Ghanbari et al. *Advances in Difference Equations* (2019) 2019:49
https://doi.org/10.1186/s13662-019-1964-0

Advances in Difference Equations
a SpringerOpen Journal

RESEARCH Open Access

The new exact solitary wave solutions and stability analysis for the (2 + 1)-dimensional Zakharov–Kuznetsov equation

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Abstract

In this paper, a new generalized exponential rational function method is employed to extract new solitary wave solutions for the Zakharov–Kuznetsov equation (ZKE). The ZKE exhibits the behavior of weakly nonlinear ion-acoustic waves in incorporated hot isothermal electrons and cold ions in the presence of a uniform magnetic field. Furthermore, the stability for the governing equations is investigated via the aspect of linear stability analysis. Numerical simulations are made to shed light on the characteristics of the obtained solutions.

Keywords: ZKE; GERFM; Exact solitary wave solutions; Stability analysis

1 Introduction

Nonlinear evolution equations (NLEEs) have been very important aspects owing to their very wide range of applicability in nonlinear science. In science nonlinear physical phenomena are one of the most significant areas of study and they appear in various fields of science and engineering, such as plasma physics, fluid mechanics, gas dynamics, elasticity, relativity, chemical reactions, ecology, optical fiber, solid state physics, biomechanics, to mention few. All these equations are fundamentally controlled by NLEEs [1–6]. NLEEs are often used to illustrate the motion of separated waves. Ever since the arrival of solitary wave in scientific work, it has been getting more concentration. Thus, it is vital to extract exact traveling wave solutions to NLEEs. This is because obtaining exact solutions to NLEEs gives us the liberty to present information on the characteristics of a complex physical phenomenon. Thus, the construction of exact traveling wave solutions to NLEEs has become a priority in the analysis of nonlinear physical phenomenon. A lot of analytical approaches have been used to establish traveling wave solutions for NLEEs [7–13]. On solitons, nonlinear physical phenomena and other novel solutions of NLEEs, there has been a variety of theoretical work [14–24].

Moreover, it is well known that stability analysis (SA) is very important in the investigation of integrability, internal properties, existence and uniqueness of a differential equation [14–17]. In this paper, new traveling wave solutions using a relatively new technique, namely the new generalized exponential rational function method (GERFM) [25] and stability analysis via the concept of linear stability analysis are constructed for the (2 + 1)-

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dimensional ZK equation given by [26–31]

$$\phi_t + \mu\phi\phi_x + \frac{1}{2}\phi_{xxx} + \frac{1}{2}(1+\delta)\phi_{yyx} = 0, \tag{1.1}$$

where μ and δ are nonzero arbitrary constants and $\phi = \phi(x,y,t)$. $\phi(x,y,t)$ is for the electrostatic wave potential in plasmas, that is, a function of the spatial variables x, y and the temporal variable t. The ZK equation determines the behavior of weakly nonlinear ion-acoustic waves incorporating hot isothermal electrons and cold ions in the presence of a uniform magnetic field [32–37]. The ZK equation also involves an anisotropic two-dimensional generalization of the KdV equation and can be analyzed in magnetized plasma for a tiny amplitude Alfvén wave at a critical angle to the uninterrupted magnetic field [32–37]. There has not been a lot of studies on this special form (1.1) of the ZK equation in the literature. The main aim of the current work is to establish new solutions to this less studied equation by means of GERFM.

2 Description of the GERFM

The GERFM may be described as follows [25]:

Step 1. Surmise that there is a nonlinear partial differential equation express by

$$N(\psi, \psi_x, \psi_t, \psi_{xx}, \psi_{yyx}, \ldots) = 0. \tag{2.1}$$

Applying $\psi = \psi(\xi)$ along with $\xi = kx + my - \omega t$, we obtain

$$N(\psi, \psi', \psi'', \ldots) = 0, \tag{2.2}$$

where k, m and ω are constants that will be computed later.

Step 2. Next, we surmise that Eq. (2.2) has the formal solution

$$\Phi(\xi) = \frac{p_1 e^{q_1\xi} + p_2 e^{q_2\xi}}{p_5 e^{q_3\xi} + p_6 e^{q_4\xi}}, \tag{2.3}$$

where p_1, p_2, p_3, p_4, and q_1, q_2, q_3, q_4 represent the complex (or real) numbers provided that Eq. (2.1) is written as

$$\psi(\xi) = A_0 + \sum_{k=1}^{N} A_k \Phi(\xi)^k + \sum_{k=1}^{N} B_k \Phi(\xi)^{-k}. \tag{2.4}$$

Meanwhile the coefficients $A_0, A_k, B_k (1 \le k \le N)$ and $p_n, q_n (1 \le n \le 4)$ will be obtained such that (2.4) satisfies (2.2). In addition, N is a positive integer that can be obtained by applying the homogeneous balance principle.

Step 3. Plugging (2.4) into Eq. (2.2) and organizing all terms yield the polynomial equation $P(e^{q_1\xi}, e^{q_2\xi}, e^{q_3\xi}, e^{q_4\xi}) = 0$. Equating every coefficient of P to zero, a set of algebraic equations for $p_n, q_n (1 \le n \le 4)$, and $k, m, \omega, A_0, A_1, B_1$ will be derived with the help of Maple.

Step 4. Solving the outcomes in Step 3 and then putting non-trivial solutions in (2.4), we obtain the soliton solutions of Eq. (1.1).

3 Application of GERFM to ZK

In order to construct explicit traveling wave solutions to Eq. (1.1), we propose the following traveling wave transformation:

$$\phi(x,y,t) = u(\xi), \qquad \xi = kx + my - \omega t. \tag{3.1}$$

Applying Eq. (3.1) to Eq. (1.1) and integrating once with respect to ξ yield

$$-\omega u + \left(\frac{k^3}{2} + \frac{km^2}{2}(1+\delta)\right)u'' + \frac{\mu k}{2}u^2 = 0. \tag{3.2}$$

Applying the balance principle to the terms of u^2 and u'' in Eq. (3.2) gives $2N = N + 2$ so $N = 2$. So (2.4) will turn into

$$u(\xi) = A_0 + A_1\Phi(\xi) + A_2\Phi^2(\xi) + \frac{B_1}{\Phi(\xi)} + \frac{B_2}{\Phi^2(\xi)}, \tag{3.3}$$

where $\Phi(\xi)$ is giving by (2.3). Substituting (3.3) into (3.2) and, following the method described in Sect. 2, we obtain non-trivial solutions of (1.1) as follows.

Family 1: For $p = [i, -i, 1, 1]$ and $q = [i, -i, i, -i]$, (2.3) turns into

$$\Phi(\xi) = -\frac{\sin(\xi)}{\cos(\xi)}. \tag{3.4}$$

Case 1:

$$k = k, \qquad m = -\frac{\sqrt{6}\sqrt{6k^2 + \mu B_2}}{6\sqrt{-\delta - 1}}, \qquad \omega = \frac{4}{3}\mu k B_2,$$
$$A_0 = 2B_2, \qquad A_1 = 0, \qquad A_2 = B_2, \qquad B_1 = 0, \qquad B_2 = B_2.$$

Putting these results in Eqs. (3.3) and (3.4), the exact solution of Eq. (1.1) is obtained:

$$\phi_1(x,y,t) = \frac{B_2}{\cos^2(\xi)\sin^2(\xi)},$$

where, for $\delta < -1$, we have

$$\xi = kx - \frac{\sqrt{6}\sqrt{6k^2 + \mu B_2}}{6\sqrt{-\delta - 1}}y - \frac{4}{3}\mu k B_2 t.$$

Case 2:

$$k = k, \qquad m = -\frac{\sqrt{2}\sqrt{2k^3 - \omega}}{2\sqrt{k}\sqrt{-\delta - 1}}, \qquad \omega = \omega, \qquad A_0 = -\frac{\omega}{k\mu},$$
$$A_1 = 0, \qquad A_2 = -\frac{3\omega}{k\mu}, \qquad B_1 = 0, \qquad B_2 = 0.$$

Putting these results in Eqs. (3.3) and (3.4), the exact solution of Eq. (1.1) is obtained:

$$\phi_2(x,y,t) = \frac{(2\cos^2(\xi) - 3)\omega}{k\mu\cos^2(\xi)},$$

where, for $\delta < -1$, we have

$$\xi = kx - \frac{\sqrt{2}\sqrt{2k^3 - \omega}}{2\sqrt{k}\sqrt{-\delta - 1}} y - \omega t.$$

Case 3:

$$k = k, \qquad m = -\frac{\sqrt{6}\sqrt{6k^2 + \mu B_2}}{6\sqrt{-\delta - 1}}, \qquad \omega = -\frac{1}{3}\mu k B_2,$$
$$A_0 = \frac{1}{3}B_2, \qquad A_1 = 0, \qquad A_2 = 0, \qquad B_1 = 0, \qquad B_2 = B_2.$$

Putting these results in Eqs. (3.3) and (3.4), the exact solution of Eq. (1.1) is obtained:

$$\phi_3(x, y, t) = \frac{(2\cos^2(\xi) + 1)B_2}{3\sin^2(\xi)},$$

where, for $\delta < -1$, we have

$$\xi = kx - \frac{\sqrt{6}\sqrt{6k^2 + \mu B_2}}{6\sqrt{-\delta - 1}}, \qquad \omega = -\frac{1}{3}\mu k B_2 y + \frac{1}{3}\mu k B_2 t.$$

Case 4:

$$k = k, \qquad m = -\frac{\sqrt{2}\sqrt{2k^3 + \omega}}{2\sqrt{k}\sqrt{-\delta - 1}}, \qquad \omega = \omega, \qquad A_0 = \frac{3\omega}{k\mu},$$
$$A_1 = 0, \qquad A_2 = \frac{3\omega}{k\mu}, \qquad B_1 = 0, \qquad B_2 = 0.$$

Putting these results in Eqs. (3.3) and (3.4), the exact solution of Eq. (1.1) is obtained:

$$\phi_4(x, y, t) = \frac{3\omega}{k\mu\cos^2(\xi)},$$

where, for $\delta < -1$, we have

$$\xi = kx - \frac{\sqrt{2}\sqrt{2k^3 + \omega}}{2\sqrt{k}\sqrt{-\delta - 1}} y - \omega t.$$

Family 2: For $p = [1 + i, 1 - i, 1, 1]$ and $q = [i, -i, i, -i]$, (2.3) turns into

$$\Phi(\xi) = \frac{-\sin(\xi) + \cos(\xi)}{\cos(\xi)}. \tag{3.5}$$

Case 1:

$$k = k, \qquad m = -\frac{\sqrt{3}\sqrt{12k^2 - A_1\mu}}{6\sqrt{-\delta - 1}}, \qquad \omega = \frac{1}{6}\mu k A_1,$$
$$A_0 = -\frac{2}{3}A_1, \qquad A_1 = A_1, \qquad A_2 = -\frac{1}{2}A_1, \qquad B_1 = 0, \qquad B_2 = 0.$$

Putting these results in Eqs. (3.3) and (3.5), the exact solution of Eq. (1.1) is obtained:

$$\phi_5(x,y,t) = \frac{(2\cos^2(\xi) - 3)A_1}{6\cos^2(\xi)},$$

where, for $\delta < -1$, we have

$$\xi = kx - \frac{\sqrt{3}\sqrt{12k^2 - A_1\mu}}{6\sqrt{-\delta - 1}}y - \frac{1}{6}\mu k A_1 t.$$

Case 2:

$$k = k, \qquad m = -\frac{\sqrt{2}\sqrt{2k^3 + \omega}}{2\sqrt{k}\sqrt{-\delta - 1}}, \qquad \omega = \omega,$$
$$A_0 = \frac{6\omega}{k\mu}, \qquad A_1 = 0, \qquad A_2 = 0, \qquad B_1 = -\frac{12\omega}{k\mu}, \qquad B_2 = \frac{12\omega}{k\mu}.$$

Putting these results in Eqs. (3.3) and (3.5), the exact solution of Eq. (1.1) is obtained:

$$\phi_6(x,y,t) = -\frac{6\omega}{k\mu(2\sin(\xi)\cos(\xi) - 1)},$$

where, for $\delta < -1$, we have

$$\xi = kx - \frac{\sqrt{2}\sqrt{2k^3 + \omega}}{2\sqrt{k}\sqrt{-\delta - 1}}y - \omega t.$$

Case 3:

$$k = k, \qquad m = -\frac{\sqrt{2}\sqrt{2k^3 - \omega}}{2\sqrt{k}\sqrt{-\delta - 1}}, \qquad \omega = \omega,$$
$$A_0 = -\frac{4\omega}{k\mu}, \qquad A_1 = 0, \qquad A_2 = 0, \qquad B_1 = \frac{12\omega}{k\mu}, \qquad B_2 = -\frac{12\omega}{k\mu}.$$

Putting these results in Eqs. (3.3) and (3.5), the exact solution of Eq. (1.1) is obtained:

$$\phi_7(x,y,t) = \frac{4\omega(\sin(\xi)\cos(\xi) + 1)}{k\mu(2\sin(\xi)\cos(\xi) - 1)},$$

where, for $\delta < -1$, we have

$$\xi = kx - \frac{\sqrt{2}\sqrt{2k^3 - \omega}}{2\sqrt{k}\sqrt{-\delta - 1}}y - \omega t.$$

Family 3: For $p = [1,-1,1,1]$ and $q = [-1,1,-1,1]$, (2.3) turns into

$$\Phi(\xi) = -\tanh(\xi). \tag{3.6}$$

Case 1:

$$k = \frac{1}{6}\sqrt{-6\mu A_2 - 36(\delta + 1)m^2}, \qquad m = m,$$

$$\omega = -\frac{2}{9}\mu A_2\sqrt{-6\mu A_2 - 36(\delta+1)m^2},$$
$$A_0 = -2A_2, \qquad A_1 = 0, \qquad A_2 = A_2, \qquad B_1 = 0, \qquad B_2 = A_2.$$

Putting these results in Eqs. (3.3) and (3.6), the exact solution of Eq. (1.1) is obtained:

$$\phi_8(x,y,t) = \frac{A_2}{\cosh^2(\xi)\sinh^2(\xi)},$$

where

$$\xi = \frac{1}{6}\sqrt{-6\mu A_2 - 36(\delta+1)m^2}x + my + \frac{2}{9}\mu A_2\sqrt{-6\mu A_2 - 36(\delta+1)m^2}t.$$

Case 2:

$$k = \frac{1}{6}\sqrt{-6\mu B_2 - 36(\delta+1)m^2}, \qquad m = m,$$
$$\omega = \frac{1}{18}\sqrt{-6\mu B_2 - 36(\delta+1)m^2}\mu B_2,$$
$$A_0 = -\frac{1}{3}B_2, \qquad A_1 = 0, \qquad A_2 = 0, \qquad B_1 = 0, \qquad B_2 = B_2.$$

Putting these results in Eqs. (3.3) and (3.6), the exact solution of Eq. (1.1) is obtained:

$$\phi_9(x,y,t) = -\frac{B_2(\tanh^2(\xi) - 3)}{3\tanh^2(\xi)},$$

where

$$\xi = \frac{1}{6}\sqrt{-6\mu B_2 - 36(\delta+1)m^2}x + my - \frac{1}{18}\sqrt{-6\mu B_2 - 36(\delta+1)m^2}\mu B_2 t.$$

Case 3:

$$k = \frac{1}{6}\sqrt{-6\mu B_2 - 36(\delta+1)m^2}, \qquad m = m,$$
$$\omega = -\frac{1}{18}\sqrt{-6\mu B_2 - 36(\delta+1)m^2}\mu B_2,$$
$$A_0 = -B_2, \qquad A_1 = 0, \qquad A_2 = 0, \qquad B_1 = 0, \qquad B_2 = B_2.$$

Putting these results in Eqs. (3.3) and (3.7), the exact solution of Eq. (1.1) is obtained:

$$\phi_{10}(x,y,t) = \frac{B_2}{\sinh^2(\xi)},$$

where

$$\xi = \frac{1}{6}\sqrt{-6\mu B_2 - 36(\delta+1)m^2}x + my + \frac{1}{18}\sqrt{-6\mu B_2 - 36(\delta+1)m^2}\mu B_2 t.$$

Family 4: For $p = [-2,-3,1,1]$ and $q = [1,0,1,0]$, (2.3) turns into

$$\Phi(\xi) = \frac{-2e^{\xi} - 3}{1 + e^{\xi}}. \tag{3.7}$$

Case 1:

$$k = k, \qquad m = -\frac{\sqrt{6}\sqrt{6k^2 + \mu A_2}}{6\sqrt{-\delta - 1}}, \qquad \omega = \frac{1}{12}\mu k A_2,$$
$$A_0 = \frac{37A_2}{6}, \qquad A_1 = 5A_2, \qquad A_2 = A_2, \qquad B_1 = 0, \qquad B_2 = 0.$$

Putting these results in Eqs. (3.3) and (3.7), the exact solution of Eq. (1.1) is obtained:

$$\phi_{11}(x,y,t) = -\frac{A_2(-e^{2\xi} + 4e^{\xi} - 1)}{6(1 + e^{\xi})^2},$$

where, for $\delta < -1$, we have

$$\xi = kx - \frac{\sqrt{6}\sqrt{6k^2 + \mu A_2}}{6\sqrt{-\delta - 1}}y - \frac{1}{12}\mu k A_2 t.$$

Case 2:

$$k = k, \qquad m = -\frac{\sqrt{k^3 - 2\omega}}{\sqrt{k}\sqrt{-\delta - 1}}, \qquad \omega = \omega,$$
$$A_0 = -\frac{72\omega}{\mu k}, \qquad A_1 = 0, \qquad A_2 = 0, \qquad B_1 = -\frac{360\omega}{\mu k}, \qquad B_2 = -\frac{432\omega}{\mu k}.$$

Putting these results in Eqs. (3.3) and (3.7), the exact solution of Eq. (1.1) is obtained:

$$\phi_{12}(x,y,t) = \frac{72\omega e^{\xi}}{\mu k(2e^{\xi} + 3)^2},$$

where, for $\delta < -1$, we have

$$\xi = kx - \frac{\sqrt{k^3 - 2\omega}}{\sqrt{k}\sqrt{-\delta - 1}}y - \omega t.$$

Family 5: For $p = [-3,-1,1,1]$ and $q = [1,-1,1,-1]$, (2.3) turns into

$$\Phi(\xi) = \frac{-2\cosh(\xi) - \sinh(\xi)}{\cosh(\xi)}. \tag{3.8}$$

Case 1:

$$k = k, \qquad m = -\frac{\sqrt{2}\sqrt{2k^3 - \omega}}{2\sqrt{k}\sqrt{-\delta - 1}}, \qquad \omega = \omega,$$
$$A_0 = -\frac{9\omega}{\mu k}, \qquad A_1 = 0, \qquad A_2 = 0, \qquad B_1 = -\frac{36\omega}{\mu k}, \qquad B_2 = -\frac{27\omega}{\mu k}.$$

Putting these results in Eqs. (3.3) and (3.8), the exact solution of Eq. (1.1) is obtained:

$$\phi_{13}(x,y,t)=\frac{9\omega}{k\mu(5\cosh^2(\xi)+4\cosh(\xi)\sinh(\xi)-1)},$$

where, for $\delta<-1$, we have

$$\xi=kx-\frac{\sqrt{2}\sqrt{2k^3-\omega}}{2\sqrt{k}\sqrt{-\delta-1}}y-\omega t.$$

Case 2:

$$k=k,\qquad m=-\frac{\sqrt{2}\sqrt{2k^3+\omega}}{2\sqrt{k}\sqrt{-\delta-1}},\qquad \omega=\omega,$$
$$A_0=\frac{11\omega}{\mu k},\qquad A_1=0,\qquad A_2=0,\qquad B_1=\frac{36\omega}{\mu k},\qquad B_2=\frac{27\omega}{\mu k}.$$

Putting these results in Eqs. (3.3) and (3.8), the exact solution of Eq. (1.1) is obtained:

$$\phi_{14}(x,y,t)=\frac{\omega(18\cosh^4(\xi)-33\cosh^2(\xi)+36\cosh(\xi)\sinh(\xi)+11)}{k\mu(3\cosh^2(\xi)+1)^2},$$

where, for $\delta<-1$, we have

$$\xi=kx-\frac{\sqrt{2}\sqrt{2k^3+\omega}}{2\sqrt{k}\sqrt{-\delta-1}}y-\omega t.$$

Family 6: For $p=[1,1,1,-1]$ and $q=[1,-1,1,-1]$, (2.3) turns into

$$\Phi(\xi)=\frac{\cosh(\xi)}{\sinh(\xi)}. \tag{3.9}$$

Case 1:

$$k=\frac{1}{6}\sqrt{-6\mu A_2-36(\delta+1)m^2},\qquad m=m,$$
$$\omega=\frac{2}{9}\mu A_2\sqrt{-6\mu A_2-36(\delta+1)m^2},$$
$$A_0=\frac{2}{3}A_2,\qquad A_1=0,\qquad A_2=A_2,\qquad B_1=0,\qquad B_2=A_2.$$

Putting these results in Eqs. (3.3) and (3.9), the exact solution of Eq. (1.1) is obtained:

$$\phi_{15}(x,y,t)=\frac{A_2(3\coth^4(\xi)+2\coth^2(\xi)+3)}{3\coth^2(\xi)},$$

where, for $\delta<-1$, we have

$$\xi=\frac{1}{6}\sqrt{-6\mu A_2-36(\delta+1)m^2}x+my-\frac{2}{9}\mu A_2\sqrt{-6\mu A_2-36(\delta+1)m^2}t.$$

Case 2:

$$k = k, \qquad m = -\frac{\sqrt{2}\sqrt{2k^3 - \omega}}{2\sqrt{k}\sqrt{-\delta - 1}}, \qquad \omega = \omega,$$
$$A_0 = \frac{3\omega}{k\mu}, \qquad A_1 = 0, \qquad A_2 = 0, \qquad B_1 = 0, \qquad B_2 = -\frac{3\omega}{k\mu}.$$

Putting these results in Eqs. (3.3) and (3.9), the exact solution of Eq. (1.1) is obtained:

$$\phi_{16}(x,y,t) = \frac{3\omega}{k\mu \cosh^2(\xi)},$$

where, for $\delta < -1$, we have

$$\xi = kx - \frac{\sqrt{2}\sqrt{2k^3 - \omega}}{2\sqrt{k}\sqrt{-\delta - 1}}y - \omega t.$$

Family 7: For $p = [-2, -1, 1, 1]$, $q = [0, 1, 0, 1]$, and $r = [0, 0]$, (2.3) turns into

$$\Phi(\xi) = \frac{-e^{\xi} - 2}{1 + e^{\xi}}. \tag{3.10}$$

Case 1:

$$k = k, \qquad m = -\frac{\sqrt{k^3 + 2\omega}}{\sqrt{k}\sqrt{-\delta - 1}}, \qquad \omega = \omega,$$
$$A_0 = \frac{26\omega}{k\mu}, \qquad A_1 = 0, \qquad A_2 = 0, \qquad B_1 = \frac{72\omega}{k\mu}, \qquad B_2 = \frac{48\omega}{k\mu}.$$

Putting these results in Eqs. (3.3) and (3.10), the exact solution of Eq. (1.1) is obtained:

$$\phi_{17}(x,y,t) = \frac{2\omega(e^{2\xi} - 8e^{\xi} + 4)}{k\mu(e^{\xi} + 2)^2},$$

where, for $\delta < -1$, we have

$$\xi = kx - \frac{\sqrt{k^3 + 2\omega}}{\sqrt{k}\sqrt{-\delta - 1}}y - \omega t.$$

Case 2:

$$k = k, \qquad m = -\frac{\sqrt{k^3 - 2\omega}}{\sqrt{k}\sqrt{-\delta - 1}}, \qquad \omega = \omega,$$
$$A_0 = -\frac{24\omega}{k\mu}, \qquad A_1 = 0, \qquad A_2 = 0, \qquad B_1 = -\frac{72\omega}{k\mu}, \qquad B_2 = -\frac{48\omega}{k\mu}.$$

Putting these results in Eqs. (3.3) and (3.10), the exact solution of Eq. (1.1) is obtained:

$$\phi_{18}(x,y,t) = \frac{24\omega e^{\xi}}{k\mu(e^{\xi} + 2)^2},$$

where, for $\delta < -1$, we have

$$\xi = kx - \frac{\sqrt{k^3 - 2\omega}}{\sqrt{k}\sqrt{-\delta - 1}}y - \omega t.$$

Family 8: For $p = [2, 0, -1, 1]$ and $q = [-1, 0, -1, 1]$, (2.3) turns into

$$\Phi(\xi) = \frac{\cosh(\xi) - \sinh(\xi)}{\cosh(\xi)}. \tag{3.11}$$

Case 1:

$$k = k, \qquad m = -\frac{\sqrt{6}\sqrt{6k^2 + \mu A_2}}{6\sqrt{-\delta - 1}}, \qquad \omega = \frac{1}{3}\mu k A_2,$$
$$A_0 = \frac{2}{3}A_2, \qquad A_1 = -2A_2, \qquad A_2 = A_2, \qquad B_1 = 0, \qquad B_2 = 0.$$

Putting these results in Eqs. (3.3) and (3.11), the exact solution of Eq. (1.1) is obtained:

$$\phi_{19}(x, y, t) = \frac{(2\cosh^2(\xi) - 3)A_2}{3\cosh^2(\xi)},$$

where, for $\delta < -1$, we have

$$\xi = kx - \frac{\sqrt{6}\sqrt{6k^2 + \mu A_2}}{6\sqrt{-\delta - 1}}y - \frac{1}{3}\mu k A_2 t.$$

Family 9: For $p = [2, 0, 1, -1]$ and $q = [1, 0, i, -i]$, (2.3) turns into

$$\Phi(\xi) = \frac{-\sin(\xi) + \cos(\xi)}{\sin(\xi)}. \tag{3.12}$$

Case 1:

$$k = k, \qquad m = -\frac{\sqrt{6}\sqrt{6k^2 + \mu A_2}}{6\sqrt{-\delta - 1}}, \qquad \omega = -\frac{1}{3}\mu k A_2,$$
$$A_0 = \frac{4}{3}A_2, \qquad A_1 = 2A_2, \qquad A_2 = A_2, \qquad B_1 = 0, \qquad B_2 = 0.$$

Putting these results in Eqs. (3.3) and (3.12), the exact solution of Eq. (1.1) is obtained:

$$\phi_{20}(x, y, t) = \frac{(2\cos^2(\xi) + 1)A_2}{3\sin^2(\xi)},$$

where, for $\delta < -1$, we have

$$\xi = kx - \frac{\sqrt{6}\sqrt{6k^2 + \mu A_2}}{6\sqrt{-\delta - 1}}y + \frac{1}{3}\mu k A_2 t.$$

Family 10: For $p = [1 + i, 1 - i, 1, 1]$, $q = [-i, i, -i, i]$, and $r = [0, 0]$, (2.3) turns into

$$\Phi(\xi) = \frac{\cosh(\xi) + \sinh(\xi)}{\sinh(\xi)}. \tag{3.13}$$

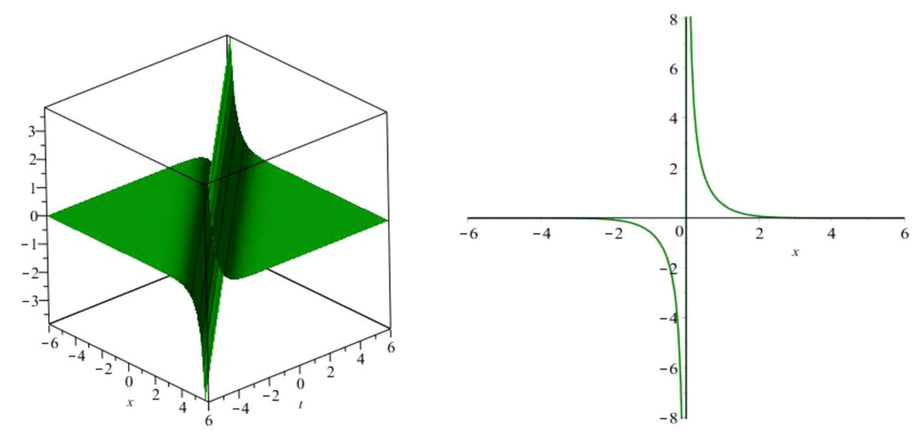

Figure 1 3D plot for ϕ_8 at $y = 1$ with $A_2 = \mu = \delta = 1$ and 2D at $t = 0$

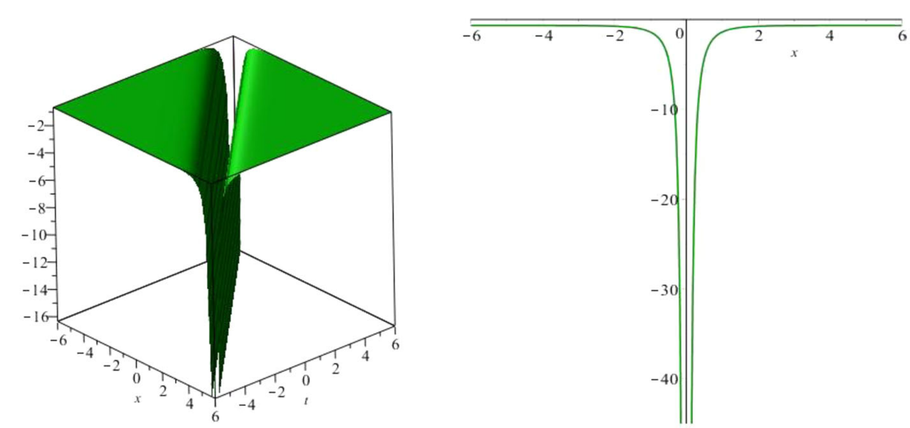

Figure 2 3D plot for ϕ_8 at $y = 1$ with $B_2 = \mu = \delta = 1$ and 2D at $t = 0$

Case 1:

$$k = k, \qquad m = -\frac{\sqrt{6}\sqrt{6k^2 + \mu A_2}}{6\sqrt{-\delta - 1}}, \qquad \omega = \frac{1}{3}\mu k A_2,$$

$$A_0 = \frac{2}{3}A_2, \qquad A_1 = -2A_2, \qquad A_2 = A_2, \qquad B_1 = 0, \qquad B_2 = 0.$$

Putting these results in Eqs. (3.3) and (3.13), the exact solution of Eq. (1.1) is obtained and the physical features of some of the solutions are depicted in Figs. 1 to 5:

$$\phi_{21}(x, y, t) = \frac{(2\cosh^2(\xi) + 1)A_2}{3\sinh^2(\xi)},$$

where, for $\delta < -1$, we have

$$\xi = kx - \frac{\sqrt{6}\sqrt{6k^2 + \mu A_2}}{6\sqrt{-\delta - 1}}y - \frac{1}{3}\mu k A_2 t.$$

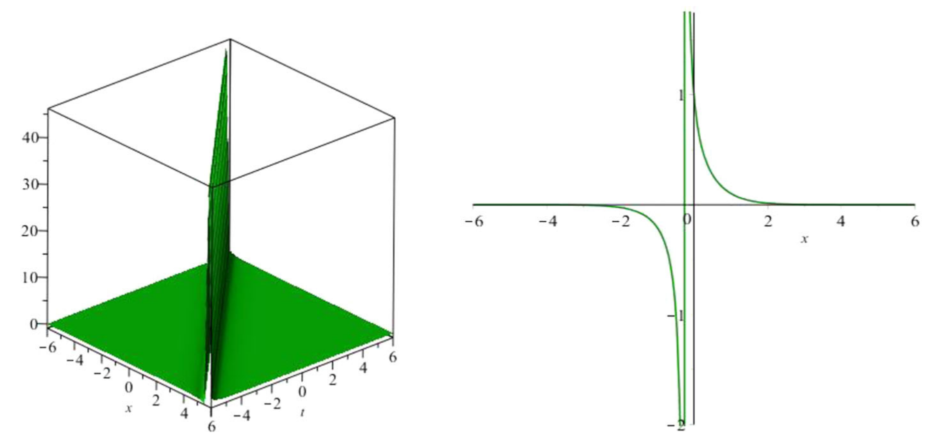

Figure 3 3D plot for ϕ_{13} at $y = 1$ with $\sigma = \mu = k = \delta = 1$ and 2D at $t = 0$

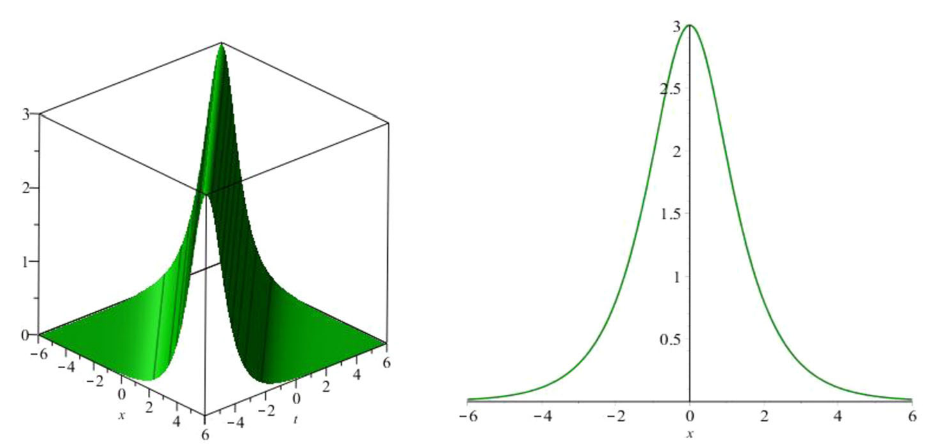

Figure 4 3D plot for ϕ_{16} at $y = 1$ with $\sigma = \mu = k = \delta = 1$ and 2D at $t = 0$

4 Stability analysis of Eq. (1.1)

The connection for the dispersion in Eq. (1.1) will be analyzed [14–17]. The features of the real part (Re) of σ show whether the outcome will become larger or disappear in a given period. When the real part of $\sigma(k)$ is negative for every k values, the superposition of solutions of the form $e^{(i\sigma t+ikx)}$ may subsequently disappear. Put differently, if the the real part is positive for some values of k, some components of a superposition will subsequently become huge. The former is regarded as the stable case, while the latter is regarded as the unstable case. Furthermore, if the maximum of the real part is zero, the case is regarded to be the marginally stable case. It is burdensome to evaluate the long term behavior in this case. Moreover, in Fig. 6 we plot frequency of the pertubation against the wave number.

Consider the perturbed solution of the form

$$\phi(x,t,y) = P_0 + \epsilon w(x,t,y). \tag{4.1}$$

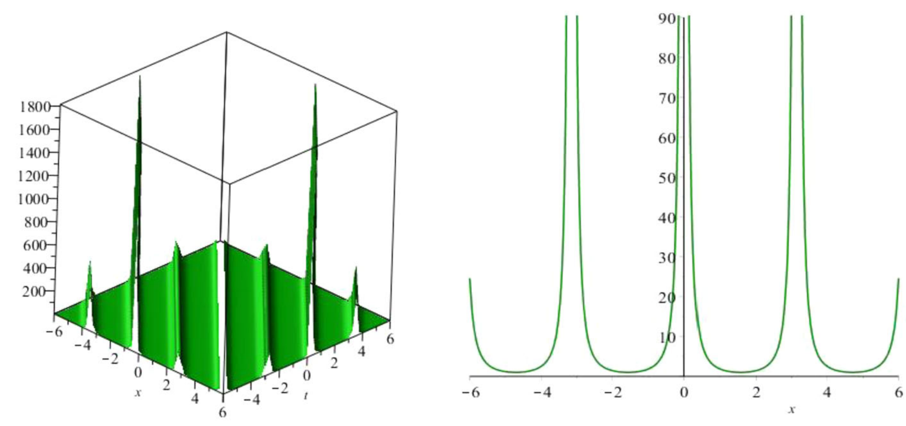

Figure 5 3D plot for ϕ_{20} at $y = 1$ with $A_2 = k = \delta = 1$ and 2D at $t = 0$

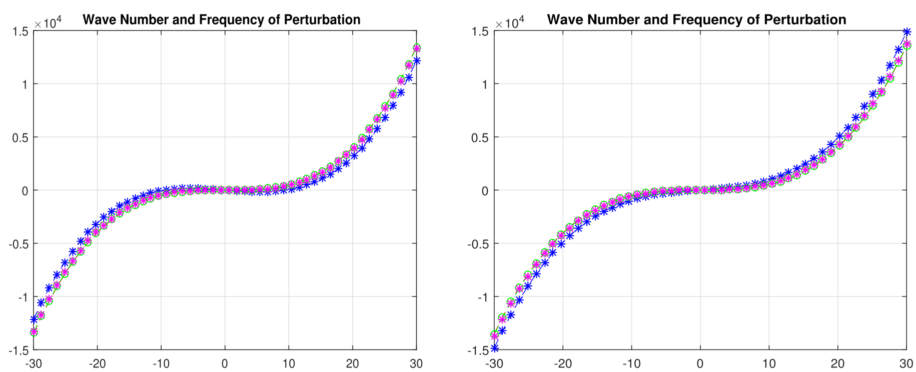

Figure 6 Frequency of the perturbation against the wave number when $P_0 < 0$ and $P_0 > 0$, respectively

It is easy to see that any constant P_0 is a steady state solution of Eq. (1.1). Inserting Eq. (4.1) into Eq. (1.1), one gets

$$2\epsilon w_t + 2\mu\epsilon P_0 w_x + 2\mu\epsilon^2 w w_x + \epsilon w_{xyy} + \delta\epsilon w_{xyy} + \epsilon w_{xxx} = 0, \tag{4.2}$$

linearizing (4.2) in ϵ gives

$$2\epsilon w_t + 2\mu\epsilon P_0 w_x + \epsilon w_{xyy} + \delta\epsilon w_{xyy} + \epsilon w_{xxx} = 0. \tag{4.3}$$

Suppose that Eq. (4.3) has a solution of the form

$$w(x,t,y) = e^{i(k_1 x + k_2 y + \sigma t)}, \tag{4.4}$$

where k is the normalized wave number; substituting Eq. (4.4) into Eq. (4.3), we get

$$\left(k_1^3 + k_1 k_2^2(1+\delta) - 2\sigma - 2k_1\mu P_0\right) = 0. \tag{4.5}$$

Solving for σ from the above equation yields

$$\sigma(k_1,k_2)=\frac{1}{2}\bigl(k_1^3+k_1k_2^2+k_1k_2^2\delta-2k_1\mu P_0\bigr). \tag{4.6}$$

From Eq. (4.5), one can see that the real part is negative for all k values, then any superposition of the solutions will appear to decay. Therefore, the dispersion is stable.

5 Conclusion

This research applied GERFM to extracting new solitary wave solutions for the ZK equation. The ZK equation exhibits the behavior of weakly nonlinear ion-acoustic waves incorporating hot isothermal electrons and cold ions in the presence of a uniform magnetic field. There have not been a lot of studies on this special form (1.1) of the ZK equation in the literature. Owing to this, it is of great importance to establish different types of solutions to this equation. We successfully obtained solutions such as exact solutions, exact periodic wave solutions, soliton solutions and exponential function solutions. GERFM has the capacity to generate several types of solutions in different form unlike some of the classic methods that could only generate a small number of solutions. Thus, GERFM is very efficient and effective in extracting new types of solutions to varieties of NLEEs. Graphical features of some of the obtained solutions are presented in order to shed more light on the characteristics of the obtained solutions. Furthermore, the stability of the governing equations was investigated via a linear stability analysis.

Acknowledgements
The authors are grateful to the anonymous reviewers for their valuable feedback to improve the quality of the present paper.

Funding
Not applicable.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors participated equally to reach the final version of the paper. All authors read and approved the final version of the manuscript.

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Received: 28 September 2018 Accepted: 15 January 2019 Published online: 06 February 2019

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