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Delay-dependent stabilization of a class of time-delay nonlinear systems: LMI approach

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Abstract

This paper deals with the state and output feedback stabilization problems for a family of nonlinear time-delay systems satisfying some relaxed triangular-type condition. A new delay-dependent stabilization condition using a controller is formulated in terms of linear matrix inequalities (LMIs). Based on the Lyapunov-Krasovskii functionals, global asymptotical stability of the closed-loop systems is achieved. Finally, simulation results are shown to illustrate the feasibility of the proposed strategy.

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Keywords: input delays; state delays; delay-dependent stability; Lyapunov Krasovskii functional; linear matrix inequalities (LMIs)

1 Introduction

Time delays are important components of many dynamical systems that describe coupling or interconnection between dynamics, propagation, or transport phenomena in shared environments, economic models [1, 2], biological systems [3], and in competition in population dynamics [4]. In the literature, there are two categories of criteria: delay-dependent [5, 6] and delay-independent criteria. For criteria-dependent delay gives information on the length of delays, this model is used more frequently than delay-independent ones. For delay-dependent criteria, see [7–14] and the references therein. For a delay-free system, under control laws with high gain observers, asymptotic stability has been achieved by [15–17]. The analysis of nonlinear systems with time delays is typically more difficult than systems without time delays [18, 19]. The study of the stabilization of the system with delay was the object of much research (see for example [20–22] and the references therein). In [23], using the Lyapunov-Krasovskii functional approach and the linear matrix inequality (LMI)-based design method, the stability problem for time-delay systems is discussed. In [5], a principle of separation has been established in a class of systems, inspired by [24], which covers the class of systems considered by [20]. Under an output feedback controller the global asymptotic stability is obtained. In [25] and [26], under linear growth conditions, global stabilization by state feedback and output feedback has been studied for a class of nonlinear time-delay systems. In this regard, [27] and [17] used a linear high gain observer to achieve global stabilization by output feedback for a class of nonlinear systems under the same conditions as [25] and [26]. For a system without delay, [24] describes a

new condition to ensure overall stabilization by a linear output reaction. In [11], an algorithm for the design of a time-dependent state feedback controller to stabilize the system under LMI constraints has been presented. To solve the problem of synthesis for control systems with varying time delays, we use the result of [13] as well as the algorithm of [11].

The dynamic behavior of neural networks is studied for instance in [28–30] and the references therein. In [30], extended dissipativity conditions for generalized neural networks with interval time delays were investigated. To solve the problem, extended dissipativity conditions were established in the form of linear matrix inequalities by constructing a suitable Lyapunov-Krasovskii functional. Using a new weighted integral inequality technique [29], proposed conditions were expressed in terms of linear matrix inequalities, and used to examine the exponential stability problem for delayed generalized neural networks. The problem of robust finite-time stabilization and guaranteed cost control for neural networks with varying interval time delay has been achieved by [28]. By constructing a set of improved Lyapunov-Krasovskii functionals, a design of memoryless state feedback guaranteed cost controllers has been presented for the system in terms of linear matrix inequalities.

Under constructing a set of improved Lyapunov-Krasovskii functionals and a Newton-Leibniz formula, the conditions for the exponential stability of the systems have been established in terms of LMIs. [31] proposed delay-dependent conditions for the exponential stability of linear systems with non-differentiable varying interval time delay.

Under delay-dependent conditions, global exponential stability of a class of nonlinear time-delay systems has been achieved by [6]. The condition on the nonlinearity to cover the time-delay systems, given by [6], is a generalization of conditions considered by [5, 20, 25, 26]. Moreover, the generalized conditions cover the systems given by [24, 27] for a class of nonlinear delay-free systems.

In this paper, we investigate the problem of output feedback stabilization of a class of nonlinear time-delay systems, which cover the systems considered by [6]. Motivated by [11] and [6], we use appropriate Lyapunov-Krasovskii functionals to establish global asymptotical stability of the closed-loop systems. Then they are used to obtain a new state and input delay-dependent criterion that ensures the stability of the closed-loop system with a state feedback controller. The rest of this paper is organized as follows. In the next section, some preliminary results are summarized and the system description is given. Our main results are stated in Section 3. First, a parameter-dependent linear state and output feedback controllers are synthesized to ensure global asymptotical stability of the nonlinear time-delay system. Finally, an illustrative example is discussed to demonstrate the effectiveness of the obtained results.

2 System description and preliminary

Consider a time-delay system of the form

$$\begin{cases} \dot{x}(t) = f(x(t), x(t - \tau)), \\ x(\theta) = \varphi(\theta), \end{cases} \quad (1)$$

where $\tau > 0$ denotes the time delay, and $\varphi \in \mathcal{C}$ is the initial function where \mathcal{C} denotes the Banach space of continuous functions mapping the interval $[-\tau, 0] \rightarrow \mathbb{R}^n$ equipped with

the supremum-norm

$$\|\varphi\|_\infty = \max_{\theta \in [-\tau, 0]} \|\varphi(\theta)\|,$$

with $\|\cdot\|$ being the Euclidean norm. The map $f : \mathbb{R}^n \times \mathbb{R}^n$ is smooth and satisfies $f(0, 0) = 0$. The function segment x_t is defined by $x_t(\theta) = x(t + \theta)$, $\theta \in [-\tau, 0]$. For $\varphi \in \mathcal{C}$, we denote by $x(t, \varphi)$ or in short $x(t)$ the solution of (1) that satisfies $x_0 = \varphi$. The segment of this solution is denoted by $x_t(\varphi)$ or in short x_t .

Definition 1 The zero solution of (1) is called

- stable, if for any $\varepsilon > 0$ there exists $\delta > 0$ such that

$$\|\varphi\|_\infty < \delta \implies \|x(t)\| < \varepsilon, \quad \forall t \geq 0,$$

- attractive, if there exists $\sigma > 0$ such that

$$\|\varphi\|_\infty < \sigma \implies \lim_{t \rightarrow +\infty} x(t) = 0, \tag{2}$$

- asymptotically stable, if it is stable and attractive,
- globally asymptotically stable, if it is stable and δ can be chosen arbitrarily large for sufficiently large ε , and (2) is satisfied for all $\sigma > 0$.

Sufficient conditions for stability of a functional differential equation are provided by the theory of Lyapunov-Krasovskii functionals [32], a generalization of the classical Lyapunov theory of ordinary differential equations [33]. Let us recall here that a function $\alpha : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is of class \mathcal{K} if it is continuous and increasing and $\alpha(0) = 0$, and of class \mathcal{K}_∞ if it is of class \mathcal{K} and it is unbounded. The following theorem provides sufficient Lyapunov-Krasovskii conditions for global asymptotic stability of the zero solution of system (1) (see [34]).

Theorem 2 Assume that there exists a locally Lipschitz functional $V : \mathcal{C} \rightarrow \mathbb{R}_+$, functions α_1 and α_2 of class \mathcal{K}_∞ , and function α_3 of class \mathcal{K} , such that

- (i) $\alpha_1(\|x(t)\|) \leq V(x_t) \leq \alpha_2(\|x_t\|_\infty)$,
- (ii) $\dot{V}(x_t) \leq -\alpha_3(\|x(t)\|)$.

Then the zero solution of system (1) is globally asymptotically stable.

Notation 3 Throughout the paper, the time argument is omitted and the delayed state vector $x(t - \tau)$ is denoted by x^τ . A^T means the transpose of A . $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ denote the maximal and minimal eigenvalue of a matrix A , respectively. $P > 0$ means that the matrix P is symmetric positive definite. I is an appropriately dimensioned identity matrix, and $\text{diag}[\dots]$ denotes a block-diagonal matrix.

Lemma 4 See the Schur complement from [23]. Let M, P, Q be the given matrices such that $Q > 0$. Then

$$\begin{bmatrix} P & M^T \\ M & -Q \end{bmatrix} < 0 \iff P + M^T Q^{-1} M < 0.$$

Lemma 5 For any vector $a, b \in \mathbb{R}^n$ and scalar $\varepsilon > 0$, we have

$$2a^T b \leq \varepsilon a^T a + \varepsilon^{-1} b^T b.$$

In this paper, we consider the time-delay nonlinear system

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + f(x(t), x(t - \tau), u(t)), \\ y(t) = Cx(t), \end{cases} \tag{3}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}$ is the input of the system, $y \in \mathbb{R}$ is the measured output, and τ is a positive known scalar that denotes the time delay affecting the state variables. The matrices A, B and C are given by

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \quad C = [1 \ 0 \ \cdots \ 0 \ 0],$$

and the perturbed term is

$$f(x(t), x(t - \tau), u(t)) = [f_1(x(t), x(t - \tau), u(t)), \dots, f_n(x(t), x(t - \tau), u(t))]^T.$$

The mappings $f_i : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{R}, i = 1, \dots, n$, are smooth and satisfy the following assumption.

Assumption 1 There exist functions $\gamma_1(\varepsilon) > 0$ and $\gamma_2(\varepsilon) > 0$ such that for $\varepsilon > 0$,

$$\sum_{i=1}^n \varepsilon^{i-1} |f_i(x, y, u)| \leq \gamma_1(\varepsilon) \sum_{i=1}^n \varepsilon^{i-1} |x_i| + \gamma_2(\varepsilon) \sum_{i=1}^n \varepsilon^{i-1} |y_i|. \tag{4}$$

3 Main results

3.1 Global stabilization by state feedback

This section presents the delay-dependent stabilization conditions obtained by means of the LMI method. The state feedback controller is given by

$$u = K(\varepsilon)x, \tag{5}$$

where $K(\varepsilon) = [\frac{k_1}{\varepsilon^n}, \dots, \frac{k_n}{\varepsilon}]$ and $K = [k_1, \dots, k_n]$ such that $A_K := A + BK$ is Hurwitz.

Theorem 6 Suppose that Assumption 1 is satisfied. Then there exist symmetric positive definite matrices S, Q, Z and there exists a positive constant ε such that the following LMIs hold:

$$\frac{1}{\varepsilon} \Psi + a(\varepsilon)I < 0, \tag{6}$$

$$\frac{-1}{\varepsilon} Q + b(\varepsilon)I < 0, \tag{7}$$

where

$$\Psi = \begin{bmatrix} A_K^T S + SA_K + Q & S & \bar{\tau} A_K^T Z & \bar{\tau} A_K^T Z \\ S & -I & 0 & 0 \\ \bar{\tau} Z A_K & 0 & -\bar{\tau} I & 0 \\ \bar{\tau} Z A_K & 0 & 0 & -\bar{\tau} Z \end{bmatrix},$$

$$a(\varepsilon) = \varepsilon n^2 (\bar{\tau} (\|Z\| + 1) + 1) \gamma_1(\varepsilon) (\gamma_1(\varepsilon) + \gamma_2(\varepsilon)),$$

$$b(\varepsilon) = \varepsilon n^2 (\bar{\tau} (\|Z\| + 1) + 1) \gamma_2(\varepsilon) (\gamma_1(\varepsilon) + \gamma_2(\varepsilon)).$$

Then the closed loop time-delay system (3)-(5) is asymptotically stable for any time delay τ satisfying $0 \leq \tau \leq \bar{\tau}$.

Proof The closed loop system is given by

$$\dot{x} = (A + BK(\varepsilon))x + f(x, x^\tau, u).$$

For $\varepsilon > 0$, let $D(\varepsilon) = \text{diag}[1, \varepsilon, \dots, \varepsilon^{n-1}]$ and $\chi = D(\varepsilon)x$.

Using the fact that $A + BK(\varepsilon) = \frac{1}{\varepsilon} D(\varepsilon)^{-1} A_K D(\varepsilon)$, we get

$$\dot{\chi} = \frac{1}{\varepsilon} A_K \chi + D(\varepsilon) f(x, x^\tau, u).$$

Let us choose a Lyapunov-Krasovskii functional candidate as follows:

$$W(\chi_t) = W_1(\chi_t) + W_2(\chi_t) + W_3(\chi_t), \tag{8}$$

where

$$W_1(\chi_t) = \chi^T S \chi,$$

$$W_2(\chi_t) = \varepsilon \int_{-\tau}^0 \int_{t+\beta}^t \dot{\chi}^T(s) Z \dot{\chi}(s) ds d\beta,$$

$$W_3(\chi_t) = \frac{1}{\varepsilon} \int_{t-\tau}^t \chi^T(s) Q \chi(s) ds.$$

Since S is symmetric positive definite, for all $\chi \in \mathbb{R}^n$,

$$\lambda_{\min}(S) \|\chi\|^2 \leq \chi^T S \chi \leq \lambda_{\max}(S) \|\chi\|^2.$$

This implies that, on the one hand,

$$W(\chi_t) \geq \lambda_{\min}(S) \|\chi\|^2,$$

and on the other hand,

$$\begin{aligned}
 W(\chi_t) &= W_1(\chi_t) + W_2(\chi_t) + W_3(\chi_t) \\
 &= \chi^T S \chi + \varepsilon \int_{-\tau}^0 \int_{t+\beta}^t \dot{\chi}^T(s) Z \dot{\chi}(s) ds d\beta + \frac{1}{\varepsilon} \int_{t-\tau}^t \chi^T(s) Q \chi(s) ds \\
 &= \chi^T S \chi + \varepsilon \int_{-\tau}^0 \int_{\beta}^0 \dot{\chi}^T(s+t) Z \dot{\chi}(s+t) ds d\beta + \frac{1}{\varepsilon} \int_{-\tau}^0 \chi^T(s+t) Q \chi(s+t) ds \\
 &\leq \lambda_{\max}(S) \|\chi\|^2 + \varepsilon \int_{-\tau}^0 \lambda_{\max}(Z) \int_{\beta}^0 \|\dot{\chi}_t(s)\|^2 ds d\beta + \frac{1}{\varepsilon} \lambda_{\max}(Q) \int_{-\tau}^0 \|\chi_t(s)\|^2 ds \\
 &\leq \lambda_{\max}(S) \|\chi\|^2 - \varepsilon \tau \lambda_{\max}(Z) \int_0^{\beta} \|\dot{\chi}_t(s)\|^2 ds + \frac{1}{\varepsilon} \lambda_{\max}(Q) \int_{-\tau}^0 \|\chi_t\|_{\infty}^2 ds \\
 &\leq \left(\lambda_{\max}(S) + \frac{\tau}{\varepsilon} \lambda_{\max}(Q) \right) \|\chi_t\|_{\infty}^2.
 \end{aligned}$$

The time derivative of W_1 is

$$\dot{W}_1(\chi_t) = \frac{1}{\varepsilon} \chi^T (A_K^T S + S A_K) \chi + 2 \chi^T S D(\varepsilon) f(x, x^{\tau}, u).$$

So by Assumption 1 we get

$$\begin{aligned}
 \|D(\varepsilon) f(x, x^{\tau}, u)\| &\leq \sum_{i=1}^n \varepsilon^{i-1} |f_i(x, x^{\tau}, u)| \\
 &\leq \gamma_1(\varepsilon) \sum_{i=1}^n \varepsilon^{i-1} |x_i| + \gamma_2(\varepsilon) \sum_{i=1}^n \varepsilon^{i-1} |x_i^{\tau}| \\
 &\leq n \gamma_1(\varepsilon) \|D(\varepsilon) x\| + n \gamma_2(\varepsilon) \|D(\varepsilon) x^{\tau}\|,
 \end{aligned}$$

which implies that

$$\|D(\varepsilon) f(x, x^{\tau}, u)\| \leq n \gamma_1(\varepsilon) \|\chi\| + n \gamma_2(\varepsilon) \|\chi^{\tau}\|. \tag{9}$$

Using Lemma 5 we deduce that

$$\begin{aligned}
 \dot{W}_1(\chi_t) &\leq \frac{1}{\varepsilon} \chi^T (A_K^T S + S A_K) \chi + \frac{1}{\varepsilon} \chi^T S S \chi + \varepsilon \|D(\varepsilon) f(x, x^{\tau}, u)\|^2 \\
 &\leq \frac{1}{\varepsilon} \chi^T (A_K^T S + S A_K) \chi + \frac{1}{\varepsilon} \chi^T S S \chi + \varepsilon (n \gamma_1(\varepsilon) \|\chi\| + n \gamma_2(\varepsilon) \|\chi^{\tau}\|)^2 \\
 &\leq \frac{1}{\varepsilon} \chi^T (A_K^T S + S A_K) \chi + \frac{1}{\varepsilon} \chi^T S S \chi + \varepsilon n^2 \gamma_1^2(\varepsilon) \|\chi\|^2 + \varepsilon n^2 \gamma_2^2(\varepsilon) \|\chi^{\tau}\|^2 \\
 &\quad + \varepsilon n^2 \gamma_1(\varepsilon) \gamma_2(\varepsilon) (\|\chi\|^2 + \|\chi^{\tau}\|^2) \\
 &\leq \frac{1}{\varepsilon} \chi^T (A_K^T S + S A_K) \chi + \frac{1}{\varepsilon} \chi^{\tau} S S \chi + \varepsilon n^2 \gamma_1(\varepsilon) (\gamma_1(\varepsilon) + \gamma_2(\varepsilon)) \|\chi\|^2 \\
 &\quad + \varepsilon n^2 \gamma_2(\varepsilon) (\gamma_1(\varepsilon) + \gamma_2(\varepsilon)) \|\chi^{\tau}\|^2.
 \end{aligned}$$

Using Lemma 5 and (9), the time derivative of W_2 is

$$\begin{aligned} \dot{W}_2(\chi_t) &= \varepsilon \left(\int_{-\tau}^0 (\dot{\chi}^T(t)Z\dot{\chi}(t) - \dot{\chi}^T(t+\beta)Z\dot{\chi}(t+\beta)) d\beta \right) \\ &= \varepsilon \tau \dot{\chi}^T(t)Z\dot{\chi}(t) - \varepsilon \int_{t-\tau}^t \dot{\chi}^T(s)Z\dot{\chi}(s) ds \\ &\leq \varepsilon \bar{\tau} \left[\frac{1}{\varepsilon} A_K \chi + D(\varepsilon)f(x, x^\tau, u) \right]^T Z \left[\frac{1}{\varepsilon} A_K \chi + D(\varepsilon)f(x, x^\tau, u) \right] \\ &\leq \frac{\bar{\tau}}{\varepsilon} \chi^T (A_K^T Z A_K) \chi + 2\bar{\tau} \chi^T A_K^T Z D(\varepsilon)f(x, x^\tau, u) \\ &\quad + \varepsilon \bar{\tau} \|Z\| \|D(\varepsilon)f(x, x^\tau, u)\|^2 \\ &\leq \frac{\bar{\tau}}{\varepsilon} \chi^T (A_K^T Z A_K) \chi + \frac{\bar{\tau}}{\varepsilon} \chi^T A_K^T Z Z A_K \chi + \varepsilon \bar{\tau} \|D(\varepsilon)f(x, x^\tau, u)\|^2 \\ &\quad + \varepsilon \bar{\tau} \|Z\| \|D(\varepsilon)f(x, x^\tau, u)\|^2 \\ &\leq \frac{\bar{\tau}}{\varepsilon} \chi^T (A_K^T Z A_K + A_K^T Z Z A_K) \chi + \varepsilon \bar{\tau} (\|Z\| + 1) n^2 \gamma_1(\varepsilon) (\gamma_1(\varepsilon) + \gamma_2(\varepsilon)) \|\chi\|^2 \\ &\quad + \varepsilon \bar{\tau} (\|Z\| + 1) n^2 \gamma_2(\varepsilon) (\gamma_1(\varepsilon) + \gamma_2(\varepsilon)) \|\chi^\tau\|^2. \end{aligned}$$

The time derivative of W_3 is

$$\dot{W}_3(\chi_t) = \frac{1}{\varepsilon} \chi^T Q \chi^T - \frac{1}{\varepsilon} (\chi^\tau)^T Q \chi^\tau.$$

Hence, we have

$$\begin{aligned} \dot{W}(\chi_t) &\leq \frac{1}{\varepsilon} \chi^T \{ (A_K^T S + S A_K + S S) + \bar{\tau} (A_K^T Z A_K + A_K^T Z Z A_K) + Q \} \chi \\ &\quad - \frac{1}{\varepsilon} (\chi^\tau)^T Q \chi^\tau + a(\varepsilon) \|\chi\|^2 + b(\varepsilon) \|\chi^\tau\|^2. \end{aligned} \tag{10}$$

Then, using the Lyapunov-Krasovskii stability Theorem 2 and the Schur complement Lemma 4, we conclude that the closed loop time-delay system (3)-(5) is asymptotically stable if (6) and (7) hold. \square

3.2 Global stabilization by output feedback

In [6], under Assumption 1, if the conditions does not depend on the delay τ , global exponential stability by the dynamic output feedback control is achieved. In this subsection, we study the problem of global asymptotic stability by output feedback control under Assumption 1 and delay-dependent conditions. The following system is proposed:

$$\dot{\hat{x}}(t) = A \hat{x} + B u(t) - L(\varepsilon)(y - C \hat{x}), \tag{11}$$

where $L(\varepsilon) = [\frac{l_1}{\varepsilon}, \dots, \frac{l_n}{\varepsilon^n}]^T$ and $L = [l_1, \dots, l_n]^T$ such that $A_L := A + LC$ is Hurwitz. The output feedback controller is given by

$$u = K(\varepsilon)\hat{x}. \tag{12}$$

Theorem 7 *Suppose that Assumption 1 is satisfied. Then there exist symmetric positive definite matrices P, M, N and there exists a positive constant ε such that the following LMIs hold:*

$$\Phi = \begin{bmatrix} A_L^T P + P A_L + N & P & \bar{\tau} A_L^T M & \bar{\tau} A_L^T M \\ P & -I & 0 & 0 \\ \bar{\tau} M A_L & 0 & -\bar{\tau} I & 0 \\ \bar{\tau} M A_L & 0 & 0 & -\bar{\tau} M \end{bmatrix} < 0, \tag{13}$$

$$\frac{1}{\varepsilon} \Psi + (a(\varepsilon) + c(\varepsilon)) I < 0, \tag{14}$$

$$\frac{-1}{\varepsilon} Q + (b(\varepsilon) + d(\varepsilon)) I < 0, \tag{15}$$

where

$$c(\varepsilon) = \varepsilon n^2 (\bar{\tau} (\|M\| + 1) + 1) \gamma_1(\varepsilon) (\gamma_1(\varepsilon) + \gamma_2(\varepsilon)),$$

$$d(\varepsilon) = \varepsilon n^2 (\bar{\tau} (\|M\| + 1) + 1) \gamma_2(\varepsilon) (\gamma_1(\varepsilon) + \gamma_2(\varepsilon)).$$

Then the closed loop time-delay system (3)-(12) is asymptotically stable for any time delay τ satisfying $0 \leq \tau \leq \bar{\tau}$.

Proof Define $e = x - \hat{x}$. We have

$$\dot{e} = (A + L(\varepsilon)C)e + f(x, x^\tau, u). \tag{16}$$

For $\varepsilon > 0$, let $D(\varepsilon) = \text{diag}[1, \varepsilon, \dots, \varepsilon^{n-1}]$ and $\eta = D(\varepsilon)e$.

Using the fact that $A + L(\varepsilon)C = \frac{1}{\varepsilon} D(\varepsilon)^{-1} A_L D(\varepsilon)$, we get

$$\dot{\eta} = \frac{1}{\varepsilon} A_L \eta + D(\varepsilon) f(x, x^\tau, u). \tag{17}$$

Let us choose a Lyapunov-Krasovskii functional candidate as follows:

$$V(\eta_t) = V_1(\eta_t) + V_2(\eta_t) + V_3(\eta_t), \tag{18}$$

where

$$V_1(\eta_t) = \eta^T P \eta,$$

$$V_2(\eta_t) = \varepsilon \int_{-\tau}^0 \int_{t+\beta}^t \dot{\eta}^T(s) M \dot{\eta}(s) ds d\beta,$$

$$V_3(\eta_t) = \frac{1}{\varepsilon} \int_{t-\tau}^t \eta^T(s) N \eta(s) ds.$$

The time derivative of V_1 is

$$\begin{aligned} \dot{V}_1(\eta_t) &= \frac{1}{\varepsilon} \chi^T (A_L^T P + PA_L) \eta + 2\eta^T PD(\varepsilon)f(x, x^\tau, u) \\ &\leq \frac{1}{\varepsilon} \eta^T (A_L^T P + PA_L) \eta + \frac{1}{\varepsilon} \eta^T PP\eta + \varepsilon \|D(\varepsilon)f(x, x^\tau, u)\|^2 \\ &\leq \frac{1}{\varepsilon} \eta^T (A_L^T P + PA_L) \eta + \frac{1}{\varepsilon} \eta^T PP\eta + \varepsilon (n\gamma_1(\varepsilon)\|\chi\| + n\gamma_2(\varepsilon)\|\chi^\tau\|)^2 \\ &\leq \frac{1}{\varepsilon} \eta^T (A_L^T P + PA_L) \eta + \frac{1}{\varepsilon} \eta^T PP\eta + \varepsilon n^2 \gamma_1^2(\varepsilon)\|\chi\|^2 + \varepsilon n^2 \gamma_2^2(\varepsilon)\|\chi^\tau\|^2 \\ &\quad + \varepsilon n^2 \gamma_1(\varepsilon)\gamma_2(\varepsilon)(\|\chi\|^2 + \|\chi^\tau\|^2) \\ &\leq \frac{1}{\varepsilon} \eta^T (A_L^T P + PA_L) \eta + \frac{1}{\varepsilon} \eta^T PP\eta + \varepsilon n^2 \gamma_1(\varepsilon)(\gamma_1(\varepsilon) + \gamma_2(\varepsilon))\|\chi\|^2 \\ &\quad + \varepsilon n^2 \gamma_2(\varepsilon)(\gamma_1(\varepsilon) + \gamma_2(\varepsilon))\|\chi^\tau\|^2. \end{aligned}$$

The time derivative of V_2 is

$$\begin{aligned} \dot{V}_2(\eta_t) &= \varepsilon \left(\int_{-\tau}^0 (\dot{\eta}^T(t)M\dot{\eta}(t) - \dot{\eta}^T(t+\beta)M\dot{\eta}(t+\beta)) d\beta \right) \\ &= \varepsilon \tau \dot{\eta}^T(t)M\dot{\eta}(t) - \varepsilon \int_{t-\tau}^t \dot{\eta}^T(s)M\dot{\eta}(s) ds \\ &\leq \varepsilon \bar{\tau} \left[\frac{1}{\varepsilon} A_L \eta + D(\varepsilon)f(x, x^\tau, u) \right]^T M \left[\frac{1}{\varepsilon} A_L \eta + D(\varepsilon)f(x, x^\tau, u) \right] \\ &\leq \frac{\bar{\tau}}{\varepsilon} \eta^T (A_L^T M A_L) \eta + 2\bar{\tau} \eta^T A_L^T M D(\varepsilon)f(x, x^\tau, u) \\ &\quad + \varepsilon \bar{\tau} \|M\| \|D(\varepsilon)f(x, x^\tau, u)\|^2 \\ &\leq \frac{\bar{\tau}}{\varepsilon} \eta^T (A_L^T M A_L) \eta + \frac{\bar{\tau}}{\varepsilon} \eta^T A_L^T M M A_L \eta + \varepsilon \bar{\tau} \|D(\varepsilon)f(x, x^\tau, u)\|^2 \\ &\quad + \varepsilon \bar{\tau} \|M\| \|D(\varepsilon)f(x, x^\tau, u)\|^2 \\ &\leq \frac{\bar{\tau}}{\varepsilon} \eta^T (A_L^T M A_L + A_L^T M M A_L) \eta + \varepsilon \bar{\tau} (1 + \|M\|) n^2 \gamma_1(\varepsilon)(\gamma_1(\varepsilon) + \gamma_2(\varepsilon))\|\chi\|^2 \\ &\quad + \varepsilon \bar{\tau} (1 + \|M\|) n^2 \gamma_2(\varepsilon)(\gamma_1(\varepsilon) + \gamma_2(\varepsilon))\|\chi^\tau\|^2. \end{aligned}$$

The time derivative of V_3 is

$$\dot{V}_3(\eta_t) = \frac{1}{\varepsilon} \eta^T N \eta - \frac{1}{\varepsilon} (\eta^\tau)^T N \eta^\tau.$$

So we have

$$\begin{aligned} \dot{V} &\leq \frac{1}{\varepsilon} \eta^T (A_L^T P + PA_L + PP + \bar{\tau} A_L^T M A_L + \bar{\tau} A_L^T M M A_L + N) \eta \\ &\quad - \frac{1}{\varepsilon} (\eta^\tau)^T N \eta^\tau + c(\varepsilon)\|\chi\|^2 + d(\varepsilon)\|\chi^\tau\|^2. \end{aligned} \tag{19}$$

Let

$$U(\eta_t, \chi_t) = \alpha V(\eta_t) + W(\chi_t),$$

where W is given by (8). Using (10) and (19), we get

$$\begin{aligned} \dot{U}(\eta_t, \chi_t) \leq & \frac{\alpha}{\varepsilon} \eta^T (A_L^T P + P A_L + P P + \bar{\tau} A_L^T M A_L + \bar{\tau} A_L^T M M A_L + N) \eta \\ & - \frac{\alpha}{\varepsilon} (\eta^\tau)^T N \eta^\tau + \{ \alpha c(\varepsilon) + a(\varepsilon) \} \| \chi \|^2 + \{ \alpha d(\varepsilon) + b(\varepsilon) \} \| \chi^\tau \|^2 \\ & + \frac{1}{\varepsilon} \chi^T \{ (A_K^T S + S A_K + S S) + \bar{\tau} (A_K^T Z A_K + A_K^T Z Z A_K) + Q \} \chi \\ & - \frac{1}{\varepsilon} (\chi^\tau)^T Q \chi^\tau. \end{aligned}$$

Finally, we select α such that

$$\alpha < \min \left(-\frac{1}{\varepsilon} \frac{\lambda_{\min}(\Psi) + \varepsilon a(\varepsilon)}{c(\varepsilon)}, \frac{1}{\varepsilon} \frac{\lambda_{\max}(Q) - \varepsilon b(\varepsilon)}{d(\varepsilon)} \right). \quad \square$$

Remark 8 The nonlinear matrix inequalities which appeared in the criteria are successfully transformed into the LMIs to be solved easily by various effective optimization algorithms [23] or using the MATLAB LMI Control Toolbox [35].

Remark 9 Compared with [11] and [10], our new criteria overcome some of the main sources of conservatism, and contain the criteria in [11] and [10] as a special case of a class of linear delay systems. Furthermore, the new criteria also contain the well-known delay-independent stability condition in [6] and [25].

Remark 10 In [6], state feedback and output controllers for a certain class of nonlinear timing systems cover the class of systems satisfying a linear growth condition [17], using the Lyapunov-Krasovskii functions. Authors derived delay-independent conditions to ensure global exponential stability of the closed-loop systems. In this paper, in order to reduce the conservatism, a new delay-dependent stability criterion is obtained in Theorem 6 and Theorem 7 by constructing a new Lyapunov-Krasovskii functional given by (8) and (18).

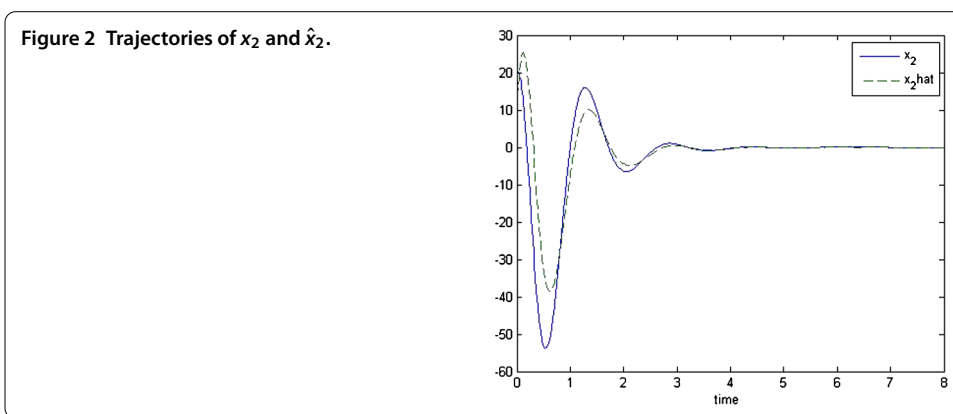
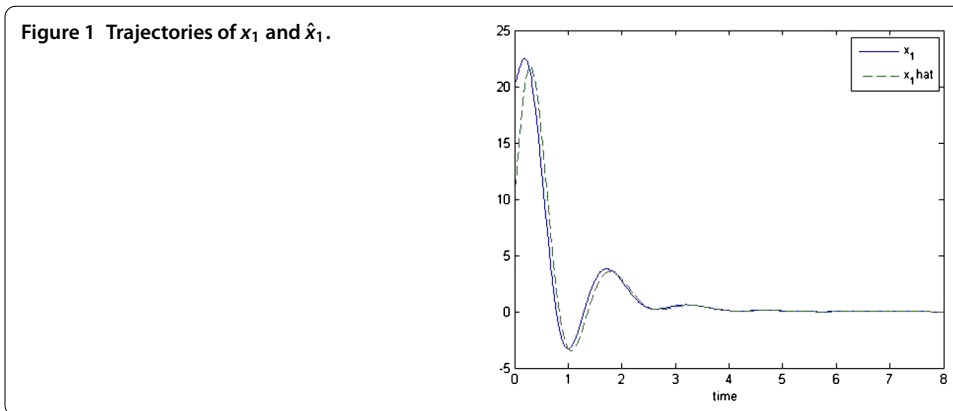
3.3 Numerical example

To check the effectiveness of the result, consider the following system:

$$\begin{aligned} \dot{x}_1 &= x_2(t) + \frac{1}{10} x_2 \sin x_3 \cos u + \frac{1}{10} x_2(t - \tau) \cos u, \\ \dot{x}_2 &= x_3(t), \\ \dot{x}_3 &= u. \end{aligned} \tag{20}$$

Following the notation used throughout the paper, let $f_1(x, x^\tau, u) = \frac{1}{10} x_2 \sin x_3 \cos u + \frac{1}{10} x_2(t - \tau) \cos u$, and $f_2(x, x^\tau, u) = f_3(x, x^\tau, u) = 0$. Since f_1 depends on x_3 and x_2^τ , the output feedback scheme in [2, 20] is not applicable. It is easy to check that system (20) satisfies Assumption 1 with $\gamma_1(\varepsilon) = \gamma_2(\varepsilon) = \frac{1}{10\varepsilon}$.

Select $K = [-4 \ -9 \ -4]$ and $L = [-2 \ -4 \ -2]^T$. A_K and A_L are Hurwitz. Applying Theorem 7 and the MATLAB LMI Control Toolbox, we find that conditions (6) and (13) are given,



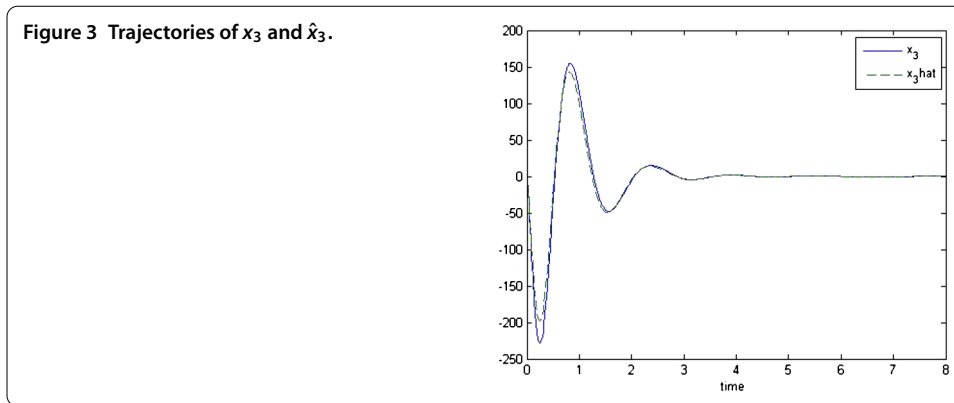
respectively, by

$$\begin{aligned}
 S &= \begin{bmatrix} 7.2525 & 6.2371 & 1.1651 \\ 6.2371 & 9.1658 & 1.6812 \\ 1.1651 & 1.6812 & 0.9827 \end{bmatrix}, & P &= \begin{bmatrix} 2.2881 & -0.7102 & -0.0859 \\ -0.7102 & 1.0878 & -0.7938 \\ -0.0859 & -0.7938 & 1.2502 \end{bmatrix}, \\
 Z &= \begin{bmatrix} 0.1768 & 0.1126 & 0.0211 \\ 0.1126 & 0.2428 & 0.0259 \\ 0.0211 & 0.0259 & 0.0103 \end{bmatrix}, & M &= \begin{bmatrix} 0.0426 & -0.0142 & -0.0090 \\ -0.0142 & 0.0226 & -0.0294 \\ -0.0090 & -0.0294 & 0.0685 \end{bmatrix}, \\
 Q &= \begin{bmatrix} 0.8717 & 1.5881 & 0.5684 \\ 1.5881 & 3.7280 & 1.6842 \\ 0.5684 & 1.6842 & 1.8074 \end{bmatrix}, & N &= \begin{bmatrix} 0.3636 & -0.0875 & -0.0198 \\ -0.0875 & 0.1448 & -0.0873 \\ -0.0198 & -0.0873 & 0.1389 \end{bmatrix}.
 \end{aligned}$$

The above system is asymptotically stable for any τ satisfying $0 \leq \tau \leq 1.1125$ and $0 \leq \tau \leq 0.2594$. So $\|Z\| = 0.3306$ and $\|M\| = 0.0830$. This implies that condition (14) is satisfied for all $\varepsilon > 0.2279$ and condition (15) is satisfied for all $\varepsilon > 0.2189$. For our numerical simulation, we choose the delay $\tau = 0.2$, and $\varepsilon = 0.4$. Corresponding numerical simulation results are shown in Figures 1-3.

4 Conclusion

In this paper, we are concerned with the problem of global asymptotic stability for a certain class of nonlinear time-delay systems, written in triangular form, satisfying a linear



growth condition [6, 17, 25, 26]. In [6], the authors derived delay-independent conditions to ensure global exponential stability of the closed-loop systems. Using the Lyapunov-Krasovskii functionals given by (8) and (17), we have derived delay-dependent conditions, using a controller that is formulated in terms of LMIs, to ensure global asymptotical stability of the resulting closed-loop systems. The obtained result extends for global exponential stability.

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Competing interests

The authors declare that they have no competing interests.

Authors' contributions

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