quad \dots \quad 0 \quad 0 \quad 0 \quad \dots \quad 0 \quad 0 \quad 0 \right]^T, \\ V &= \left[V_1^T \quad V_2^T \quad 0 \quad \dots \quad 0 \quad 0 \quad \dots \quad 0 \quad 0 \quad 0 \quad \dots \quad 0 \quad 0 \quad 0 \right]^T, \\ \tau_a &= \frac{(\tau_1 + \tau_2)}{2}, \quad \tau_\delta = \frac{(\tau_2 - \tau_1)}{2}, \quad \tau_{12} = \tau_2 - \tau_1, \quad \tau_s = \frac{1}{2}(\tau_2^2 - \tau_1^2). \end{aligned}$$

Proof To prove the dissipativity criteria for the recurrent neural networks (51), we define the following Lyapunov–Krasovskii functional:

$$\begin{aligned} V(t, x(t)) &= V_1(t, x(t)) + V_2(t, x(t)) + V_3(t, x(t)) + V_4(t, x(t)) \\ &\quad + V_5(t, x(t)) + V_6(t, x(t)), \end{aligned} \tag{54}$$

where

$$\begin{aligned} V_1(t, x(t)) &= x^T(t)P_1x(t) + \int_{t-\tau_1}^t x^T(s)P_2x(s) ds + \int_{t-\tau(t)}^{t-\tau_1} x^T(s)P_3x(s) ds \\ &\quad + \int_{t-d(t)}^t x^T(s)P_4x(s) ds, \\ V_2(t, x(t)) &= \int_{t-\frac{\tau_a}{N}}^t \xi_1^T(s)Q\xi_1(s) ds + \int_{t-\frac{\tau_a}{N}}^t \xi_2^T(s)R\xi_2(s) ds, \\ V_3(t, x(t)) &= \int_{-\frac{\tau_a}{N}}^0 \int_{t+\theta}^t \dot{x}^T(s)S_1\dot{x}(s) ds d\theta + \int_{-\tau_2}^{-\tau_a} \int_{t+\theta}^t \dot{x}^T(s)S_2\dot{x}(s) ds d\theta, \\ V_4(t, x(t)) &= \tau_2 \int_{-\tau_2}^0 \int_{t+\theta}^t \dot{x}^T(s)S_3\dot{x}(s) ds d\theta + \tau_{12} \int_{-\tau_2}^{-\tau_1} \int_{t+\theta}^t \dot{x}^T(s)S_4\dot{x}(s) ds d\theta, \\ V_5(t, x(t)) &= \tau_2 \int_{-\tau_2}^0 \int_{t+\theta}^t f^T(x(s))S_5f(x(s)) ds d\theta + \tau_{12} \int_{-\tau_2}^{-\tau_1} \int_{t+\theta}^t f^T(x(s))S_6f(x(s)) ds d\theta \\ &\quad + d \int_{-d}^0 \int_{t+\theta}^t f^T(x(s))S_7f(x(s)) ds d\theta, \\ V_6(t, x(t)) &= \frac{\tau_2^2}{2} \int_{-\tau_2}^0 \int_{\theta}^0 \int_{t+\lambda}^t \dot{x}^T(s)T_1\dot{x}(s) ds d\lambda d\theta \\ &\quad + \tau_s \int_{-\tau_2}^{-\tau_1} \int_{\theta}^0 \int_{t+\lambda}^t \dot{x}^T(s)T_2\dot{x}(s) ds d\lambda d\theta. \end{aligned}$$

Then, using the same proof as in Theorem 3.1, we get the result. □

Remark 5 If the distributed delay is not considered in system (51), then the recurrent neural network is rewritten as

$$\begin{aligned} \dot{x}(t) &= -Ax(t) + W_1f(x(t)) + W_2f(x(t - \tau(t))) + u(t), \quad t > 0, t \neq t_k, \\ y(t) &= f(x(t)). \end{aligned} \tag{55}$$

The dissipative condition of delayed neural network (55) is constructed as follows.

Corollary 3.4 *Under Assumption (H1) and given scalars τ_1, τ_2 , and μ_1 , the neural network (55) is (Q, S, \mathcal{R}) - ϑ -dissipative if there exist positive definite matrices P_1, P_i ($i = 2, \dots, 3$), Q, R, S_i ($i = 1, 2, \dots, 6$), T_i ($i = 1, 2$), positive diagonal matrices U_1, U_2 , and matrices O, L_i, M_i, V_i ($i = 1, 2$) of appropriate dimensions such that the following LMIs hold:*

$$\begin{bmatrix} \Phi & \sqrt{\frac{\tau_a}{N}}L & \sqrt{\tau_\delta}M \\ * & -S_1 & 0 \\ * & * & -S_2 \end{bmatrix} < 0, \tag{56}$$

$$\begin{bmatrix} \Phi & \sqrt{\frac{\tau_a}{N}}L & \sqrt{\tau_\delta}V \\ * & -S_1 & 0 \\ * & * & -S_2 \end{bmatrix} < 0, \tag{57}$$

and

$$Q = \begin{bmatrix} Q_{11} & Q_{12} & \cdots & Q_{1N} \\ * & Q_{22} & \cdots & Q_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ * & * & Q_{N-1,N-1} & Q_{N-1,N} \\ * & * & * & Q_{NN} \end{bmatrix} \geq 0,$$

$$R = \begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1N} \\ * & R_{22} & \cdots & R_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ * & * & R_{N-1,N-1} & R_{N-1,N} \\ * & * & * & R_{NN} \end{bmatrix} \geq 0,$$

where

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} & \Phi_{15} & 0 & \Phi_{17} & \Phi_{18} & 0 & 0 & 0 \\ * & \Phi_{22} & \Phi_{23} & 0 & 0 & F_2U_2 & 0 & 0 & 0 & 0 & 0 \\ * & * & \Phi_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Phi_{44} & OW_1 & OW_2 & 0 & 0 & 0 & 0 & O \\ * & * & * & * & \Phi_{55} & 0 & 0 & 0 & 0 & 0 & -S \\ * & * & * & * & * & -U_2 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -T_1 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -T_2 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -S_5 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -S_6 & 0 \\ * & * & * & * & * & * & * & * & * & * & \gamma I - \mathcal{R} \end{bmatrix},$$

$$\Phi_{11} = P_2 + Q_{11} + L_1 + L_1^T + R_{11} - S_3 - \tau_2^2 T_1 + \tau_{12}^2 T_2 - F_1 U_1,$$

$$\Phi_{55} = \tau_2^2 S_5 + \tau_{12}^2 S_6 - U_1 - Q,$$

and the other elements are as in Corollary 3.3.

$$\begin{aligned} \Phi_1 &= P_2 + Q_{11} + L_1 + L_1^T + R_{11} - S_3 - \tau_2^2 T_1 + \tau_{12}^2 T_2 - F_1 U_1, \\ \Phi_{55} &= \tau_2^2 S_5 + \tau_{12}^2 S_6 - U_1. \end{aligned}$$

Proof The proof directly follows from Corollary 3.4. □

4 Numerical examples

In this section, we analyze several numerical examples to exploit the effectiveness of the proposed methods.

Example 1 Consider the fuzzy impulsive neural network (7) with two-mode Markovian jumping process with the following parameters:

Mode 1:

$$\begin{aligned} A_{11} &= \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, & A_{21} &= \begin{bmatrix} 2.3 & 0 \\ 0 & 2.3 \end{bmatrix}, & W_{111} &= \begin{bmatrix} 0.40 & 0.12 \\ 0.22 & 0.72 \end{bmatrix}, \\ W_{121} &= \begin{bmatrix} 0.53 & 0.10 \\ 0.21 & 0.73 \end{bmatrix}, & W_{211} &= \begin{bmatrix} 0.6 & 0.5 \\ 0.2 & 0.1 \end{bmatrix}, & W_{221} &= \begin{bmatrix} 0.71 & 0.41 \\ 0.21 & 0.11 \end{bmatrix}, \\ W_{311} &= \begin{bmatrix} -0.3 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}, & W_{321} &= \begin{bmatrix} -0.31 & 0.11 \\ 0.11 & 0.12 \end{bmatrix}. \end{aligned}$$

Mode 2:

$$\begin{aligned} A_{12} &= \begin{bmatrix} 2.5 & 0 \\ 0 & 2.5 \end{bmatrix}, & A_{22} &= \begin{bmatrix} 2.15 & 0 \\ 0 & 2.15 \end{bmatrix}, & W_{112} &= \begin{bmatrix} 0.4 & 0.15 \\ 0.1 & 0.5 \end{bmatrix}, \\ W_{122} &= \begin{bmatrix} 0.48 & 0.17 \\ 0.12 & 0.3 \end{bmatrix}, & W_{212} &= \begin{bmatrix} 0.6 & 0.4 \\ 0.1 & 0.15 \end{bmatrix}, & W_{222} &= \begin{bmatrix} 0.65 & 0.35 \\ 0.12 & 0.14 \end{bmatrix}, \\ W_{312} &= \begin{bmatrix} -0.3 & 0.2 \\ 0.3 & 0.1 \end{bmatrix}, & W_{322} &= \begin{bmatrix} -0.15 & 0.21 \\ 0.15 & 0.12 \end{bmatrix}. \end{aligned}$$

We choose

$$Q = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}, \quad S = \begin{bmatrix} 0.3 & 0 \\ 0.2 & 1 \end{bmatrix}, \quad R = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}, \quad I_k = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix}.$$

We consider the activation functions $f_1(x) = f_2(x) = \tanh(x)$. Assumption (H1) is satisfied with $F_1^- = 0, F_1^+ = 1, F_2^- = 0,$ and $F_2^+ = 1$. Thus

$$F_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad F_2 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}.$$

Let $r(t)$ be a right-continuous Markov chain taking values in $S = \{1, 2\}$ with generator $\pi = \begin{bmatrix} -7 & 7 \\ 5 & -5 \end{bmatrix}$, and let the membership functions for rules 1 and 2 be $\mu_1(\theta(t)) = \sin^2(x_1 - 0.5)$ and $\mu_2(\theta(t)) = \cos^2(x_1 - 0.5)$. Then via the Matlab LMI control toolbox, for $N = 2$, we can see that the LMIs given in Theorem 3.1 are feasible. Thus we observe from Theorem 3.1 that the neural network (7) subject to leakage delays and impulsive effect is dissipative. The simulation results for the state responses of system (7) with two Markovian jumping

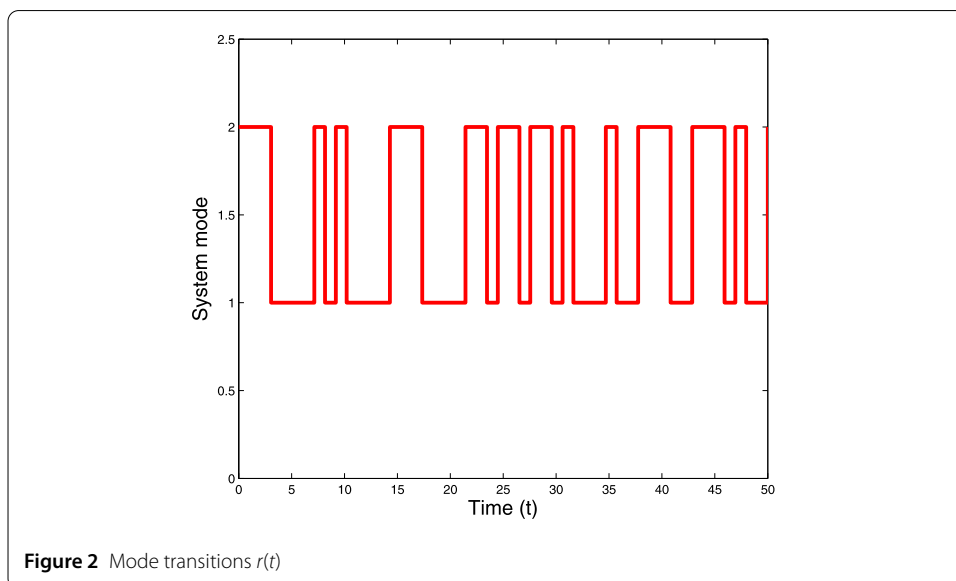
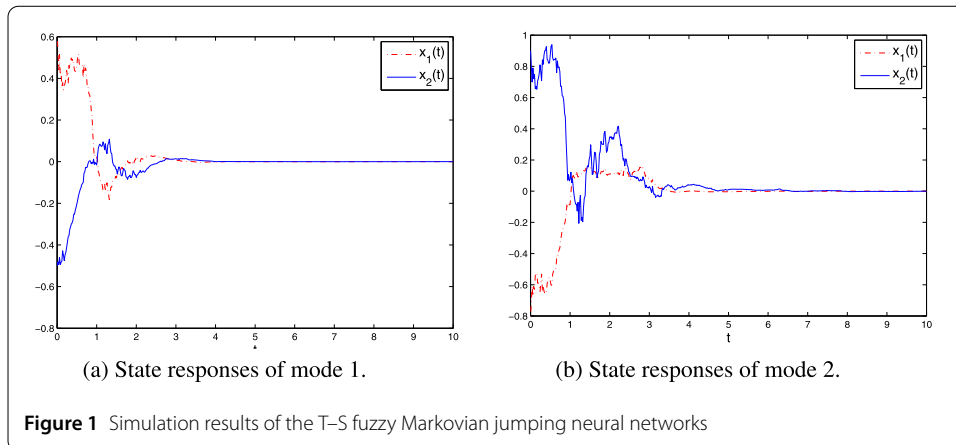


Table 1 Maximum upper bound for delays $\tau_2 = d$ with different μ ($\mu_1 = \mu_2$)

	Theorem 3.1					
	$\mu = 0$	$\mu = 0.25$	$\mu = 0.5$	$\mu = 0.75$	$\mu = 0.8$	$\mu = 0.9$
$\tau_1 = 0.1$	0.4864	0.4531	0.4091	0.3336	0.3117	0.2363
$\tau_1 = 0.2$	0.4931	0.4552	0.4298	0.3534	0.3312	0.2823
$\tau_1 = 0.3$	0.5013	0.4685	0.4362	0.3827	0.3564	0.3495
$\tau_1 = 0.4$	0.5052	0.4852	0.4431	0.4316	0.4206	0.4062
$\tau_1 = 0.5$	0.5074	0.5044	0.4835	0.4416	0.4301	0.4189

modes ($i = 1, 2$) are given in Fig. 1. Also, Fig. 2 illustrates the mode transition rates. In Table 1, we mention the maximum allowable upper bound for delays $\tau_2 = d$ with different values of μ_1, μ_2 .

Example 2 Consider the Markovian jumping neural network (48) with parameters

$$A_1 = \begin{bmatrix} 2.1 & 0 \\ 0 & 2.3 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 2.2 & 0 \\ 0 & 2.3 \end{bmatrix}, \quad W_{11} = \begin{bmatrix} 0.3 & 0.6 \\ 0.2 & 0.4 \end{bmatrix},$$

Table 2 Maximum upper bound of $\tau_2 = d$ when $\mu_1 = 0.5, \mu_2 = 0.1$

τ_1 :	0	0.3	0.5	0.8	0.9	1.5
	0.9044	0.8422	0.7512	0.7468	0.7215	0.7136

Table 3 Allowable upper bound of τ_2 and d for different τ_1 and μ ($\mu_1 = \mu_2$)

τ_1	$\mu = 0.1$	$\mu = 0.3$	$\mu = 0.5$	$\mu = 0.7$	$\mu = 0.9$
$\tau_1 = 0$	$\tau_2 = 0.5806$ $d = 0.5264$	$\tau_2 = 0.5369$ $d = 0.5036$	$\tau_2 = 0.5324$ $d = 0.5031$	$\tau_2 = 0.5296$ $d = 0.5028$	$\tau_2 = 0.5294$ $d = 0.5025$
$\tau_1 = 0.2$	$\tau_2 = 0.6049$ $d = 0.6588$	$\tau_2 = 0.5687$ $d = 0.6459$	$\tau_2 = 0.5538$ $d = 0.6426$	$\tau_2 = 0.5525$ $d = 0.6415$	$\tau_2 = 0.5506$ $d = 0.6412$
$\tau_1 = 0.4$	$\tau_2 = 0.7834$ $d = 0.6744$	$\tau_2 = 0.7426$ $d = 0.6703$	$\tau_2 = 0.7354$ $d = 0.6693$	$\tau_2 = 0.7328$ $d = 0.6685$	$\tau_2 = 0.7321$ $d = 0.6680$

$$\begin{aligned}
 W_{12} &= \begin{bmatrix} 0.3 & -0.1 \\ 0.2 & 0.5 \end{bmatrix}, & W_{21} &= \begin{bmatrix} 0.2 & 0.7 \\ 0.4 & 0.3 \end{bmatrix}, & W_{22} &= \begin{bmatrix} 0.1 & 0.8 \\ -0.6 & 1.1 \end{bmatrix}, \\
 W_{31} &= \begin{bmatrix} 0.4 & -0.4 \\ 0.2 & 0.6 \end{bmatrix}, & W_{32} &= \begin{bmatrix} 0.3 & 0.2 \\ -0.5 & 0.4 \end{bmatrix}, & Q &= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \\
 S &= \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, & R &= \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, & \pi &= \begin{bmatrix} -2 & 2 \\ 3 & -3 \end{bmatrix},
 \end{aligned}$$

and the activation function $f_1(x) = f_2(x) = \tanh(x)$. Choosing $N = 2, \mu_1 = 0.5,$ and $\mu_2 = 0.1$ and using Matlab LMI toolbox and Corollary 3.2, we obtained the maximum allowable upper bound of τ_2 and d for various values of τ_1 listed in Table 2. This implies that the Markovian jumping neural network (48) is dissipative in the sense of Definition 1.

Example 3 Consider the neural network (51) with the following parameters:

$$\begin{aligned}
 A &= \begin{bmatrix} 4 & 0 \\ 0 & 3 \end{bmatrix}, & W_1 &= \begin{bmatrix} 3.2 & -0.4 \\ -4 & 3.6 \end{bmatrix}, & W_2 &= \begin{bmatrix} -2.2 & -1.2 \\ -1.2 & -4 \end{bmatrix}, \\
 W_3 &= \begin{bmatrix} 1 & -0.4 \\ 0.6 & -1 \end{bmatrix}, & Q &= \begin{bmatrix} -0.1 & 0 \\ 0 & -0.3 \end{bmatrix}, & S &= \begin{bmatrix} 5 & 0 \\ 0 & 6 \end{bmatrix}, \\
 R &= \begin{bmatrix} 0.6 & 0.4 \\ 2.1 & -0.3 \end{bmatrix}, & f_1(x) &= f_2(x) = \tanh(x).
 \end{aligned}$$

For this neural network, we would like to have the dissipativity for the allowable maximum time delay value of τ_2 and d for different values of μ and given τ_1 . We can see from Table 3 that the condition presented in Corollary 3.3 still ensures the dissipativity of this model. The results in [12, 35, 37, 41] are not applicable to this system as the time-varying distributed delay is involved in this system.

Example 4 Consider the neural networks (55) with the following coefficient matrices:

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 1.5 \end{bmatrix}, \quad W_1 = \begin{bmatrix} 1.2 & 1 \\ -0.2 & 0.3 \end{bmatrix}, \quad W_2 = \begin{bmatrix} 0.8 & 0.4 \\ -0.2 & 0.1 \end{bmatrix},$$

Table 4 Optimal dissipativity performance γ for different μ_1

	μ_1				
	0.2	0.4	0.6	0.8	1
[35]	1.6740	1.5871	1.4500	1.2802	1.2461
[41]	1.7186	1.7183	1.7180	1.7176	1.7174
[37]	1.8570	1.8566	1.8561	1.8559	1.8558
[12]	1.9281	1.9138	1.9065	1.8759	1.8613
Corollary 3.4 [$N = 2$]	2.2436	2.2374	2.2269	2.1865	2.1754

Table 5 Maximum upper bound of τ_2 for different values of μ_1 (Example 5)

Methods	$\mu_1 = 0$	$\mu_1 = 0.5$	$\mu_1 = 0.7$
[39]	0.4528	0.3638	0.3575
[36]	0.6837	0.5285	0.4377
[42]	0.7340	0.6834	0.6355
[25]	0.8541	0.7438	0.6821
Corollary 3.5 [$N = 2$]	1.1284	1.0236	0.9532

$$Q = \begin{bmatrix} -0.9 & 0 \\ 0 & -0.9 \end{bmatrix}, \quad S = \begin{bmatrix} 0.5 & 0 \\ 0.3 & 1 \end{bmatrix}, \quad R = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}.$$

Here we choose $F_1^- = -0.1, F_1^+ = 0.9, F_2^- = -0.1,$ and $F_2^+ = 0.9.$ Thus

$$F_1 = \begin{bmatrix} -0.9 & 0 \\ 0 & -0.9 \end{bmatrix}, \quad F_2 = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.4 \end{bmatrix}.$$

We assume that $\tau_1 = 0$ and $\tau_2 = 0.4$ for different values of $\mu_1.$ The optimum dissipativity performances γ are calculated by the methods in Corollary 3.4 and are listed in Table 4. We can observe that our considered dissipativity condition provides a less conservative result in comparison to the existing works [12, 35, 37, 41].

Example 5 Consider the neural networks (55) with the following parameters:

$$A = \begin{bmatrix} 1.4 & 0 \\ 0 & 1.5 \end{bmatrix}, \quad W_1 = \begin{bmatrix} 1.2 & 1 \\ -1.2 & 1.3 \end{bmatrix}, \quad W_2 = \begin{bmatrix} -0.2 & 0.5 \\ 0.3 & -0.8 \end{bmatrix}.$$

Moreover, the activation function is chosen as $g_i(x_i) = 0.5(|x_i + 1| - |x_i - 1|), i = 1, 2.$ The allowable upper bounds of τ_2 when $\tau_1 = 0$ for various values of μ_1 obtained by Corollary 3.5 are listed in Table 5. We easily see that the obtained passivity-based results in our work are more general than the others [25, 36, 39, 42].

Example 6 Consider the neural networks (55), with the following parameters:

$$A = \begin{bmatrix} 2.2 & 0 \\ 0 & 1.8 \end{bmatrix}, \quad W_1 = \begin{bmatrix} 1.2 & 1 \\ -0.2 & 0.3 \end{bmatrix}, \quad W_2 = \begin{bmatrix} 0.8 & 0.4 \\ -0.2 & 0.1 \end{bmatrix},$$

$F_1^- = F_2^- = 0,$ and $F_1^+ = F_2^+ = 1.$ For different values of $\mu_1,$ the allowable upper bounds of τ_2 when $\tau_1 = 0$ computed by Corollary 3.5 and with the results presented in [16, 35, 37, 38, 40, 41] are listed in Table 6.

Table 6 Maximum upper bound of τ_2 for different values of μ_1 (Example 6)

	μ_1		
	0.5	0.9	≥ 1
[38]	1.4693	1.4243	1.4240
[35]	1.8450	1.7647	1.7313
[41]	2.2058	2.0366	2.000
[40]	3.0430	2.8428	2.8036
[37]	3.2019	3.0620	3.0612
[16]	3.4092	3.1568	3.1243
Corollary 3.5 [N = 2]	3.8564	3.5586	3.5024

Table 7 Maximum upper bound of τ_2 for different values of μ_1 (Example 7)

Methods	$\mu_1 = 0.4$	$\mu_1 = 0.45$	$\mu_1 = 0.50$	$\mu_1 = 0.55$
[30]	3.9972	3.2760	3.0594	2.9814
[33]	4.3814	3.6008	3.3377	3.2350
[34]	5.2420	4.4301	4.1055	3.9231
[28]	5.4036	4.6017	4.3121	4.1582
[13]	6.1305	5.8231	5.6357	5.3208
Corollary 3.5 [N = 2]	8.6421	7.7543	7.4216	7.1754

Example 7 Consider the neural networks (55) with the following parameters studied in [13, 28, 30, 33, 34].

$$A = \begin{bmatrix} 1.5 & 0 \\ 0 & 0.7 \end{bmatrix}, \quad W_1 = \begin{bmatrix} 0.0503 & 0.0454 \\ 0.0987 & 0.2075 \end{bmatrix}, \quad W_2 = \begin{bmatrix} 0.2381 & 0.9320 \\ 0.0388 & 0.5062 \end{bmatrix},$$

$f_1(x) = 0.3 \tanh(x)$, $f_2(x) = 0.8 \tanh(x)$, $F_1^- = F_2^- = 0$, $F_1^+ = 0.3$, and $F_2^+ = 0.8$. By using Corollary 3.5 and solving MATLAB LMI tool box the corresponding results for the maximum allowable upper bounds of τ_2 for different values of μ_1 when $\tau_1 = 0$ are computed and listed in Table 7. We can observe from Table 7 that the passivity condition proposed in this paper provides less conservative results than the others [13, 28, 30, 33, 34].

5 Conclusion

In this paper, we have studied the problem of dissipative conditions for Takagi–Sugeno fuzzy Markovian jumping neural networks with impulsive perturbations using the delay partition method. By constructing a proper LKF and LMI approach together with delay fractioning approach, we have established a set of sufficient conditions ensuring that the considered fuzzy Markovian neural networks are $(\mathcal{Q}, \mathcal{S}, \mathcal{R})-\vartheta$ -dissipative. Finally, several numerical examples are given to illustrate the effectiveness of the proposed dissipative theory. Moreover, our results show that the developed method yields less conservative results than some other works. Furthermore, the problem of finite-time extended dissipative conditions for stochastic T–S fuzzy singular Markovian jump systems with randomly occurring uncertainties and time delay using the delay portioning approach is an untreated work and will be the topic of our future work.

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Availability of data and materials

Data sharing not applicable to this paper as no data sets were generated or analyzed during the current study.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All three authors contributed equally to this work. They all read and approved the final version of the manuscript.

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