Advances in Difference Equations
a SpringerOpen Journal

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Dispersive analytical soliton solutions of some nonlinear waves dynamical models via modified mathematical methods

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Abstract

We have employed a modified form of the F-expansion technique on four nonlinear waves models and obtained numerous exact and soliton wave solutions in a different form. New constructed solutions showed the significance of these models. The constructed results have plenty applications in nonlinear research.

Keywords: Complex fractional Kundu–Eckhaus equation; Van der Waals normal form for fluidized granular matter equation; Benney–Luke equation; The (3 + 1)-dimensional modified Korteweg–de Vries–Zakharov–Kuznetsov equation; Riccati equation; Modified F-expansion method; Exact and solitary wave solutions

1 Introduction

The exact solutions to NLEEs lead to basic knowledge of the structure of physical phenomena. Therefore researchers have been interested in studying and seeking the derived exact solutions and made a great effort in it due to the great importance in nonlinear science. It is a fact that there is no unique method which is fruitful to solve all kinds of nonlinear wave problems. Exact traveling wave solutions to nonlinear systems of PDEs are essential for analyzing natural phenomena in wide areas of the physical sciences [1–15].

The exploitation of a symbolic computation package will make it realistic to propose a number of direct analytical methods. The research of traveling wave solutions of some nonlinear evolution equations derived from such fields played an important role in the analysis of some phenomena such as the $\exp(-\varphi(\xi))$ method, Bernoulli's sub-ODE method, the homogeneous balance method, the modified simple equation method, the modified extended direct algebraic method, the modified extended mapping method, the Kudryashov method, the extended sinh–cosh and sin–cos methods, the Lie symmetry method, the soliton ansatz method and many more methods [15–40].

Previous authors in [24] applied a new auxiliary technique on the complex fractional Kundu–Eckhaus equation and in [23] applied six methods on the van der Waals normal form for fluidized granular matter wave equations. Further the authors in [26] derived results on a Benney-like equation with the help of a modified simple equation method. Similarly, in [28] one employed an auxiliary equation method for the (3 + 1)-dimensional modified Korteweg–de Vries–Zakharov–Kuznetsov equation. But here our decision is to investigate a novel soliton of models in Eq. (1), Eq. (23), Eq. (58) and Eq. (100) by employing the modified F-expansion method. Our obtained solutions are more powerful than those in the previous existing literature. These solutions have potential applications to handle nonlinear problems in mathematics and physics.

This article is organized as follows: Applications of the modified F-expansion method are described in Sect. 2, Discussion of our results with other results is in Sect. 3. Our conclusion is in Sect. 4.

2 Applications

2.1 The nonlinear integer order Kundu–Eckhaus equation

Consider the general form of the complex fractional Kundu–Eckhaus equation [24],

$$i\frac{\partial^\alpha \nu}{\partial t^\alpha} + \nu_{xx} - 2\beta|\nu|^2\nu + \gamma^2|\nu|^4\nu + 2i\gamma\left(|\nu|^2\right)_x\nu = 0, \quad 0 < \alpha < 1. \tag{1}$$

The above model having applications in optical fiber, quantum field theory and in dispersive water waves. Let a wave transformation be $\nu(x,t) = u(\xi)e^{i\eta}$ and, moreover, $\xi = ik(x - \frac{2\mu t^\alpha}{\alpha})$, $\eta = (\mu x + \frac{\epsilon t^\alpha}{\alpha})$ in Eq. (1) for conversion of the complex fractional Kundu–Eckhaus equation to integer order; for further details about the conformable fractional derivative see [20–22]. We have

$$\begin{aligned} &\frac{\partial^\alpha \nu}{\partial t^\alpha} = i\left(\epsilon u - 2\mu k u'\right)e^{i\eta}, \qquad \frac{\partial^2 \nu}{\partial x^2} = -\left(\mu^2 u + 2\mu k u' + k^2 u''\right)e^{i\eta}, \\ &\frac{\partial}{\partial x}\left(|\nu|^2\nu\right) = 2iku^2u'e^{i\eta}. \end{aligned} \tag{2}$$

Substituting (2) in (1), this action gives an ODE such that

$$-\left(\epsilon + \mu^2\right)u - k^2u'' - 2\beta u^3 + \gamma^2u^5 - 4\gamma k u^2 u' = 0. \tag{3}$$

Balancing between u'' and u^5 in (3), we obtain $N = \frac{1}{2}$. Now using another transformation $u(\xi) = \Psi^{\frac{1}{2}})$ on (3), we obtain

$$-4\left(\epsilon + \mu^2\right)\Psi^2 + k^2\Psi'^2 - 2k^2\Psi\Psi'' - 8\beta\Psi^3 + 4\gamma^2\Psi^4 - 8k\Psi^2\Psi' = 0. \tag{4}$$

Balancing between the highest derivative and nonlinear term in (4), we obtain N=1. Assuming (4) has a solution [25],

$$\Psi(\xi) = a_0 + a_1F(\xi) + b_1F^{-1}(\xi), \tag{5}$$

where F satisfies the Ricatti equation [25]

$$F'(\xi) = B_1 + B_2F + B_3F^2. \tag{6}$$

Put (5) with (6) in (4), we obtain numerous equations involving these parameters, a_0, a_1, b_1, B_1, B_2, B_3, β, γ, k, μ and ϵ, after solving this equation system, we have the following solution possibilities.

2.1.1 The soliton-like solutions of Eq. (1)

If $B_1 = 0$, $B_2 = 1$, $B_3 = -1$, then we have the following results:

$$\begin{aligned} &k = \frac{\sqrt{7}\beta + \beta}{2\gamma}, \qquad \mu = \pm\frac{\sqrt{-\frac{\sqrt{7}\beta^2}{\gamma^2} - \frac{4\beta^2}{\gamma^2} - 8\epsilon}}{2\sqrt{2}}, \\ &‘a_1 = \frac{5\beta - \sqrt{7}\beta}{4\gamma^2}, \qquad a_0 = 0, \qquad b_1 = 0. \end{aligned} \tag{7}$$

Put (7) in (5), then the solution of (1) becomes

$$\Psi_1(\xi) = \left(\frac{5\beta - \sqrt{7}\beta}{8\gamma^2}\left(1 + \tanh\left(\frac{1}{2}\xi\right)\right)\right)^{\frac{1}{2}} e^{i\eta}. \tag{8}$$

If $B_1 = 0$, $B_2 = -1$, $B_3 = 1$, then we have the following results:

$$\begin{aligned} &k = \frac{-\sqrt{7}\beta - \beta}{2\gamma}, \qquad \mu = \pm\frac{\sqrt{-\frac{\sqrt{7}\beta^2}{\gamma^2} - \frac{4\beta^2}{\gamma^2} - 8\epsilon}}{2\sqrt{2}}, \\ &a_1 = \frac{5\beta - \sqrt{7}\beta}{4\gamma^2}, \qquad a_0 = 0, \qquad b_1 = 0. \end{aligned} \tag{9}$$

Substitute (9) in (5)

$$\Psi_2(\xi) = \left(\frac{5\beta - \sqrt{7}\beta}{8\gamma^2}\left(1 - \coth\left(\frac{1}{2}\xi\right)\right)\right)^{\frac{1}{2}} e^{i\eta}. \tag{10}$$

If $B_1 = \frac{1}{2}$, $B_2 = 0$, $B_3 = -\frac{1}{2}$, then we have the following results:

$$\begin{aligned} &k = \frac{3(\sqrt{7}\beta - 2\beta)}{(\sqrt{7} - 5)\gamma}, \qquad \mu = \pm\frac{\sqrt{8(5\sqrt{7} - 16)\gamma^2\epsilon + 9(4\sqrt{7} - 11)\beta^2}}{2(\sqrt{7} - 5)\gamma}, \\ &a_1 = -\frac{9\beta}{4(\sqrt{7} - 5)\gamma^2}, \qquad a_0 = \frac{\sqrt{7}\beta + 5\beta}{8\gamma^2}, \qquad b_1 = 0. \end{aligned} \tag{11}$$

Put (11) in (5), we obtain

$$\Psi_3(\xi) = \left(\frac{\sqrt{7}\beta + 5\beta}{8\gamma^2} - \frac{9\beta}{4(\sqrt{7} - 5)\gamma^2}\big(\coth(\xi) \pm csch(\xi)\big)\right)^{\frac{1}{2}} e^{i\eta}. \tag{12}$$

If $B_1 = 1$, $B_2 = 0$, $B_3 = -1$, then we have the following results:

$$\begin{aligned} &k = -\frac{3(\sqrt{7}\beta - 2\beta)}{(\sqrt{7} - 5)\gamma}, \qquad \mu = \pm\frac{\sqrt{8(5\sqrt{7} - 16)\gamma^2\epsilon + 9(4\sqrt{7} - 11)\beta^2}}{2(\sqrt{7} - 5)\gamma}, \\ &a_1 = \frac{9\beta}{4(\sqrt{7} - 5)\gamma^2}, \qquad a_0 = \frac{\sqrt{7}\beta + 5\beta}{8\gamma^2}, \qquad b_1 = 0. \end{aligned} \tag{13}$$

Put (13) in (5),

$$\Psi_4 = \left(\frac{\sqrt{7}\beta + 5\beta}{8\gamma^2} + \frac{9\beta}{4(\sqrt{7}-5)\gamma^2}\big(\tanh(\xi)\big)\right)^{\frac{1}{2}} e^{i\eta}. \tag{14}$$

2.1.2 The trigonometric function solutions of Eq. (1)

If $B_1 = \frac{1}{2}$, $B_2 = 0$, $B_3 = \frac{1}{2}$, then we have the following results:

$$\begin{aligned} &k = \frac{(\sqrt{7}-2)\sqrt{32+10\sqrt{7}}\sqrt{-\beta^2}}{6\gamma}, \qquad \mu = \pm\frac{\sqrt{-8\gamma^2\epsilon + \sqrt{7}\beta^2 - 4\beta^2}}{2\sqrt{2}\gamma}, \\ &a_1 = \frac{\sqrt{-5\sqrt{7}\beta^2 - 16\beta^2}}{4\sqrt{2}\gamma^2}, \qquad a_0 = \frac{\sqrt{7}\beta + 5\beta}{8\gamma^2}, \qquad b_1 = 0. \end{aligned} \tag{15}$$

Putting (15) in (5), then solution (1) is

$$\Psi_5(\xi) = \left(\frac{\sqrt{7}\beta + 5\beta}{8\gamma^2} + \frac{\sqrt{-5\sqrt{7}\beta^2 - 16\beta^2}}{4\sqrt{2}\gamma^2}\big(\sec(\xi) + \tan(\xi)\big)\right)^{\frac{1}{2}} e^{i\eta}. \tag{16}$$

If $B_1 = -\frac{1}{2}$, $B_2 = 0$, $B_3 = -\frac{1}{2}$, then we have the following results:

$$\begin{aligned} &k = -\frac{(\sqrt{7}-2)\sqrt{32+10\sqrt{7}}\sqrt{-\beta^2}}{6\gamma}, \qquad \mu = \pm\frac{\sqrt{-8\gamma^2\epsilon + \sqrt{7}\beta^2 - 4\beta^2}}{2\sqrt{2}\gamma}, \\ &a_1 = \frac{\sqrt{-5\sqrt{7}\beta^2 - 16\beta^2}}{4\sqrt{2}\gamma^2}, \qquad a_0 = \frac{\sqrt{7}\beta + 5\beta}{8\gamma^2}, \qquad b_1 = 0. \end{aligned} \tag{17}$$

Put (17) in Eq. (5),

$$\Psi_6(\xi) = \left(\frac{\sqrt{7}\beta + 5\beta}{8\gamma^2} + \frac{\sqrt{-5\sqrt{7}\beta^2 - 16\beta^2}}{4\sqrt{2}\gamma^2}\big(\sec(\xi) - \tan(\xi)\big)\right)^{\frac{1}{2}} e^{i\eta}. \tag{18}$$

If $B_1 = 1(-1)$, $B_2 = 0$, $B_3 = 1(-1)$, then we have the following results:

$$\begin{aligned} &k = -\frac{(\sqrt{7}-2)\sqrt{8+\frac{5\sqrt{7}}{2}}\sqrt{-\beta^2}}{6\gamma}, \qquad \mu = \pm\frac{\sqrt{-8\gamma^2\epsilon + \sqrt{7}\beta^2 - 4\beta^2}}{2\sqrt{2}\gamma}, \\ &a_1 = \frac{\sqrt{-5\sqrt{7}\beta^2 - 16\beta^2}}{4\sqrt{2}\gamma^2}, \qquad a_0 = \frac{\sqrt{7}\beta + 5\beta}{8\gamma^2}, \qquad b_1 = 0. \end{aligned} \tag{19}$$

Replace (19) in (5),

$$\Psi_7(\xi) = \left(\frac{\sqrt{7}\beta + 5\beta}{8\gamma^2} + \frac{\sqrt{-5\sqrt{7}\beta^2 - 16\beta^2}}{4\sqrt{2}\gamma^2}\big(\tan(\xi)\big(\cot(\xi)\big)\big)\right)^{\frac{1}{2}} e^{i\eta}. \tag{20}$$

2.1.3 The rational function solutions of Eq. (1)

If $B_3 = 0$, then we have the following results:

$$\begin{aligned} &k = \frac{\sqrt{7}\beta - \beta}{2B_2\gamma}, \qquad \mu \pm \frac{\sqrt{\frac{\sqrt{7}\beta^2}{\gamma^2} - \frac{4\beta^2}{\gamma^2} - 8\epsilon}}{2\sqrt{2}}, \\ &a_1 = 0, \qquad a_0 = 0, \qquad b_1 = \frac{-\sqrt{7}B_1\beta - 5B_1\beta}{4B_2\gamma^2}. \end{aligned} \tag{21}$$

Transfer (21) in (5),

$$\Psi_8(\xi) = \left(\frac{-\sqrt{7}B_1\beta - 5B_1\beta}{4\gamma^2}\left(\frac{1}{(\exp(B_2\xi) - B_1)}\right)\right)^{\frac{1}{2}} e^{i\eta}. \tag{22}$$

2.2 The van der Waals normal form for fluidized granular matter wave equation

Consider the generalized form of the nonlinear the van der Waals normal form for fluidized granular matter wave model [23] given by

$$\frac{\partial^2 v}{\partial t^2} + \frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 v}{\partial x^2} - \eta\frac{\partial v}{\partial t} - v^3 - \epsilon v\right) = 0, \tag{23}$$

where η is the bulk viscosity, ϵ is for the bifurcation parameter. The extreme advantage of this model is that it is used to explain the basic physical phase phenomenon in nonlinear science.

Let us make a traveling wave transformation for (23),

$$v(x,t) = \Psi(\xi), \qquad \xi = kx + \omega t. \tag{24}$$

Substituting (24) in (23) and twice integrating with zero constant, we obtain

$$\Psi + k^4\frac{1}{\omega^2(1 - \frac{k\epsilon}{\omega^2})}\Psi'' + \frac{\eta k^2\Psi'}{\omega(1 - \frac{k\epsilon}{\omega^2})} + \frac{k^2\Psi^3}{\omega^2(1 - \frac{k\epsilon}{\omega^2})} = 0. \tag{25}$$

Suppose that (25) has the same solution form (5), we obtain collections of equations involving these parameters, $a_0, a_1, b_1, B_1, B_2, B_3, k$ and ω. After solving we have the following results.

2.2.1 The soliton-like solutions of Eq. (23)

If $B_1 = 0$, $B_2 = 1$, $B_3 = -1$, then we have the following results:

$$\begin{aligned} &k = \frac{(-1)^{2/3}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt[3]{9 - 2\eta^2}}, \qquad \omega = \frac{3\sqrt[3]{-1}\sqrt[3]{\eta}\epsilon^{2/3}}{(9 - 2\eta^2)^{2/3}}, \\ &a_1 = \pm\frac{\sqrt[6]{-1}\sqrt{2}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt[3]{9 - 2\eta^2}}, \qquad a_0 = \pm\frac{\sqrt[6]{-1}\sqrt{2}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt[3]{9 - 2\eta^2}}, \qquad b_1 = 0. \end{aligned} \tag{26}$$

Put (26) in (5),

$$\Psi_9(\xi) = \frac{\sqrt[6]{-1}\sqrt{2}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt[3]{9 - 2\eta^2}}\left(\pm 1 \pm \frac{1}{2}\left(1 + \tanh\left(\frac{1}{2}\xi\right)\right)\right). \tag{27}$$

If $B_1 = 0$, $B_2 = -1$, $B_3 = 1$, then we have the following results:

$$\begin{aligned}
&k = \frac{(-1)^{2/3}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt[3]{9-2\eta^2}}, \qquad \omega = \frac{3\sqrt[3]{-1}\sqrt[3]{\eta}\epsilon^{2/3}}{(9-2\eta^2)^{2/3}}, \qquad a_1 = \pm\frac{\sqrt[6]{-1}\sqrt{2}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt[3]{9-2\eta^2}}, \\
&a_0 = \pm\frac{\sqrt[6]{-1}\sqrt{2}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt[3]{9-2\eta^2}}, \qquad b_1 = 0.
\end{aligned} \tag{28}$$

Put (28) in (5), then the solution of (23) becomes

$$\Psi_{10}(\xi) = \frac{\sqrt[6]{-1}\sqrt{2}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt[3]{9-2\eta^2}}\left(\pm 1 \pm \frac{1}{2}\left(1 - \coth\left(\frac{1}{2}\xi\right)\right)\right). \tag{29}$$

If $B_1 = \frac{1}{2}$, $B_2 = 0$, $B_3 = -\frac{1}{2}$, then we have the following results:

$$\begin{aligned}
&k = \pm\frac{(-1)^{2/3}\eta^{2/3}\sqrt[3]{\epsilon}}{2^{2/3}\sqrt[3]{9-2\eta^2}}, \qquad \omega \pm \frac{3(-1)^{5/6}\sqrt[3]{\eta}\epsilon^{2/3}}{\sqrt[3]{2}\sqrt[6]{9-2\eta^2}\sqrt{2\eta^2-9}}, \\
&a_1 = \pm\frac{(-1)^{2/3}\eta^{2/3}\sqrt[6]{9-2\eta^2}\sqrt[3]{\epsilon}}{2\sqrt[6]{2}\sqrt{2\eta^2-9}}, \\
&a_0 = \pm\frac{\sqrt[6]{-\frac{1}{2}}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt[3]{9-2\eta^2}}, \qquad b_1 = \pm\frac{(-1)^{2/3}\eta^{2/3}\sqrt[6]{9-2\eta^2}\sqrt[3]{\epsilon}}{2\sqrt[6]{2}\sqrt{2\eta^2-9}}.
\end{aligned} \tag{30}$$

Replace (30) in Eq. (5),

$$\begin{aligned}
\Psi_{11}(\xi) = {}& \frac{\sqrt[6]{-\frac{1}{2}}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt[3]{9-2\eta^2}} \\
&\pm \frac{(-1)^{2/3}\eta^{2/3}\sqrt[6]{9-2\eta^2}\sqrt[3]{\epsilon}}{2\sqrt[6]{2}\sqrt{2\eta^2-9}}\left(\coth(\xi) \pm csch(\xi) + \frac{1}{\coth(\xi) \pm csch(\xi)}\right).
\end{aligned} \tag{31}$$

If $B_1 = 1$, $B_2 = 0$, $B_3 = -1$, then we have the following results:
Family I

$$\begin{aligned}
&k = \frac{(-1)^{2/3}\eta^{2/3}\sqrt[3]{\epsilon}}{2^{2/3}\sqrt[3]{9-2\eta^2}}, \qquad \omega = \pm\frac{3(-1)^{5/6}\sqrt[3]{\eta}\epsilon^{2/3}}{\sqrt[3]{2}\sqrt[6]{9-2\eta^2}\sqrt{2\eta^2-9}}, \qquad a_1 = 0, \\
&a_0 = -\frac{\sqrt[6]{-\frac{1}{2}}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt[3]{9-2\eta^2}}, \qquad b_1 = \pm\frac{(-1)^{2/3}\eta^{2/3}\sqrt[6]{9-2\eta^2}\sqrt[3]{\epsilon}}{\sqrt[6]{2}\sqrt{2\eta^2-9}}.
\end{aligned} \tag{32}$$

Substitute (32) in (5),

$$\Psi_{12}(\xi) = -\frac{\sqrt[6]{-\frac{1}{2}}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt[3]{9-2\eta^2}} \pm \frac{(-1)^{2/3}\eta^{2/3}\sqrt[6]{9-2\eta^2}\sqrt[3]{\epsilon}}{\sqrt[6]{2}\sqrt{2\eta^2-9}}\left(\frac{1}{\tanh(\xi)}\right). \tag{33}$$

Family II

$$k = \frac{(-1)^{2/3}\eta^{2/3}\sqrt[3]{\epsilon}}{2^{2/3}\sqrt[3]{9-2\eta^2}}, \qquad \omega \pm \frac{3(-1)^{5/6}\sqrt[3]{\eta}\epsilon^{2/3}}{\sqrt[3]{2}\sqrt[6]{9-2\eta^2}\sqrt{2\eta^2-9}},$$
$$a_1 = \pm\frac{(-1)^{2/3}\eta^{2/3}\sqrt[6]{9-2\eta^2}\sqrt[3]{\epsilon}}{\sqrt[6]{2}\sqrt{2\eta^2-9}}, \qquad a_0 = \frac{\sqrt[6]{-\frac{1}{2}}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt[3]{9-2\eta^2}}, \qquad b_1 = 0. \tag{34}$$

Put (34) in (5),

$$\Psi_{13} = \frac{\sqrt[6]{-\frac{1}{2}}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt[3]{9-2\eta^2}} \pm \frac{(-1)^{2/3}\eta^{2/3}\sqrt[6]{9-2\eta^2}\sqrt[3]{\epsilon}}{\sqrt[6]{2}\sqrt{2\eta^2-9}}\bigl(\tanh(\xi)\bigr). \tag{35}$$

Family III

$$k = \frac{(-1)^{2/3}\eta^{2/3}\sqrt[3]{\epsilon}}{2\sqrt[3]{2}\sqrt[3]{9-2\eta^2}}, \qquad \omega = \pm\frac{3(-1)^{5/6}\sqrt[3]{\eta}\epsilon^{2/3}}{2^{2/3}\sqrt[6]{9-2\eta^2}\sqrt{2\eta^2-9}},$$
$$a_1 = \pm\frac{(-1)^{2/3}\eta^{2/3}\sqrt[6]{9-2\eta^2}\sqrt[3]{\epsilon}}{2^{5/6}\sqrt{2\eta^2-9}},$$
$$a_0 = \frac{\sqrt[6]{-2}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt[3]{9-2\eta^2}}, \qquad \text{‘}b_1 = \pm\frac{(-1)^{2/3}\eta^{2/3}\sqrt[6]{9-2\eta^2}\sqrt[3]{\epsilon}}{2^{5/6}\sqrt{2\eta^2-9}}. \tag{36}$$

Transfer (36) in Eq. (5),

$$\Psi_{14} = \frac{\sqrt[6]{-2}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt[3]{9-2\eta^2}} \pm \frac{(-1)^{2/3}\eta^{2/3}\sqrt[6]{9-2\eta^2}\sqrt[3]{\epsilon}}{2^{5/6}\sqrt{2\eta^2-9}}\left(\tanh(\xi) + \frac{1}{\tanh(\xi)}\right). \tag{37}$$

2.2.2 The trigonometric function solutions of Eq. (23)

If $B_1 = \frac{1}{2}$, $B_2 = 0$, $B_3 = \frac{1}{2}$, then we have the following results:

Family I

$$k = -\frac{\sqrt[3]{-1}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt[3]{2\eta^2-9}}, \qquad \omega = \pm\frac{3\sqrt[6]{-1}\sqrt[3]{\eta}\epsilon^{2/3}}{(2\eta^2-9)^{2/3}}, \qquad a_1 = \pm\frac{(-1)^{5/6}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt{2}\sqrt[3]{2\eta^2-9}},$$
$$a_0 = -\frac{\sqrt[3]{-1}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt{2}\sqrt[3]{2\eta^2-9}}, \qquad b_1 = 0. \tag{38}$$

Substitute (38) in (5),

$$\Psi_{15} = -\frac{\sqrt[3]{-1}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt{2}\sqrt[3]{2\eta^2-9}} \pm \frac{(-1)^{5/6}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt{2}\sqrt[3]{2\eta^2-9}}\bigl(\sec(\xi) + \tan(\xi)\bigr). \tag{39}$$

Family II

$$k = \frac{(-1)^{2/3}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt[3]{2\eta^2-9}}, \qquad \omega = \pm\frac{3(-1)^{5/6}\sqrt[3]{\eta}\epsilon^{2/3}}{(2\eta^2-9)^{2/3}}, \qquad a_1 = 0,$$
$$a_0 = \pm\frac{(-1)^{2/3}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt{2}\sqrt[3]{2\eta^2-9}}, \qquad b_1 = -\frac{\sqrt[6]{-1}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt{2}\sqrt[3]{2\eta^2-9}}. \tag{40}$$

Put (40) in (5),

$$\Psi_{16} = \pm\frac{(-1)^{2/3}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt{2}\sqrt[3]{2\eta^2-9}} - \frac{\sqrt[6]{-1}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt{2}\sqrt[3]{2\eta^2-9}}\left(\frac{1}{\sec(\xi)+\tan(\xi)}\right). \tag{41}$$

Family III

$$\begin{aligned} &k = -\frac{\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt[3]{36-8\eta^2}}, \qquad \omega = \pm\frac{3\sqrt[3]{\eta}\epsilon^{2/3}}{\sqrt[6]{36-8\eta^2}\sqrt{2\eta^2-9}}, \qquad a_1 = \frac{\eta^{2/3}\sqrt[6]{36-8\eta^2}\sqrt[3]{\epsilon}}{2\sqrt{2}\sqrt{2\eta^2-9}}, \\ &a_0 = \pm\frac{\sqrt{2}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt[3]{36-8\eta^2}}, \qquad b_1 = -\frac{\eta^{2/3}\sqrt[6]{9-2\eta^2}\sqrt[3]{\epsilon}}{2\sqrt[6]{2}\sqrt{2\eta^2-9}}. \end{aligned} \tag{42}$$

Replace (42) in (5),

$$\begin{aligned} \Psi_{17} = {} & \pm\frac{\sqrt{2}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt[3]{36-8\eta^2}} + \frac{\eta^{2/3}\sqrt[6]{36-8\eta^2}\sqrt[3]{\epsilon}}{2\sqrt{2}\sqrt{2\eta^2-9}}\bigl(\sec(\xi)+\tan(\xi)\bigr) \\ & - \frac{\eta^{2/3}\sqrt[6]{9-2\eta^2}\sqrt[3]{\epsilon}}{2\sqrt[6]{2}\sqrt{2\eta^2-9}}\left(\frac{1}{\sec(\xi)+\tan(\xi)}\right). \end{aligned} \tag{43}$$

If $B_1 = -\frac{1}{2}$, $B_2 = 0$, $B_3 = -\frac{1}{2}$, then we have the following results:
Family I

$$\begin{aligned} &k = \frac{(-1)^{2/3}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt[3]{2\eta^2-9}}, \qquad `\omega = \pm\frac{3(-1)^{5/6}\sqrt[3]{\eta}\epsilon^{2/3}}{(2\eta^2-9)^{2/3}}, \qquad a_1 = \frac{\sqrt[6]{-1}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt{2}\sqrt[3]{2\eta^2-9}}, \\ &a_0 = \frac{(-1)^{2/3}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt{2}\sqrt[3]{2\eta^2-9}}, \qquad b_1 = 0. \end{aligned} \tag{44}$$

Put (44) in (5),

$$\Psi_{18} = \frac{(-1)^{2/3}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt{2}\sqrt[3]{2\eta^2-9}} + \frac{\sqrt[6]{-1}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt{2}\sqrt[3]{2\eta^2-9}}\bigl(\sec(\xi)-\tan(\xi)\bigr). \tag{45}$$

Family II

$$\begin{aligned} &k = \frac{(-1)^{2/3}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt[3]{2\eta^2-9}}, \qquad \omega = \pm\frac{3(-1)^{5/6}\sqrt[3]{\eta}\epsilon^{2/3}}{(2\eta^2-9)^{2/3}}, \qquad a_1 = 0, \\ &a_0 = \pm\frac{(-1)^{2/3}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt{2}\sqrt[3]{2\eta^2-9}}, \qquad b_1 = \frac{\sqrt[6]{-1}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt{2}\sqrt[3]{2\eta^2-9}}. \end{aligned} \tag{46}$$

Substitute (46) in Eq. (5),

$$\Psi_{19}(\xi) = \pm\frac{(-1)^{2/3}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt{2}\sqrt[3]{2\eta^2-9}} + \frac{\sqrt[6]{-1}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt{2}\sqrt[3]{2\eta^2-9}}\left(\frac{1}{\sec(\xi)-\tan(\xi)}\right). \tag{47}$$

Family III

$$k = -\frac{\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt[3]{36-8\eta^2}}, \qquad \omega = \pm\frac{3\sqrt[3]{\eta}\epsilon^{2/3}}{\sqrt[6]{36-8\eta^2}\sqrt{2\eta^2-9}}, \qquad a_1 = \pm\frac{\eta^{2/3}\sqrt[6]{9-2\eta^2}\sqrt[3]{\epsilon}}{2\sqrt[6]{2}\sqrt{2\eta^2-9}},$$
$$a_0 = \pm\frac{\sqrt{2}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt[3]{36-8\eta^2}}, \qquad b_1 = \frac{\eta^{2/3}\sqrt[6]{36-8\eta^2}\sqrt[3]{\epsilon}}{2\sqrt{2}\sqrt{2\eta^2-9}}. \tag{48}$$

Put (48) in (5),

$$\Psi_{20}(\xi) = \pm\frac{\sqrt{2}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt[3]{36-8\eta^2}} + \frac{\eta^{2/3}\sqrt[6]{36-8\eta^2}\sqrt[3]{\epsilon}}{2\sqrt{2}\sqrt{2\eta^2-9}}\left(\frac{1}{\sec(\xi)-\tan(\xi)}\right)$$
$$\pm\frac{\eta^{2/3}\sqrt[6]{9-2\eta^2}\sqrt[3]{\epsilon}}{2\sqrt[6]{2}\sqrt{2\eta^2-9}}\bigl(\sec(\xi)-\tan(\xi)\bigr). \tag{49}$$

If $B_1 = 1(-1)$, $B_2 = 0$, $B_3 = 1(-1)$, then we have the following results:

Family I

$$k = -\frac{(-1)^{2/3}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt[3]{36-8\eta^2}}, \qquad \omega = \pm\frac{3\sqrt[3]{-1}\sqrt[3]{\eta}\epsilon^{2/3}}{\sqrt[6]{36-8\eta^2}\sqrt{2\eta^2-9}},$$
$$a_1 = \pm\frac{(-1)^{2/3}\eta^{2/3}\sqrt[6]{9-2\eta^2}\sqrt[3]{\epsilon}}{\sqrt[6]{2}\sqrt{2\eta^2-9}}, \qquad a_0 = -\frac{(-1)^{2/3}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt[6]{2}\sqrt[3]{9-2\eta^2}}, \qquad b_1 = 0. \tag{50}$$

Replace (50) in (5),

$$\Psi_{21}(\xi) = -\frac{(-1)^{2/3}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt[6]{2}\sqrt[3]{9-2\eta^2}} \pm \frac{(-1)^{2/3}\eta^{2/3}\sqrt[6]{9-2\eta^2}\sqrt[3]{\epsilon}}{\sqrt[6]{2}\sqrt{2\eta^2-9}}\bigl(\tan(\xi)\bigl(\cot(\xi)\bigr)\bigr). \tag{51}$$

Family II

$$k = -\frac{(-1)^{2/3}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt[3]{36-8\eta^2}}, \qquad \omega \pm \frac{3\sqrt[3]{-1}\sqrt[3]{\eta}\epsilon^{2/3}}{\sqrt[6]{36-8\eta^2}\sqrt{2\eta^2-9}}, \qquad a_1 = 0,$$
$$a_0 \pm \frac{(-1)^{2/3}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt[6]{2}\sqrt[3]{9-2\eta^2}}, \qquad b_1 = -\frac{(-1)^{2/3}\eta^{2/3}\sqrt[6]{9-2\eta^2}\sqrt[3]{\epsilon}}{\sqrt[6]{2}\sqrt{2\eta^2-9}}. \tag{52}$$

Substituting (52) in (5),

$$\Psi_{22}(\xi) = \pm\frac{(-1)^{2/3}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt[6]{2}\sqrt[3]{9-2\eta^2}} - \frac{(-1)^{2/3}\eta^{2/3}\sqrt[6]{9-2\eta^2}\sqrt[3]{\epsilon}}{\sqrt[6]{2}\sqrt{2\eta^2-9}}\left(\frac{1}{\tan(\xi)(\cot(\xi))}\right). \tag{53}$$

Family III

$$k = -\frac{\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt[3]{144-32\eta^2}}, \qquad \omega = \pm\frac{3\sqrt[3]{\eta}\epsilon^{2/3}}{\sqrt[6]{144-32\eta^2}\sqrt{2\eta^2-9}},$$
$$a_1 = -\frac{\eta^{2/3}\sqrt[6]{9-2\eta^2}\sqrt[3]{\epsilon}}{2^{5/6}\sqrt{2\eta^2-9}},$$
$$a_0 = \pm\frac{2\sqrt{2}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt[3]{144-32\eta^2}}, \qquad b_1 = \pm\frac{\eta^{2/3}\sqrt[6]{9-2\eta^2}\sqrt[3]{\epsilon}}{2^{5/6}\sqrt{2\eta^2-9}}. \tag{54}$$

Put (54) in (5),

$$\Psi_{23}(\xi) = \pm\frac{2\sqrt{2}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt[3]{144-32\eta^2}} - \frac{\eta^{2/3}\sqrt[6]{9-2\eta^2}\sqrt[3]{\epsilon}}{2^{5/6}\sqrt{2\eta^2-9}}\left(\tan(\xi)\big(\cot(\xi)\big) + \frac{1}{\tan(\xi)(\cot(\xi))}\right). \tag{55}$$

2.2.3 The rational function solutions of Eq. (23)

If $B_3 = 0$, then we have the following results:

$$\begin{aligned} &k = \frac{(-1)^{2/3}\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt[3]{9B_2^2-2B_2^2\eta^2}}, \qquad \omega = \frac{3\sqrt[3]{-1}B_2\sqrt[3]{\eta}\epsilon^{2/3}}{(-B_2^2(2\eta^2-9))^{2/3}}, \qquad a_1 = 0, \\ &a_0 = \pm\frac{\sqrt[6]{-1}\sqrt{2}B_2\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt[3]{-B_2^2(2\eta^2-9)}}, \qquad b_1 = \pm\frac{\sqrt[6]{-1}\sqrt{2}B_1\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt[3]{-B_2^2(2\eta^2-9)}}. \end{aligned} \tag{56}$$

Put (56) in (5),

$$\Psi_{24}(\xi) = \pm\frac{\sqrt[6]{-1}\sqrt{2}B_2\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt[3]{-B_2^2(2\eta^2-9)}} \pm \frac{\sqrt[6]{-1}\sqrt{2}B_1\eta^{2/3}\sqrt[3]{\epsilon}}{\sqrt[3]{-B_2^2(2\eta^2-9)}}\left(\frac{B_2}{(\exp(B_2\xi)-B_1)}\right). \tag{57}$$

2.3 The Benney–Luke equation

Let us consider the general form of the Benney–Luke equation in [26],

$$u_{tt} - u_{xx} + \beta u_{xxxx} - \gamma u_{xxtt} + u_t u_{xx} + 2u_x u_{xt} = 0. \tag{58}$$

This equation is also called a water wave equation, it is used to describe two-way water wave propagation. Let $u(x,t) = \Psi(\xi)$, $\xi = x - kt$; substituting in (58), integrating and by removal of the constant of integration,

$$\big(k^2-1\big)\Psi' + \Psi'''\big(\beta - \gamma k^2\big) - \frac{3}{2}k\Psi'^2 = 0. \tag{59}$$

We assumed that (5) is the solution of (59). Substitute Eq. (5) with (6) in (59), After solving these equations system, we have following solution cases.

2.3.1 The soliton-like solutions of Eq. (58)

If $B_1 = 0$, $B_2 = 1$, $B_3 = -1$, then we have the following results:

$$k = \pm\frac{\sqrt{\beta-1}}{\sqrt{\gamma-1}}, \qquad a_1 = \frac{4(\gamma-\beta)}{\sqrt{(\beta-1)(\gamma-1)}}, \qquad b_1 = 0. \tag{60}$$

Put (60) in (5),

$$\Psi_{25} = a_0 + \frac{2(\gamma-\beta)}{\sqrt{(\beta-1)(\gamma-1)}}\left(1 + \tanh\left(\frac{1}{2}\xi\right)\right), \quad \beta > 1, \gamma > 0. \tag{61}$$

If $B_1 = 0$, $B_2 = -1$, $B_3 = 1$, then we have the following results:

$$k = \pm\frac{\sqrt{\beta-1}}{\sqrt{\gamma-1}}, \qquad a_1 = \frac{4(\beta-\gamma)}{\sqrt{(\beta-1)(\gamma-1)}}, \qquad b_1 = 0. \tag{62}$$

Replace (62) in (5),

$$\Psi_{26} = a_0 + \frac{2(\beta-\gamma)}{\sqrt{(\beta-1)(\gamma-1)}}\left(1-\coth\left(\frac{1}{2}\xi\right)\right), \quad \beta > 1, \gamma > 0. \tag{63}$$

If $B_1 = \frac{1}{2}$, $B_2 = 0$, $B_3 = -\frac{1}{2}$, then we have the following results:

Family I

$$k = \pm\frac{\sqrt{\beta-1}}{\sqrt{\gamma-1}}, \qquad a_1 = 0, \qquad b_1 = \frac{2(\beta-\gamma)}{\sqrt{(\beta-1)(\gamma-1)}}. \tag{64}$$

Put (64) in (5),

$$\Psi_{27} = a_0 - \frac{2(\beta-\gamma)}{\sqrt{(\beta-1)(\gamma-1)}}\left(\frac{1}{\coth(\xi) \pm csch(\xi)}\right), \quad \beta > 1, \gamma > 0. \tag{65}$$

Family II

$$k = \pm\frac{\sqrt{\alpha-1}}{\sqrt{\beta-1}}, \qquad b_1 = 0, \qquad a_1 = \frac{2(\beta-\gamma)}{\sqrt{(\beta-1)(\gamma-1)}}. \tag{66}$$

Transfer (66) in (5),

$$\Psi_{28} = a_0 + \frac{2(\beta-\gamma)}{\sqrt{(\beta-1)(\gamma-1)}}\big(\coth(\xi) \pm csch(\xi)\big), \quad \beta > 1, \gamma > 1. \tag{67}$$

Family III

$$k = \pm\frac{\sqrt{1-4\beta}}{\sqrt{1-4\gamma}}, \qquad a_1 = \frac{2(\beta-\gamma)}{\sqrt{1-4\beta}\sqrt{1-4\gamma}}, \qquad b_1 = \frac{2(\beta-\gamma)}{\sqrt{1-4\beta}\sqrt{1-4\gamma}}. \tag{68}$$

Replace (68) in (5),

$$\Psi_{29} = a_0 + \frac{2(\beta-\gamma)}{\sqrt{1-4\beta}\sqrt{1-4\gamma}}\left(\big(\coth(\xi) \pm csch(\xi)\big) + \frac{1}{\coth(\xi) \pm csch(\xi)}\right),$$
$$\beta < \frac{1}{4}, \gamma < \frac{1}{4}. \tag{69}$$

If $B_1 = 1$, $B_2 = 0$, $B_3 = -1$, then we have the following results:

Family I

$$k = \pm\frac{\sqrt{1-4\beta}}{\sqrt{1-4\gamma}}, \qquad a_1 = 0, \qquad b_1 = -\frac{4(\beta-\gamma)}{\sqrt{1-4\beta}\sqrt{1-4\gamma}}. \tag{70}$$

Put (70) in (5),

$$\Psi_{30} = a_0 - \frac{4(\beta-\gamma)}{\sqrt{1-4\beta}\sqrt{1-4\gamma}}\left(\frac{1}{\tanh(\xi)}\right), \quad \beta < \frac{1}{4}, \gamma < \frac{1}{4}. \tag{71}$$

Family II

$$k = \frac{\sqrt{1-4\beta}}{\sqrt{1-4\gamma}}, \qquad b_1 = 0, \qquad a_1 = -\frac{4(\beta-\gamma)}{\sqrt{1-4\beta}\sqrt{1-4\gamma}}. \tag{72}$$

Replace (72) in (5),

$$\Psi_{31} = a_0 - \frac{4(\beta - \gamma)}{\sqrt{1-4\beta}\sqrt{1-4\gamma}}\big(\tanh(\xi)\big), \quad \beta < \frac{1}{4}, \gamma < \frac{1}{4}. \tag{73}$$

Family III

$$k = \pm\frac{\sqrt{1-16\beta}}{\sqrt{1-16\gamma}}, \qquad a_1 = \frac{4(\beta-\gamma)}{\sqrt{1-16\beta}\sqrt{1-16\gamma}}, \qquad b_1 = \frac{4(\beta-\gamma)}{\sqrt{1-16\beta}\sqrt{1-16\gamma}}. \tag{74}$$

Put (60) in (5),

$$\Psi_{32} = a_0 + \frac{4(\beta-\gamma)}{\sqrt{1-16\beta}\sqrt{1-16\gamma}}\left(\tanh(\xi) + \frac{1}{\tanh(\xi)}\right), \quad \beta < \frac{1}{16}, \gamma < \frac{1}{16}. \tag{75}$$

2.3.2 The trigonometric function solutions of Eq. (58)

If $B_1 = \frac{1}{2}$, $B_2 = 0$, $B_3 = \frac{1}{2}$, then we have the following results:

Family I

$$k = \pm\frac{\sqrt{\beta+1}}{\sqrt{\gamma+1}}, \qquad a_1 = \frac{2(\beta-\gamma)}{\sqrt{\beta+1}\sqrt{\gamma+1}}, \qquad b_1 = 0, \tag{76}$$

Substitute (76) in (5),

$$\Psi_{33}(\xi) = a_0 + \frac{2(\beta-\gamma)}{\sqrt{\beta+1}\sqrt{\gamma+1}}\big(\sec(\xi) + \tan(\xi)\big), \quad \beta > -1, \gamma > -1. \tag{77}$$

Family II

$$k = \pm\frac{\sqrt{\beta+1}}{\sqrt{\gamma+1}}, \qquad b_1 = \frac{2(\gamma-\beta)}{\sqrt{\beta+1}\sqrt{\gamma+1}}, \qquad a_1 = 0. \tag{78}$$

Put (78) in (6),

$$\Psi_{34}(\xi) = a_0 + \frac{2(\gamma-\beta)}{\sqrt{\beta+1}\sqrt{\gamma+1}}\left(\frac{1}{\sec(\xi)+\tan(\xi)}\right), \quad \beta > -1, \gamma > -1. \tag{79}$$

Family III

$$k = \pm\frac{\sqrt{-4\beta-1}}{\sqrt{-4\gamma-1}}, \qquad a_1 = \frac{2(\beta-\gamma)}{\sqrt{4\beta+1}\sqrt{4\gamma+1}}, \qquad b_1 = -\frac{2(\beta-\gamma)}{\sqrt{4\beta+1}\sqrt{4\gamma+1}}. \tag{80}$$

Put (80) in (5),

$$\Psi_{35} = a_0 + \frac{2(\beta-\gamma)}{\sqrt{4\beta+1}\sqrt{4\gamma+1}}\left(\sec(\xi) + \tan(\xi) - \frac{1}{\sec(\xi)+\tan(\xi)}\right),$$
$$\beta > \frac{-1}{4}, \gamma > \frac{-1}{4}. \tag{81}$$

If $B_1 = -\frac{1}{2}$, $B_2 = 0$, $B_3 = -\frac{1}{2}$, then we have the following results:

Family I

$$k = \pm\frac{\sqrt{\beta+1}}{\sqrt{\gamma+1}}, \qquad a_1 = \frac{2(\gamma-\beta)}{\sqrt{\beta+1}\sqrt{\gamma+1}}, \qquad b_1 = 0. \tag{82}$$

Put (82) in(6),

$$\Psi_(\xi) = a_0 + \frac{2(\gamma-\beta)}{\sqrt{\beta+1}\sqrt{\gamma+1}}\bigl(\sec(\xi)-\tan(\xi)\bigr), \quad \beta > -1, \gamma - 1. \tag{83}$$

Family II

$$k = \pm\frac{\sqrt{\beta+1}}{\sqrt{\gamma+1}}, \qquad b_1 = \frac{2(\beta-\gamma)}{\sqrt{\beta+1}\sqrt{\gamma+1}}, \qquad a_1 = 0. \tag{84}$$

Put (84) in (5),

$$\Psi_{37} = a_0 + \frac{2(\beta-\gamma)}{\sqrt{\beta+1}\sqrt{\gamma+1}}\left(\frac{1}{\sec(\xi)-\tan(\xi)}\right), \quad \beta > -1, \gamma - 1. \tag{85}$$

Family III

$$k = \pm\frac{\sqrt{-4\beta-1}}{\sqrt{-4\gamma-1}}, \qquad a_1 = -\frac{2(\beta-\gamma)}{\sqrt{4\beta+1}\sqrt{4\gamma+1}}, \qquad b_1 = \frac{2(\beta-\gamma)}{\sqrt{4\beta+1}\sqrt{4\gamma+1}}. \tag{86}$$

Replace (86) in (5),

$$\Psi_{38} = a_0 - \frac{2(\beta-\gamma)}{\sqrt{4\beta+1}\sqrt{4\gamma+1}}\left(\bigl(\sec(\xi)-\tan(\xi)\bigr) - \frac{1}{\sec(\xi)-\tan(\xi)}\right),$$
$$\beta > -\frac{1}{4}, \gamma > -\frac{1}{4}. \tag{87}$$

If $B_1 = 1(-1)$, $B_2 = 0$, $B_3 = 1(-1)$, then we have the following results:
Family I

$$k \pm \frac{\sqrt{4\beta+1}}{\sqrt{4\gamma+1}}, \qquad a_1 = \frac{4(\beta-\gamma)}{\sqrt{4\beta+1}\sqrt{4\gamma+1}}, \qquad b_1 = 0, \tag{88}$$

Put (88) in (5),

$$\Psi_{39} = a_0 + \frac{4(\beta-\gamma)}{\sqrt{4\beta+1}\sqrt{4\gamma+1}}\bigl(\tan(\xi)\bigl(\cot(\xi)\bigr)\bigr), \quad \beta > -\frac{1}{4}, \gamma > -\frac{1}{4}. \tag{89}$$

Family II

$$k = \pm\frac{\sqrt{4\beta+1}}{\sqrt{4\gamma+1}}, \qquad b_1 = -\frac{4(\beta-\gamma)}{\sqrt{4\beta+1}\sqrt{4\gamma+1}}, \qquad a_1 = 0. \tag{90}$$

Replace (90) in (5),

$$\Psi_{40}(\xi) = a_0 - \frac{4(\beta-\gamma)}{\sqrt{4\beta+1}\sqrt{4\gamma+1}}\left(\frac{1}{\tan(\xi)(\cot(\xi)}\right), \quad \beta > -\frac{1}{4}, \gamma > -\frac{1}{4}. \tag{91}$$

Family III

$$k=\pm\frac{\sqrt{16\beta+1}}{\sqrt{16\gamma+1}},\qquad a_1=-\frac{4(\beta-\gamma)}{\sqrt{16\beta+1}\sqrt{16\gamma+1}},\qquad b_1=\frac{4(\beta-\gamma)}{\sqrt{16\beta+1}\sqrt{16\gamma+1}}. \tag{92}$$

Transfer (92) in (5),

$$\begin{aligned}\Psi_{41}&=a_0-\frac{4(\beta-\gamma)}{\sqrt{16\beta+1}\sqrt{16\gamma+1}}(\tan(\xi)\left(\cot(\xi)-\frac{1}{\tan(\xi)(\cot(\xi)}\right),\\ &\beta>-\frac{1}{16},\gamma>-\frac{1}{16}.\end{aligned} \tag{93}$$

2.3.3 The rational function solutions of Eq. (58)

If $B_1=0$, $B_2=0$, then we have the following results:

$$k=\pm1,\qquad a_1=\pm4B_3(\beta-\gamma),\qquad b_1=0, \tag{94}$$

Substitute (94) in (5),

$$\Psi_{42}=a_0+4B_3\left(\frac{(\beta-\gamma)}{A_3\xi+\epsilon}\right). \tag{95}$$

If $B_2=0$, $B_3=0$, then we have the following results:

$$k=\pm1,\qquad b_1=\pm4B_1(\beta-\gamma),\qquad a_1=0. \tag{96}$$

Put (96) in (5),

$$\Psi_{43}=a_0\pm4\left(\frac{(\beta-\gamma)}{\xi}\right). \tag{97}$$

If $B_1\neq0$, $B_2\neq0$, $B_3=0$, then we have the following results:

$$k=\frac{\sqrt{\beta B_2^2-1}}{\sqrt{\gamma B_2^2-1}},\qquad b_1=\frac{4B_1(\beta-\gamma)}{\sqrt{\beta B_2^2-1}\sqrt{\gamma B_2^2-1}},\qquad a_1=0. \tag{98}$$

Replace (98) in (5),

$$\Psi_{44}=a_0+\frac{4B_1(\beta-\gamma)}{\sqrt{\beta B_2^2-1}\sqrt{\gamma B_2^2-1}}\left(\frac{B_2}{(\exp(B_2\xi)-B_1)}\right). \tag{99}$$

2.4 The (3 + 1)-dimensional modified Korteweg–de Vries–Zakharov–Kuznetsov equation

Let us consider the general form in [28],

$$u_t+\alpha u^2u_x+\beta u_{xxx}+\gamma(u_{yy}+u_{zz})_x=0. \tag{100}$$

Consider the wave transformation for Eq. (100),

$$u(x,y,z,t)=\Psi(\xi),\quad \xi=x+y+z-\eta t. \tag{101}$$

Put Eq. (101) in Eq. (100) and after integrating we have the following ODE form:

$$-\eta\Psi + \alpha\frac{\Psi^3}{3} + (\beta + 2\gamma)\Psi'' = 0. \tag{102}$$

Let (5) be a solution of (102). Substitute (5) with (6) in (102); after solving we have the following solution possibilities.

2.4.1 The soliton-like solutions of Eq. (100)

If $B_1 = 0$, $B_2 = 1$, $B_3 = -1$, then we have the following results:

$$a_1 = \pm\frac{\sqrt{-6\beta - 12\gamma}}{\sqrt{\alpha}}, \qquad \eta = \frac{1}{2}(-\beta - 2\gamma), \qquad a_0 = \pm\frac{\sqrt{-3\beta - 6\gamma}}{\sqrt{2\alpha}}, \qquad b_1 = 0. \tag{103}$$

Put (103) in (5), we have

$$\Psi_{45} = \pm\frac{\sqrt{-3\beta - 6\gamma}}{\sqrt{2\alpha}} \pm \frac{\sqrt{-6\beta - 12\gamma}}{2\sqrt{\alpha}}\left(1 + \tanh\left(\frac{1}{2}\xi\right)\right). \tag{104}$$

If $B_1 = 0$, $B_2 = -1$, $B_3 = 1$, then we have the following results:

$$\begin{aligned} &a_1 = \pm\frac{\sqrt{-6(\beta + 2\gamma)}}{\sqrt{\alpha}}, \qquad \eta = \frac{1}{2}(-\beta - 2\gamma), \\ &a_0 = \pm\frac{\sqrt{-3(\beta + 2\gamma)}}{\sqrt{2\alpha}}, \qquad b_1 = 0. \end{aligned} \tag{105}$$

Replace (105) in (5),

$$\Psi_{46} = \pm\frac{\sqrt{-3(\beta + 2\gamma)}}{\sqrt{2\alpha}} \pm \frac{\sqrt{-6(\beta + 2\gamma)}}{2\sqrt{\alpha}}\left(1 - \coth\left(\frac{1}{2}\xi\right)\right). \tag{106}$$

If $B_1 = \frac{1}{2}$, $B_2 = 0$, $B_3 = -\frac{1}{2}$, then we have the following results:

Family I

$$a_1 = 0, \qquad \eta = -\frac{1}{2}(\beta + 2\gamma), \qquad a_0 = 0, \qquad b_1 = \pm\frac{\sqrt{-3\beta - 6\gamma}}{\sqrt{2\alpha}}. \tag{107}$$

Substitute (107) in (5),

$$\Psi_{47} = \pm\frac{\sqrt{-3\beta - 6\gamma}}{\sqrt{2\alpha}}\left(\frac{1}{\coth(\xi) \pm csch(\xi)}\right). \tag{108}$$

Family II

$$b_1 = 0, \qquad \eta = -\frac{1}{2}(\beta + 2\gamma), \qquad a_0 = 0, \qquad a_1 = \pm\frac{\sqrt{-3\beta - 6\gamma}}{\sqrt{2\alpha}}. \tag{109}$$

Put (109) in (5),

$$\Psi_{48} = \pm\frac{\sqrt{-3\beta - 6\gamma}}{\sqrt{2\alpha}}\big(\coth(\xi) \pm csch(\xi)\big). \tag{110}$$

Family III

$$b_1 = \pm\frac{\sqrt{-3\beta-6\gamma}}{\sqrt{2\alpha}}, \qquad \eta = (\beta+2\gamma), \qquad a_0 = 0, \qquad a_1 = \pm\frac{\sqrt{-3\beta-6\gamma}}{\sqrt{2\alpha}}. \tag{111}$$

Putting (111) in (5),

$$\Psi_{49} = \pm\frac{\sqrt{-3\beta-6\gamma}}{\sqrt{2\alpha}}\left(\left(\coth(\xi)\pm csch(\xi)\right)+\frac{1}{\coth(\xi)\pm csch(\xi)}\right). \tag{112}$$

If $B_1 = 1$, $B_2 = 0$, $B_3 = -1$, then we have the following results:
Family I

$$a_1 = 0, \qquad \eta = -2(\beta+2\gamma), \qquad a_0 = 0, \qquad b_1 = \pm\frac{\sqrt{-6\beta-12\gamma}}{\sqrt{\alpha}}. \tag{113}$$

Replacing (113) in (5),

$$\Psi_{50} = \pm\frac{\sqrt{-6\beta-12\gamma}}{\sqrt{\alpha}}\left(\frac{1}{\tanh(\xi)}\right). \tag{114}$$

Family II

$$b_1 = 0, \qquad \eta = -2(\beta+2\gamma), \qquad a_0 = 0, \qquad a_1 = \pm\frac{\sqrt{-6\beta-12\gamma}}{\sqrt{\alpha}}. \tag{115}$$

Put (115) in (5),

$$\Psi_{51} = \pm\frac{\sqrt{-6\beta-12\gamma}}{\sqrt{\alpha}}\left(\tanh(\xi)\right). \tag{116}$$

Family III

$$b_1 = \pm\frac{\sqrt{-6\beta-12\gamma}}{\sqrt{\alpha}}, \qquad \eta = -8(\beta+2\gamma), \qquad a_0 = 0, \qquad a_1 = \pm\frac{\sqrt{-6\beta-12\gamma}}{\sqrt{\alpha}}. \tag{117}$$

Substitute (117) in (5),

$$\Psi_{52} = \pm\frac{\sqrt{-6\beta-12\gamma}}{\sqrt{\alpha}}\left(\tanh(\xi)+\frac{1}{\tanh(\xi)}\right). \tag{118}$$

2.4.2 The trigonometric function solutions of Eq. (100)
If $B_1 = \frac{1}{2}$, $B_2 = 0$, $B_3 = \frac{1}{2}$, then we have the following results:
Family I

$$a_1 = \pm\frac{\sqrt{-3\beta-6\gamma}}{\sqrt{2\alpha}}, \qquad \eta = \frac{1}{2}(\beta+2\gamma), \qquad a_0 = 0, \qquad b_1 = 0. \tag{119}$$

Put (119) in (5),

$$\Psi_{53} = \pm\frac{\sqrt{-3\beta-6\gamma}}{\sqrt{2\alpha}}\left(\sec(\xi)+\tan(\xi)\right). \tag{120}$$

Family II

$$b_1 = \pm\frac{\sqrt{-3\beta - 6\gamma}}{\sqrt{2\alpha}}, \qquad \eta = \frac{1}{2}(\beta + 2\gamma), \qquad a_0 = 0, \qquad a_1 = 0. \tag{121}$$

Put (121) in (5),

$$\Psi_{54} = \pm\frac{\sqrt{-3\beta - 6\gamma}}{\sqrt{2\alpha}}\left(\frac{1}{\sec(\xi) + \tan(\xi)}\right). \tag{122}$$

Family III

$$b_1 = \pm\frac{\sqrt{-3\beta - 6\gamma}}{\sqrt{2\alpha}}, \qquad \eta = (2\beta + 4\gamma), \qquad a_0 = 0, \qquad a_1 = \pm\frac{\sqrt{-3\beta - 6\gamma}}{\sqrt{2\alpha}}. \tag{123}$$

Replace (123) in Eq. (5),

$$\Psi_{55} = \pm\frac{\sqrt{-3\beta - 6\gamma}}{\sqrt{2\alpha}}\left(\left(\sec(\xi) + \tan(\xi)\right) + \frac{1}{\sec(\xi) + \tan(\xi)}\right). \tag{124}$$

If $B_1 = -\frac{1}{2}$, $B_2 = 0$, $B_3 = -\frac{1}{2}$, then we have the following results:
Family I

$$a_1 = \pm\frac{\sqrt{-3\beta - 6\gamma}}{\sqrt{2\alpha}}, \qquad \eta = \frac{1}{2}(\beta + 2\gamma), \qquad a_0 = 0, \qquad b_1 = 0. \tag{125}$$

Substitute (125) in (5),

$$\Psi_{56} = \pm\frac{\sqrt{-3\beta - 6\gamma}}{\sqrt{2\alpha}}\left(\sec(\xi) - \tan(\xi)\right). \tag{126}$$

Family II

$$b_1 = \pm\frac{\sqrt{-3\beta - 6\gamma}}{\sqrt{2\alpha}}, \qquad \eta = \frac{1}{2}(\beta + 2\gamma), \qquad a_0 = 0, \qquad a_1 = 0. \tag{127}$$

Put (127) in (5),

$$\Psi_{57} = \pm\frac{\sqrt{-3\beta - 6\gamma}}{\sqrt{2\alpha}}\left(\frac{1}{\sec(\xi) - \tan(\xi)}\right). \tag{128}$$

Family III

$$b_1 = \pm\frac{\sqrt{-3\beta - 6\gamma}}{\sqrt{2\alpha}}, \qquad \eta = (-\beta - 2\gamma), \qquad a_0 = 0, \qquad a_1 = \pm\frac{\sqrt{-3\beta - 6\gamma}}{\sqrt{2\alpha}}. \tag{129}$$

Put (129) in (5),

$$\Psi_{58} = \pm\frac{\sqrt{-3\beta - 6\gamma}}{\sqrt{2\alpha}}\left(\left(\sec(\xi) - \tan(\xi)\right) + \frac{1}{\sec(\xi) - \tan(\xi)}\right). \tag{130}$$

If $B_1 = 1(-1)$, $B_2 = 0$, $B_3 = 1(-1)$, then we have the following results:

Family I

$$a_1 = \pm\frac{\sqrt{-6\beta-12\gamma}}{\sqrt{\alpha}}, \qquad \eta = 2(\beta+2\gamma), \qquad a_0 = 0, \qquad b_1 = 0. \tag{131}$$

Put (131) in (5),

$$\Psi_{59} = \pm\frac{\sqrt{-6\beta-12\gamma}}{\sqrt{\alpha}}\bigl(\tan(\xi)\bigl(\cot(\xi)\bigr)\bigr). \tag{132}$$

Family II

$$b_1 = \pm\frac{\sqrt{-6\beta-12\gamma}}{\sqrt{\alpha}}, \qquad \eta = 2(\beta+2\gamma), \qquad a_0 = 0, \qquad a_1 = 0. \tag{133}$$

Replace (133) in (5),

$$\Psi_{60} = \pm\frac{\sqrt{-6\beta-12\gamma}}{\sqrt{\alpha}}\left(\frac{1}{\tan(\xi)(\cot(\xi)}\right). \tag{134}$$

Family III

$$b_1 = \pm\frac{\sqrt{-6\beta-12\gamma}}{\sqrt{\alpha}}, \qquad \eta = -4(\beta+2\gamma), \qquad a_0 = 0, \qquad a_1 = \pm\frac{\sqrt{-6\beta-12\gamma}}{\sqrt{\alpha}}. \tag{135}$$

Substitute (135) in (5),

$$\Psi_{61} = \pm\frac{\sqrt{-6\beta-12\gamma}}{\sqrt{\alpha}}(\tan(\xi)\left(\cot(\xi)+\frac{1}{\tan(\xi)(\cot(\xi)}\right). \tag{136}$$

2.4.3 The rational function solution of Eq. (100)

If $B_1 \neq 0$, $B_2 \neq 0$, $B_3 = 0$, then we have the following results:

$$\begin{aligned} &a_1 = 0, \qquad \eta = \frac{1}{2}\bigl(-\beta B_2^2 - 2B_2^2\gamma\bigr), \\ &a_0 = \pm\frac{B_2\sqrt{-\beta-2\gamma}}{\sqrt{\alpha}}, \qquad b_1 = \pm\frac{B_1\sqrt{-6(\beta+2\gamma)}}{\sqrt{\alpha}}. \end{aligned} \tag{137}$$

Put (137) in (5),

$$\Psi_{62} = \pm\frac{B_2\sqrt{-\beta-2\gamma}}{\sqrt{\alpha}} \pm \frac{B_1\sqrt{-6(\beta+2\gamma)}}{\sqrt{\alpha}}\left(\frac{B_2}{(\exp(B_2\xi)-B_1)}\right). \tag{138}$$

3 Discussion of the results

After successfully employing the modified F-expansion method on four important models, now in this section we shall discuss the similarities and dissimilarities of our novel constructed results with other results in the previous literature. By choosing different values of a_i and b_i in Eq. (5) with Eq. (6), we have obtained a collection of different solutions. Few of our results are similar to others, our solution (15) and (19) approximate in the same way as (2.37), (2.59) in [24], respectively. In the same way our constructed solutions

(39) and (39) are similar to the solutions (22) and (26) in [23], respectively. Furthermore our solutions (49) and (81) are approximately similar solutions to (3.16) and (3.14) in [27]. The results of all our constructed solutions of both models are new and have not been investigated before in any previous research literature. The discussion of the results and graphically illustration of some solution demonstrate that our methods are more efficient, and a reliable and powerful tool to solve nonlinear problems.

4 Conclusion

From this study, we have seen the analytical structure of the complex fractional Kundu–Eckhaus, the van der Waals normal form for fluidized granular matter, the Benny–Luke and the (3 + 1)-dimensional modified Korteweg–de Vries–Zakharov–Kuznetsov waves models for constructing the exact, solitary wave solutions by employing the modified F-expansion method. The constructed solutions are novel and more general. These results facilitate us to explore the physical phenomena of these nonlinear models. The obtained results having potential applications in mathematical physics.

Acknowledgements
The author thanks the referees for their suggestions and comments.

Funding
Funding information is not applicable / No funding was received.

Competing interests
This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors. The authors did not have any competing interests in this research.

Authors' contributions
All parts contained in the research were carried out by the authors through hard work and a review of the various references and contributions in the field of mathematics and physics. All authors read and approved the final manuscript.

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Received: 21 June 2018 Accepted: 6 September 2018 Published online: 20 September 2018

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