Zhao *Advances in Difference Equations* (2017) 2017:328
DOI 10.1186/s13662-017-1381-1

Advances in Difference Equations
a SpringerOpen Journal

RESEARCH Open Access

Positive periodic solutions of Lotka-Volterra-like impulsive functional differential equations with infinite distributed time delays on time scales

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Abstract
This paper is concerned with the Lotka-Volterra-like functional differential equations with impulses and infinite distributed time delays on time scales. By applying a fixed point theorem of strict-set contractions, some sufficient conditions have been established to guarantee the existence of periodic solutions. As applications, the existence of positive periodic solutions is given for some common Lotka-Volterra systems on time scales.

MSC: 34K13; 34N05; 26E70

Keywords: positive periodic solution; Lotka-Volterra-like system; impulsive and time-delay functional differential equation; theorem of strict-set contractions; time scale

1 Introduction

In this article we consider the Lotka-Volterra-like functional differential equations with impulses and infinite distributed time delays on time scales as follows:

$$\begin{cases} u_i^{\Delta}(t) = u_i(t)[r_i(t) - X_i(t,u(t),v(t)) - Y_i(t,u(t),v(t))], & t \neq t_k, t \in \mathbb{T}, \\ u_i(t_k^+) = u_i(t_k^-) - I_{ik}(u(t_k),v(t_k)), & k = 1,2,\ldots, \\ v_j^{\Delta}(t) = v_j(t)[-d_j(t) + \hat{X}_j(t,u(t),v(t)) + \hat{Y}_j(t,u(t),v(t))], & t \neq t_k, t \in \mathbb{T}, \\ v_j(t_k^+) = v_j(t_k^-) + \hat{I}_{jk}(u(t_k),v(t_k)), & k = 1,2,\ldots, \end{cases} \tag{1.1}$$

where $i = 1,2,\ldots,n$; $j = 1,2,\ldots,m$. $\mathbb{T}$ is an ω-periodic time scale, $\omega > 0$ is a constant. We have

$$\begin{aligned} X_i\big(t,u(t),v(t)\big) &= X_i\big(t,u_1\big(t-\tau_{i1}(t)\big),\ldots,u_n\big(t-\tau_{in}(t)\big),v_1\big(t-\sigma_{i1}(t)\big),\ldots,v_m\big(t-\sigma_{im}(t)\big)\big), \\ \hat{X}_j\big(t,u(t),v(t)\big) &= \hat{X}_j\big(t,u_1\big(t-\hat{\tau}_{j1}(t)\big),\ldots,u_n\big(t-\hat{\tau}_{jn}(t)\big),v_1\big(t-\hat{\sigma}_{j1}(t)\big),\ldots,v_m\big(t-\hat{\sigma}_{jm}(t)\big)\big), \\ Y_i\big(t,u(t),v(t)\big) &= Y_i\bigg(t,\int_{-\infty}^{0} K_{i1}(s)u_1(t+s)\Delta s,\ldots,\int_{-\infty}^{0} K_{in}(s)u_n(t+s)\Delta s, \\ &\qquad \int_{-\infty}^{0} L_{i1}(s)v_1(t+s)\Delta s,\ldots,\int_{-\infty}^{0} L_{im}(s)v_m(t+s)\Delta s\bigg), \end{aligned}$$

$$\hat{Y}_j\big(t,u(t),v(t)\big) = \hat{Y}_j\bigg(t,\int_{-\infty}^0 \hat{K}_{j1}(s)u_1(t+s)\Delta s,\dots,\int_{-\infty}^0 \hat{K}_{jn}(s)u_n(t+s)\Delta s,$$
$$\int_{-\infty}^0 \hat{L}_{j1}(s)v_1(t+s)\Delta s,\dots,\int_{-\infty}^0 \hat{L}_{jm}(s)v_m(t+s)\Delta s\bigg).$$

For each interval I of $\mathbb{R}$, we denote $I_{\mathbb{T}} = I \cap \mathbb{T}$. $u_i(t_k^+)$, $u_i(t_k^-)$, $v_j(t_k^+)$ and $v_j(t_k^-)$ represent the right and the left limit $u_i(t_k)$ and $v_j(t_k)$ in the sense of time scales, respectively. In addition, if t_k is right-scattered, then $u_i(t_k^+) = u_i(t_k)$, $v_j(t_k^+) = v_j(t_k)$, whereas, if t_k left-scattered, then $u_i(t_k^-) = u_i(t_k)$, $v_j(t_k^-) = v_j(t_k)$. $r_i, d_j \in C_{rd}(\mathbb{T},(0,\infty))$, $\tau_{il}, \sigma_{ij}, \hat{\tau}_{jl}, \sigma_{jh} \in C_{rd}(\mathbb{T},\mathbb{T})$ $(i,l = 1,2,\dots,n; j,h = 1,2,\dots,m)$ are ω-periodic functions. X_i, $\hat{X}_j$, Y_i, $\hat{Y}_j \in C_{rd}(\mathbb{T}\times\mathbb{R}^{m+n},\mathbb{R})$ $(i = 1,2,\dots,n; j = 1,2,\dots,m)$ are ω-periodic with respect to their first arguments, respectively. We have $K_{il}, \hat{K}_{jl}, L_{ij}, \hat{L}_{jh} \in C_{rd}((-\infty,0]_{\mathbb{T}},(0,\infty))$ with $\int_{-\infty}^0 K_{il}(s)\Delta s = \int_{-\infty}^0 \hat{K}_{jl}(s)\Delta s = \int_{-\infty}^0 L_{ij}(s)\Delta s = \int_{-\infty}^0 \hat{L}_{jh}(s)\Delta s = 1$. $I_{ik}, \hat{I}_{jk} \in C([0,\infty)^{n+m},[0,\infty))$. There exists a positive integer p such that $t_{k+p} = t_k + \omega$, $I_{i,k+p} = I_{ik}$, $\hat{I}_{j,k+p} = I_{jk}$, $k \in \mathbb{Z}$. Without loss of generality, we also assume that $[0,\omega)_{\mathbb{T}} \cap \{t_k : k \in \mathbb{Z}\} = \{t_1,t_2,\dots,t_p\}$.

System (1.1) contain many mathematical population models of differential equations and difference equations. For example, if time scale $\mathbb{T} = \mathbb{R}$, some specific models of system (1.1) are enumerated as follows: the Lotka-Volterra competition system with impulses and time delays [1–3]

$$\begin{cases} u_i'(t) = u_i(t)[a_i(t) - \sum_{j=1}^n a_{ij}(t)\int_{-T_{ij}}^0 K_{ij}(s)u_j(t+s)\,ds], & t \neq t_k, i = 1,2,\dots,n, \\ u_i(t_k^+) = u_i(t_k^-) - I_{ik}(u_i(t_k)), & k = 1,2,\dots, \end{cases}$$

and

$$\begin{cases} u_i'(t) = u_i(t)[a_i(t) - \sum_{j=1}^n a_{ij}(t)u_j(t) - \sum_{j=1}^n b_{ij}(t)u_j(t-\tau_j(t))], & t \neq t_k, i = 1,2,\dots,n, \\ u_i(t_k^+) = u_i(t_k^-) - I_{ik}(u_i(t_k)), & k = 1,2,\dots, \end{cases}$$

the following predator-prey delay Lotka-Volterra system with impulses [2, 4]:

$$\begin{cases} u_i'(t) = u_i(t)[a_i(t) - \sum_{l=1}^n a_{il}(t)u_l(t-\sigma_{il}(t)) - \sum_{j=1}^m b_{ij}(t)v_j(t-\tau_{ij}(t))], \\ \quad t \neq t_k, i = 1,\dots,n, \\ u_i(t_k^+) = u_i(t_k^-) - I_{ik}(u_i(t_k)), \quad k = 1,2,\dots, \\ v_j'(t) = v_j(t)[-r_j(t) - \sum_{l=1}^n d_{jl}(t)u_l(t-\delta_{jl}(t)) - \sum_{h=1}^m e_{jh}(t)v_h(t-\theta_{jh}(t))], \\ \quad t \neq t_k, j = 1,\dots,m, \\ v_j(t_k^+) = v_j(t_k^-) + \hat{I}_{jk}(v_j(t_k)), \quad k = 1,2,\dots. \end{cases}$$

It is well known that the application of theories of functional differential equations in mathematical ecology has developed rapidly. The Lotka-Volterra system described by functional differential equations is one of the most famous and important population dynamics models. Owing to its theoretical and practical significance, Lotka-Volterra systems have been studied extensively [5–17]. However, dynamics in each equally spaced time interval may vary continuously. So it may be more realistic to assume that the population dynamics involves the hybrid discrete-continuous processes. For example, Gamarra and Solé pointed out that such hybrid processes appear in the population dynamics of certain species that feature non-overlapping generations: the change in population from one generation to the next is discrete and so is modeled by a difference equation, while within-generation dynamics vary continuously (due to mortality rates, resource consumption, predation, interaction, etc.) and thus are described by a differential equation [18]. However, it is often difficult to study discrete and continuous differential systems in a unified way. Fortunately, the theory of calculus on time scales (see [19, 20] and references cited therein) proposed by Hilger in his Ph.D. thesis [21] can unify continuous and discrete analysis, and it has become an effective approach to the study of mathematical models involving the hybrid discrete-continuous processes. There are many achievements in the study of the hybrid discrete-continuous mathematical models (see [22–30]).

To the best of our knowledge, few papers have been published on the existence of positive periodic solutions of system (1.1). Our main purpose of this paper is by using a fixed point theorem of strict-set contraction to establish some sufficient conditions to guarantee the existence of positive periodic solutions of system (1.1) on time scales.

2 Preliminaries on time scales

In this section, we briefly recall some basic definitions and lemmas on time scales which are used in what follows. For more details, see [19–21].

Let $\mathbb{T}$ be a nonempty closed subset (time scale) of $\mathbb{R}$. The forward and backward jump operators $\sigma, \rho : \mathbb{T} \to \mathbb{T}$ and the graininess $\mu : \mathbb{T} \to \mathbb{R}^+$ are defined, respectively, by

$$\sigma(t) = \inf\{s \in \mathbb{T} : s > t\}, \qquad \rho(t) = \sup\{s \in \mathbb{T} : s < t\} \quad \text{and} \quad \mu(t) = \sigma(t) - t.$$

A point $t \in \mathbb{T}$ is called left-dense if $t > \inf \mathbb{T}$ and $\rho(t) = t$, left-scattered if $\rho(t) < t$, right-dense if $t < \sup \mathbb{T}$ and $\sigma(t) = t$, and right-scattered if $\sigma(t) > t$. If $\mathbb{T}$ has a left-scattered maximum m, then $\mathbb{T}^k = \mathbb{T} \setminus \{m\}$; otherwise $\mathbb{T}^k = \mathbb{T}$. If $\mathbb{T}$ has a right-scattered minimum m, then $\mathbb{T}_k = \mathbb{T} \setminus \{m\}$; otherwise $\mathbb{T}^k = \mathbb{T}$.

Let $\omega > 0$. Throughout this paper, the time scale T is assumed to be ω-periodic, that is, $\forall t \in \mathbb{T}$ implies $t + \omega \in \mathbb{T}$ and $\mu(t + \omega) = \mu(t)$. In particular, the time scale $\mathbb{T}$ under consideration is unbounded above and below.

Definition 2.1 A function $f : \mathbb{T} \to \mathbb{R}$ is called *regulated* provided its right-side limits exist (finite) at all right-side points in $\mathbb{T}$ and its left-side limits exist (finite) at all left-side points in $\mathbb{T}$.

Definition 2.2 A function $f : \mathbb{T} \to \mathbb{R}$ is called rd-continuous provided it is continuous at right-dense point in $\mathbb{T}$ and its left-side limits exist (finite) at left-dense points in $\mathbb{T}$. The set of rd-continuous functions $f : \mathbb{T} \to \mathbb{R}$ will be denoted by $C_{rd} = C_{rd}(\mathbb{T}) = C_{rd}(\mathbb{T}, \mathbb{R})$.

Definition 2.3 Assume $f : \mathbb{T} \to \mathbb{R}$ and $t \in \mathbb{T}^k$. Then we define $f^{\Delta}(t)$ to be to be the number (if it exists) with the property that given any $\varepsilon > 0$ there exists a neighborhood U of t (*i.e.*, $U = (t - \delta, t + \delta) \cap \mathbb{T}$ for some $\delta > 0$) such that

$$\left|\left[f\left(\sigma(t)\right) - f(s)\right] - f^{\Delta}(t)\left[\sigma(t) - s\right]\right| < \varepsilon\left|\sigma(t) - s\right|$$

for all $s \in U$. we call $f^{\Delta}(t)$ the delta (or Hilger) derivative of f at t. The set of functions $f : \mathbb{T} \to \mathbb{R}$ that are differentiable and whose derivative is rd-continuous is denoted by $C^1_{rd} = C^1_{rd}(\mathbb{T}) = C^1_{rd}(\mathbb{T}, \mathbb{R})$.

If f is continuous, then f is rd-continuous. If f is rd-continuous, the f is regulated. If f is delta differentiable at t, then f is continuous at t.

Lemma 2.1 *Let f be regulated, then there exists a function F which is delta differentiable with region of differentiation D such that*

$$F^{\Delta}(t) = f(t) \quad \textit{for all } t \in D.$$

Definition 2.4 Assume $f : \mathbb{T} \to \mathbb{R}$ is a regulated function. Any function F as in Lemma 2.1 is called a Δ-antiderivative of f. We define the indefinite integral of a regulated function f by

$$\int f(t)\Delta t = F(t) + C,$$

where C is an arbitrary constant and F is a Δ-antiderivative of f. We define the Cauchy integral by

$$\int_a^b f(s)\Delta s = F(b) - F(a) \quad \text{for all } a, b \in \mathbb{T}.$$

A function $F : \mathbb{T} \to \mathbb{R}$ is called an antiderivative of $f : \mathbb{T} \to \mathbb{R}$ provided

$$F^{\Delta}(t) = f(t) \quad \text{for all } t \in \mathbb{T}^k.$$

Lemma 2.2 *If $a, b \in \mathbb{T}$, $\alpha, \beta \in \mathbb{R}$ and $f, g \in C(\mathbb{T}, \mathbb{R})$, then*

(i) $\int_a^b [\alpha f(t) + \beta g(t)]\Delta t = \alpha \int_a^b f(t)\Delta t + \beta \int_a^b g(t)\Delta t$;

(ii) *if $f(t) \geq 0$ for all $a \leq t < b$, then $\int_a^b f(t)\Delta t \geq 0$;*

(iii) *if $|f(t)| \leq g(t)$ on $[a, b) := \{t \in \mathbb{T} : a \leq t < b\}$, then $|\int_a^b f(t)\Delta t| \leq \int_a^b g(t)\Delta t$.*

Definition 2.5 ([31]) A time scale $\mathbb{T}$ is called periodic if there exists $p > 0$ such that if $\forall t \in \mathbb{T}$, then $t \pm p \in \mathbb{T}$. For $\mathbb{T} \neq \mathbb{R}$, the smallest positive p is called the period of the time scale $\mathbb{T}$.

Definition 2.6 ([31]) Let $\mathbb{T} \neq \mathbb{R}$ be a periodic time scale with period $p > 0$. The function $f : \mathbb{T} \to \mathbb{R}$ is called periodic with period ω if there exists a natural number n such that $\omega = np$, $f(t + \omega) = f(t)$ for all $t \in \mathbb{T}$ and ω is the smallest number such that $f(t + \omega) = f(t)$.

If $\mathbb{T} = \mathbb{R}$, we say that f is periodic with period $\omega > 0$ if ω is the smallest positive number such that $f(t + \omega) = f(t)$ for all $t \in \mathbb{R}$.

A function $p : \mathbb{T} \to \mathbb{R}$ is called regressive if $1 + \mu(t)p(t) \neq 0$ for all $t \in \mathbb{T}^k$. The set of all regressive and rd-continuous functions $f : \mathbb{T} \to \mathbb{R}$ will be denoted by $\mathcal{R} = \mathcal{R}(\mathbb{T}) = \mathcal{R}(\mathbb{T}, \mathbb{R})$. We define the set $\mathcal{R}^+$ of all positively regressive elements of $\mathcal{R}$ by $\mathcal{R}^+ = \mathcal{R}^+(\mathbb{T}, \mathbb{R}) = \{p \in \mathcal{R} : 1 + \mu(t)p(t) > 0 \text{ for all } t \in \mathbb{T}\}$. If p is a regressive function, then the generalized exponential function e_p is defined by $e_p(t, s) = \exp\{\int_s^t \xi_{\mu(\tau)}(p(\tau))\Delta\tau\}$ for $s, t \in \mathbb{T}$, with the cylinder transformation

$$\xi_h(z) = \begin{cases} \frac{\operatorname{Log}(1+hz)}{h}, & \text{if } h \neq 0, \\ z, & \text{if } h = 0. \end{cases}$$

Let $p, q : \mathbb{T} \to \mathbb{R}$ be two regressive functions, we define

$$p \oplus q = p + q + \mu pq, \qquad \ominus p = -\frac{p}{1+\mu p}, \qquad p \ominus q = p \oplus p(\ominus q).$$

The generalized exponential function has the following properties.

Lemma 2.3 ([19]) *Assume that $p, q : \mathbb{T} \to \mathbb{R}$ are two regressive functions, then*

(1) $e_0(t,s) \equiv 1$ *and* $e_p(t,t) \equiv 1$;
(2) $e_p(\sigma(t),s) = (1+\mu(t)p(t))e_p(t,s)$;
(3) $\frac{1}{e_p(t,s)} = e_{\ominus p}(t,s)$;
(4) $e_p(t,s) = \frac{1}{e_p(s,t)} = e_{\ominus p}(s,t)$;
(5) $e_p(t,s)e_p(s,r) = e_p(t,r)$;
(6) $e_p(t,s)e_q(t,s) = e_{p\oplus q}(t,s)$;
(7) $\frac{e_p(t,s)}{e_q(t,s)} = e_{p\ominus q}(t,s)$;
(8) $[e_p(t,s)]^\Delta = p(t)e_p(t,s)$;
(9) $[e_p(c,\cdot)]^\Delta = -p[e_p(c,\cdot)]^\sigma$ *for* $c \in \mathbb{T}$;
(10) $\frac{d}{dz}[e_z(t,s)] = (\int_s^t \frac{1}{1+\mu(\tau)z}\Delta\tau)e_z(t,s)$.

For convenience, we now introduce some notations as follows.

$$B(0,R) = \{(x_1,x_2,\ldots,x_{n+m})^T \in \mathbb{R}^{n+m} : \|(x_1,x_2,\ldots,x_{n+m})\| \le R\}, \qquad f^M = \max_{t\in[0,\omega]_{\mathbb{T}}}\{f(t)\},$$

$$X_i^0 = \limsup_{\sum_{i=1}^{n+m} x_i \to 0} \max_{t\in[0,\omega]_{\mathbb{T}}} \frac{X_i(t,x_1,x_2,\ldots,x_{n+m})}{\sum_{i=1}^{n+m} x_i},$$

$$\hat{X}_j^0 = \limsup_{\sum_{i=1}^{n+m} x_i \to 0} \max_{t\in[0,\omega]_{\mathbb{T}}} \frac{\hat{X}_j(t,x_1,x_2,\ldots,x_{n+m})}{\sum_{i=1}^{n+m} x_i},$$

$$Y_i^0 = \limsup_{\sum_{i=1}^{n+m} x_i \to 0} \max_{t\in[0,\omega]_{\mathbb{T}}} \frac{Y_i(t,x_1,x_2,\ldots,x_{n+m})}{\sum_{i=1}^{n+m} x_i},$$

$$\hat{Y}_j^0 = \limsup_{\sum_{i=1}^{n+m} x_i \to 0} \max_{t\in[0,\omega]_{\mathbb{T}}} \frac{\hat{Y}_j(t,x_1,x_2,\ldots,x_{n+m})}{\sum_{i=1}^{n+m} x_i},$$

$$X_i^\infty = \liminf_{\sum_{i=1}^{n+m} x_i \to \infty} \min_{t\in[0,\omega]_{\mathbb{T}}} \frac{X_i(t,x_1,x_2,\ldots,x_{n+m})}{\sum_{i=1}^{n+m} x_i},$$

$$\hat{X}_j^\infty = \liminf_{\sum_{i=1}^{n+m} x_i \to \infty} \min_{t\in[0,\omega]_{\mathbb{T}}} \frac{\hat{X}_j(t,x_1,x_2,\ldots,x_{n+m})}{\sum_{i=1}^{n+m} x_i},$$

$$Y_i^\infty = \liminf_{\sum_{i=1}^{n+m} x_i \to \infty} \min_{t\in[0,\omega]_{\mathbb{T}}} \frac{Y_i(t,x_1,x_2,\ldots,x_{n+m})}{\sum_{i=1}^{n+m} x_i},$$

$$\hat{Y}_j^\infty = \liminf_{\sum_{i=1}^{n+m} x_i \to \infty} \min_{t\in[0,\omega]_{\mathbb{T}}} \frac{\hat{Y}_j(t,x_1,x_2,\ldots,x_{n+m})}{\sum_{i=1}^{n+m} x_i},$$

$$\gamma_i = \limsup_{\sum_{i=1}^{n+m} x_i \to 0} \frac{\sum_{k=1}^p I_{ik}(x_1,x_2,\ldots,x_{n+m})}{\sum_{i=1}^{n+m} x_i},$$

$$\hat{\gamma}_j = \limsup_{\sum_{i=1}^{n+m} x_i \to 0} \frac{\sum_{k=1}^p \hat{I}_{jk}(x_1,x_2,\ldots,x_{n+m})}{\sum_{i=1}^{n+m} x_i},$$

where $i = 1,2,\ldots,n$, $j = 1,2,\ldots,m$, f is a rd-continuous ω-periodic function.

Lemma 2.4 ([19]) *Let $r : \mathbb{T} \to \mathbb{R}$ be right-dense continuous and regressive. For $a \in \mathbb{T}$ and $y_a \in \mathbb{R}$, the unique solution of the initial value problem*

$$y^{\Delta}(t) = r(t)y(t) + h(t), \quad y(a) = y_a,$$

is given by

$$y(t) = e_r(t,a)y_a + \int_a^t e_r\bigl(t,\sigma(s)\bigr)h(s)\Delta s.$$

The existence of periodic solutions of system (1.1) is equivalent to the existence of periodic solutions of the corresponding integral system. So the following lemma is important in our discussion.

Lemma 2.5 $x(t) = (u(t),v(t))^T = (u_1(t),\dots,u_n(t),v_1(t),\dots,v_m(t))^T$ *is an ω-periodic solution of* (1.1) *is equivalent to $x(t)$ is an ω-periodic solution of the following integral system*:

$$\begin{cases} u_i(t) = \int_t^{t+\omega} G_i(t,s)u_i(s)[X_i(s,u(s),v(s)) + Y_i(s,u(s),v(s))]\Delta s \\ \qquad + \sum_{t_k\in[t,t+\omega)_{\mathbb{T}}} G_i(t,t_k)e_{r_i}(\sigma(t_k),t_k)I_{ik}(u(t_k),v(t_k)), \quad i = 1,2,\dots,n, \\ v_j(t) = \int_t^{t+\omega} \hat{G}_j(t,s)v_j(s)[\hat{X}_j(s,u(s),v(s)) + \hat{Y}_j(s,u(s),v(s))]\Delta s \\ \qquad + \sum_{t_k\in[t,t+\omega)_{\mathbb{T}}} \hat{G}_j(t,t_k)e_{(-d_j)}(\sigma(t_k),t_k)\hat{I}_{jk}(u(t_k),v(t_k)), \quad j = 1,2,\dots,m, \end{cases} \tag{2.1}$$

here

$$G_i(t,s) = \frac{e_{r_i}(t,\sigma(s))}{1 - e_{r_i}(0,\omega)}, \quad s \in [t,t+\omega]_{\mathbb{T}}, i = 1,2,\dots,n, \tag{2.2}$$

and

$$\hat{G}_j(t,s) = \frac{e_{(-d_j)}(t,\sigma(s))}{e_{(-d_j)}(0,\omega) - 1}, \quad s \in [t,t+\omega]_{\mathbb{T}}, j = 1,2,\dots,m. \tag{2.3}$$

Proof If $x(t)$ is an ω-periodic solution of (1.1), $\forall t \in \mathbb{T}$, there exists $k \in \mathbb{N}^+$ such that t_k is the first impulsive point after t. By applying Lemma 2.4 and the first equation of (1.1), for $s \in [t,t_k]_{\mathbb{T}}$, we have

$$u_i(s) = e_{r_i}(s,t)u_i(t) - \int_t^s e_{r_i}\bigl(s,\sigma(\tau)\bigr)u_i(\tau)\bigl[X_i\bigl(\tau,u(\tau),v(\tau)\bigr) + Y_i\bigl(\tau,u(\tau),v(\tau)\bigr)\bigr]\Delta\tau,$$

then

$$u_i(t_k) = e_{r_i}(t_k,t)u_i(t) - \int_t^{t_k} e_{r_i}\bigl(t_k,\sigma(\tau)\bigr)u_i(\tau)\bigl[X_i\bigl(\tau,u(\tau),v(\tau)\bigr) + Y_i\bigl(\tau,u(\tau),v(\tau)\bigr)\bigr]\Delta\tau.$$

Again using Lemma 2.4, for $s \in (t_k,t_{k+1}]_{\mathbb{T}}$, then

$$\begin{aligned} u_i(s) &= e_{r_i}(s,t_k)u_i\bigl(t_k^+\bigr) - \int_{t_k}^s e_{r_i}\bigl(s,\sigma(\tau)\bigr)u_i(\tau)\bigl[X_i\bigl(\tau,u(\tau),v(\tau)\bigr) + Y_i\bigl(\tau,u(\tau),v(\tau)\bigr)\bigr]\Delta\tau \\ &= e_{r_i}(s,t_k)u_i(t_k) - \int_{t_k}^s e_{r_i}\bigl(s,\sigma(\tau)\bigr)u_i(\tau)\bigl[X_i\bigl(\tau,u(\tau),v(\tau)\bigr) + Y_i\bigl(\tau,u(\tau),v(\tau)\bigr)\bigr]\Delta\tau \\ &\quad - e_{r_i}(s,t_k)I_{ik}\bigl(u(t_k),v(t_k)\bigr). \end{aligned}$$

Thus, for $s \in [t, t_{k+1}]_{\mathbb{T}}$, we get

$$\begin{aligned} u_i(s) &= e_{r_i}(s,t)u_i(t) - \int_t^s e_{r_i}\big(s,\sigma(\tau)\big)u_i(\tau)\big[X_i\big(\tau,u(\tau),v(\tau)\big) + Y_i\big(\tau,u(\tau),v(\tau)\big)\big]\Delta\tau \\ &\quad - e_{r_i}(s,t_k)I_{ik}\big(u(t_k),v(t_k)\big). \end{aligned}$$

Repeating the above process for $s \in [t, t+\omega]_{\mathbb{T}}$, we obtain

$$\begin{aligned} u_i(s) &= e_{r_i}(s,t)u_i(t) - \int_t^s e_{r_i}\big(s,\sigma(\tau)\big)u_i(\tau)\big[X_i\big(\tau,u(\tau),v(\tau)\big) + Y_i\big(\tau,u(\tau),v(\tau)\big)\big]\Delta\tau \\ &\quad - \sum_{t_k \in [t,t+\omega)_{\mathbb{T}}} e_{r_i}(s,t_k)I_{ik}\big(u(t_k),v(t_k)\big). \end{aligned}$$

Let $s = t+\omega$ in the above equality and notice that $u_i(t) = u_i(t+\omega)$, $e_{r_i}(t,t+\omega) = e_{r_i}(0,\omega)$, $e_{r_i}(t+\omega,\sigma(\tau)) = e_{r_i}(t,\sigma(\tau))e_{r_i}(t+\omega,t)$, $e_{r_i}(t,t_k) = e_{r_i}(t,\sigma(t_k))e_{r_i}(\sigma(t_k),t_k)$ and $e_{r_i}(t,t+\omega) \times e_{r_i}(t+\omega,t) = 1$, we have

$$\begin{aligned} u_i(t) &= u_i(t+\omega) \\ &= e_{r_i}(t+\omega,t)u_i(t) \\ &\quad - \int_t^{t+\omega} e_{r_i}\big(t+\omega,\sigma(\tau)\big)u_i(\tau)\big[X_i\big(\tau,u(\tau),v(\tau)\big) + Y_i\big(\tau,u(\tau),v(\tau)\big)\big]\Delta\tau \\ &\quad - \sum_{t_k \in [t,t+\omega)_{\mathbb{T}}} e_{r_i}(t+\omega,t_k)I_{ik}\big(u(t_k),v(t_k)\big) \\ &= e_{r_i}(\omega,0)u_i(t) \\ &\quad - \int_t^{t+\omega} e_{r_i}\big(t,\sigma(\tau)\big)e_{r_i}(\omega,0)u_i(\tau)\big[X_i\big(\tau,u(\tau),v(\tau)\big) + Y_i\big(\tau,u(\tau),v(\tau)\big)\big]\Delta\tau \\ &\quad - \sum_{t_k \in [t,t+\omega)_{\mathbb{T}}} e_{r_i}\big(t,\sigma(t_k)\big)e_{r_i}(\omega,0)e_{r_i}\big(\sigma(t_k),t_k\big)I_{ik}\big(u(t_k),v(t_k)\big), \end{aligned}$$

which implies that

$$\begin{aligned} u_i(t) &= \int_t^{t+\omega} G_i(t,\tau)u_i(\tau)\big[X_i\big(\tau,u(\tau),v(\tau)\big) + Y_i\big(\tau,u(\tau),v(\tau)\big)\big]\Delta\tau \\ &\quad + \sum_{t_k \in [t,t+\omega)_{\mathbb{T}}} G_i(t,t_k)e_{r_i}\big(\sigma(t_k),t_k\big)I_{ik}\big(u(t_k),v(t_k)\big). \end{aligned}$$

In like manner, one has

$$\begin{aligned} v_j(t) &= \int_t^{t+\omega} \hat{G}_j(t,\tau)v_j(\tau)\big[\hat{X}_j\big(\tau,u(\tau),v(\tau)\big) + \hat{Y}_j\big(\tau,u(\tau),v(\tau)\big)\big]\Delta\tau \\ &\quad + \sum_{t_k \in [t,t+\omega)_{\mathbb{T}}} \hat{G}_j(t,t_k)e_{(-d_j)}\big(\sigma(t_k),t_k\big)\hat{I}_{jk}\big(u(t_k),v(t_k)\big). \end{aligned}$$

Thus, we conclude that $x(t)$ satisfies (2.1).

Let $x(t)$ be an ω-periodic solution of (2.1), noting that the above reduction is completely reversible, we know that $x(t)$ is also an ω-periodic solution of (1.1). This completes the proof of Lemma 2.5. □

Throughout this paper, we assume that

(H_1) $\sup_{t\in[0,\omega]_{\mathbb{T}}}\{\mu(t)d_j(t)\} < 1$, $j = 1,2,\ldots,m$.

Lemma 2.6 *If the condition* (H_1) *holds, then* $G_i(t,s)$ $(i = 1,2,\ldots,n)$ *and* $\hat{G}_j(t,s)$ $(j = 1,2,\ldots,m)$ *defined by* (2.2) *and* (2.3) *satisfy the following*:

(1) $\frac{\sigma_i}{1-\sigma_i} \le G_i(t,s) \le \frac{1}{1-\sigma_i}$, $\forall s \in [t,t+\omega]_{\mathbb{T}}$, *where* $\sigma_i = e_{r_i}(0,\omega)$, $i = 1,2,\ldots,n$;

(2) $\frac{1}{\hat{\sigma}_j-1} \le \hat{G}_j(t,s) \le \frac{\hat{\sigma}_j}{\hat{\sigma}_j-1}$, $\forall s \in [t,t+\omega]_{\mathbb{T}}$, *where* $\hat{\sigma}_j = e_{(-d_j)}(0,\omega)$, $j = 1,2,\ldots,m$;

(3) $G_i(t+\omega,s+\omega) = G_i(t,s)$, $i = 1,2,\ldots,n$, $\hat{G}_j(t+\omega,s+\omega) = \hat{G}_j(t,s)$, $j = 1,2,\ldots,m$.

Proof According to the condition (H_1) and $\mu(t) = \sigma(t) - t \ge 0$, $r_i(t), d_j(t) > 0$, we have $1 + \mu(t)r_i(t) > 1$, $0 < 1 - \mu(t)d_j(t) < 1$. In addition, in the light of the definitions of the generalized exponential function, we get

$$0 < \sigma_i = e_{r_i}(0,\omega) < 1, \qquad \hat{\sigma}_j = e_{(-d_j)}(0,\omega) > 1, \quad i = 1,2,\ldots,n, j = 1,2,\ldots,m.$$

Noticing that $t \le s \le \sigma(s) \le t+\omega$, we have

$$\frac{\sigma_i}{1-\sigma_i} = \frac{e_{r_i}(t,t+\omega)}{1-\sigma_i} \le G_i(t,s) \le \frac{e_{r_i}(t,t)}{1-\sigma_i} = \frac{1}{1-\sigma_i},$$
$$\frac{1}{\hat{\sigma}_j-1} = \frac{e_{(-d_j)}(t,t)}{\hat{\sigma}_j-1} \le \hat{G}_j(t,s) \le \frac{e_{(-d_j)}(t,t+\omega)}{\hat{\sigma}_j-1} = \frac{\hat{\sigma}_j}{\hat{\sigma}_j-1}.$$

Thus, the assertions (1) and (2) hold. Now we show that the assertion (3) holds too. Indeed, by $\sigma(t+\omega) = \sigma(t)+\omega$ and the integration by substitution, we have

$$G_i(t+\omega,s+\omega) = \frac{e_{r_i}(t+\omega,\sigma(s+\omega))}{1-e_{r_i}(0,\omega)} = \frac{e_{r_i}(t+\omega,\sigma(s)+\omega)}{1-e_{r_i}(0,\omega)} = \frac{e_{r_i}(t,\sigma(s))}{1-e_{r_i}(0,\omega)} = G_i(t,s).$$

Similarly one shows that $\hat{G}_j(t+\omega,s+\omega) = \hat{G}_j(t,s)$. The proof of Lemma 2.6 is complete. □

For the sake of obtaining the existence of a periodic solution of system (2.1), we need the following preparations.

Let X be a real Banach space, and K a closed, nonempty subset of X. Then K is a cone provided

(i) $k\alpha + l\beta \in K$ for all $\alpha,\beta \in K$ and all $k,l \ge 0$;

(ii) $\alpha, -\alpha \in K$ imply $\alpha = \theta$, here θ is the zero element of X.

Let E be a Banach space and K be a cone in E. The semi-order induced by the cone K is denoted by $\le$. That is, $x \le y$ if and only if $y - x \in K$. In addition, for a bounded subset $A \subset E$, let $\alpha_E(A)$ denote the (Kuratowski) measure of non-compactness defined by

$$\alpha_E(A) = \inf\bigl\{\delta > 0 : A \text{ admits a finite cover by subsets of } A_i \subset A \text{ such that } \operatorname{diam}(A_i) \le \delta\bigr\},$$

where $\operatorname{diam}(A_i)$ denotes the diameter of the set A_i.

Let E, F be two Banach spaces and $D \subset E$, a continuous and bounded map $\Phi : \overline{\Omega} \to F$ is called k-set contractive if for any bounded set $S \subset D$, we have

$$\alpha_F\bigl(\Phi(S)\bigr) \le k\alpha_E\bigl(\Phi(S)\bigr).$$

Φ is called strict-set contractive if it is k-set contractive for some $0 \leq k < 1$. Particularly, completely continuous operators are 0-set contractive.

The following lemma is useful for the proofs of our main results of this paper.

Lemma 2.7 ([32, 33]) *Let K be a cone in the real Banach space X and $K_{r,R} = \{x \in K : r \leq \|x\| \leq R\}$ with $R > r > 0$. Suppose that $\Phi : K_{r,R} \to K$ is strict-set contractive such that one of the following two conditions is satisfied*:

(i) $\Phi x \not\leq x$, $\forall x \in K$, $\|x\| = r$ *and* $\Phi x \not\geq x$, $\forall x \in K$, $\|x\| = R$.

(ii) $\Phi x \not\geq x$, $\forall x \in K$, $\|x\| = r$ *and* $\Phi x \not\leq x$, $\forall x \in K$, $\|x\| = R$.

Then Φ has at least one fixed point in $K_{r,R}$.

Define

$$PC(\mathbb{T}) = \bigl\{x = (x_1, \dots, x_{n+m}) : \mathbb{T} \to \mathbb{R}^{n+m}, x|_{(t_k,t_{k+1})} \in C_{rd}\bigl((t_k, t_{k+1}), \mathbb{R}^{n+m}\bigr),$$
$$\exists x\bigl(t_k^-\bigr) = x(t_k), x\bigl(t_k^+\bigr), k \in \mathbb{N}^+\bigr\}.$$

Set

$$X = \bigl\{x : x \in PC(\mathbb{T}), x(t + \omega) = x(t), t \in \mathbb{T}\bigr\}$$

equipped with the norm defined by $\|x\| = \sum_{i=1}^{n+m} |x_i|_0$, where $|x_i|_0 = \sup_{t\in[0,\omega]_{\mathbb{T}}}\{|x_i(t)|\}$, $i = 1, 2, \dots, n + m$. Then X is a Banach space. In view of Lemma 2.6, we define the cone K in X as

$$K = \left\{x = (u_1, \dots, u_n, v_1, \dots, v_m) \in X : u_i(t) \geq \sigma_i |u_i|_0, v_j(t) \geq \frac{1}{\hat{\sigma}_j} |v_j|_0, t \in [0, \omega]_{\mathbb{T}}\right\}.$$

Let the map Φ be defined by

$$(\Phi x)(t) = \bigl((\Phi_1 x)(t), \dots, (\Phi_n x)(t), (\Psi_1 x)(t), \dots, (\Psi_m x)(t)\bigr)^T, \tag{2.4}$$

where $x \in K$, $t \in \mathbb{T}$,

$$(\Phi_i x)(t) = \int_t^{t+\omega} G_i(t,s)u_i(s)\bigl[X_i\bigl(s, u(s), v(s)\bigr) + Y_i\bigl(s, u(s), v(s)\bigr)\bigr]\Delta s$$
$$+ \sum_{t_k\in[t,t+\omega)_{\mathbb{T}}} G_i(t, t_k)e_{r_i}\bigl(\sigma(t_k), t_k\bigr)I_{ik}\bigl(u(t_k), v(t_k)\bigr), \quad i = 1, 2, \dots, n,$$
$$(\Psi_j x)(t) = \int_t^{t+\omega} \hat{G}_j(t,s)v_j(s)\bigl[\hat{X}_j\bigl(s, u(s), v(s)\bigr) + \hat{Y}_j\bigl(s, u(s), v(s)\bigr)\bigr]\Delta s$$
$$+ \sum_{t_k\in[t,t+\omega)_{\mathbb{T}}} \hat{G}_j(t, t_k)e_{(-d_j)}\bigl(\sigma(t_k), t_k\bigr)\hat{I}_{jk}\bigl(u(t_k), v(t_k)\bigr), \quad j = 1, 2, \dots, m,$$

and $G_i(t,s)$ $(i = 1, 2, \dots, n)$, $\hat{G}_j(t,s)$ $(j = 1, 2, \dots, m)$ defined by (2.2) and (2.3), respectively.

Lemma 2.8 *Assume that* (H_1) *holds, then $\Phi : K \to K$ defined by* (2.4) *is well defined, that is*, $\Phi(K) \subset K$.

Proof For any $x \in K$, it is clear that $\Phi x \in PC(\mathbb{T})$. In view of Lemma 2.6 and (2.4), we obtain

$$\begin{aligned}
&(\Phi_i x)(t+\omega) \\
&\quad = \int_{t+\omega}^{t+2\omega} G_i(t+\omega,s)u_i(s)\big[X_i\big(s,u(s),v(s)\big)+Y_i\big(s,u(s),v(s)\big)\big]\Delta s \\
&\qquad + \sum_{t_k\in[t+\omega,t+2\omega)_{\mathbb{T}}} G_i(t+\omega,t_k)e_{r_i}\big(\sigma(t_k),t_k\big)I_{ik}\big(u(t_k),v(t_k)\big) \\
&\quad = \int_t^{t+\omega} G_i(t+\omega,\tau+\omega)u_i(\tau+\omega)\big[X_i\big(\tau+\omega,u(\tau+\omega),v(\tau+\omega)\big)+Y_i\big(\tau+\omega,u(\tau+\omega), \\
&v(\tau+\omega)\big)\big]\Delta\tau + \sum_{t_l\in[t,t+\omega)_{\mathbb{T}}} G_i(t+\omega,t_l+\omega)e_{r_i}\big(\sigma(t_l+\omega),t_l+\omega\big)I_{il}\big(u(t_l+\omega),v(t_l+\omega)\big) \\
&\quad = \int_t^{t+\omega} G_i(t,\tau)u_i(\tau)\big[X_i\big(\tau,u(\tau),v(\tau)\big)+Y_i\big(\tau,u(\tau),v(\tau)\big)\big]\Delta\tau \\
&\qquad + \sum_{t_l\in[t,t+\omega)_{\mathbb{T}}} G_i(t,t_l)e_{r_i}\big(\sigma(t_l),t_l\big)I_{il}\big(u(t_l),v(t_l)\big) = (\Phi_i x)(t),
\end{aligned}$$

that is, $(\Phi_i x)(t+\omega) = (\Phi_i x)(t)$, $\forall t \in \mathbb{T}$, $i = 1,2,\ldots,n$. Similarly, we have $(\Psi_j x)(t+\omega) = (\Psi_j x)(t)$, $\forall t \in \mathbb{T}$, $j = 1,2,\ldots,m$. So $\Phi x \in X$. For any $x \in K$, we have

$$\begin{aligned}
|\Phi_i x|_0 &\le \frac{1}{1-\sigma_i}\left[\int_0^{\omega} u_i(s)\big[X_i\big(s,u(s),v(s)\big)+Y_i\big(s,u(s),v(s)\big)\big]\Delta s\right. \\
&\qquad \left. + \sum_{k=1}^{p} e_{r_i}\big(\sigma(t_k),t_k\big)I_{ik}\big(u(t_k),v(t_k)\big)\right], \quad i=1,2,\ldots,n, \\
|\Psi_j x|_0 &\le \frac{\hat{\sigma}_j}{\hat{\sigma}_j-1}\left[\int_0^{\omega} v_j(s)\big[\hat{X}_j\big(s,u(s),v(s)\big)+\hat{Y}_j\big(s,u(s),v(s)\big)\big]\Delta s\right. \\
&\qquad \left. + \sum_{k=1}^{p} e_{(-d_j)}\big(\sigma(t_k),t_k\big)\hat{I}_{jk}\big(u(t_k),v(t_k)\big)\right], \quad j=1,2,\ldots,m,
\end{aligned}$$

and

$$\begin{aligned}
(\Phi_i x)(t) &\ge \frac{\sigma_i}{1-\sigma_i}\left[\int_t^{t+\omega} u_i(s)\big[X_i\big(s,u(s),v(s)\big)+Y_i\big(s,u(s),v(s)\big)\big]\Delta s\right. \\
&\qquad \left. + \sum_{k=1}^{p} e_{r_i}\big(\sigma(t_k),t_k\big)I_{ik}\big(u(t_k),v(t_k)\big)\right] \\
&= \frac{\sigma_i}{1-\sigma_i}\left[\int_0^{\omega} u_i(s)\big[X_i\big(s,u(s),v(s)\big)+Y_i\big(s,u(s),v(s)\big)\big]\Delta s\right. \\
&\qquad \left. + \sum_{k=1}^{p} e_{r_i}\big(\sigma(t_k),t_k\big)I_{ik}\big(u(t_k),v(t_k)\big)\right] \\
&\ge \sigma_i|\Phi_i x|_0, \quad i=1,2,\ldots,n,
\end{aligned}$$

$$\begin{aligned}(\Psi_j x)(t) &\geq \frac{1}{\hat{\sigma}_j - 1}\Bigg[\int_t^{t+\omega} v_j(s)\big[\hat{X}_j\big(s,u(s),v(s)\big) + \hat{Y}_j\big(s,u(s),v(s)\big)\big]\Delta s \\ &\quad + \sum_{k=1}^{p} e_{(-d_j)}\big(\sigma(t_k),t_k\big)\hat{I}_{jk}\big(u(t_k),v(t_k)\big)\Bigg] \\ &= \frac{1}{\hat{\sigma}_j - 1}\Bigg[\int_0^{\omega} v_j(s)\big[\hat{X}_j\big(s,u(s),v(s)\big) + \hat{Y}_j\big(s,u(s),v(s)\big)\big]\Delta s \\ &\quad + \sum_{k=1}^{p} e_{(-d_j)}\big(\sigma(t_k),t_k\big)\hat{I}_{jk}\big(u(t_k),v(t_k)\big)\Bigg] \\ &\geq \frac{1}{\hat{\sigma}_j}|\Psi_j x|_0, \quad j = 1,2,\ldots,m.\end{aligned}$$

So $\Phi x \in K$. This completes the proof of Lemma 2.8. □

Lemma 2.9 *Assume that* (H_1) *holds, then* $\Phi : K \to K$ *defined by* (2.4) *is completely continuous.*

Proof It is easy to see that Φ is continuous and bounded. Now we show that Φ maps bounded sets into relatively compact sets. Let $\Omega \subset K$ be an arbitrary open bounded set in K, then there exists a number $R > 0$ such that $\|x\| < R$ for any $x = (u_1,\ldots,u_n, v_1,\ldots,v_m)^T \in \Omega$. We prove that $\overline{\Phi(\Omega)}$ is compact. In fact, for any $x \in \Omega$ and $t \in [0,\omega]_{\mathbb{T}}$, we have

$$\begin{aligned}\big|(\Phi_i x)(t)\big| &= \int_t^{t+\omega} G_i(t,s)u_i(s)\big[X_i\big(s,u(s),v(s)\big) + Y_i\big(s,u(s),v(s)\big)\big]\Delta s \\ &\quad + \sum_{t_k\in[t,t+\omega)_{\mathbb{T}}} G_i(t,t_k)e_{r_i}\big(\sigma(t_k),t_k\big)I_{ik}\big(u(t_k),v(t_k)\big) \\ &\leq \frac{1}{1-\sigma_i}\Bigg[\int_0^{\omega} u_i(s)\big[X_i\big(s,u(s),v(s)\big) + Y_i\big(s,u(s),v(s)\big)\big]\Delta s \\ &\quad + \sum_{k=1}^{p} e_{r_i}\big(\sigma(t_k),t_k\big)I_{ik}\big(u(t_k),v(t_k)\big)\Bigg] \\ &\leq \frac{1}{1-\sigma_i}\Bigg[R\omega\Big(\max_{s\in[0,\omega]_{\mathbb{T}},x\in B(0,R)}\big\{X_i(s,u,v)\big\} + \max_{s\in[0,\omega]_{\mathbb{T}},x\in B(0,R)}\big\{Y_i(s,u,v)\big\}\Big) \\ &\quad + \frac{1}{\sigma_i}\sum_{k=1}^{p}\max_{x\in B(0,R)}\big\{I_{ik}(u,v)\big\}\Bigg] \\ &\triangleq A_i, \quad i = 1,2,\ldots,n,\end{aligned}$$

and

$$\begin{aligned}\big|(\Phi_i x)^{\Delta}(t)\big| &= \big|r_i(t)(\Phi_i x)(t) - u_i(t)\big[X_i\big(t,u(t),v(t)\big) + Y_i\big(t,u(t),v(t)\big)\big]\big| \\ &\leq r_i^M A_i + R\Big(\max_{t\in[0,\omega]_{\mathbb{T}},x\in B(0,R)}\big\{X_i(t,u,v)\big\} + \max_{t\in[0,\omega]_{\mathbb{T}},x\in B(0,R)}\big\{Y_i(t,u,v)\big\}\Big) \\ &\triangleq B_i, \quad i = 1,2,\ldots,n.\end{aligned}$$

Similarly, for any $x \in \Omega$ and $t \in [0,\omega]_{\mathbb{T}}$, we have

$$\begin{aligned}
\bigl|(\Psi_j x)(t)\bigr| &= \int_t^{t+\omega} \hat{G}_j(t,s)v_j(s)\bigl[\hat{X}_j\bigl(s,u(s),v(s)\bigr) + \hat{Y}_j\bigl(s,u(s),v(s)\bigr)\bigr]\Delta s \\
&\quad + \sum_{t_k\in[t,t+\omega)_{\mathbb{T}}} \hat{G}_j(t,t_k)e_{(-d_j)}\bigl(\sigma(t_k),t_k\bigr)\hat{I}_{jk}\bigl(u(t_k),v(t_k)\bigr) \\
&\le \frac{\hat{\sigma}_j}{\hat{\sigma}_j - 1}\Biggl[R\omega\Bigl(\max_{s\in[0,\omega]_{\mathbb{T}},x\in B(0,R)}\bigl\{\hat{X}_j(s,u,v)\bigr\} + \max_{s\in[0,\omega]_{\mathbb{T}},x\in B(0,R)}\bigl\{\hat{Y}_j(s,u,v)\bigr\}\Bigr) \\
&\quad + \hat{\sigma}_j\sum_{k=1}^{p}\max_{x\in B(0,R)}\bigl\{\hat{I}_{jk}(u,v)\bigr\}\Biggr] \\
&\triangleq \hat{A}_j, \quad j = 1,2,\ldots,m,
\end{aligned}$$

and

$$\begin{aligned}
\bigl|(\Psi_j x)^{\Delta}(t)\bigr| &= \bigl|-d_j(t)(\Psi_j x)(t) + v_j(s)\bigl[\hat{X}_j\bigl(t,u(t),v(t)\bigr) + \hat{Y}_j\bigl(t,u(t),v(t)\bigr)\bigr]\bigr| \\
&\le d_j^M\hat{A}_j + R\Bigl(\max_{t\in[0,\omega]_{\mathbb{T}},x\in B(0,R)}\bigl\{\hat{X}_j(t,u,v)\bigr\} + \max_{t\in[0,\omega]_{\mathbb{T}},x\in B(0,R)}\bigl\{\hat{Y}_j(t,u,v)\bigr\}\Bigr) \\
&\triangleq \hat{B}_j, \quad j = 1,2,\ldots,m.
\end{aligned}$$

Hence,

$$\bigl\|(\Phi x)\bigr\| \le \sum_{i=1}^{n} A_i + \sum_{j=1}^{m}\hat{A}_j, \qquad \bigl\|(\Phi x)^{\Delta}\bigr\| \le \sum_{i=1}^{n} B_i + \sum_{j=1}^{m}\hat{B}_j.$$

It follows from Lemma 2.4 in [34] that $\Phi(\bar{\Omega})$ is relatively compact in X. The proof of Lemma 2.9 is complete. □

3 Main results

In this section, we shall give our main results.

Theorem 3.1 *Assume that*

(H_2) $\max\{\max_{1\le i\le n}\{\frac{\gamma_i}{\sigma_i(1-\sigma_i)}\}, \max_{1\le j\le m}\{\frac{\hat{\gamma}_j\hat{\sigma}_j^2}{\hat{\sigma}_j-1}\}\} < 1$.

(H_3) $X_i^0 < \infty$, $\hat{X}_j^0 < \infty$, $Y_i^0 < \infty$, $\hat{Y}_j^0 < \infty$, $X_i^\infty > 0$, $\hat{X}_j^\infty > 0$, $Y_i^\infty > 0$, $\hat{Y}_j^\infty > 0$, $i = 1,2,\ldots,n$, $j = 1,2,\ldots,m$.

If (H_1)-(H_3) *hold, then system* (1.1) *has at least one* ω*-periodic solution.*

Proof By the assumptions (H_2) and (H_3) of the theorem, there exists a positive number δ such that

$$\max\biggl\{\max_{1\le i\le n}\biggl\{\frac{\gamma_i}{\sigma_i(1-\sigma_i)}\biggr\}, \max_{1\le j\le m}\biggl\{\frac{\hat{\gamma}_j\hat{\sigma}_j^2}{\hat{\sigma}_j - 1}\biggr\}\biggr\} + \delta < 1$$

and for any

$$0 < \epsilon < \min\biggl\{\frac{1}{2}, \frac{1}{2}\min_{1\le i\le n}\bigl\{X_i^\infty + Y_i^\infty\bigr\}, \frac{1}{2}\min_{1\le j\le m}\bigl\{\hat{X}_j^\infty + \hat{Y}_j^\infty\bigr\}\biggr\},$$

there exist positive numbers $r_0 < R_0$ such that, for $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$,

$$\begin{aligned}
&I_{ik}(x_1, \ldots, x_{n+m}) < (\gamma_i + \epsilon) \sum_{l=1}^{n+m} x_l, \quad \text{for } 0 < \sum_{l=1}^{n+m} x_l < r_0, \\
&\hat{I}_{jk}(x_1, \ldots, x_{n+m}) < (\hat{\gamma}_j + \epsilon) \sum_{l=1}^{n+m} x_l, \quad \text{for } 0 < \sum_{l=1}^{n+m} x_l < r_0, \\
&X_i(t, x_1, \ldots, x_{n+m}) < \left(X_i^0 + \epsilon\right) \sum_{l=1}^{n+m} x_l, \quad \text{for } 0 < \sum_{l=1}^{n+m} x_l < r_0, \\
&\hat{X}_j(t, x_1, \ldots, x_{n+m}) < \left(\hat{X}_j^0 + \epsilon\right) \sum_{l=1}^{n+m} x_l, \quad \text{for } 0 < \sum_{l=1}^{n+m} x_l < r_0, \\
&Y_i(t, x_1, \ldots, x_{n+m}) < \left(Y_i^0 + \epsilon\right) \sum_{l=1}^{n+m} x_l, \quad \text{for } 0 < \sum_{l=1}^{n+m} x_l < r_0, \\
&\hat{Y}_j(t, x_1, \ldots, x_{n+m}) < \left(\hat{Y}_j^0 + \epsilon\right) \sum_{l=1}^{n+m} x_l, \quad \text{for } 0 < \sum_{l=1}^{n+m} x_l < r_0, \\
&X_i(t, x_1, \ldots, x_{n+m}) > \left(X_i^\infty - \epsilon\right) \sum_{l=1}^{n+m} x_l, \quad \text{for } \sum_{l=1}^{n+m} x_l > R_0, \\
&\hat{X}_j(t, x_1, \ldots, x_{n+m}) > \left(\hat{X}_j^\infty - \epsilon\right) \sum_{l=1}^{n+m} x_l, \quad \text{for } \sum_{l=1}^{n+m} x_l > R_0, \\
&Y_i(t, x_1, \ldots, x_{n+m}) < \left(Y_i^\infty - \epsilon\right) \sum_{l=1}^{n+m} x_l, \quad \text{for } \sum_{l=1}^{n+m} x_l > R_0, \\
&\hat{Y}_j(t, x_1, \ldots, x_{n+m}) > \left(\hat{Y}_j^\infty - \epsilon\right) \sum_{l=1}^{n+m} x_l, \quad \text{for } \sum_{l=1}^{n+m} x_l > R_0.
\end{aligned}$$

Take

$$0 < r < \min\left\{ \min_{1 \le i \le n} \left\{ \frac{\delta(1 - \sigma_i)}{\omega(X_i^0 + Y_i^0 + 1)} \right\}, \min_{1 \le j \le m} \left\{ \frac{\delta(\hat{\sigma}_j - 1)}{\omega\hat{\sigma}_j(\hat{X}_j^0 + \hat{Y}_j^0 + 1)} \right\}, r_0 \right\}$$

and

$$\begin{aligned}
R = \max\Bigg\{ &\Big[\min_{1 \le i \le n, 1 \le j \le m} \left\{\sigma_i, \hat{\sigma}_j^{-1}\right\} \Big]^{-1} R_0, \\
&\Big[\omega \min_{1 \le i \le n, 1 \le j \le m} \left\{\sigma_i, \hat{\sigma}_j^{-1}\right\} \\
&\times \min\left\{ \min_{1 \le i \le n} \left\{ \frac{\sigma_i^2(X_i^\infty + Y_i^\infty - 2\epsilon)}{1 - \sigma_i} \right\}, \min_{1 \le j \le m} \left\{ \frac{\hat{X}_j^\infty + \hat{Y}_j^\infty - 2\epsilon}{\hat{\sigma}_j(\hat{\sigma}_j - 1)} \right\} \right\} \Big]^{-1} \Bigg\}.
\end{aligned}$$

Then we have $0 < r < R$. It follows from Lemmas 2.8 and 2.9 that Φ is strict-set contractive on $K_{r,R}$. By Lemma 2.5, it is easy to see that if there exists $x^* \in K$ such that $\Phi x^* = x^*$, then x^* is one positive ω-periodic solution of system (1.1). Now, we shall prove that condition (ii) of Lemma 2.7 holds.

First, we prove that $\Phi x \ngeq x$, $\forall x \in K$, $\|x\| = r$. Otherwise, there exists $x \in K$, $\|x\| = r$ such that $\Phi x \neq x$. So $\|x\| > 0$ and $\Phi x - x \in K$, which implies that

$$(\Phi_i x)(t) - u_i(t) \geq \sigma_i |\Phi_i x - u_i|_0 \geq 0, \quad \forall t \in [0,\omega]_{\mathbb{T}}, i = 1,2,\dots,n, \tag{3.1}$$

and

$$(\Psi_j x)(t) - v_j(t) \geq \frac{1}{\hat{\sigma}_j} |\Psi_j x - v_j|_0 \geq 0, \quad \forall t \in [0,\omega]_{\mathbb{T}}, j = 1,2,\dots,m. \tag{3.2}$$

Moreover, for $t \in [0,\omega]_{\mathbb{T}}$, we have

$$\begin{aligned}
(\Phi_i x)(t) &= \int_t^{t+\omega} G_i(t,s) u_i(s) \big[X_i\big(s,u(s),v(s)\big) + Y_i\big(s,u(s),v(s)\big)\big] \Delta s \\
&\quad + \sum_{t_k \in [t,t+\omega)_{\mathbb{T}}} G_i(t,t_k) e_{r_i}\big(\sigma(t_k),t_k\big) I_{ik}\big(u(t_k),v(t_k)\big) \\
&\leq \frac{1}{1-\sigma_i} \left[\int_0^{\omega} u_i(s) \big[X_i\big(s,u(s),v(s)\big) + Y_i\big(s,u(s),v(s)\big)\big] \Delta s \right. \\
&\quad \left. + \sum_{k=1}^{p} e_{r_i}\big(\sigma(t_k),t_k\big) I_{ik}\big(u(t_k),v(t_k)\big) \right] \\
&\leq \frac{\omega |u_i|}{1-\sigma_i} \left[\big(X_i^0 + \epsilon\big) \left(\sum_{i=1}^{n} |u_i|_0 + \sum_{j=1}^{m} |v_j|_0 \right) + \big(Y_i^0 + \epsilon\big) \left(\sum_{i=1}^{n} |u_i|_0 + \sum_{j=1}^{m} |v_j|_0 \right) \right] \\
&\quad + \frac{1}{\sigma_i(1-\sigma_i)} (\gamma_i + \epsilon) \left(\sum_{i=1}^{n} |u_i|_0 + \sum_{j=1}^{m} |v_j|_0 \right) \\
&\leq \frac{\omega(X_i^0 + Y_i^0 + 2\epsilon)}{1-\sigma_i} r^2 + \frac{\gamma_i + \epsilon}{\sigma_i(1-\sigma_i)} r \\
&< \left[\frac{\omega(X_i^0 + Y_i^0 + 2\epsilon)}{1-\sigma_i} \times \frac{\delta(1-\sigma_i)}{\omega(X_i^0 + Y_i^0 + 1)} + \frac{\gamma_i + \epsilon}{\sigma_i(1-\sigma_i)} \right] r \\
&= \left[\frac{\delta(X_i^0 + Y_i^0 + 2\epsilon)}{X_i^0 + Y_i^0 + 1} + \frac{\gamma_i + \epsilon}{\sigma_i(1-\sigma_i)} \right] r, \quad i = 1,2,\dots,n.
\end{aligned} \tag{3.3}$$

Similarly, for $t \in [0,\omega]_{\mathbb{T}}$, we have

$$\begin{aligned}
(\Psi_j x)(t) &= \int_t^{t+\omega} \hat{G}_j(t,s) v_j(s) \big[\hat{X}_j\big(s,u(s),v(s)\big) + \hat{Y}_j\big(s,u(s),v(s)\big)\big] \Delta s \\
&\quad + \sum_{t_k \in [t,t+\omega)_{\mathbb{T}}} \hat{G}_j(t,t_k) e_{(-d_j)}\big(\sigma(t_k),t_k\big) \hat{I}_{jk}\big(u(t_k),v(t_k)\big) \\
&\leq \frac{\omega \hat{\sigma}_j |v_j|}{\hat{\sigma}_j - 1} \left[\big(\hat{X}_j^0 + \epsilon\big) \left(\sum_{i=1}^{n} |u_i|_0 + \sum_{j=1}^{m} |v_j|_0 \right) + \big(\hat{Y}_j^0 + \epsilon\big) \left(\sum_{i=1}^{n} |u_i|_0 + \sum_{j=1}^{m} |v_j|_0 \right) \right] \\
&\quad + \frac{\hat{\sigma}_j^2}{\hat{\sigma}_j - 1} (\hat{\gamma}_j + \epsilon) \left(\sum_{i=1}^{n} |u_i|_0 + \sum_{j=1}^{m} |v_j|_0 \right) \\
&\leq \frac{\omega \hat{\sigma}_j (\hat{X}_j^0 + \hat{Y}_j^0 + 2\epsilon)}{\hat{\sigma}_j - 1} r^2 + \frac{\hat{\sigma}_j^2 (\hat{\gamma}_j + \epsilon)}{\hat{\sigma}_j - 1} r
\end{aligned}$$

$$< \left[\frac{\omega\hat{\sigma}_j(\hat{X}_j^0 + \hat{Y}_j^0 + 2\epsilon)}{\hat{\sigma}_j - 1} \times \frac{\delta(\hat{\sigma}_j - 1)}{\omega\hat{\sigma}_j(\hat{X}_j^0 + \hat{Y}_j^0 + 1)} + \frac{\hat{\sigma}_j^2(\hat{\gamma}_j + \epsilon)}{\hat{\sigma}_j - 1}\right] r$$

$$= \left[\frac{\delta(\hat{X}_j^0 + \hat{Y}_j^0 + 2\epsilon)}{\hat{X}_j^0 + \hat{Y}_j^0 + 1} + \frac{\hat{\sigma}_j^2(\hat{\gamma}_j + \epsilon)}{\hat{\sigma}_j - 1}\right] r, \quad j = 1, 2, \ldots, m. \tag{3.4}$$

From (3.1)-(3.4) and the arbitrariness of ϵ, we get

$$\|x\| \le \|\Phi x\| \le \left(\max\left\{\max_{1\le i\le n}\left\{\frac{\gamma_i}{\sigma_i(1-\sigma_i)}\right\}, \max_{1\le j\le m}\left\{\frac{\hat{\gamma}_j\hat{\sigma}_j^2}{\hat{\sigma}_j - 1}\right\}\right\} + \delta\right) r < r = \|x\|,$$

which is a contradiction. Next, we prove that $\Phi x \nleq x$, $\forall x \in K$, $\|x\| = R$ also holds. Indeed, we only need to prove that $\Phi x \nless x$, $\forall x \in K$, $\|x\| = R$. For the sake of contradiction, suppose that there exists $x \in K$ and $\|x\| = R$ such that $\Phi x < x$. Thus $x - \Phi x \in K \setminus \{\theta = (0, 0, \ldots, 0)^T\}$. Furthermore, for any $t \in [0, \omega]_{\mathbb{T}}$, we have

$$u_i(t) - (\Phi x)(t) \ge \sigma_i |u_i - \Phi_i x|_0 \ge 0, \quad i = 1, 2, \ldots, n, \tag{3.5}$$

and

$$v_j(t) - (\Psi x)(t) \ge \hat{\sigma}_j^{-1} |v_j - \Psi_j x|_0 \ge 0, \quad j = 1, 2, \ldots, m. \tag{3.6}$$

Since $x \in K$ and $\|x\| = R$, we find $\forall s \in [0, \omega]_{\mathbb{T}}$,

$$\begin{aligned}
&\sum_{l=1}^{n} u_l\bigl(s - \tau_{il}(s)\bigr) + \sum_{j=1}^{m} v_j\bigl(s - \tau_{ij}(s)\bigr) \\
&\quad \ge \sum_{l=1}^{n} \sigma_l |u_l|_0 + \sum_{j=1}^{m} \hat{\sigma}_j^{-1} |v_j|_0 \\
&\quad \ge \min_{1\le l\le n, 1\le j\le m}\bigl\{\sigma_l, \hat{\sigma}_j^{-1}\bigr\}\left(\sum_{l=1}^{n} |u_l|_0 + \sum_{j=1}^{m} |v_j|_0\right) \\
&\quad = \min_{1\le l\le n, 1\le j\le m}\bigl\{\sigma_l, \hat{\sigma}_j^{-1}\bigr\} R \ge R_0, \quad i = 1, 2, \ldots, n,
\end{aligned}$$

and

$$\begin{aligned}
&\sum_{l=1}^{n} \int_{-\infty}^{0} K_{il}(\tau) u_l(s + \tau)\Delta\tau + \sum_{j=1}^{m} \int_{-\infty}^{0} L_{ij}(\tau) v_j(s + \tau)\Delta\tau \\
&\quad \ge \sum_{l=1}^{n} \sigma_l |u_l|_0 \int_{-\infty}^{0} K_{il}(\tau)\Delta\tau + \sum_{j=1}^{m} \hat{\sigma}_j^{-1} |v_j|_0 \int_{-\infty}^{0} L_{ij}(\tau)\Delta\tau \\
&\quad \ge \min_{1\le l\le n, 1\le j\le m}\bigl\{\sigma_l, \hat{\sigma}_j^{-1}\bigr\}\left\{\sum_{l=1}^{n} |u_l|_0 + \sum_{j=1}^{m} |v_j|_0\right\} \\
&\quad = \min_{1\le l\le n, 1\le j\le m}\bigl\{\sigma_l, \hat{\sigma}_j^{-1}\bigr\} R \ge R_0, \quad i = 1, 2, \ldots, n.
\end{aligned}$$

In addition, for any $t \in [0,\omega]_{\mathbb{T}}$, we have

$$\begin{aligned}
(\Phi_i x)(t) &= \int_t^{t+\omega} G_i(t,s)u_i(s)\bigl[X_i\bigl(s,u(s),v(s)\bigr) + Y_i\bigl(s,u(s),v(s)\bigr)\bigr]\Delta s \\
&\quad + \sum_{t_k\in[t,t+\omega)_{\mathbb{T}}} G_i(t,t_k)e_{r_i}\bigl(\sigma(t_k),t_k\bigr)I_{ik}\bigl(u(t_k),v(t_k)\bigr) \\
&\geq \frac{\sigma_i^2|u_i|_0}{1-\sigma_i}\int_0^{\omega}\bigl[X_i\bigl(s,u(s),v(s)\bigr) + Y_i\bigl(s,u(s),v(s)\bigr)\bigr]\Delta s \\
&\geq \frac{\sigma_i^2\omega|u_i|_0}{1-\sigma_i}\Biggl[\bigl(X_i^{\infty}-\epsilon\bigr)\Biggl(\sum_{i=1}^{n}\sigma_i|u_i|_0 + \sum_{j=1}^{m}\sigma_j^{-1}|v_j|_0\Biggr) \\
&\quad + \bigl(Y_i^{\infty}-\epsilon\bigr)\Biggl(\sum_{l=1}^{n}\int_{-\infty}^{0}K_{il}(\tau)\sigma_l|u_l|_0\Delta\tau + \sum_{j=1}^{m}\int_{-\infty}^{0}L_{ij}(\tau)\sigma_j^{-1}|v_j|_0\Delta\tau\Biggr)\Biggr] \\
&\geq \frac{\sigma_i^2\omega|u_i|_0}{1-\sigma_i}\bigl(X_i^{\infty}+Y_i^{\infty}-2\epsilon\bigr)\min_{1\leq i\leq n,1\leq j\leq m}\bigl\{\sigma_i,\hat{\sigma}_j^{-1}\bigr\}R, \quad i=1,2,\ldots,n.
\end{aligned} \tag{3.7}$$

Similarly, for any $t \in [0,\omega]_{\mathbb{T}}$, we derive

$$\begin{aligned}
(\Psi_j x)(t) &= \int_t^{t+\omega} \hat{G}_j(t,s)v_j(s)\bigl[\hat{X}_j\bigl(s,u(s),v(s)\bigr) + \hat{Y}_j\bigl(s,u(s),v(s)\bigr)\bigr]\Delta s \\
&\quad + \sum_{t_k\in[t,t+\omega)_{\mathbb{T}}} \hat{G}_j(t,t_k)e_{(-d_j)}\bigl(\sigma(t_k),t_k\bigr)\hat{I}_{jk}\bigl(u(t_k),v(t_k)\bigr) \\
&\geq \frac{|v_j|_0}{\hat{\sigma}_j(\hat{\sigma}_j-1)}\int_0^{\omega}\bigl[\hat{X}_j\bigl(s,u(s),v(s)\bigr) + \hat{Y}_j\bigl(s,u(s),v(s)\bigr)\bigr]\Delta s \\
&\geq \frac{\omega|v_j|_0}{\hat{\sigma}_j(\hat{\sigma}_j-1)}\Biggl[\bigl(\hat{X}_j^{\infty}-\epsilon\bigr)\Biggl(\sum_{i=1}^{n}\sigma_i|u_i|_0 + \sum_{j=1}^{m}\sigma_j^{-1}|v_j|_0\Biggr) \\
&\quad + \bigl(\hat{Y}_j^{\infty}-\epsilon\bigr)\Biggl(\sum_{l=1}^{n}\int_{-\infty}^{0}\hat{K}_{jl}(\tau)\sigma_l|u_l|_0\Delta\tau + \sum_{i=1}^{m}\int_{-\infty}^{0}\hat{L}_{ij}(\tau)\sigma_j^{-1}|v_j|_0\Delta\tau\Biggr)\Biggr] \\
&\geq \frac{\omega|v_j|_0}{\hat{\sigma}_j(\hat{\sigma}_j-1)}\bigl(\hat{X}_j^{\infty}+\hat{Y}_j^{\infty}-2\epsilon\bigr)\min_{1\leq i\leq n,1\leq j\leq m}\bigl\{\sigma_i,\hat{\sigma}_j^{-1}\bigr\}R, \quad j=1,2,\ldots,m.
\end{aligned} \tag{3.8}$$

It follows from (3.7) and (3.8) that

$$\begin{aligned}
\|\Phi x\| &= \sum_{i=1}^{n}|\Phi_i x|_0 + \sum_{j=1}^{m}|\Psi_j x|_0 \\
&\geq \sum_{i=1}^{n}\frac{\sigma_i^2\omega|u_i|_0}{1-\sigma_i}\bigl(X_i^{\infty}+Y_i^{\infty}-2\epsilon\bigr)\min_{1\leq i\leq n,1\leq j\leq m}\bigl\{\sigma_i,\hat{\sigma}_j^{-1}\bigr\}R \\
&\quad + \sum_{j=1}^{m}\frac{\omega|v_j|_0}{\hat{\sigma}_j(\hat{\sigma}_j-1)}\bigl(\hat{X}_j^{\infty}+\hat{Y}_j^{\infty}-2\epsilon\bigr)\min_{1\leq i\leq n,1\leq j\leq m}\bigl\{\sigma_i,\hat{\sigma}_j^{-1}\bigr\}R \\
&\geq \min\Biggl\{\min_{1\leq i\leq n}\Biggl\{\frac{\sigma_i^2(X_i^{\infty}+Y_i^{\infty}-2\epsilon)}{1-\sigma_i}\Biggr\}, \min_{1\leq j\leq m}\Biggl\{\frac{\hat{X}_j+\hat{Y}_j-2\epsilon}{\hat{\sigma}_j(\hat{\sigma}_j-1)}\Biggr\}\Biggr\} \\
&\quad \times \min_{1\leq i\leq n,1\leq j\leq m}\bigl\{\sigma_i,\hat{\sigma}_j^{-1}\bigr\}R^2\omega \\
&\geq R.
\end{aligned} \tag{3.9}$$

From (3.5)-(3.9), we obtain $\|x\| > \|\Phi x\| \geq R$, which is a contradiction. Therefore, condition (ii) of Lemma 2.7 holds. By Lemma 2.7, we see that Φ has at least one nonzero fixed point in $K_{r,R}$. Therefore, system (1.1) has at least one positive ω-periodic solution. The proof of Theorem 3.1 is complete. □

Similar to the proof of Theorem 3.1, one can show that the existence of positive ω-periodic solutions for the impulsive system without infinite distributed time delays or with pure infinite distributed delays on time scales.

Theorem 3.2 *In system* (1.1), *assume that* $Y_i(t,\cdot,\cdot) \equiv 0$, $\hat{Y}_j(t,\cdot,\cdot) \equiv 0$, $X_i^0 < \infty$, $\hat{X}_j^0 < \infty$, $X_i^\infty > 0$, $\hat{X}_j^\infty > 0$ $(i = 1,\dots,n; j = 1,\dots,m)$ *and* (H_1)-(H_2) *hold. Then system* (1.1) *has at least one positive* ω*-periodic solution.*

Theorem 3.3 *In system* (1.1), *assume that* $X_i(t,\cdot,\cdot) \equiv 0$, $\hat{X}_j(t,\cdot,\cdot) \equiv 0$, $Y_i^0 < \infty$, $\hat{Y}_j^0 < \infty$, $Y_i^\infty > 0$, $\hat{Y}_j^\infty > 0$ $(i = 1,\dots,n; j = 1,\dots,m)$ *and* (H_1)-(H_2) *hold. Then system* (1.1) *has at least one positive* ω*-periodic solution.*

In system (1.1), if $I_{ik} \equiv 0$, $\hat{I}_{jk} \equiv 0$ $(i = 1,2,\dots,n; j = 1,2,\dots,m)$, then system (1.1) changes into the following nonimpulsive system:

$$\begin{cases} u_i^\Delta(t) = u_i(t)[r_i(t) - X_i(t,u(t),v(t)) - Y_i(t,u(t),v(t))], & t \in \mathbb{T}, \\ v_j^\Delta(t) = v_j(t)[-d_j(t) + \hat{X}_j(t,u(t),v(t)) + \hat{Y}_j(t,u(t),v(t))], & t \in \mathbb{T}, \end{cases} \tag{3.10}$$

where $\mathbb{T}$, r_i, d_j, X_i, Y_i, $\hat{X}_j$, $\hat{Y}_j$ $(i = 1,2,\dots,n; j = 1,2,\dots,m)$ are the same as those in system (1.1). We have the following.

Theorem 3.4 *Assume that* $X_i^0 < \infty$, $\hat{X}_j^0 < \infty$, $Y_i^\infty > 0$, $\hat{Y}_j^\infty > 0$ $(i = 1,\dots,n; j = 1,\dots,m)$ *and* (H_1) *hold. Then system* (3.10) *has at least one positive* ω*-periodic solution.*

Proof Let $X = \{x : x \in C_{rd}(\mathbb{T},\mathbb{R}^{n+m}), x(t+\omega) = x(t), t \in \mathbb{T}\}$ with the norm defined by $\|x\| = \sum_{i=1}^{n+m} |x|_0|$, here $|x|_0 = \sup_{t\in[0,\omega]_{\mathbb{T}}}\{|x_i(t)|\}$, $i = 1,2,\dots,n+m$. Then X is a Banach space. Define the cone K in X by

$$\begin{aligned} K = \bigl\{ & x : x = (u_1,\dots,u_n,v_1,\dots,v_m)^T \in X, u_i(t) \geq \sigma_i|u_i|_0, v_j(t) \geq \hat{\sigma}_j^{-1}|v_j|_0, \\ & t \in [0,\omega]_{\mathbb{T}}, i = 1,\dots,n, j = 1,\dots,m\bigr\}. \end{aligned}$$

The map Φ be defined by

$$(\Phi x)(t) = \bigl((\Phi_1 x)(t),\dots,(\Phi_n x)(t),(\Psi_1 x)(t),\dots,(\Psi_m x)(t)\bigr),$$

where $x \in K$, $t \in \mathbb{T}$,

$$\begin{aligned} (\Phi_i x)(t) &= \int_t^{t+\omega} G_i(t,s)u_i(s)\bigl[X_i\bigl(s,u(s),v(s)\bigr) + Y_i\bigl(s,u(s),v(s)\bigr)\bigr]\Delta s, \quad i = 1,2,\dots,n, \\ (\Psi_j x)(t) &= \int_t^{t+\omega} \hat{G}_j(t,s)v_j(s)\bigl[\hat{X}_j\bigl(s,u(s),v(s)\bigr) + \hat{Y}_j\bigl(s,u(s),v(s)\bigr)\bigr]\Delta s, \quad j = 1,2,\dots,m, \end{aligned}$$

and $G_i(t,s)$, $\hat{G}_j(t,s)$ ($i = 1,2,\ldots,n; j = 1,2,\ldots,m$) are defined by (2.2) and (2.3), respectively. The remainder of the proof is similar to the proof of Theorem 3.1 and is omitted here. The proof of Theorem 3.4 is complete. □

Similarly, we can prove the following.

Theorem 3.5 *In system* (3.10), *assume that* $Y_i(t,\cdot,\cdot) \equiv 0$, $\hat{Y}_j(t,\cdot,\cdot) \equiv 0$, $X_i^0 < \infty$, $\hat{X}_j^0 < \infty$, $X_i^\infty > 0$, $\hat{X}_j^\infty > 0$ ($i = 1,\ldots,n; j = 1,\ldots,m$) *and* ($H_1$) *hold. Then system* (3.10) *has at least one positive* ω*-periodic solution.*

Theorem 3.6 *In system* (3.10), *assume that* $X_i(t,\cdot,\cdot) \equiv 0$, $\hat{X}_j(t,\cdot,\cdot) \equiv 0$, $Y_i^0 < \infty$, $\hat{Y}_j^0 < \infty$, $Y_i^\infty > 0$, $\hat{Y}_j^\infty > 0$ ($i = 1,\ldots,n; j = 1,\ldots,m$) *and* ($H_1$) *hold. Then system* (3.10) *has at least one positive* ω*-periodic solution.*

4 Applications

In this section, as applications of our main results, we will give some existence results of positive periodic solutions for Lotka-Volterra systems with or without impulses.

Firstly, we consider two classes of Lotka-Volterra system with impulses and time delays on time scales as follows:

$$\begin{cases} u_i^\Delta(t) = u_i(t)[a_i(t) - \sum_{j=1}^n a_{ij}(t)\int_{-\infty}^0 K_{ij}(s)u_j(t+s)\Delta s], & t \neq t_k, t \in \mathbb{T}, \\ u_i(t_k^+) = u_i(t_k^-) - I_{ik}(u_i(t_k)), & k = 1,2,\ldots, \end{cases} \tag{4.1}$$

and

$$\begin{cases} u_i^\Delta(t) = u_i(t)[a_i(t) - \sum_{j=1}^n a_{ij}(t)u_j(t) - \sum_{j=1}^n b_{ij}(t)u_j(t-\tau_j(t))], & t \neq t_k, t \in \mathbb{T}, \\ u_i(t_k^+) = u_i(t_k^-) - I_{ik}(u_i(t_k)), & k = 1,2,\ldots, \end{cases} \tag{4.2}$$

where $i = 1,2,\ldots,n$. $\mathbb{T}$ is an ω-periodic time scale, $\omega > 0$ is a constant. $a_i \in C_{rd}(\mathbb{T},(0,\infty))$, $a_{ij}, b_{ij} \in C_{rd}(\mathbb{T},(0,\infty))$, $\tau_j \in C_{rd}(\mathbb{T},(0,\infty)_{\mathbb{T}})$ ($j = 1,2,\ldots,n$) are rd-continuous ω-periodic functions. $K_{ij} \in C_{rd}((-\infty,0)_{\mathbb{T}},(0,\infty))$ with $\int_{-\infty}^0 K_{ij}(s)\Delta s = 1$ ($i,j = 1,2,\ldots,n$), $I_{ik} \in C((0,\infty),(0,\infty))$. There exists a positive integer p such that $t_{i,k+p} = t_k + \omega$, $I_{i,k+p} = I_{ik}$, $k \in \mathbb{Z}$, $[0,\omega)_{\mathbb{T}} \cap \{t_k : k \in \mathbb{Z}\} = \{t_1,t_2,\ldots,t_p\}$.

Theorem 4.1 *Assume that* $\max_{1\le i\le n}\{\frac{\gamma_i}{\sigma_i(1-\sigma_i)}\} < 1$, $a_{ij}(t) > 0$ ($i,j = 1,2,\ldots,n$), *then system* (4.1) *has at least one positive* ω*-periodic solution.*

Proof In this case,

$$Y_i(t,u_1,u_2,\ldots,u_n) = \sum_{j=1}^n a_{ij}(t)u_j, \quad i = 1,2,\ldots,n,$$

$$Y_i^0 = \limsup_{\sum_{i=1}^n u_i \to 0} \max_{t\in[0,\omega]_{\mathbb{T}}} \frac{Y_i(t,u_1,\ldots,u_n)}{\sum_{i=1}^n u_i} \le \max_{1\le j\le n}\Bigl\{\max_{t\in[0,\omega]_{\mathbb{T}}}\bigl\{a_{ij}(t)\bigr\}\Bigr\} < \infty, \quad i = 1,2,\ldots,n,$$

and

$$Y_i^\infty = \liminf_{\sum_{i=1}^n u_i \to 0} \min_{t\in[0,\omega]_{\mathbb{T}}} \frac{Y_i(t,u_1,\ldots,u_n)}{\sum_{i=1}^n u_i} \ge \min_{1\le j\le n}\Bigl\{\min_{t\in[0,\omega]_{\mathbb{T}}}\bigl\{a_{ij}(t)\bigr\}\Bigr\} > 0, \quad i = 1,2,\ldots,n.$$

It follows from Theorem 3.3 that system (4.1) has at least one positive ω-periodic solution. The proof of Theorem 4.1 is complete. □

In the light of Theorem 3.1, we have the following.

Theorem 4.2 *Assume that* $\max_{1\leq i\leq n}\{\frac{\gamma_i}{\sigma_i(1-\sigma_i)}\} < 1$, $a_{ij}(t), b_{ij}(t) > 0$ $(i,j = 1,2,\ldots,n)$, *then system* (4.2) *has at least one positive ω-periodic solution.*

Next we consider the following predator-prey delay Lotka-Volterra system with impulses on time scales:

$$\begin{cases} u_i^{\Delta}(t) = u_i(t)[a_i(t) - \sum_{l=1}^{n} a_{il}(t)u_l(t-\sigma_{il}(t)) - \sum_{j=1}^{m} b_{ij}(t)v_j(t-\tau_{ij}(t))], \\ \quad t \neq t_k, t \in \mathbb{T}, \\ u_i(t_k^+) = u_i(t_k^-) - I_{ik}(u_i(t_k)), \quad k = 1,2,\ldots, \\ v_j^{\Delta}(t) = v_j(t)[-r_j(t) - \sum_{l=1}^{n} d_{jl}(t)u_l(t-\delta_{jl}(t)) - \sum_{h=1}^{m} e_{jh}(t)v_h(t-\theta_{jh}(t))], \\ \quad t \neq t_k, t \in \mathbb{T}, \\ v_j(t_k^+) = v_j(t_k^-) + \hat{I}_{jk}(v_j(t_k)), \quad k = 1,2,\ldots, \end{cases} \tag{4.3}$$

where $i = 1,2,\ldots,n$, $j = 1,2,\ldots,m$. $\mathbb{T}$ is an ω-periodic time scale, $\omega > 0$ is a constant. $a_i, r_j \in C_{rd}(\mathbb{T},(0,\infty))$, $a_{il}, b_{ij}, d_{jl}, e_{jh} \in C_{rd}(\mathbb{T},(0,\infty))$, $\sigma_{il}, \tau_{ij}, \delta_{jl}, \theta_{jh} \in C_{rd}(\mathbb{T},(0,\infty)_{\mathbb{T}})$ $(i,l = 1,2,\ldots,n;$ $j,h = 1,2,\ldots,m)$ are ω-periodic functions. $I_{ik}, \hat{I}_{jk} \in C((0,\infty),(0,\infty))$. there exists a positive integer p such that $t_{i,k+p} = t_k + \omega$, $I_{i,k+p} = I_{ik}$, $k \in \mathbb{Z}$, $[0,\omega)_{\mathbb{T}} \cap \{t_k : k \in \mathbb{Z}\} = \{t_1, t_2, \ldots, t_p\}$.

Theorem 4.3 *Assume that* $\max_{1\leq i\leq n}\{\frac{\gamma_i}{\sigma_i(1-\sigma_i)}\} < 1$, $\max_{1\leq j\leq n}\{\frac{\hat{\gamma}_j\hat{\sigma}_j}{\hat{\sigma}_j-1}\} < 1$, $a_{il}(t), b_{ij}(t), d_{jl}(t), e_{jh}(t) > 0$ $(i,l = 1,2,\ldots,n; j,h = 1,2,\ldots,m)$ *and* $\sup_{t\in[0,\omega]_{\mathbb{T}}}\{\mu(t)r_j(t)\} < 1$ $(j = 1,2,\ldots,m)$, *then system* (4.3) *has at least one positive ω-periodic solution.*

Proof In this case,

$$X_i(t,u_1,\ldots,u_n,v_1,\ldots,v_m) = \sum_{l=1}^{n} a_{il}(t)u_l + \sum_{j=1}^{m} b_{ij}(t)v_j, \quad i = 1,2,\ldots,n,$$

$$\hat{X}_j(t,u_1,\ldots,u_n,v_1,\ldots,v_m) = \sum_{l=1}^{n} d_{jl}(t)u_l + \sum_{h=1}^{m} e_{jh}(t)v_h, \quad j = 1,2,\ldots,m,$$

$$\begin{aligned} X_i^0 &= \limsup_{\sum_{i=1}^{n} u_i + \sum_{j=1}^{m} v_j \to 0} \max_{t\in[0,\omega]_{\mathbb{T}}} \frac{X_i(t,u_1,\ldots,u_n,v_1,\ldots,v_m)}{\sum_{i=1}^{n} u_i + \sum_{j=1}^{m} v_j} \\ &\leq \max_{1\leq l\leq n, 1\leq j\leq m}\Big\{\max_{t\in[0,\omega]_{\mathbb{T}}}\{a_{il}(t), b_{ij}(t)\}\Big\} < \infty, \quad i = 1,2,\ldots,n, \\ X_i^{\infty} &= \liminf_{\sum_{i=1}^{n} u_i + \sum_{j=1}^{m} v_j \to 0} \min_{t\in[0,\omega]_{\mathbb{T}}} \frac{X_i(t,u_1,\ldots,u_n,v_1,\ldots,v_m)}{\sum_{i=1}^{n} u_i + \sum_{j=1}^{m} v_j} \\ &\geq \min_{1\leq l\leq n, 1\leq j\leq m}\Big\{\min_{t\in[0,\omega]_{\mathbb{T}}}\{a_{il}(t), b_{ij}(t)\}\Big\} > 0, \quad i = 1,2,\ldots,n, \\ \hat{X}_j^0 &= \limsup_{\sum_{i=1}^{n} u_i + \sum_{j=1}^{m} v_j \to 0} \max_{t\in[0,\omega]_{\mathbb{T}}} \frac{\hat{X}_j(t,u_1,\ldots,u_n,v_1,\ldots,v_m)}{\sum_{i=1}^{n} u_i + \sum_{j=1}^{m} v_j} \\ &\leq \max_{1\leq l\leq n, 1\leq h\leq m}\Big\{\max_{t\in[0,\omega]_{\mathbb{T}}}\{d_{jl}(t), e_{jh}(t)\}\Big\} < \infty, \quad j = 1,2,\ldots,m, \end{aligned}$$

and

$$\hat{X}_j^\infty = \liminf_{\sum_{i=1}^n u_i + \sum_{j=1}^m v_j \to 0} \min_{t\in[0,\omega]_\mathbb{T}} \frac{\hat{X}_j(t, u_1, \ldots, u_n, v_1, \ldots, v_m)}{\sum_{i=1}^n u_i + \sum_{j=1}^m v_j}$$
$$\geq \min_{1\leq l\leq n, 1\leq h\leq m} \left\{ \min_{t\in[0,\omega]_\mathbb{T}} \{ d_{jl}(t), e_{jh}(t) \} \right\} > 0, \quad j = 1, 2, \ldots, m.$$

By Theorem 3.2, system (4.3) has at least one positive ω-periodic solution. The proof is complete. □

To this end, considering the existence of positive periodic solutions for systems (4.1)-(4.3) without impulses, we conclude the following assertions.

Theorem 4.4 *In system* (4.1), *assume that* $I_{ik} \equiv 0$, $a_{ij}(t) > 0$ $(i, j = 1, 2, \ldots, n)$, *then system* (4.1) *has at least one positive* ω*-periodic solution.*

Theorem 4.5 *In system* (4.2), *assume that* $I_{ik} \equiv 0$, $a_{ij}(t), b_{ij}(t) > 0$ $(i, j = 1, 2, \ldots, n)$, *then system* (4.2) *has at least one positive* ω*-periodic solution.*

Theorem 4.6 *In system* (4.3), *assume that* $I_{ik} \equiv 0$, $\hat{I}_{jk} \equiv 0$, $a_{il}(t), b_{ij}(t), d_{jl}(t), e_{jh}(t) > 0$ $(i, l = 1, 2, \ldots, n; j, h = 1, 2, \ldots, m)$ *and* $\sup_{t\in[0,\omega]_\mathbb{T}}\{\mu(t)r_j(t)\} < 1$ $(j = 1, 2, \ldots, m)$, *then system* (4.3) *has at least one positive* ω*-periodic solution.*

Acknowledgements
The author thanks the referees for a number of suggestions which have improved many aspects of this article. This work is supported by the National Natural Sciences Foundation of People's Republic of China under Grant (No. 11161025, No. 11661047).

Competing interests
The author declares to have no competing interests.

Authors' contributions
The author read and approved the final manuscript.

Publisher's Note

Received: 4 July 2017 Accepted: 28 September 2017 Published online: 13 October 2017

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