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Function matrix projective synchronization of non-dissipatively coupled heterogeneous systems with different-dimensional nodes

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Abstract

This paper regards function matrix projective synchronization of two different non-dissipatively coupled complex dynamical networks for different dimensions and different nodes. In this kind of complex dynamical networks the internal delays are different from the coupled delays. By using Lyapunov stability theory, using mathematical induction, two different hybrid feedback controllers are built to realize the function matrix projective synchronization. Compared with the existing results, the coupling matrices do not need to be symmetric or diffuse. By giving a numerical simulation we explain the validity and appropriateness of our conclusion.

Keywords: Function matrix projective synchronization; Complex networks; Hybrid feedback control; Time-varying delays

1 Introduction

Lots of large systems in the real world, for example, biological neural networks, social networks, food chains and food webs, can be depicted by complex networks. The complex dynamical networks are composed of coupled nodes, in which all the nodes form the edge-connected nonlinear dynamic system. In recent decades, the research of complex networks (CNs) has drawn wide attention from scholars of various fields [1–6]. Especially, synchronization as an important dynamic property of the coupled nonlinear systems has been extensively studied in [7–11].

In recent years, a novel synchronization, called function projective synchronization (FPS), was proposed and studied [12–15]. FPS is the coupled drive and the response systems could be synchronized to the scale function matrix. Therefore, FPS is a generalized synchronization of chaotic systems. Obviously, FPS includes complete synchronization, anti-synchronization and projective synchronization. If the scale function matrix is a unit matrix or a constant, we can obtain projective synchronization, complete synchronization or anti-synchronization. In the function matrix projective synchronization (FMPS) approach, the uncertainty of the scale function matrix can improve the security of communication [16, 17], therefore, FMS aroused wide interest of scholars.

For example, based on adaptive control, the function projective synchronization for a kind of chaotic system was considered in [18]. The function projective synchronization of complex networks by a hybrid feedback controller was proposed in [19]. Additionally, further consequences as regards the generalized matrix projective synchronization for general complex networks were studied in [20]. Even though synchronization of the complex networks for the same and different dynamic systems has been widely researched, the premise is that the dynamic dimensions of the nodes are the same. Actually, most systems are based on nonlinear dynamics, so the interactions between them may be completely different. On the other hand, during the information transmission process, because of the spatial and temporal characteristics of CNs, the time delays present in a single system and the coupled delays between the nodes may differ at different times.

Inspired by the discussion of the above issues, by constructing a Lyapunov function, applying mathematical induction, and using the matrix theory to study the complex networks for nonidentical nodes and different dimensions, we realize the function matrix projective synchronization by the hybrid feedback controller. It is worth noting that the coupling matrix is not required to obey symmetry or diffusion conditions. At last, a numerical simulation is given to explain the validity and appropriateness of our conclusion.

Notation: In this paper, $|\cdot|$ respects the absolute value. I_n denotes the n-dimensional unit matrix. $\Re^n$ is n-dimensional Euclidean space, $\Re^{n\times n}$ respects the set of the $n \times n$ real matrices. $A > B$ ($A \geq B$) denotes the matrix $A - B$ is positive definite (nonnegative). $\operatorname{diag}(\cdots)$ means the block diagonal matrix. $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ are the minimum eigenvalue of A and the maximum eigenvalue of A, respectively. The symmetric terms in a symmetric matrix are respected by $*$. $\|\cdot\|$ respects the Euclidean norm. If not clearly stated, all the matrices of this paper are assumed to have compatible dimensions.

2 Preliminaries

A typical complex dynamical networks consisting of N dynamical nodes with different intrinsic and coupled delays is described by

$$\dot{x}_i(t) = f_i\bigl(x_i(t), x_i\bigl(t-\rho(t)\bigr)\bigr) + \varepsilon \sum_{j=1}^{N} c_{ij} Q_{ij} x_j\bigl(t-\tau(t)\bigr), \tag{1}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in_i}(t))^T \in \Re^{n_i}$ stands for the state vector of the ith node, $i = 1, 2, \ldots, N$. $f_i(\cdot,\cdot) \in \Re^{n_i}$ is the vector-valued function. $\varepsilon > 0$ denotes the coupling strength, $\rho(t)$ and $\tau(t)$ are the intrinsic time-varying delay and the coupled time-varying delay, respectively. $Q_{ij} \in \Re^{n_i\times n_j}$ stands for the inner coupling matrix, $j = 1, 2, \ldots, N$. $C = (c_{ij})_{N\times N}$ represents the outer coupling matrix which satisfies $c_{ij} \neq 0$, if there exists a connection from the ith to the jth node ($i \neq j$); otherwise $c_{ij} = 0$.

Considering (1) as the drive system, then the response system under the controller can be written as follows:

$$\dot{y}_i(t) = g_i\bigl(y_i(t), y_i\bigl(t-\rho(t)\bigr)\bigr) + \varepsilon \sum_{j=1}^{N} d_{ij} G_{ij} y_j\bigl(t-\tau(t)\bigr) + u_i(t), \tag{2}$$

where $y_i(t) = (y_{i1}(t), y_{i2}(t), \ldots, y_{im_i}(t))^T \in \Re^{m_i}$ stands for the state vector of the ith node, $i = 1, 2, \ldots, N$. $g_i(\cdot) \in \Re^{m_i}$ is a vector-valued function. $\varepsilon > 0$ represents the coupling strength.

$u_i(t)$ is the controller. $G_{ij} \in \Re^{m_i \times m_j}$ stands for the inner coupling matrix, $j = 1, 2, \ldots, N$. $D = (d_{ij})_{N \times N}$ denotes the outer coupling matrix which satisfies $d_{ij} \neq 0$, if there exists a connection from the jth node to the ith node ($i \neq j$); otherwise $d_{ij} = 0$.

Definition 2.1 The function matrix projective synchronization between the drive system (1) and the response system (2) is realized, if there is a matrix $M_i(t) \in \Re^{m_i \times n_i}$ which is a continuously differentiable scaling function such that

$$\lim_{t \to \infty} \left\| y_i(t) - M_i(t) x_i(t) \right\| = 0, \quad i = 1, 2, \ldots, N.$$

Remark 2.1 By using different special expressions for the scaling function matrix, the function matrix projective synchronization will be transformed into the complete synchronization, the anti-synchronization, the hybrid synchronization and the projective synchronization.

Definition 2.2 If for a continuously differentiable scaling function matrix $M_i(t) \in \Re^{m_i \times n_i}$, such that $\|y_i(t) - M_i(t)x_i(t)\| \leq Me^{-\alpha t}$, then the exponentially FMPS between the drive system (1) and the response system (2) is called an implemented function matrix projective synchronization with the exponential rate α, where $M, \alpha \in \Re^{+}$.

Assumption 2.1 For different time-varying delays, $\rho(t)$ and $\tau(t)$ are differential expressions and satisfy $0 \leq \rho(t) \leq \rho$, $0 \leq \tau_i(t) \leq \tau_i$, $\dot{\rho}(t) \leq \tilde{\rho} < 1$ and $\dot{\tau}(t) \leq \mu < 1$, where ρ and τ_i are two constants.

Assumption 2.2 For the function $\sigma(\cdot, \cdot) \in \Re^n$ is called to satisfy the QUAD condition, that is, to say $\sigma \in \mathrm{QUAD}(L, \Delta)$, if there are two diagonal matrices $L \geq 0$ and $\Delta \geq 0$ hold on

$$(x - y)^T \bigl(\sigma(x, \tilde{x}) - \sigma(y, \tilde{y})\bigr) \leq (x - y)^T L(x - y) + (\tilde{x} - \tilde{y})^T \Delta(\tilde{x} - \tilde{y}),$$

for $\forall x, y, \tilde{x}, \tilde{y} \in \Re^n$.

Lemma 2.1 *For $\forall x, y \in \Re^n$ and a matrix $R > 0$, we have $2x^T y \leq x^T R x + y^T R^{-1} y$.*

3 FMPS of non-dissipatively coupled CNs via linear and nonlinear feedback control

In the following, we will give a linear and nonlinear feedback control methods to realize the FMPS.

The error state vector is

$$e_i(t) = y_i(t) - M_i(t) x_i(t), \tag{3}$$

where $M_i(t) \in \Re^{m_i \times n_i}$ is the time-varying scaling matrix.

Then, from (1)–(3), the error system (EDS) would be deduced

$$\begin{aligned}\dot{e}_i(t) &= \dot{y}_i(t) - \dot{M}_i(t)x_i(t) - M_i(t)\dot{x}_i(t)\\ &= g_i\big(y_i(t), y_i\big(t-\rho(t)\big)\big) + \varepsilon\sum_{j=1}^{N} d_{ij}G_{ij}y_j\big(t-\tau(t)\big)\\ &\quad + u_i(t) - \dot{M}_i(t)x_i(t) - M_i(t)\dot{x}_i(t),\\ &\quad i = 1,2,\dots,N.\end{aligned} \tag{4}$$

Next, the hybrid controller is considered as

$$u_i(t) = u_i^o(t) + u_i^c(t), \quad i = 1,2,\dots,N, \tag{5}$$

where

$$\begin{aligned}u_i^o(t) &= M_i(t)\dot{x}_i(t) + \dot{M}_i(t)x_i(t) - g_i\big(M_i(t)x_i(t), M_i\big(t-\rho(t)\big)x_i\big(t-\rho(t)\big)\big)\\ &\quad - \varepsilon\sum_{j=1}^{N} d_{ij}G_{ij}M_j\big(t-\tau(t)\big)x_j\big(t-\tau(t)\big),\\ u_i^c(t) &= -\beta_i e_i(t),\end{aligned}$$

and $\beta_i > 0$ is the feedback gain. $u_i^o(t)$ is the nonlinear controller, while $u_i^c(t)$ is the linear feedback controller.

Thus, the SDE can be rewritten by

$$\dot{e}_i(t) = \tilde{g}_i\big(e_i(t), e_i\big(t-\rho(t)\big)\big) + \varepsilon\sum_{j=1}^{N}\tilde{G}_{ij}\big(e_j\big(t-\tau(t)\big) - \beta_i e_i(t), \quad i = 1,2,\dots,N, \tag{6}$$

where $\tilde{g}_i(e_i(t), e_i(t-\rho(t))) = g_i(y_i(t), y_i(t-\rho(t))) - g_i(M_i(t)x_i(t), M_i(t-\rho(t))x_i(t-\rho(t)))$, $\tilde{G}_{ij} = d_{ij}G_{ij}$.

Theorem 3.1 *Under Assumptions 2.1–2.2, for the considered synchronization scaling function matrix $M_i(t) \in \Re^{m_i\times n_i}$, if*

$$\begin{aligned}&\frac{\varepsilon}{2}\lambda_{\max}\left(\sum_{j=1}^{N}\tilde{G}_{ij}\tilde{G}_{ij}^T\right) + (1-\tilde{\rho})^{-1}e^{\alpha\rho}\delta_{\max} + \ell_{\max}\\ &\quad + \frac{N\varepsilon}{2(1-\mu)}e^{\alpha\tau} + \frac{\alpha}{2} < \beta_i, \quad i = 1,2,\dots,N,\end{aligned} \tag{7}$$

then the drive system (1) and the response system (2) can realize FMPS by linear and nonlinear feedback control.

Proof Build a Lyapunov functional as follows:

$$V(e_t) = V_1(e_t) + V_2(e_t), \tag{8}$$

where

$$V_1(e_t) = \frac{1}{2}\sum_{i=1}^{N} e_i^T(t)e_i(t),$$

$$V_2(e_t) = \sum_{i=1}^{N}\int_{t-\rho(t)}^{t} e_i^T(s)e^{\alpha(s-t)}P_ie_i(s)\,ds + \sum_{i=1}^{N}\int_{t-\tau(t)}^{t} e_i^T(s)e^{\alpha(s-t)}R_ie_i(s)\,ds,$$

where $P_i > 0$ and $R_i > 0$ are the diagonal matrices which can be determined below.

The differential of $V(e_t)$ is obtained along the track of system (5), as follows:

$$\begin{aligned} \dot{V}_1(e_t) &= \sum_{i=1}^{N} e_i^T(t)\dot{e}_i(t) \\ &= \sum_{i=1}^{N} e_i^T(t)\tilde{g}_i\bigl(e_i(t), e_i\bigl(t-\rho(t)\bigr)\bigr) + \varepsilon\sum_{i=1}^{N}\sum_{j=1}^{N} e_i^T(t)\tilde{G}_{ij}e_j\bigl(t-\tau(t)\bigr) \\ &\quad - \sum_{i=1}^{N}\beta_ie_i^T(t)e_i(t), \end{aligned} \tag{9}$$

$$\begin{aligned} \dot{V}_2(e_t) &= -\alpha V_2(e_t) + \sum_{i=1}^{N}\bigl[e_i^T(t)P_ie_i(t) - \bigl(1-\dot{\rho}(t)\bigr)e^{-\alpha\rho(t)}e_i^T\bigl(t-\rho(t)\bigr)P_ie_i\bigl(t-\rho(t)\bigr)\bigr] \\ &\quad + \sum_{i=1}^{N}\bigl[e_i^T(t)R_ie_i(t) - \bigl(1-\dot{\tau}(t)\bigr)e^{-\alpha\tau(t)}e_i^T\bigl(t-\tau(t)\bigr)R_ie_i\bigl(t-\tau(t)\bigr)\bigr] \\ &\le \sum_{i=1}^{N} e_i^T(t)(P_i+R_i)e_i(t) - (1-\tilde{\rho})e^{-\alpha\rho}\sum_{i=1}^{N} e_i^T\bigl(t-\rho(t)\bigr)P_ie_i\bigl(t-\rho(t)\bigr) \\ &\quad - (1-\mu)e^{-\alpha\tau}\sum_{i=1}^{N} e_i^T\bigl(t-\tau(t)\bigr)R_ie_i\bigl(t-\tau(t)\bigr). \end{aligned} \tag{10}$$

It is worth noting $\tilde{g}_i(\cdot,\cdot)$ satisfies Assumption 2.2. Accordingly, there are matrices $L_i > 0$ and $\Delta_i > 0$, and we have

$$\begin{aligned} &e_i^T(t)\tilde{g}_i\bigl(e_i(t), e_i\bigl(t-\rho(t)\bigr)\bigr) \\ &\quad \le e_i^T(t)L_ie_i(t) + e_i^T\bigl(t-\rho(t)\bigr)\Delta_ie_i\bigl(t-\rho(t)\bigr), \end{aligned} \tag{11}$$

where $L_i = \operatorname{diag}(l_{i1}, l_{i2}, \ldots, l_{im_i})$, $\Delta_i = \operatorname{diag}(\delta_{i1}, \delta_{i2}, \ldots, \delta_{im_i})$, $i = 1, 2, \ldots, N$.

By Lemma 2.1, it is calculated that

$$\begin{aligned} &\varepsilon\sum_{i=1}^{N}\sum_{j=1}^{N} e_i^T(t)\tilde{G}_{ij}e_j\bigl(t-\tau(t)\bigr) \\ &\quad \le \frac{\varepsilon}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} e_i^T(t)\tilde{G}_{ij}\tilde{G}_{ij}^Te_i(t) + \frac{\varepsilon}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} e_j\bigl(t-\tau(t)\bigr)^Te_j\bigl(t-\tau(t)\bigr) \\ &\quad = \frac{\varepsilon}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} e_i^T(t)\tilde{G}_{ij}\tilde{G}_{ij}^Te_i(t) + \frac{1}{2}N\varepsilon\sum_{i=1}^{N} e_i^T\bigl(t-\tau(t)\bigr)e_i\bigl(t-\tau(t)\bigr). \end{aligned} \tag{12}$$

Then, according to (8)–(11), it follows that

$$\begin{aligned}
&\dot{V}(e_t)+\alpha V(e_t)\\
&\quad\le \frac{\alpha}{2}\sum_{i=1}^{N}e_i^T(t)e_i(t)+\sum_{i=1}^{N}e_i^T(t)L_ie_i(t)+\sum_{i=1}^{N}e_i^T\big(t-\rho(t)\big)\Delta_ie_i\big(t-\rho(t)\big)\\
&\qquad+\frac{\varepsilon}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}e_i^T(t)\tilde{G}_{ij}\tilde{G}_{ij}^Te_i(t)+\frac{1}{2}N\varepsilon\sum_{i=1}^{N}e_i^T\big(t-\tau(t)\big)e_i\big(t-\tau(t)\big)\\
&\qquad+\sum_{i=1}^{N}e_i^T(t)(P_i+R_i)e_i(t)-(1-\tilde{\rho})e^{-\alpha\rho}\sum_{i=1}^{N}e_i^T\big(t-\rho(t)\big)P_ie_i\big(t-\rho(t)\big)\\
&\qquad-(1-\mu)e^{-\alpha\tau}\sum_{i=1}^{N}e_i^T\big(t-\tau(t)\big)R_ie_i\big(t-\tau(t)\big)-\beta_i\sum_{i=1}^{N}e_i^T(t)e_i(t).
\end{aligned}\tag{13}$$

Let $P_i=(1-\tilde{\rho})^{-1}e^{\alpha\rho}\Delta_i$, $R_i=\frac{N\varepsilon}{2(1-\mu)}e^{\alpha\tau}I_{m_i}$, then

$$\begin{aligned}
\dot{V}(e_t)+\alpha V(e_t)&\le\sum_{i=1}^{N}e_i^T(t)\Bigg[L_i+\frac{\varepsilon}{2}\sum_{j=1}^{N}\tilde{G}_{ij}\tilde{G}_{ij}^T+(1-\tilde{\rho})^{-1}e^{\alpha\rho}\Delta_i+\frac{N\varepsilon}{2(1-\mu)}e^{\alpha\tau}I_{m_i}\\
&\quad+\left(\frac{\alpha}{2}-\beta_i\right)I_{m_i}\Bigg]e_i(t)\\
&\le\Bigg[\frac{\varepsilon}{2}\lambda_{\max}\Bigg(\sum_{j=1}^{N}\tilde{G}_{ij}\tilde{G}_{ij}^T\Bigg)+(1-\tilde{\rho})^{-1}e^{\alpha\rho}\delta_{\max}+\ell_{\max}+\frac{N\varepsilon}{2(1-\mu)}e^{\alpha\tau}\\
&\quad+\left(\frac{\alpha}{2}-\beta_i\right)\Bigg]\sum_{i=1}^{N}e_i^T(t)e_i(t),
\end{aligned}\tag{14}$$

where $\delta_{\max}=\max\{\delta_{ij}\}$, $\ell_{\max}=\max\{l_{ij}\}$, $i=1,2,\ldots,N$, $j=1,2,\ldots,m_i$. □

4 FMPS of non-dissipatively coupled CNs via nonlinear and adaptive feedback control

Next, we put forward a nonlinear and adaptive feedback control methods to achieve FMPS. The hybrid controller is considered as

$$u_i(t)=u_i^o(t)+u_i^c(t),\quad i=1,2,\ldots,N,\tag{15}$$

where

$$\begin{aligned}
u_i^o(t)&=M_i(t)\dot{x}_i(t)+\dot{M}_i(t)x_i(t)-g_i\big(M_i(t)x_i(t),M_i\big(t-\rho(t)\big)x_i\big(t-\rho(t)\big)\big)\\
&\quad-\varepsilon\sum_{j=1}^{N}d_{ij}G_{ij}M_j\big(t-\tau(t)\big)x_j\big(t-\tau(t)\big),\\
u_i^c(t)&=-\beta_i(t)e_i(t),\\
\dot{\beta}_i(t)&=k_ie_i^T(t)e_i(t),
\end{aligned}$$

and k_i is positive constant, $u_i^o(t)$ and $u_i^c(t)$ are the nonlinear controller and the adaptive feedback controller, respectively.

By a similar analysis to above, one has

$$\begin{aligned}\dot{e}_i(t) &= \dot{y}_i(t) - \dot{M}_i(t)x_i(t) - M_i(t)\dot{x}_i(t)\\ &= \tilde{g}_i\bigl(e_i(t), e_i\bigl(t-\rho(t)\bigr)\bigr) + \varepsilon\sum_{j=1}^{N}\tilde{G}_{ij}\bigl(e_j\bigl(t-\tau_j(t)\bigr) - e_i\bigl(t-\tau_i(t)\bigr)\bigr) - \beta_i(t)e_i(t),\\ &\quad i = 1,2,\ldots,N,\end{aligned} \tag{16}$$

where $\tilde{g}_i(e_i(t), e_i(t-\rho(t))) = g_i(y_i(t), y_i(t-\rho(t))) - g_i(M_i(t)x_i(t), M_i(t-\rho(t))x_i(t-\rho(t)))$, $\tilde{G}_{ij} = d_{ij}G_{ij}$.

Theorem 4.1 *Under the Assumptions 2.1–2.2, for the considered synchronization scaling function matrix $M_i(t) \in \Re^{m_i\times n_i}$, if there is a sufficiently large constant $\beta^* > 0$, satisfying*

$$\frac{\varepsilon}{2}\lambda_{\max}\Biggl(\sum_{j=1}^{N}\tilde{G}_{ij}\tilde{G}_{ij}^T\Biggr) + (1-\tilde{\rho})^{-1}\delta_{\max} + \ell_{\max} + \frac{N\varepsilon}{2(1-\mu)} < \beta^* \tag{17}$$

then the drive system (1) and the response system (2) can realize FMPS under nonlinear and adaptive feedback control.

Proof Build a Lyapunov functional as follows:

$$V(e_t) = V_1(e_t) + V_2(e_t), \tag{18}$$

where

$$\begin{aligned}V_1(e_t) &= \frac{1}{2}\sum_{i=1}^{N}e_i^T(t)e_i(t) + \frac{1}{2}\sum_{i=1}^{N}\frac{(\beta_i(t)-\beta^*)^2}{k_i},\\ V_2(e_t) &= \sum_{i=1}^{N}\int_{t-\rho(t)}^{t}e_i^T(s)P_ie_i(s)\,ds + \sum_{i=1}^{N}\int_{t-\tau(t)}^{t}e_i^T(s)R_ie_i(s)\,ds,\end{aligned}$$

where $\beta^* > 0$ is a constant. $P_i > 0$ and $R_i > 0$ are the diagonal matrices which can be determined as below.

The differential of $V(e_t)$ is obtained along the track of system (5), as follows:

$$\begin{aligned}\dot{V}_1(e_t) &= \sum_{i=1}^{N}e_i^T(t)\dot{e}_i(t) + \sum_{i=1}^{N}\bigl(\beta_i(t)-\beta^*\bigr)e_i^T(t)e_i(t)\\ &= \sum_{i=1}^{N}e_i^T(t)\tilde{g}_i\bigl(e_i(t), e_i\bigl(t-\rho(t)\bigr)\bigr) + \varepsilon\sum_{i=1}^{N}\sum_{j=1}^{N}e_i^T(t)\tilde{G}_{ij}e_j\bigl(t-\tau(t)\bigr)\\ &\quad - \sum_{i=1}^{N}\beta^*e_i^T(t)e_i(t).\end{aligned} \tag{19}$$

Then, similar to the proof of Theorem 3.1, and letting $P_i = (1-\tilde{\rho})^{-1}\Delta_i$, $R_i = \frac{N\varepsilon}{2(1-\mu)}I_{m_i}$, we get the following result:

$$\begin{aligned}\dot{V}(e_t) &\le \sum_{i=1}^{N} e_i^T(t)\left[L_i + \frac{\varepsilon}{2}\sum_{j=1}^{N}\tilde{G}_{ij}\tilde{G}_{ij}^T + (1-\tilde{\rho})^{-1}\Delta_i + \frac{N\varepsilon}{2(1-\mu)}I_{m_i} - \beta^* I_{m_i}\right]e_i(t)\\ &\le \left[\frac{\varepsilon}{2}\lambda_{\max}\left(\sum_{j=1}^{N}\tilde{G}_{ij}\tilde{G}_{ij}^T\right) + (1-\tilde{\rho})^{-1}\delta_{\max} + \ell_{\max}\right.\\ &\quad \left. + \frac{N\varepsilon}{2(1-\mu)} - \beta^*\right]\sum_{i=1}^{N} e_i^T(t)e_i(t),\end{aligned} \tag{20}$$

where $\delta_{\max} = \max\{\delta_{ij}\}$, $\ell_{\max} = \max\{l_{ij}\}$, $i = 1,2,\ldots,N$ and $j = 1,2,\ldots,m_i$.

In view of condition (16), one has $\dot{V}(e_t) \le 0$. Let $e(t) = (e_1^T(t), e_2^T(t),\ldots,e_N^T(t))^T$, $\tilde{\beta} = \beta^* - (\frac{\varepsilon}{2}\lambda_{\max}(\sum_{j=1}^{N}\tilde{G}_{ij}\tilde{G}_{ij}^T) + (1-\tilde{\rho})^{-1}\delta_{\max} + \ell_{\max} + \frac{N\varepsilon}{2(1-\mu)})$, since $V(e_t) > 0$, we have $\int_0^t \tilde{\beta}\|e(s)\|^2\,ds \le -\int_0^t \dot{V}(e_s)\,ds \le V(e_0) - V(e_t) \le V(e_0)$. From (18), it is clear that $V(e_0)$ is bounded. Then, by Barbalat's lemma, we have $\lim_{t\to\infty}\|e(t)\|^2 = 0$, which means the FMPS is realized. □

5 Numerical example

A numerical simulation is provided to explain the validity and correctness of the theoretical results we have given.

Consider the drive network as follows:

$$\dot{x}_i(t) = f_i\big(x_i(t), x_i\big(t-\rho(t)\big)\big) + \varepsilon\sum_{j=1}^{2} c_{ij}Q_{ij}x_j\big(t-\tau(t)\big), \tag{21}$$

where $x_i(t) = (x_{in_1}, x_{in_2})^T$, $i = 1,2$, $n_1 = 3$, $n_2 = 4$, $\rho(t) = \frac{e^t}{(1+e^t)}$, $\tau(t) = 1 - 0.2\cos(2t)$, $f_i(x_i(t), x_i(t-\rho(t))) = f_i(x_i(t)) + \frac{1}{2}\sin(x_i(t-\rho(t)))$. Here, we consider the nonlinear functions $f_i(x_i(t))$ which are composed of the hyperchaotic Rossler system and the Lorenz system with nonidentical nodes:

$$f_1\big(x_1(t)\big) = \begin{bmatrix} 36(x_{12}(t) - x_{11}(t)) \\ 20x_{12}(t) - x_{11}(t)x_{13}(t) \\ -3x_{13}(t) + x_{11}(t)x_{12}(t)\end{bmatrix},$$

$$f_2\big(x_2(t)\big) = \begin{bmatrix} 10(x_{22}(t) - x_{21}(t)) + x_{24}(t) \\ 28x_{21}(t) - x_{22}(t) - x_{21}(t)x_{23}(t) \\ x_{21}(t)x_{22}(t) - \frac{8}{3}x_{23}(t) \\ 1.3x_{24}(t) - x_{21}(t)x_{23}(t)\end{bmatrix}.$$

The coupling matrices of system (20) are defined as

$$C = \begin{pmatrix} -0.3 & -0.2 \\ 0.1 & 0.4\end{pmatrix}, \qquad Q_{11} = \begin{pmatrix} 0.4 & -0.2 & 0.3 \\ 0.2 & -0.3 & 0.4 \\ 0.3 & 0.2 & 0.1\end{pmatrix},$$

$$Q_{12} = \begin{pmatrix} 0.2 & 0 & 0.3 & -0.5 \\ 0.1 & -0.3 & 0.4 & 0 \\ 0.3 & 0.2 & 0.1 & -0.2 \end{pmatrix}, \qquad Q_{21} = \begin{pmatrix} 0.2 & -0.1 & 0.3 \\ 0 & 0.3 & 0.2 \\ 0.4 & 0.2 & 0.1 \\ 0.1 & -0.2 & 0.5 \end{pmatrix},$$

$$Q_{22} = \begin{pmatrix} 0.1 & -0.2 & 0.3 & 0.1 \\ 0.4 & -0.3 & 0 & 0.2 \\ -0.3 & 0.2 & -0.1 & 0.1 \\ 0 & 0.2 & 0.1 & -0.2 \end{pmatrix}.$$

The response system with controller $u_i(t)$ is described by

$$\dot{y}_i(t) = g_i\big(y_i(t), y_i\big(t-\rho(t)\big)\big) + \varepsilon \sum_{j=1}^{2} d_{ij} G_{ij} y_j\big(t-\tau(t)\big) + u_i(t), \tag{22}$$

$$u_i(t) = u_i^o(t) + u_i^c(t), \tag{23}$$

where

$$\begin{aligned} u_i^o(t) &= M_i(t)\dot{x}_i(t) + \dot{M}_i(t)x_i(t) - g_i\big(M_i(t)x_i(t), M_i\big(t-\rho(t)\big)x_i\big(t-\rho(t)\big)\big) \\ &\quad - \varepsilon \sum_{j=1}^{N} d_{ij} G_{ij} M_j\big(t-\tau(t)\big)x_j\big(t-\tau(t)\big), \\ u_i^c(t) &= -\beta_i(t)e_i(t), \\ \dot{\beta}_i(t) &= k_i e_i^T(t)e_i(t), \quad i = 1, 2, \end{aligned}$$

$g_i(y_i(t), y_i(t-\rho(t))) = B_i y_i(t) + \frac{1}{2}(\cos^2(y_i(t)) - y_i(t-\rho(t)))$, and the parameters of system (21) are given by

$$B_1 = \begin{pmatrix} 1 & -2 & 3 & 1 \\ 4 & 3 & -1 & 2 \\ -3 & 2 & -1 & 1 \\ 0 & 2 & 1 & -5 \end{pmatrix}, \qquad B_2 = \begin{pmatrix} 0 & 1 & -2 \\ -3 & -1 & 2 \\ 0 & 1 & -1 \end{pmatrix}, \qquad D = \begin{pmatrix} -0.5 & 0 \\ 0.3 & 0.4 \end{pmatrix},$$

$$G_{11} = \begin{pmatrix} 0.4 & 0 & 0 & -0.1 \\ 0.1 & 0.3 & 0 & -0.2 \\ 0 & 0 & 0.1 & 0 \\ -0.1 & 0.1 & -0.2 & 0.5 \end{pmatrix}, \qquad G_{12} = \begin{pmatrix} -0.2 & 0.1 & 0.1 \\ 0.1 & 0 & -0.1 \\ -0.1 & 0.1 & 0.1 \\ 0 & -0.2 & 0.1 \end{pmatrix},$$

$$G_{21} = \begin{pmatrix} 0.1 & 0.1 & 0 & -0.1 \\ 0 & 0.1 & -0.1 & 0.2 \\ 0 & -0.1 & 0.1 & 0 \end{pmatrix}, \qquad G_{22} = \begin{pmatrix} -0.1 & 0.1 & 0 \\ 0 & -0.3 & 0.1 \\ 0.2 & 0 & -0.1 \end{pmatrix}.$$

Thus, we give the time-varying scaling matrices,

$$M_1(t) = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0.5\sin 2t & -1 \\ 2\cos t & 0 & 1 \\ 0 & 0 & 1-\sin t \end{pmatrix},$$

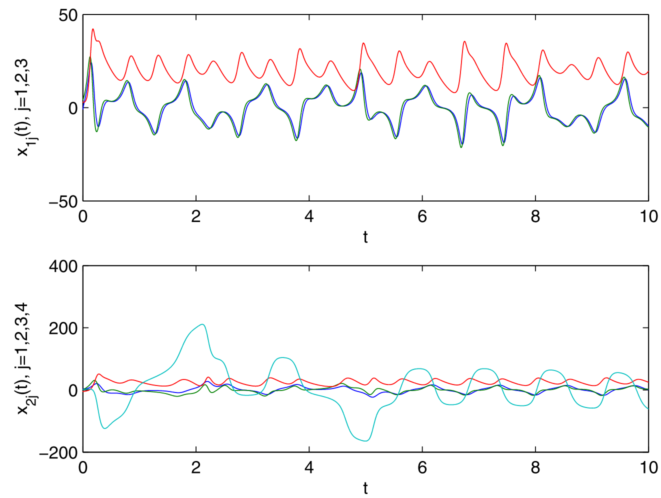

Figure 1 The state tracks of drive system (21)

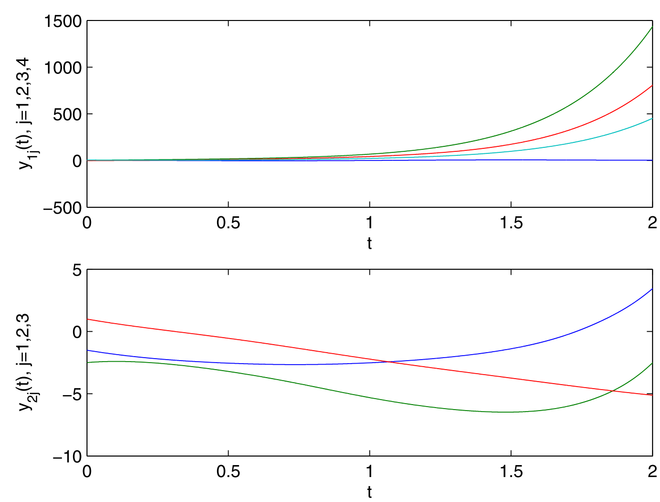

Figure 2 The state tracks of response system (22)

$$M_2(t) = \begin{pmatrix} -2 & -1 & 1 & 0 \\ 0 & -0.5\cos 2t & -1 & 0 \\ -1 & 0 & 0 & \sin t \end{pmatrix}.$$

The other parameters are taken as $\varepsilon = 0.5$, $k_1 = k_2 = 5$, $\beta_1(0) = \beta_2(0) = 0.5$. The initial values of the state variables are random. Figures 1 and 2 demonstrate the state tracks of the drive system and the response system, respectively. From Fig. 3, we know the state tracks of the drive system and the response system can be realized as a function of matrix projective synchronization with the hybrid controller (23). In addition, Fig. 4 demonstrates the track of the adaptive feedback that obtains for $\beta_i(t)$.

6 Conclusion

This paper studies the problem of the function matrix projective synchronization for different coupled complex networks for the nonidentical nodes and the different dimensions.

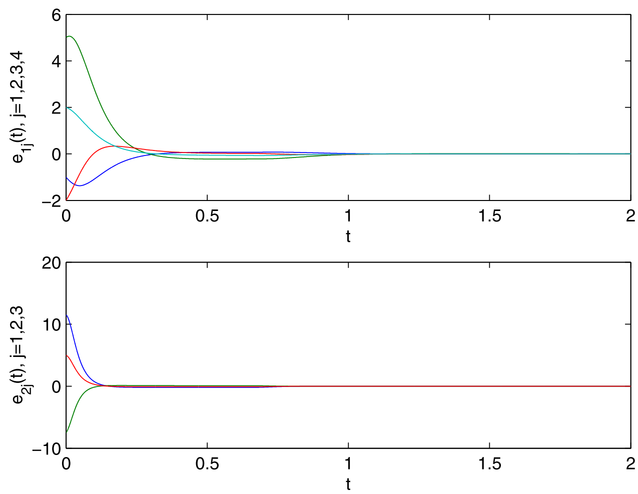

Figure 3 The tracks of synchronization errors $e_i(t)$

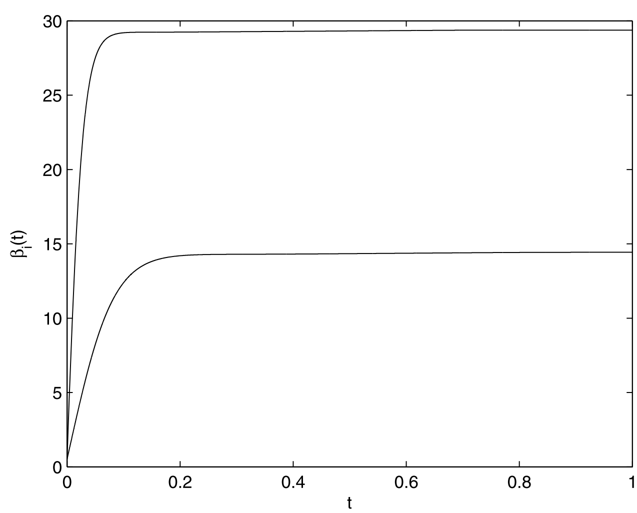

Figure 4 The tracks of adaptive feedback gains $\beta_i(t)$

In order to obtain the FMPS in which the internal time delays are different from the coupled delays, the hybrid feedback controller is given, by utilizing Lyapunov stability theory and mathematical induction. The coupling matrices are not required to satisfy the symmetry and the diffusion conditions. Finally, through presenting a numerical simulation we display the validity and appropriateness of our given scheme.

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Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally to the writing of this paper. All authors read and approved the manuscript.

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