Zhang et al. *Advances in Difference Equations* (2018) 2018:473
https://doi.org/10.1186/s13662-018-1918-y

Advances in Difference Equations
a SpringerOpen Journal

RESEARCH Open Access

Master–slave synchronization of a class of fractional-order Takagi–Sugeno fuzzy neural networks

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Abstract

In this paper, studies on the synchronization of fractional-order Takagi–Sugeno (T-S) fuzzy neural networks are performed. By employing a linear matrix inequality and constructing a skillful Lyapunov function, sufficient conditions are derived to guarantee that the master system synchronizes the slave system. Finally, an example and its simulations are presented to demonstrate the feasibility of the synchronization scheme.

Keywords: Fractional-order; Fuzzy neural networks; Synchronization

1 Introduction

Neural networks are playing a more and more important role in the reconstruction of images, signal processing, optimization problems, artificial intelligence, etc. However, neural networks can arise chaotic behaviors due to an unpredictable disturbance. To control the chaos arising in the neural networks, a variety of synchronization schemes have been proposed, including projective synchronization [22], event-based synchronization [13], exponential synchronization [8, 15], finite-time synchronization [29, 36], generalized synchronization [2, 10], pinning synchronization [9, 31], lag synchronization [26, 32], adaptive synchronization [3, 11, 18, 24, 27, 36], impulsive synchronization [14, 17, 28, 30, 31], and so on. Recently, another important topic is the fractional calculus, which depicts arbitrary non-integer-order differentiation and integration. The fractional-order neural networks have been proposed in theory and practice due to the great significance of the fractional calculus. Stability analysis of a fractional-order system with impulses was performed in [21]. Synchronization schemes were proposed for the fractional-order neural networks with delays (see, e.g., [33, 35]). Memristor-based fractional-order cellular neural networks were studied in [7, 20].

Fuzzy logic theory is a powerful tool to deal with synthesis of integer-order complex systems (see [4–6, 23, 25, 38]). However, they have not considered the effects of fuzzy logic on the fractional-order neural networks. There are few papers considering the stability and synchronization of Takagi–Sugeno (T-S) fuzzy neural networks. Recently, state estimation was given for T-S fuzzy delayed Hopfield neural networks in [1]. Adaptive fuzzy sliding mode control scheme was proposed for the uncertain fractional-order chaotic systems [16]. Finite stability analysis was performed for a memristor-based fractional-order fuzzy cellular neural networks in [37]. In [19], impulsive synchronization was proposed for fractional T-S fuzzy networks by utilizing the comparison principle. In [22], the authors studied the adaptive projective synchronization for fractional-order T-S fuzzy neural networks with uncertain parameters. In the previous works, they considered the projective synchronization and the impulsive synchronization of fractional-order T-S fuzzy neural networks. However, different from their consideration and method, we construct a different Lyapunov function and employ the linear matrix inequality. Some sufficient conditions are obtained to guarantee the master–slave synchronization of fractional-order T-S neural networks. This is the highlight of this paper.

This paper is organized as follows. Definitions and lemmas are presented in the next section. Section 3 is devoted to obtaining the sufficient conditions for synchronization of fractional-order neural networks. Finally, an example and its simulations are given.

2 Preliminaries

In this section, the assumptions, definitions, and some lemmas are given. Two definitions of the Caputo fractional-order integrals and derivatives are introduced.

Definition 2.1 For a function $x(t)$ and non-integer real number $\alpha > 0$, the Caputo fractional integral is defined as

$$I^{\alpha}x(t) = \frac{1}{\Gamma(\alpha)}\int_{t_0}^{t}(t-\tau)^{\alpha-1}x(\tau)\,d\tau,$$

where the gamma function $\Gamma(\cdot)$ satisfies $\Gamma(s) = \int_0^{\infty} t^{s-1}e^{-t}\,dt$, t_0 is the initial time, $t \geq t_0$.

Definition 2.2 For a function $x(t)$ and non-integer real number $\alpha > 0$, the Caputo fractional derivative is defined as

$$D^{\alpha}x(t) = \frac{1}{\Gamma(n-\alpha)}\int_{t_0}^{t}(t-\tau)^{n-\alpha-1}x^{(n)}(\tau)\,d\tau,$$

where t_0 is the initial time, $t \geq t_0$, $n-1 < \alpha < n \in Z^{+}$.

We need the following lemmas.

Lemma 2.1 ([12]) *For the Caputo fractional-order derivative, when* $n-1 < \alpha < n, n \in N^{+}$, *we have*

$$D^{-\alpha}\bigl(D^{\alpha}\bigr)f(t) = f(t) - \sum_{i=1}^{n-1}\frac{f^{i}(t_0)}{i!}(t-t_0)^{i}.$$

In particular, when $0 < \alpha < 1$,

$$D^{-\alpha}\bigl(D^{\alpha}\bigr)f(t) = f(t) - f(t_0), \qquad D^{-\alpha}\bigl(I^{\alpha}\bigr)f(t) = f(t).$$

Lemma 2.2 ([1]) *For any matrices* $X \in R^{m\times n}$, $Y \in R^{m\times n}$, $\Lambda = \Lambda^{T} > 0$, $\Lambda \in R^{n\times n}$, *the inequality* $X^{T}Y + Y^{T}X \leq X^{T}\Lambda X + Y^{T}\Lambda^{-1}Y$ *holds.*

Lemma 2.3 ([34]) *Given constant matrices* Ξ_1, Ξ_2, Ξ_3, *where* $\Xi_1 = \Xi_1^T$, $\Xi_2 = \Xi_2^T$, *and* $\Xi_2 > 0$, *then* $\Xi_1 + \Xi_3^T \Xi_2^{-1} \Xi_3 < 0$ *if and only if*

$$\begin{pmatrix} \Xi_1 & \Xi_3^T \\ \Xi_3 & -\Xi_2 \end{pmatrix} < 0, \quad or \quad \begin{pmatrix} -\Xi_2 & \Xi_3^T \\ \Xi_3 & \Xi_1 \end{pmatrix} < 0.$$

3 Model formulations and synchronization schemes

In this section, we discuss the master–slave synchronization of fractional-order T-S fuzzy delayed neural networks. The aim is to achieve the synchronization of the T-S fuzzy master–slave systems by using a state feedback controller. Consider a vector form of the neural network as follows:

$$D^\alpha x(t) = Cx(t) + Af\big(x(t)\big) + Bg\big(u(t-\tau)\big) + I(t). \tag{1}$$

If we take (1) as the master system, the corresponding slave system can be given as

$$D^\alpha v(t) = Cv(t) + Af\big(v(t)\big) + Bg\big(v(t-\tau)\big) - U(t) + I(t), \tag{2}$$

where $x(t) = [u_1(t), \dots, u_n(t)]^T \in R^n$ is the state vector, $v(t) = [v_1(t), \dots, v_n(t)]^T \in R^n$ is the output vector, $C = \operatorname{diag}(-c_1, \dots, -c_n)$ ($c_k > 0, k = 1, \dots, n$) is the self-feedback matrix, $U(t)$ is a suitable controller, A and $B \in R^{n\times n}$, $I(t) = [\xi_1(t), \xi_2(t), \dots, \xi_n(t)]^T \in R^n$ is the external input vector, $f(x(t)) = [f_1(u_1(t)), \dots, f_n(u_n(t))]^T$ and $g(u(t-\tau)) = [g_1(u_1(t-\tau)), \dots, g_n(u_n(t-\tau))]^T$ denotes the output vector at time t and $t-\tau$, respectively.

Motivated by [1], we define the fuzzy rule k as follows:

IF ω_1 is μ_{k1} and $\cdots$ ω_s is μ_{ks}, THEN

$$D^\alpha x(t) = C_k x(t) + A_k f\big(x(t)\big) + B_k g\big(u(t-\tau)\big) + I_k(t), \tag{3}$$

$$D^\alpha v(t) = C_i v(t) + A_k f\big(v(t)\big) + B_k g\big(v(t-\tau)\big) - U(t) + I_k(t). \tag{4}$$

The meaning of parameters ω_k, μ_{kq} ($k = 1, 2, \dots, r$, $q = 1, 2, \dots, s$), r is the same as in [1]. Using a standard fuzzy inference method, we have from (3)–(4) that

$$D^\alpha u(t) = \sum_{k=1}^{r} h_k(\omega)\big[C_k u(t) + A_k f\big(u(t)\big) + B_k g\big(u(t-\tau)\big) + I_k(t)\big], \tag{5}$$

$$D^\alpha v(t) = \sum_{k=1}^{r} h_k(\omega)\big[C_k v(t) + A_k f\big(v(t)\big) + B_k g\big(v(t-\tau)\big) - U(t) + I_k(t)\big], \tag{6}$$

where $h_k(\omega) = \frac{\omega_k(\omega)}{\sum_{k=1}^{s} \omega_k(\omega)}$ satisfies

$$h_k(\omega) \geq 0, \qquad \sum_{i=1}^{n} h_k(\omega) = 1. \tag{7}$$

Throughout this paper, we make the following assumption.

Assumption 3.1 The neuron activation functions $f_j(x)$ and $g_j(x)$ satisfy the following Lipschitz conditions:

$$\left|f_j(x) - f_j(y)\right| \le l_j|x - y|$$

and

$$\left|g_j(x) - g_j(y)\right| \le h_j|x - y|$$

for all $x, y \in \mathbb{R}$, where $l_j > 0$, $h_j > 0$ are Lipschitz constants.

Let $e(t) = v(t) - u(t)$ be the synchronization error, select the control input function

$$U(t) = \Phi\bigl(v(t) - u(t)\bigr), \tag{8}$$

where $\Phi = \operatorname{diag}(\phi_1, \phi_2, \ldots, \phi_n)$ is the controller feedback matrix.

Then we can obtain the error system as follows:

$$\begin{aligned} D^\alpha e(t) = {}& \sum_{k=1}^{r} h_k(\omega)\bigl\{(C_k - \Phi)e(t) + A_k\bigl[f\bigl(v(t)\bigr) - A_k f\bigl(u(t)\bigr)\bigr] \\ & + B_k\bigl[g\bigl(v(t-\tau)\bigr) - B_k g\bigl(u(t-\tau)\bigr)\bigr]\bigr\}. \end{aligned} \tag{9}$$

Theorem 3.1 *If there exist positive definite matrices P, Q, R, S, and V such that*

$$\begin{pmatrix} \Psi & 0 & V & PA_k & PB_k \\ 0 & H - R & -V & 0 & 0 \\ V & -V & -\frac{1}{\tau}Q & 0 & 0 \\ A_k^T P & 0 & 0 & -E & 0 \\ B_k^T P & 0 & 0 & 0 & -E \end{pmatrix} < 0 \tag{10}$$

for all k ($k = 1, 2, \ldots, r$), where $\Psi = (C_k - \Phi)^T P + P(C_k - \Phi) + L + \tau Q + R + S$, $L = \operatorname{diag}\{l_1^2, l_2^2, \ldots, l_n^2\}$, $H = \operatorname{diag}\{h_1^2, h_2^2, \ldots, h_n^2\}$, E is an identity matrix, then the fractional-order T-S fuzzy system (5) synchronizes to system (6).

Proof We define the following Lyapunov function:

$$\begin{aligned} V(t) = {}& D^{\alpha-1}\bigl[e^T(t)Pe(t)\bigr] + \int_{-\tau}^{0}\int_{t+\theta}^{t} e(\varpi)^T Q e(\varpi)\,d\varpi\,d\theta \\ & + \left[\int_{-\tau}^{0} e(t+\sigma)\,d\sigma\right]^T V \left[\int_{-\tau}^{0} e(t+\sigma)\,d\sigma\right] \\ & + \int_{-\tau}^{0} e^T(t+\sigma)Re(t+\sigma)\,d\sigma, \end{aligned} \tag{11}$$

where

$$e(t) = \bigl(e_1(t), e_2(t), \ldots, e_n(t)\bigr)^T. \tag{12}$$

Calculation on the derivative along (9) leads to

$$
\begin{aligned}
\dot{V}(t) = {} & D^{\alpha}\bigl[e^T(t)Pe(t)\bigr] + \tau e^T(t)Qe(t) \\
& - \int_{t-\tau}^{t} e^T(\sigma)Qe(\sigma)\,d\sigma + e^T(t)Re(t) \\
& - e^T(t-\tau)Re(t-\tau) + \bigl[e(t)-e(t-\tau)\bigr]^T V\biggl[\int_{t-\tau}^{t} e(\sigma)\,d\sigma\biggr] \\
& + \biggl[\int_{t-\tau}^{t} e(\sigma)\,d\sigma\biggr]^T V\bigl[e(t)-e(t-\tau)\bigr] \\
= {} & \sum_{k=1}^{r} h_k(\omega)\bigl[e^T(t)\bigl(C_k^T - \Phi^T\bigr) + \bigl(f\bigl(v(t)\bigr) - f\bigl(u(t)\bigr)\bigr)^T A_k^T + \bigl(g\bigl(v(t-\tau)\bigr) \\
& - g\bigl(u(t-\tau)\bigr)\bigr)^T B_k^T\bigr]Pe(t) + e^T(t)P\sum_{k=1}^{r} h_k(\omega)\bigl[(C_k-\Phi)e(t) \\
& + A_k\bigl(f\bigl(v(t)\bigr) - f\bigl(u(t)\bigr)\bigr) + B_k\bigl(g\bigl(v(t-\tau)\bigr) - g\bigl(u(t-\tau)\bigr)\bigr)\bigr] \\
& + \tau e^T(t)Qe(t) - \int_{t-\tau}^{t} e^T(\sigma)Qe(\sigma)\,d\sigma + e^T(t)Re(t) - e^T(t-\tau)Re(t-\tau) \\
& + \bigl[e(t)-e(t-\tau)\bigr]^T V\biggl[\int_{t-\tau}^{t} e(\sigma)\,d\sigma\biggr] \\
& + \biggl[\int_{t-\tau}^{t} e(\sigma)\,d\sigma\biggr]^T V\bigl[e(t)-e(t-\tau)\bigr] \\
= {} & \sum_{k=1}^{r} h_k(\omega)\bigl\{e^T(t)\bigl[(C_k-\Phi)^T P + P(C_k-\Phi)\bigr]e(t) + \bigl(f\bigl(v(t)\bigr) \\
& - f\bigl(u(t)\bigr)\bigr)^T A_k^T Pe(t) + e^T(t)PA_k\bigl(f\bigl(v(t)\bigr) - f\bigl(u(t)\bigr)\bigr) + \bigl(g\bigl(v(t-\tau)\bigr) \\
& - g\bigl(u(t-\tau)\bigr)\bigr)^T B_k^T Pe(t) + e^T(t)PB_k\bigl(g\bigl(v(t-\tau)\bigr) - g\bigl(u(t-\tau)\bigr)\bigr)\bigr\} \\
& + \tau e^T(t)Qe(t) - \int_{t-\tau}^{t} e^T(\sigma)Qe(\sigma)\,d\sigma + e^T(t)Re(t) - e^T(t-\tau)Re(t-\tau) \\
& + \bigl[e(t)-e(t-\tau)\bigr]^T V\biggl[\int_{t-\tau}^{t} e(\sigma)\,d\sigma\biggr] \\
& + \biggl[\int_{t-\tau}^{t} e(\sigma)\,d\sigma\biggr]^T V\bigl[e(t)-e(t-\tau)\bigr].
\end{aligned}
$$

In view of Lemma 2.2, we obtain

$$
\begin{aligned}
& \bigl(f\bigl(v(t)\bigr) - f\bigl(u(t)\bigr)\bigr)^T A_k^T Pe(t) + e^T(t)PA_k\bigl(f\bigl(v(t)\bigr) - f\bigl(u(t)\bigr)\bigr) \\
& \quad \le \sum_{j=1}^{n} l_j^2 e_j^2(t) + e^T(t)PA_kA_k^T Pe(t) \\
& \quad \le e^T(t)\bigl(L + PA_iA_k^T P\bigr)e(t),
\end{aligned}
$$

and

$$\begin{aligned}
&\big(g\big(v(t-\tau)\big)-g\big(u(t-\tau)\big)\big)^T B_k^T Pe(t) + e^T(t)PB_k\big(g\big(v(t-\tau)\big)-g\big(u(t-\tau)\big)\big) \\
&\quad \le \sum_{j=1}^{n} h_j^2 e_j^2(t-\tau) + e^T(t)PB_kB_k^T Pe(t) \\
&\quad \le e^T(t-\tau)He(t-\tau) + e^T(t)PB_kB_k^T Pe(t),
\end{aligned}$$

where $L = \operatorname{diag}\{l_1^2, l_2^2, \ldots, l_n^2\}$, $H = \operatorname{diag}\{h_1^2, h_2^2, \ldots, h_n^2\}$. Thus, we have

$$\begin{aligned}
\dot{V}(t) \le{}& \sum_{k=1}^{r} h_k(\omega)\big\{e^T(t)\big[(C_k-\Phi)^T P + P(C_k-\Phi)\big]e(t) + e^T(t)Le(t) \\
&+ e^T(t)PA_kA_k^T Pe(t) + e^T(t-\tau)He(t-\tau) + e^T(t)PB_kB_k^T Pe(t)\big\} \\
&+ \tau e^T(t)Qe(t) - \int_{t-\tau}^{t} e^T(\sigma)Qe(\sigma)\,d\sigma + e^T(t)Re(t) - e^T(t-\tau)Re(t-\tau) \\
&+ \big[e(t)-e(t-\tau)\big]^T V\left[\int_{t-\tau}^{t} e(\sigma)\,d\sigma\right] + \left[\int_{t-\tau}^{t} e(\sigma)\,d\sigma\right]^T V\big[e(t)-e(t-\tau)\big].
\end{aligned}$$

By using the inequality

$$\left[\int_{t-\tau}^{t} e(\sigma)\,d\sigma\right]^T Q\left[\int_{t-\tau}^{t} e(\sigma)\,d\sigma\right] \le \tau \int_{t-\tau}^{t} e(\sigma)^T Qe(\sigma)\,d\sigma,$$

we have

$$\int_{t-\tau}^{t} e(\sigma)^T Qe(\sigma)\,d\sigma \ge \frac{1}{\tau}\left[\int_{t-\tau}^{t} e(\sigma)\,d\sigma\right]^T Q\left[\int_{t-\tau}^{t} e(\sigma)\,d\sigma\right].$$

Thus,

$$\begin{aligned}
\dot{V}(t) \le{}& \sum_{k=1}^{r} h_k(\omega)\big\{e^T(t)\big[(C_k-\Phi)^T P + P(C_k-\Phi)\big]e(t) + e^T(t)Le(t) + e^T(t)PA_kA_k^T Pe(t) \\
&+ e^T(t-\tau)He(t-\tau) + e^T(t)PB_kB_k^T Pe(t)\big\} + \tau e^T(t)Qe(t) \\
&- \frac{1}{\tau}\left[\int_{t-\tau}^{t} e(\sigma)\,d\sigma\right]^T Q\left[\int_{t-\tau}^{t} e(\sigma)\,d\sigma\right] + e^T(t)Re(t) - e^T(t-\tau)Re(t-\tau) \\
&+ \big[e(t)-e(t-\tau)\big]^T V\left[\int_{t-\tau}^{t} e(\sigma)\,d\sigma\right] + \left[\int_{t-\tau}^{t} e(\sigma)\,d\sigma\right]^T V\big[e(t)-e(t-\tau)\big] \\
={}& \sum_{k=1}^{r} h_k(\omega)\big\{e^T(t)\big[(C_k-\Phi)^T P + P(C_k-\Phi) + L + PA_kA_k^T P \\
&+ PB_kB_k^T P + \tau Q + R\big]e(t)\big\} + e^T(t-\tau)(H-R)e(t-\tau) \\
&- \frac{1}{\tau}\left[\int_{t-\tau}^{t} e(\sigma)\,d\sigma\right]^T Q\left[\int_{t-\tau}^{t} e(\sigma)\,d\sigma\right] + \big[e(t)-e(t-\tau)\big]^T V\left[\int_{t-\tau}^{t} e(\sigma)\,d\sigma\right] \\
&+ \left[\int_{t-\tau}^{t} e(\sigma)\,d\sigma\right]^T V\big[e(t)-e(t-\tau)\big]
\end{aligned}$$

$$= \sum_{k=1}^{r} h_k(\omega) \left\{ \begin{pmatrix} e(t) \\ e(t-\tau) \\ \int_{t-\tau}^{t} e(\sigma)\,d\sigma \end{pmatrix}^T \begin{pmatrix} \Pi & 0 & V \\ 0 & H-R & -V \\ V & -V & -\frac{1}{\tau}Q \end{pmatrix} \begin{pmatrix} e(t) \\ e(t-\tau) \\ \int_{t-\tau}^{t} e(\sigma)\,d\sigma \end{pmatrix} - e(t)^T Se(t) \right\},$$

where

$$\Pi = (C_k - \Phi)^T P + P(C_k - \Phi) + L + PA_k A_k^T P + PB_k B_k^T P + \tau Q + R + S.$$

Note that, by Lemma 2.3, the matrix inequality

$$\begin{pmatrix} \Psi & 0 & V & PA_k & PB_k \\ 0 & -R & -V & 0 & 0 \\ V & -V & -\frac{1}{\tau}Q & 0 & 0 \\ A_k^T P & 0 & 0 & -E & 0 \\ B_k^T P & 0 & 0 & 0 & -E \end{pmatrix} < 0$$

implies that the following inequality holds:

$$\begin{pmatrix} \Pi & 0 & V \\ 0 & H-R & -V \\ V & -V & -\frac{1}{\tau}Q \end{pmatrix} < 0. \tag{13}$$

Therefore,

$$\dot{V}(t) < \sum_{k=1}^{r} h_k(\omega)\{-e(t)^T Se(t)\} = -e(t)^T Se(t) < 0.$$

This implies that the fractional-order T-S fuzzy neuron system (5) synchronizes to system (6). □

Remark 3.1 We construct a skillful Lyapunov function with the Caputo fractional-order integral, definite integral, and double integral in the proof of Theorem 3.1.

4 Numerical example

In this section, as an example, we consider a fractional-order T-S fuzzy delayed neural networks with two neurons.

Fuzzy Rule 1 IF ω_1 is μ_{11} and $\cdots$ ω_s is μ_{1s}, THEN

$$D^\alpha u(t) = C_1 u(t) + A_1 f\big(u(t)\big) + B_1 g\big(u(t-\tau)\big) + I_1(t), \tag{14}$$

$$D^\alpha v(t) = C_1 v(t) + A_1 f\big(v(t)\big) + B_1 g\big(v(t-\tau)\big) - \Phi e(t) + I_1(t). \tag{15}$$

Fuzzy Rule 2 IF ω_1 is μ_{21} and $\cdots$ ω_s is μ_{2s}, THEN

$$D^\alpha u(t) = C_2 u(t) + A_2 f\big(u(t)\big) + B_2 g\big(u(t-\tau)\big) + I_2(t), \tag{16}$$

$$D^\alpha v(t) = C_2 v(t) + A_2 f\big(v(t)\big) + B_2 g\big(v(t-\tau)\big) - \Phi e(t) + I_2(t). \tag{17}$$

Using a standard fuzzy inference method, system (12)–(13) is inferred as follows:

$$D^\alpha u(t) = \sum_{k=1}^{2} h_k(\omega)\big[C_k u(t) + A_k f\big(u(t)\big) + B_k g\big(u(t-\tau)\big) + I_k(t)\big], \tag{18}$$

$$D^\alpha v(t) = \sum_{k=1}^{2} h_k(\omega)\big[C_k v(t) + A_k f\big(v(t)\big) + B_k g\big(v(t-\tau)\big) - \Phi e(t) + I_k(t)\big], \tag{19}$$

with $r = 2$,

$$f_1\big(u_1(t)\big) = g_1\big(u_1(t)\big) = \frac{1}{1+e^{-u_1(t)}},$$
$$f_2\big(u_2(t)\big) = g_2\big(u_2(t)\big) = \frac{1}{1+e^{-u_2(t)}}.$$

Correspondingly,

$$u(t) = \begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix}, \qquad v(t) = \begin{pmatrix} v_1(t) \\ v_2(t) \end{pmatrix},$$
$$A_1 = B_1 = \begin{pmatrix} -1 & 0.4 \\ 0 & -0.1 \end{pmatrix}, \qquad C_1 = \begin{pmatrix} -4.5 & 0 \\ 0 & -0.5 \end{pmatrix},$$
$$A_2 = B_2 = \begin{pmatrix} 1 & -0.8 \\ 0.4 & 0.5 \end{pmatrix}, \qquad C_2 = \begin{pmatrix} -2.1 & 0 \\ 0 & -2.8 \end{pmatrix},$$
$$\Phi = \begin{pmatrix} 11 & 0 \\ 0 & 11 \end{pmatrix}, \qquad I_1(t) = \begin{pmatrix} \sin 2t \\ -\cos t \end{pmatrix}, \qquad I_2(t) = \begin{pmatrix} \sin t \\ \cos t \end{pmatrix}.$$

Taking $h_1(\omega) = \sin^2(10\tanh\frac{\pi(t+2)}{2})$, $h_2(\omega) = \cos^2(10\tanh\frac{\pi(t+2)}{2})$. Select $\tau = 1$, $\alpha = 0.9$, and the initial conditions of $u(t)$ and $v(t)$

$$x(0) = \begin{pmatrix} -4.2 \\ 1.2 \end{pmatrix}, \qquad y(0) = \begin{pmatrix} 2.1 \\ -1.7 \end{pmatrix}.$$

Based on these parameters, we obtain the solution of the linear matrix inequality (10) by using Matlab LMI toolbox:

$$P = \begin{pmatrix} 0.1490 & 0.0001 \\ 0.0001 & 0.1490 \end{pmatrix}, \qquad Q = \begin{pmatrix} 1.0569 & 0.003 \\ 0.003 & 1.0569 \end{pmatrix},$$
$$R = \begin{pmatrix} 1.0986 & 0.0003 \\ 0.0003 & 1.1114 \end{pmatrix}, \qquad S = \begin{pmatrix} 0.9998 & 0.0009 \\ 0.009 & 1.0384 \end{pmatrix},$$

$$V = \begin{pmatrix} 0.1206 & 0 \\ 0 & 0.1206 \end{pmatrix}.$$

Obviously, P, Q, R, S, and V are positive definite matrices. The simulation results for the synchronization of our drive-master systems are shown in Figs. 1–3. In this numerical example, we employed the first type of Lagrange interpolation approximation to draw the image of the fractional-order Caputo derivative.

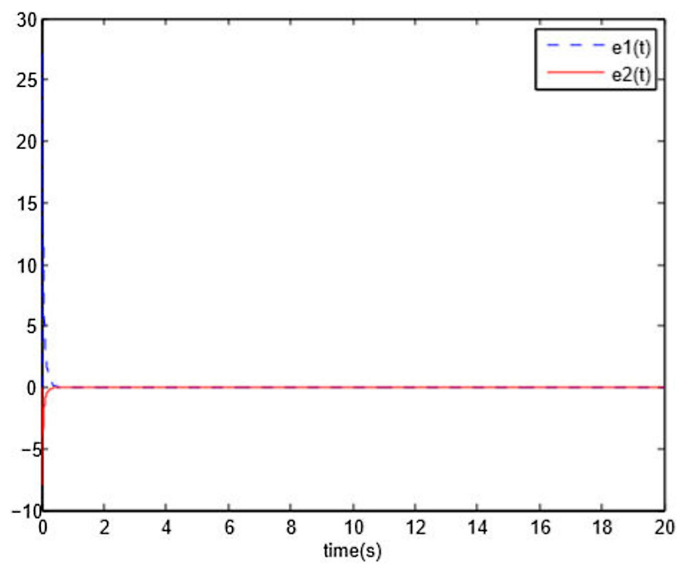

Figure 1 Dynamics of states $u_1(t)$ and $v_1(t)$

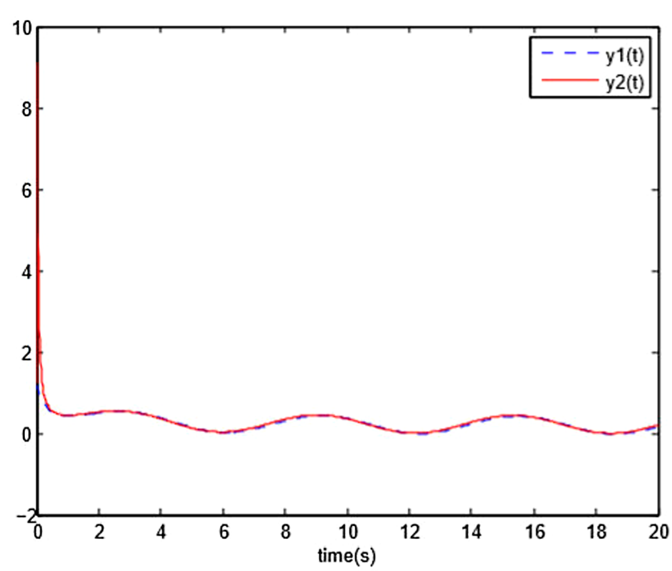

Figure 2 Dynamics of states $u_2(t)$ and $v_2(t)$

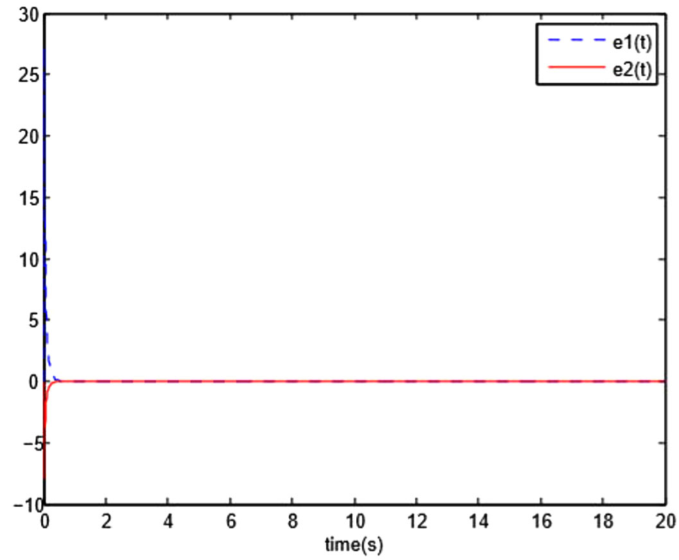

Figure 3 Dynamics of states $e_1(t)$ and $e_2(t)$

Acknowledgements
The authors thank the editor and anonymous referees for their valuable suggestions and comments, which improved the presentation of this paper.

Funding
This work was supported in part by the National Natural Science Foundation of China under grants No. 11671176 and No. 61573004, the Natural Science Foundation of Zhejiang Province under grant No. LY15A010007, the Natural Science Foundation of Fujian Province under grant No. 2018J01001, the Start-up Fund of Huaqiao University, the Natural Science Foundation of Shandong Province (China) (grant No. ZR2018MA018), Subsidized Project for Postgraduates' Innovative Fund in Scientific Research of Huaqiao University, partially supported by the Youth Foundation Ability Promotion Plan in Guangxi (KY2016YB404).

Availability of data and materials
All data are fully available without restriction.

Competing interests
The authors declare that they have no conflict of interests.

Authors' contributions
BZ carried out the computations in the proof, participated in the sequence alignment, and drafted the manuscript. JZ carried out the simulations and figures. HL helped to draft the manuscript. JC participated in the discussion of the study. YX conceived of the study, proposed the project, and drafted the manuscript. All authors read and approved the final manuscript.

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Publisher's Note

Received: 17 August 2018 Accepted: 4 December 2018 Published online: 29 December 2018

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