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Fault detection filter design for continuous-time nonlinear Markovian jump systems with mode-dependent delay and time-varying transition probabilities

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Abstract

This paper focuses on fault detection filter (FDF) design for continuous-time nonlinear Markovian jump systems (NMJSs) with mode-dependent delay and time-varying transition probabilities (TPs). By using a novel Lyapunov-Krasovskii function and based on convex polyhedron technique, a new FDF, as the residual generator, is constructed to guarantee the mean-square exponential stability and a prescribed level of disturbance attenuation for admissible perturbations of NMJSs. Finally, the numerical simulation is carried out to demonstrate the effectiveness of our method.

Keywords: Markovian jump system; time-varying; transition probabilities; fault detection; nonlinear

1 Introduction

Subject to the random abrupt variations, Markovian jump systems (MJSs) are assumed to be a framework to model dynamic systems, and they can be found in economic systems, communication systems, robot manipulator systems and so on. During the past decades, many efforts have been devoted to MJSs, which can be possibly used in the field of system stability [1–6], system control [7–13] and filtering [14–16].

For MJSs, fault detection is an important research topic. In the framework of fault detection, a threshold on residual signals is set. Once the value of residual evaluation function goes beyond the predefined threshold, the alert is triggered [17]. Up to now many results on fault detection of MJSs have been published, see [18–28] and the references therein. Generally, the fault detection method can be divided into three groups. The first group is the filter-based method. In [29], a filter is used to generate the residual signals to detect the fault. The second group is the statistic method. In [30], Bayesian theory and the likelihood method are used to evaluate the fault. The third group is the geometric method. In [31], a geometric approach is employed to find the fault. However in general, TPs are assumed to be time invariant. It is meaningful to focus on the case that TPs are time variant for the possible application in real engineering. In addition, time delays are mode-dependent sometimes, and usually the existence of nonlinear terms makes the real fault detection problem more complicated. To our best knowledge, the studies on fault detection for continuous-time nonlinear MJSs (NMJSs) with mode-dependent delay and time-varying TPs have been seldom carried out up to now, which motivates this paper. In addition, some techniques and lemmas will be included to improve the conservatism of theoretical results.

The remainder of this paper is organized as follows. The mathematical model under consideration and some preliminaries are provided in Section 2. A FDF for continuous-time NMJSs with mode-dependent delay and time-varying TPs is designed in Section 3. The illustrative example is included to verify the correctness of obtained theoretical results in Section 4, and finally the paper is concluded in Section 5.

Notations used in this paper are fairly standard. Let R^n be the n-dimensional Euclidean space, $R^{n\times m}$ represents the set of $n \times m$ real matrix, the symbol * denotes the elements below the main diagonal of a symmetric block matrix, $A > 0$ means that A is a real symmetric positive definitive matrix, I denotes the identity matrix with appropriate dimensions. diag$\{\cdot\}$ denotes the diagonal matrix. E$\{\cdot\}$ refers to the expectation operator with respect to some probability measure P. $\|\cdot\|$ refers to the Euclidean vector norm or the induced matrix 2-norm. The superscript T stands for matrix transposition, $L_{n,h} = L([-h,0],R^n)$ denotes the Banach space of continuous functions mapping the interval $[-h,0]$ into R^n with the topology of uniform convergence.

2 Model description and preliminaries

In this paper, (Ω,Υ,P) denotes the probability space, where Ω is the sample space, Υ is σ-algebra of a subset of the sample space, and P is the probability measure defined on Υ. The process $\{r_t, t \in [0,+\infty)\}$ is described by a Markovian chain with finite state space $S_1 = \{1,2,\dots,N\}$, and its transition probability matrix $\Pi^{(\sigma_{t+\Delta})} = [\pi_{il}^{(\sigma_{t+\Delta})}]_{N\times N}$ $(i,l \in S_1)$ is governed by

$$P\{r_{t+\Delta t} = l|r_t = i\} = \begin{cases} \pi_{il}\Delta t + o(\Delta t), & l \neq i, \\ 1 + \pi_{ii}\Delta t + o(\Delta t), & l = i, \end{cases}$$

where $\pi_{ii} = -\sum_{l=1,l\neq i}^{N} \pi_{il}$, $\lim_{\Delta t\to 0} o(\Delta t)/\Delta t = 0$, and $\pi_{il} \geq 0$, $l \neq i$ is the transition rate from mode i at time t to mode l at time $t + \Delta t$.

In real engineering $\Pi^{(\sigma_{t+\Delta})}$ is not invariable. Hence, in this paper, we assume that σ_t varies in another finite set $S_2 = \{1,2,\dots,M\}$, and the variations are considered as the stochastic variation. The variation of σ_t is governed by a higher-level transition probability (HTP) matrix $\Lambda = [\lambda_{jk}]_{M\times M}$ $(j,k \in S_2)$ and the transition probability of Markov chain satisfies

$$P\{\sigma_{t+\Delta t} = k|\sigma_t = j\} = \begin{cases} \lambda_{jk}\Delta t + o(\Delta t), & k \neq j, \\ 1 + \lambda_{jj}\Delta t + o(\Delta t), & k = j, \end{cases}$$

where $\lambda_{jj} = -\sum_{k=1,k\neq j}^{M} \lambda_{jk}$, and $\lambda_{jk} \geq 0$, $k \neq j$ is the transition rate from mode j at t to mode k at $t + \Delta$. The stochastic processes r_t and σ_t are assumed to be independent throughout this paper.

First, consider the Markov jump system with time-varying TPs as follows:

$$\begin{cases} \dot{x}(t) = A(r_t,\sigma_t)x(t) + B(r_t,\sigma_t)x(t-\tau(t,r_t,\sigma_t)) + D(r_t,\sigma_t)G(t) + E(r_t,\sigma_t)u(t) \\ \qquad + E_d(r_t,\sigma_t)d(t) + E_f(r_t,\sigma_t)f(t), \\ l(t) = A_l(r_t,\sigma_t)x(t) + B_l(r_t,\sigma_t)x(t-\tau(t,r_t,\sigma_t)) + D_l(r_t,\sigma_t)G(t) + E_{dl}(r_t,\sigma_t)d(t) \\ \qquad + E_{fl}(r_t,\sigma_t)f(t), \\ x(t_0+\theta) = \psi(t_0+\theta), \quad \theta \in [-h,0], \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the state vector of the system, $\tau(t,r_t,\sigma_t)$ is the mode-dependent time-varying delay of the system, which satisfies $h_1 \leq \tau(t,r_t,\sigma_t) \leq h$ and $\dot{\tau}(t,r_t,\sigma_t) \leq d_n$, $h_{12} = h - h_1$ is the change region of delay. $l(t) \in R^p$ is the measurable output, $u(t) \in R^q$ is the control input, $d(t) \in R^q$ is the unknown disturbance input, $f(t) \in R^q$ is the fault, $d(t)$ and $f(t)$ are assumed to be L_2 norm bound, $\psi(t_0+\theta) \in L_{n,h}$ is the initial condition of the state vector, $G(t) \in R^n$ is the nonlinear term, such that

$$Mx(t) \leq G(t) \leq Nx(t).$$

To enhance the feasible region of the criteria, we can divide the bounding into two subintervals

$$\frac{M+N}{2}x(t) \leq G(t) \leq Nx(t),$$
$$Mx(t) \leq G(t) \leq \frac{M+N}{2}x(t).$$

Model (1) can be represented as

$$\begin{cases} \dot{x}(t) = y(t), \\ y(t) = A(r_t,\sigma_t)x(t) + B(r_t,\sigma_t)x(t-\tau(t,r_t,\sigma_t)) + D(r_t,\sigma_t)G(t) + E(r_t,\sigma_t)u(t) \\ \qquad + E_d(r_t,\sigma_t)d(t) + E_f(r_t,\sigma_t)f(t), \\ l(t) = A_l(r_t,\sigma_t)x(t) + B_l(r_t,\sigma_t)x(t-\tau(t,r_t,\sigma_t)) + D_l(r_t,\sigma_t)G(t) + E_{dl}(r_t,\sigma_t)d(t) \\ \qquad + E_{fl}(r_t,\sigma_t)f(t). \end{cases} \tag{2}$$

In this paper, the following linear filter is designed:

$$\begin{cases} \dot{x}_f(t) = y_f(t), \\ y_f(t) = A_f(r_t,\sigma_t)x_f(t) + B_f(r_t,\sigma_t)l(t), \\ r_f(t) = L_f(r_t,\sigma_t)x_f(t), \\ x_f(t_0+\theta) = \psi_f(t_0+\theta), \quad \theta \in [-h,0], \end{cases} \tag{3}$$

where $x_f(t) \in R^n$ is the state vector of the filter.

To improve the sensitiveness of residual to fault, we add a weighting matrix function $W_f(s)$ into the fault $f(t)$, that is, $r_w(s) = W_f(s)f(s)$, where $r_w(s)$ and $f(s)$ refer to the Laplace transform of $r_w(t)$ and $f(t)$. The minimal realization of $r_w(s) = W_f(s)f(s)$ is assumed to be

$$\begin{cases} \dot{x}_w(t) = y_w(t), \\ y_w(t) = A_w(r_t,\sigma_t)x_w(t) + E_w(r_t,\sigma_t)f(t), \\ r_w(t) = L_w(r_t,\sigma_t)x_w(t), \\ x_w(t_0+\theta) = \psi_w(t_0+\theta), \quad \theta \in [-h,0], \end{cases} \tag{4}$$

where $r_w(t)$ is the reference residual, and our objective is to design a fault detection filter (FDF) which can result in the minimal difference between the reference model and the fault detection filter.

For simplicity, for each possible $r_t = r_i$, $\sigma_t = \sigma_j$, $i \in S_1$, $j \in S_2$, the matrix $A(r_t, \sigma_t)$ will be denoted by A_{ij}, and so on.

Define $r(t) = r_f(t) - r_w(t)$, we get the filtering error system as follows:

$$\begin{cases} \dot{\bar{x}}(t) = \bar{y}(t), \\ \dot{\bar{y}}(t) = \bar{A}_{ij}\bar{x}(t) + \bar{B}_{ij}K^T\bar{x}(t - \tau_{ij}(t)) + \bar{D}_{ij}G(x) + \bar{E}_{ij}w(t), \\ r(t) = \bar{L}_{ij}^T\bar{x}(t), \\ \bar{x}(t_0 + \theta) = \bar{\psi}_w(t_0 + \theta), \quad \theta \in [-h, 0], \end{cases} \tag{5}$$

where

$$\bar{x}(t) = \bigl[x(t), x_f(t), x_w(t)\bigr]^T, \qquad w(t) = \bigl[u(t), d(t), f(t)\bigr]^T,$$

$$\bar{A}_{ij} = \begin{bmatrix} A_{ij} & 0 & 0 \\ B_{fij}A_{lij} & A_{fij} & 0 \\ 0 & 0 & A_{wij} \end{bmatrix}, \qquad \bar{B}_{ij} = \begin{bmatrix} B_{ij} \\ B_{fij}B_{lij} \\ 0 \end{bmatrix},$$

$$\bar{D}_{ij} = \begin{bmatrix} D_{ij} \\ B_{fij}D_{lij} \\ 0 \end{bmatrix}, \qquad \bar{E}_{ij} = \begin{bmatrix} E_{uij} & E_{ldij} & E_{lfij} \\ 0 & B_{fij}E_{ldij} & B_{fij}E_{lfij} \\ 0 & 0 & E_{wij} \end{bmatrix},$$

$$\bar{L}_{ij} = \begin{bmatrix} 0 & L_{ij} & -L_{fij} \end{bmatrix}^T, \qquad K = [I \quad 0 \quad 0]^T.$$

The problem of fault detection can be transformed into H_∞ filtering problem for the system, that is, to determine all matrices such that the filtering error system (5) is robustly mean-square exponentially stable with H_∞ performance γ as follows:

$$\sup_{w(t)\neq 0} \frac{\|r(t)\|}{\|w(t)\|} < \gamma, \tag{6}$$

where $\|r(t)\| = \sqrt{\{\int_0^\infty r(t)r(t)\,dt\}}$, $\|w(t)\| = \sqrt{\{\int_0^\infty w(t)w(t)\}\,dt}$.

In this paper, the residual evaluation function $J(r)$ and threshold J_{th} are chosen as follows:

$$J(r) = \int_{t_0}^{t_0+T} r^T(t)r(t)\,dt < \gamma, \tag{7}$$

$$J_{\text{th}} = \sup_{f(t)=0} E\left\{\int_{t_0}^{t_0+T} r^T(t)r(t)\,dt\right\}, \tag{8}$$

where $[t_0, t_0 + T]$ is the finite-time window, T denotes the timeslot, and t_0 denotes the initial evaluation time. The occurrence of fault can be detected by comparing $J(r)$ and J_{th} based on the relationship as follows:

$$J(r) > J_{\text{th}} \Rightarrow \text{with fault} \Rightarrow \text{alarm}, \tag{9}$$

$$J(r) \leq J_{\text{th}} \Rightarrow \text{without fault}. \tag{10}$$

Before ending the section, we give the following notations, definitions and lemmas, which will be used in the proof of our main results.

$$e_1 = \left[I \cdot K^T, 0_2, \ldots, 0_m\right]^T, \qquad e_k = \left[0 \cdot K^T, 0_2, \ldots, 0_{k-1}, I_k, 0_{k+1}, \ldots, 0_m\right]^T,$$
$$w_k = [0_1, \ldots, 0_{k-1}, I_k, 0_{k+1}, \ldots, 0_n]^T.$$

Definition 1 The filtering error system (4) with $w(t) = 0$ is mean-square exponentially stable if there exist scalars $\alpha > 0$ and $\beta > 0$ such that

$$\mathrm{E}\left\|\bar{x}(t)\right\|_2^2 \leq \alpha e^{-\beta t}\|\bar{\psi}\|_h^2, \tag{11}$$

where $\|\bar{\psi}\|_h = \max\{\sup_{h\leq\theta\leq 0}\|\bar{\psi}(\theta)\|, \sup_{h\leq\theta\leq 0}\|\dot{\bar{\psi}}(\theta)\|\}$.

Definition 2 Given a positive scalar γ, the filtering error system (5) is mean-square exponentially stable with H_∞ performance γ if, for every system mode and delay mode, the filtering error system (5) with $w(t) = 0$ is mean-square exponentially stable, and under zero initial condition it satisfies $\|r(t)\|_2 \leq \gamma\|w(t)\|_2$ for any non-zero $w(t) \in L_2[0, +\infty]$.

Lemma 1 ([32]) *Let $\Phi_1, \Phi_1, \ldots, \Phi_N : R^m \to R^n$ be a given finite number of functions such that they have positive values in an open subset D of R^m. Then a reciprocally convex combination of these functions over D is a function of the form*

$$\frac{1}{\alpha_1}\Phi_1 + \frac{1}{\alpha_2}\Phi_2 + \cdots + \frac{1}{\alpha_N}\Phi_N : D \to R^n, \tag{12}$$

where the real numbers α_i satisfy $\alpha_i > 0$ and $\sum_i \alpha_i = 1$.

Lemma 2 ([33]) *For any constant matrices E, G and F with appropriate dimensions, $F^T F \leq kI$, k is a positive scalar, then*

$$2x^T EFGy \leq cxEE^T x + \frac{k}{c}y^T G^T Gy, \tag{13}$$

where c is a positive scalar, $x \in R^n$, and $y \in R^n$.

Lemma 3 ([34]) *For any positive definite matrix $\Phi \in R^{n\times n}$, scalar $\gamma > 0$, vector function $w : [0, \gamma] \to R^n$ such that the integrations concerned are well defined, then*

$$\left(\int_0^\gamma w(s)\,ds\right)^T \Phi\left(\int_0^\gamma w(s)\,ds\right) \leq \gamma\int_0^\gamma w^T(s)\Phi w(s)\,ds. \tag{14}$$

3 Main results

In this section, based on the Lyapunov method and linear matrix inequality techniques, the following theoretical results can be derived.

Theorem 1 *For $d_n < 1$, given positive scalars h, h_1 and k, if there exist R_1, R_2, R_3, S_{12}, M, Q_1, Q_2, U, U_1, U_2, W, M_{ij}, F_{ij} with appropriate dimension, such that*

$$\begin{bmatrix} T_{ij} & \Xi_{2ij} \\ * & -\gamma^2 I \end{bmatrix} < 0, \tag{15a}$$

$$\begin{bmatrix} \bar{T}_{ij} & \Xi_{2ij} \\ * & -\gamma^2 I \end{bmatrix} < 0, \tag{15b}$$

$$\begin{bmatrix} W & U \\ * & R_1 \end{bmatrix} > 0, \tag{15c}$$

$$\begin{bmatrix} W & U_2 \\ * & R_1 \end{bmatrix} > 0, \tag{15d}$$

$$\begin{bmatrix} W & U_3 \\ * & R_1 \end{bmatrix} > 0, \tag{15e}$$

$$\begin{bmatrix} R_2 & S_{12} \\ S_{12} & R_2 \end{bmatrix} > 0, \tag{15f}$$

$$-M + \sum_{l \in S_1} \pi_{il}^{(j)} M_{lj} + \sum_{k \in S_2} \lambda_{jk} M_{ik} < 0, \tag{15g}$$

where

$$\begin{aligned} T_{ij} = T_{ij0} &- \left\{ e_5 - e_7 - \frac{M+N}{2}(e_1 - e_3) \right\} \left\{ e_5 - e_7 - N(e_1 - e_3) \right\}^T \\ &- \left\{ e_7 - e_8 - \frac{M+N}{2}(e_3 - e_2) \right\} \left\{ e_7 - e_8 - N(e_3 - e_2) \right\}^T \\ &- \left\{ e_8 - e_9 - \frac{M+N}{2}(e_2 - e_4) \right\} \left\{ e_8 - e_9 - N(e_2 - e_4) \right\}^T, \end{aligned}$$

$$T_{ij0} = \Sigma_{ij} + \Theta + \Theta^T - (1 - d_n) e_2^T M_{ij} e_2 + h e^{kh} W + p_{ij} p_{ij}^T,$$

$$\Theta = \left[U K^T, U_3 - U_2, U_2 - U, -U_3, 0, 0, 0, 0, 0 \right],$$

$$p_{ij} = [\bar{L}_{ij}, 0, 0, 0, 0, 0, 0, 0, 0]^T, \qquad H = h e^{kh} R_1 + h_{12}^2 e^{kh_{12}} R_2,$$

$$\Xi_{2ij} = \left[\bar{E}_{ij}^T F_{ij}^T, 0, 0, 0, 0, \bar{E}_{ij}^T K V, 0, 0, 0 \right]^T,$$

$$\begin{aligned} \bar{T}_{ij} = \bar{T}_{ij0} &- \left\{ e_5 - e_7 - M(e_1 - e_3) \right\} \left\{ e_5 - e_7 - \frac{M+N}{2}(e_1 - e_3) \right\}^T \\ &- \left\{ e_7 - e_8 - M(e_3 - e_2) \right\} \left\{ e_7 - e_8 - \frac{M+N}{2}(e_3 - e_2) \right\}^T \\ &- \left\{ e_8 - e_9 - M(e_2 - e_4) \right\} \left\{ e_8 - e_9 - \frac{M+N}{2}(e_2 - e_4) \right\}^T, \end{aligned}$$

$$\Sigma_{1,1} = F_{ij} \bar{A}_{ij} + \bar{A}_{ij}^T F_{ij} + k F_{ij} + \sum_{l \in S_1} \pi_{il}^{(j)} F_{lj} + \sum_{k \in S_2} \lambda_{jk} F_{ik} + K(M_{ij} + hM + Q_1 + Q_2) K^T,$$

$$\Sigma_{1,2} = F_{ij} \bar{B}_{ij},$$

$$\Sigma_{2,2} = -2R_2 + S_{12} + S_{12}^T,$$

$$\Sigma_{2,3} = 2R_2 - 2S_{12},$$

$$\Sigma_{3,3} = -Q_1 e^{-kh_1} - R_2,$$

$$\Sigma_{2,4} = 2R_2 - 2S_{12},$$

$$\Sigma_{3,4} = 2S_{12},$$
$$\Sigma_{4,4} = -Q_2 e^{-kh} - R_2,$$
$$\Sigma_{1,5} = F_{ij}\bar{D}_{ij},$$
$$\Sigma_{1,6} = \bar{A}_{ij}^T KV,$$
$$\Sigma_{2,6} = \bar{B}_{ij}^T KV,$$
$$\Sigma_{5,6} = \bar{D}_{ij}^T KV,$$
$$\Sigma_{6,6} = H - V.$$

For $d_n \geq 1$, given positive scalars h, h_1 and k, if there exist R_1, R_2, R_3, S_{12}, Q_1, Q_2, U, U_1, U_2, W, F_{ij} with appropriate dimension, such that

$$\begin{bmatrix} H_{ij} & \Xi_{2ij} \\ * & -\gamma^2 I \end{bmatrix} < 0, \tag{15h}$$

$$\begin{bmatrix} \bar{H}_{ij} & \Xi_{2ij} \\ * & -\gamma^2 I \end{bmatrix} < 0, \tag{15i}$$

$$\begin{bmatrix} W & U \\ * & R_1 \end{bmatrix} > 0, \tag{15j}$$

$$\begin{bmatrix} W & U_2 \\ * & R_1 \end{bmatrix} > 0, \tag{15k}$$

$$\begin{bmatrix} W & U_3 \\ * & R_1 \end{bmatrix} > 0, \tag{15l}$$

$$\begin{bmatrix} R_2 & S_{12} \\ S_{12} & R_2 \end{bmatrix} > 0, \tag{15m}$$

where

$$\begin{aligned} H_{ij} = H_{ij0} &- \left\{e_5 - e_7 - \frac{M+N}{2}(e_1 - e_3)\right\}\left\{e_5 - e_7 - N(e_1 - e_3)\right\}^T \\ &- \left\{e_7 - e_8 - \frac{M+N}{2}(e_3 - e_2)\right\}\left\{e_7 - e_8 - N(e_3 - e_2)\right\}^T \\ &- \left\{e_8 - e_9 - \frac{M+N}{2}(e_2 - e_4)\right\}\left\{e_8 - e_9 - N(e_2 - e_4)\right\}^T, \end{aligned}$$

$$H_{ij0} = \bar{\Sigma}_{ij} + \Theta + \Theta^T + he^{kh}W + p_{ij}p_{ij}^T,$$

$$\begin{aligned} \bar{H}_{ij} = \bar{H}_{ij0} &- \left\{e_5 - e_7 - M(e_1 - e_3)\right\}\left\{e_5 - e_7 - \frac{M+N}{2}(e_1 - e_3)\right\}^T \\ &- \left\{e_7 - e_8 - M(e_3 - e_2)\right\}\left\{e_7 - e_8 - \frac{M+N}{2}(e_3 - e_2)\right\}^T \\ &- \left\{e_8 - e_9 - M(e_2 - e_4)\right\}\left\{e_8 - e_9 - \frac{M+N}{2}(e_2 - e_4)\right\}^T, \end{aligned}$$

$$\bar{\Sigma}_{1,1} = F_{ij}\bar{A}_{ij} + \bar{A}_{ij}^T F_{ij} + kF_{ij} + \sum_{l\in S_1} \pi_{il}^{(j)} F_{lj} + \sum_{k\in S_2} \lambda_{jk} F_{ik} + K(Q_1 + Q_2)K^T,$$

$$\bar{\Sigma}_{1,2} = F_{ij}\bar{B}_{ij},$$

$$\bar{\Sigma}_{2,2} = -2R_2 + S_{12} + S_{12}^T,$$

$$\bar{\Sigma}_{2,3} = 2R_2 - 2S_{12},$$

$$\bar{\Sigma}_{3,3} = -Q_1 e^{-kh_1} - R_2,$$

$$\bar{\Sigma}_{2,4} = 2R_2 - 2S_{12},$$

$$\bar{\Sigma}_{3,4} = 2S_{12},$$

$$\bar{\Sigma}_{4,4} = -Q_2 e^{-kh} - R_2,$$

$$\bar{\Sigma}_{1,5} = F_{ij}\bar{D}_{ij},$$

$$\bar{\Sigma}_{1,6} = \bar{A}_{ij}^T KV,$$

$$\bar{\Sigma}_{2,6} = \bar{B}_{ij}^T KV,$$

$$\bar{\Sigma}_{5,6} = \bar{D}_{ij}^T KV,$$

$$\bar{\Sigma}_{6,6} = H - V.$$

Then the filtering error system (5) *is mean-square exponentially stable with* H_∞ *performance* γ.

Proof For $d_n < 1$, choosing the following Lyapunov function candidate

$$V(t,i,j) = \sum_{i=1}^{8} V_i(t,i,j), \tag{16}$$

where

$$V_1(t,i,j) = \bar{x}^T(t) F_{ij}\bar{x}(t),$$

$$V_2(t,i,j) = \int_{t-h_1}^{t} \bar{x}^T(s) K e^{k(s-t)} Q_1 K^T \bar{x}(s)\, ds,$$

$$V_3(t,i,j) = \int_{t-h}^{t} \bar{x}^T(s) K e^{k(s-t)} Q_2 K^T \bar{x}(s)\, ds,$$

$$V_4(t,i,j) = \int_{-h}^{0}\int_{t+\beta}^{t} \bar{y}^T(\alpha) K e^{k(\alpha-t+h)} R_1 K^T \bar{y}(\alpha)\, d\alpha\, d\beta,$$

$$V_5(t,i,j) = \int_{-h}^{0}\int_{t+\beta}^{t} \xi^T(\alpha) K e^{k(\alpha-t+h)} W K^T \xi(\alpha)\, d\alpha\, d\beta,$$

$$V_6(t,i,j) = h_{12}\int_{-h}^{-h_1}\int_{t+\beta}^{t} \bar{y}^T(\alpha) K e^{k(\alpha-t+h)} R_2 K^T \bar{y}(\alpha)\, d\alpha\, d\beta,$$

$$V_7(t,i,j) = \int_{t-\tau_{ij}(t)}^{t} \bar{x}^T(s) K e^{k(s-t)} M_{ij} K^T \bar{x}(s)\, ds,$$

$$V_8(t,i,j) = \int_{-h}^{0}\int_{t+\beta}^{t} \bar{x}^T(\alpha) K e^{k(\alpha-t+h)} M K^T \bar{x}(\alpha)\, d\alpha\, d\beta$$

with

$$\xi(t) = \bigl[\bar{x}^T(t), \bar{x}^T\bigl(t-\tau(t)\bigr)K, \bar{x}^T(t-h_1)K, \bar{x}^T(t-h)K, G(t), \bar{y}^T(t)K, \\ G(t-h_1), G\bigl(t-\tau(t)\bigr), G(t-h)\bigr]^T.$$

Let L be the infinitesimal generator of random process. Then we have

$$LV(t,i,j) = \sum_{i=1}^{8} LV_i(t,i,j), \tag{17}$$

where

$$\begin{aligned}
LV_1(t,i,j) &= 2\bar{x}^T(t)F_{ij}\bigl(\bar{A}_{ij}\bar{x}(t) + \bar{B}_{ij}K^T\bar{x}\bigl(t-\tau_{ij}(t)\bigr) + \bar{D}_{ij}G(t) + \bar{E}_{ij}w(t)\bigr) \\
&\quad + \bar{x}^T(t)\Bigl(\sum_{l\in S_1}\pi_{il}^{(j)}F_{lj} + \sum_{k\in S_2}\lambda_{jk}F_{ik}\Bigr)\bar{x}(t), \\
LV_2(t,i,j) &= \bar{x}^T(t)KQ_1K^T\bar{x}(t) - e^{-kh_1}\bar{x}^T(t-h_1)KQ_1K^T\bar{x}(t-h_1) - kV_2(t,i,j), \\
LV_3(t,i,j) &= \bar{x}^T(t)KQ_2K^T\bar{x}(t) - e^{-kh}\bar{x}^T(t-h)KQ_2K^T\bar{x}(t-h) - kV_3(t,i,j), \\
LV_4(t,i,j) &= he^{kh}\bar{y}^T(t)KR_1K^T\bar{y}(t) - \int_{t-h_1}^{t} e^{k(s-t+h)}\bar{y}^T(s)KR_1K^T\bar{y}(s)\,ds - kV_4(t,i,j) \\
&\le he^{kh}\bar{y}^T(t)KR_1K^T\bar{y}(t) - \int_{t-h}^{t}\bar{y}^T(s)KR_1K^T\bar{y}(s)\,ds - kV_4(t,i,j), \\
LV_5(t,i,j) &= he^{kh}\xi^T(t)W\xi(t) - \int_{t-h}^{t} e^{k(s-t+h)}\xi^T(s)W\xi(s)\,ds - kV_5(t,i,j) \\
&\le he^{kh}\xi^T(t)W\xi(t) - \int_{t-h}^{t}\xi^T(s)W\xi(s)\,ds - kV_5(t,i,j), \\
LV_6(t,i,j) &\le h_{12}^2e^{kh_{12}}\bar{y}^T(t)KR_2K^T\bar{y}(t) - h_{12}\int_{t-\tau_{ij}(t)}^{t-h_1}\bar{y}^T(s)KR_2K^T\bar{y}(s)\,ds \\
&\quad - h_{12}\int_{t-h}^{t-\tau_{ij}(t)}\bar{y}^T(s)KR_2K^T\bar{y}(s)\,ds - kV_6(t,i,j) \\
&\le h_{12}^2e^{kh_{12}}\bar{y}^T(t)KR_2K^T\bar{y}(t) - \frac{h_{12}}{\tau_{ij}(t)-h_1}\xi^T(t)(e_3-e_2)R_2(e_3-e_2)^T\xi(t) \\
&\quad - \frac{h_{12}}{h-\tau_{ij}(t)}\xi^T(t)(e_2-e_4)R_2(e_2-e_4)^T\xi(t) - kV_6(t,i,j) \\
&\le h_{12}^2e^{kh_{12}}\bar{y}^T(t)KR_2K^T\bar{y}(t) - (1+\gamma_1)\xi^T(t)(e_3-e_2)R_2(e_3-e_2)^T\xi(t) \\
&\quad - (1+\gamma_2)\xi^T(t)(e_2-e_4)R_2(e_2-e_4)^T\xi(t) - kV_6(t,i,j),
\end{aligned}$$

where $0 \le \gamma_1 = \frac{\tau_{ij}(t)-h_1}{h-\tau_{ij}(t)} \le 1$, $0 \le \gamma_2 = \frac{h-\tau_{ij}(t)}{\tau_{ij}(t)-h_1} \le 1$.

For matrix $\left[\begin{smallmatrix} R_2 & S_{12} \\ S_{12} & R_2 \end{smallmatrix}\right] > 0$, it holds that

$$-\xi^T(t)\begin{bmatrix}\sqrt{\gamma_1}(e_3^T-e_2^T) \\ \sqrt{\gamma_2}(e_2^T-e_4^T)\end{bmatrix}^T\begin{bmatrix} R_2 & S_{12} \\ S_{12} & R_2\end{bmatrix}\begin{bmatrix}\sqrt{\gamma_1}(e_3^T-e_2^T) \\ \sqrt{\gamma_2}(e_2^T-e_4^T)\end{bmatrix}\xi(t) \le 0. \tag{18}$$

Hence

$$-\gamma_1\xi^T(t)(e_3-e_2)R_2(e_3-e_2)^T\xi(t)-\gamma_2\xi^T(t)(e_2-e_4)R_2(e_2-e_4)^T\xi(t)$$
$$\le -\xi^T(t)(e_3-e_2)R_2(e_2-e_4)^T\xi(t)-\xi^T(t)(e_2-e_4)R_2(e_3-e_2)^T\xi(t).$$

We can obtain

$$LV_6(t,i,j)\le h_{12}^2e^{kh_{12}}\bar{y}^T(t)KR_2K^T\bar{y}(t) - \xi^T(t)\begin{bmatrix} e_3^T-e_2^T \\ e_2^T-e_4^T\end{bmatrix}^T\begin{bmatrix} R_2 & S_{12} \\ S_{12} & R_2\end{bmatrix}\begin{bmatrix} e_3^T-e_2^T \\ e_2^T-e_4^T\end{bmatrix}\xi(t)-kV_6(t,i,j).$$

Remark 1 When $\tau_{ij}(t)=h$ or $\tau_{ij}(t)=h_1$, it can be derived that $\xi^T(t)(e_3-e_2)=0$ or $\xi^T(t)(e_2-e_4)=0$, respectively. Hence the inequality holds.

$$\begin{aligned}LV_7(t,i,j)\le{}& \bar{x}^T(t)KM_{ij}K^T\bar{x}(t)\\ &-\bigl(1-\dot{\tau}_{ij}(t)\bigr)e^{-kh}\bar{x}^T\bigl(t-\tau_{ij}(t)\bigr)KM_{ij}K^T\bar{x}\bigl(t-\tau_{ij}(t)\bigr)-kV_7(t,i,j)\\ &+\sum_{l\in S_1}\pi_{il}^{(j)}\int_{t-\tau_{lj}(t)}^t\bar{x}^T(s)KM_{lj}K^T\bar{x}(s)\,ds\\ &+\sum_{k\in S_2}\lambda_{jk}\int_{t-\tau_{lj}(t)}^t\bar{x}^T(s)KM_{ik}K^T\bar{x}(s)\,ds,\end{aligned}$$

$$LV_8(t,i,j)\le he^{kh}\bar{x}^T(t)KMK^T\bar{x}(t)-\int_{t-h}^t\bar{x}^T(s)KMK^T\bar{x}(s)\,ds-kV_8(t,i,j).$$

Remark 2 For $d_n<1$, it can be concluded that $-(1-\dot{\tau}_{ij}(t))<0$, which means $V_7(t,i,j)$ and $V_8(t,i,j)$ can be used to improve the conservatism of criteria.

According to the Leibniz-Newton formula,

$$\begin{aligned}&2\xi^T(t)UK^T\biggl[\bar{x}(t)-\bar{x}(t-h_1)-\int_{t-h_1}^t\bar{y}(s)\,ds\biggr]=0,\\ &2\xi^T(t)U_2K^T\biggl[\bar{x}(t-h_1)-\bar{x}\bigl(t-\tau_{ij}(t)\bigr)-\int_{t-\tau_{ij}(t)}^{t-h_1}\bar{y}(s)\,ds\biggr]=0,\\ &2\xi^T(t)U_3K^T\biggl[\bar{x}\bigl(t-\tau_{ij}(t)\bigr)-\bar{x}(t-h)-\int_{t-h}^{t-\tau_{ij}(t)}\bar{y}(s)\,ds\biggr]=0,\\ &2\bar{y}^T(t)KVK^T\bigl[-\bar{y}(t)+\bar{A}_{ij}\bar{x}(t)+\bar{B}_{ij}K^T\bar{x}\bigl(t-\tau(t)\bigr)+\bar{D}_{ij}G(t)+\bar{E}_{ij}w(t)\bigr]=0.\end{aligned}\tag{19}$$

Then the following inequality can be concluded:

$$\begin{aligned}LV(t,i,j)\le{}&\eta^T(t)\Xi_{ij0}\eta(t)-\int_{t-h_1}^t\zeta^T\Phi_1\zeta\,ds+\int_{t-\tau_{ij}(t)}^t\bar{x}^T(s)K\Phi_2K^T\bar{x}(s)\,ds\\ &-\int_{t-\tau_{ij}(t)}^{t-h_1}\zeta^T\Phi_3\zeta\,ds-\int_{t-h}^{t-\tau_{ij}(t)}\zeta^T\Phi_4\zeta\,ds-kV(t,i,j),\end{aligned}\tag{20}$$

where

$$\Xi_{ij0} = \begin{bmatrix} \Upsilon_{ij0} & \Xi_{2ij} \\ * & 0 \end{bmatrix},$$

$$\Phi_1 = \begin{bmatrix} W & U \\ * & R_1 \end{bmatrix}, \qquad \Phi_3 = \begin{bmatrix} W & U_2 \\ * & R_1 \end{bmatrix}, \qquad \Phi_4 = \begin{bmatrix} W & U_3 \\ * & R_1 \end{bmatrix},$$

$$\eta(t) = \left[\xi^T(t), w^T(t)\right]^T, \qquad \zeta = \left[\xi^T(t), y^T(s)\right]^T,$$

$$\Upsilon_{ij0} = \Sigma_{ij} + \Theta + \Theta^T - (1-d_n)e_2^T M_{ij} e_2 + he^{kh}W,$$

$$\Theta = \left[UK^T, U_3 - U_2, U_2 - U, -U_3, 0, 0, 0, 0, 0\right],$$

$$\Xi_{2ij} = \left[\bar{E}_{ij}^T F_{ij}^T, 0, 0, 0, 0, K^T \bar{E}_{ij} V^T, 0, 0, 0\right]^T,$$

$$\Phi_2 = -M + \sum_{l\in S_1} \pi_{il}^{(j)} M_{lj} + \sum_{k\in S_2} \lambda_{jk} M_{ik}.$$

For case 1: $\frac{M+N}{2}x(t) \le G(t) \le Nx(t)$.
Consider

$$\frac{M+N}{2}\big(x(t) - x(t-h_1)\big) \le G(t) - G(t-h_1) \le N\big(x(t) - x(t-h_1)\big),$$
$$\frac{M+N}{2}\big(x(t-h_1) - x\big(t-\tau_{ij}(t)\big)\big) \le G(t-h_1) - G\big(t-\tau_{ij}(t)\big) \le N\big(x(t-h_1) - x\big(t-\tau_{ij}(t)\big)\big),$$
$$\frac{M+N}{2}\big(x\big(t-\tau_{ij}(t)\big) - x(t-h)\big) \le G\big(t-\tau_{ij}(t)\big) - G\big(x(t-h)\big) \le N\big(x\big(t-\tau_{ij}(t)\big) - x(t-h)\big).$$

We get

$$0 \le -\left\{G(t) - G(t-h_1) - \frac{M+N}{2}\big(x(t) - x(t-h_1)\big)\right\} \times \big\{f(t) - f(t-h_1) - N\big(x(t) - x(t-h_1)\big)\big\},$$
$$0 \le -\left\{G(t-h_1) - G\big(t-\tau_{ij}(t)\big) - \frac{M+N}{2}\big(x(t-h_1) - x\big(t-\tau_{ij}(t)\big)\big)\right\} \times \big\{G(t-h_1) - G\big(t-\tau_{ij}(t)\big) - N\big(x(t-h_1) - x\big(t-\tau_{ij}(t)\big)\big)\big\},$$
$$0 \le -\left\{G\big(t-\tau_{ij}(t)\big) - G(t-h) - \frac{M+N}{2}\big(x\big(t-\tau_{ij}(t)\big) - x(t-h)\big)\right\} \times \big\{G\big(t-\tau_{ij}(t)\big) - G(t-h) - N\big(x\big(t-\tau_{ij}(t)\big) - x(t-h)\big)\big\}.$$

The following inequality can be concluded:

$$LV(t,i,j) \le \eta^T(t)\Xi_{ij}\eta(t) - \int_{t-h_1}^{t} \zeta^T \Phi_1 \zeta \, ds + \int_{t-\tau_{ij}(t)}^{t} \bar{x}^T(s) K \Phi_2 K^T \bar{x}(s)\, ds - \int_{t-\tau_{ij}(t)}^{t-h_1} \zeta^T \Phi_3 \zeta \, ds - \int_{t-h}^{t-\tau_{ij}(t)} \zeta^T \Phi_4 \zeta \, ds - kV(t,i,j), \tag{21}$$

where

$$\begin{aligned}\Upsilon_{ij} &= \Upsilon_{ij0} - \left\{e_5 - e_7 - \frac{M+N}{2}(e_1 - e_3)\right\}\left\{e_5 - e_7 - N(e_1 - e_3)\right\}^T \\ &\quad - \left\{e_7 - e_8 - \frac{M+N}{2}(e_3 - e_2)\right\}\left\{e_7 - e_8 - N(e_3 - e_2)\right\}^T \\ &\quad - \left\{e_8 - e_9 - \frac{M+N}{2}(e_2 - e_4)\right\}\left\{e_8 - e_9 - N(e_2 - e_4)\right\}^T, \\ \Xi_{ij} &= \begin{bmatrix} \Upsilon_{ij} & \Xi_{2ij} \\ * & 0 \end{bmatrix}.\end{aligned}$$

Consider the following performance index:

$$\begin{aligned}J &= E\left\{\int_{t_0}^{t}\left[r^T(s)r(s) - \gamma^2 w^T(s)w(s)\right] ds\right\} \\ &= E\left\{\int_{t_0}^{t}\left[r^T(s)r(s) - \gamma^2 w^T(s)w(s) + LV(s,i,j)\right] ds\right\} + E\left\{V(t_0,i,j)\right\} - E\left\{V(t,i,j)\right\}.\end{aligned}$$

For $E\{V(t_0,i,j)\} = 0$ and $E\{V(t,i,j)\} \geq 0$, we have

$$\begin{aligned}J &\leq E\left\{\int_{t_0}^{t}\left[r^T(s)r(s) - \gamma^2 w^T(s)w(s) + LV(s,i,j)\right] ds\right\} \\ &= E\left\{\int_{t_0}^{t}\left[\eta^T(s)\Pi_{ij}\eta(s) - \int_{s-h_1}^{s}\zeta^T(s)\Phi_1\zeta(s)\,du + \int_{t-\tau_{ij}(s)}^{t}\bar{x}^T(u)K\Phi_2K^T\bar{x}(u)\,du \right.\right. \\ &\quad \left.\left. - \int_{t-\tau_{ij}(t)}^{t-h_1}\zeta^T(s)\Phi_3\zeta(s)\,du - \int_{t-h}^{t-\tau_{ij}(t)}\zeta^T(s)\Phi_4\zeta(s)\,du - kV(s,i,j)\right] ds\right\},\end{aligned}$$

where

$$\begin{aligned}\Pi_{ij} &= \begin{bmatrix} T_{ij} & \Xi_{2ij} \\ * & -\gamma^2 I \end{bmatrix}, \\ T_{ij} &= \Upsilon_{ij} + p_{ij}p_{ij}^T.\end{aligned}$$

With (15a)-(15m), it can be derived that $\Pi_{ij} < 0$.

For case 2: $Mx(t) \leq G(t) \leq \frac{M+N}{2}x(t)$.

Consider

$$\begin{aligned}&M\big(x(t) - x(t-h_1)\big) \leq G(t) - G(t-h_1) \leq \frac{M+N}{2}\big(x(t) - x(t-h_1)\big), \\ &M(x(t-h_1) - x\big(t-\tau_{ij}(t)\big) \leq G(t-h_1) - G\big(t-\tau_{ij}(t)\big) \leq \frac{M+N}{2}\big(x(t-h_1) - x\big(t-\tau_{ij}(t)\big)\big), \\ &M\big(x\big(t-\tau_{ij}(t)\big) - x(t-h)\big) \leq G\big(t-\tau_{ij}(t)\big) - G\big(x(t-h)\big) \\ &\qquad \leq \frac{M+N}{2}\big(x\big(t-\tau_{ij}(t)\big) - x(t-h)\big).\end{aligned}$$

We get

$$
\begin{aligned}
0 &\le -\{G(t) - G(t-h_1) - M(x(t) - x(t-h_1))\} \\
&\quad \times \left\{f(t) - f(t-h_1) - \frac{M+N}{2}(x(t) - x(t-h_1))\right\}, \\
0 &\le -\{G(t-h_1) - G(t-\tau_{ij}(t)) - M(x(t-h_1) - x(t-\tau_{ij}(t)))\} \\
&\quad \times \left\{G(t-h_1) - G(t-\tau_{ij}(t)) - \frac{M+N}{2}(x(t-h_1) - x(t-\tau_{ij}(t)))\right\}, \\
0 &\le -\{G(t-\tau_{ij}(t)) - G(t-h) - M(x(t-\tau_{ij}(t)) - x(t-h))\} \\
&\quad \times \left\{G(t-\tau_{ij}(t)) - G(t-h) - \frac{M+N}{2}(x(t-\tau_{ij}(t)) - x(t-h))\right\}.
\end{aligned}
$$

Then the following inequality can be concluded:

$$
\begin{aligned}
LV(t,i,j) &\le \eta^T(t)\bar{\Xi}_{ij}\eta(t) - \int_{t-h_1}^{t} \zeta^T\Phi_1\zeta\,ds + \int_{t-\tau_{ij}(t)}^{t} \bar{x}^T(s)K\Phi_2K^T\bar{x}(s)\,ds \\
&\quad - \int_{t-\tau_{ij}(t)}^{t-h_1} \zeta^T\Phi_3\zeta\,ds - \int_{t-h}^{t-\tau_{ij}(t)} \zeta^T\Phi_4\zeta\,ds - kV(t,i,j),
\end{aligned}
\tag{22}
$$

where

$$
\begin{aligned}
\bar{\Xi}_{ij} &= \begin{bmatrix} \bar{\Upsilon}_{ij} & \Xi_{2ij} \\ * & 0 \end{bmatrix}, \\
\bar{\Upsilon}_{ij} &= \Upsilon_{ij0} - \{e_5 - e_7 - M(e_1 - e_3)\}\left\{e_5 - e_7 - \frac{M+N}{2}(e_1 - e_3)\right\}^T \\
&\quad - \{e_7 - e_8 - M(e_3 - e_2)\}\left\{e_7 - e_8 - \frac{M+N}{2}(e_3 - e_2)\right\}^T \\
&\quad - \{e_8 - e_9 - M(e_2 - e_4)\}\left\{e_8 - e_9 - \frac{M+N}{2}(e_2 - e_4)\right\}^T.
\end{aligned}
$$

Consider the following performance index:

$$
\begin{aligned}
J &= E\left\{\int_{t_0}^{t} [r^T(s)r(s) - \gamma^2 w^T(s)w(s)]\,ds\right\} \\
&= E\left\{\int_{t_0}^{t} [r^T(s)r(s) - \gamma^2 w^T(s)w(s) + LV(s,i,j)]\,ds\right\} + E\{V(t_0,i,j)\} - E\{V(t,i,j)\}.
\end{aligned}
$$

For $E\{V(t_0,i)\} = 0$ and $E\{V(t,i)\} \ge 0$, we have

$$
\begin{aligned}
J &\le E\left\{\int_{t_0}^{t} [r^T(s)r(s) - \gamma^2 w^T(s)w(s) + LV(s,i,j)]\,ds\right\} \\
&= E\left\{\int_{t_0}^{t} \left[\eta^T(s)\bar{\Pi}_{ij}\eta(s) - \int_{s-h_1}^{s} \zeta^T(s)\Phi_1\zeta(s)\,du + \int_{t-\tau_{ij}(s)}^{t} \bar{x}^T(u)K\Phi_2K^T\bar{x}(u)\,du \right.\right. \\
&\quad \left.\left. - \int_{t-\tau_{ij}(t)}^{t-h_1} \zeta^T(s)\Phi_3\zeta(s)\,du - \int_{t-h}^{t-\tau_{ij}(t)} \zeta^T(s)\Phi_4\zeta(s)\,du - kV(s,i,j)\right]ds\right\},
\end{aligned}
$$

where

$$\bar{\Pi}_{ij} = \begin{bmatrix} \bar{T}_{ij} & \Xi_{2ij} \\ * & -\gamma^2 I \end{bmatrix},$$

$$\bar{T}_{ij} = \bar{\Upsilon}_{ij} + p_{ij}p_{ij}^T.$$

With (15a)-(15m), it can be derived that $\bar{\Pi}_{ij} < 0$.

Next, we discuss the stability of the filtering error system (5) with $w(t) = 0$, which is equivalent to the stability of the filtering error system (5) without $w(t)$.

For case 1 and case 2, with (21) and (22), we can get the following inequalities respectively:

$$\begin{aligned} LV(t,i,j) &\leq \xi^T(t)\Upsilon_{ij}\xi(t) - \int_{t-h_1}^{t} \zeta^T\Phi_1\zeta\,ds + \int_{t-\tau_{ij}(t)}^{t} \bar{x}^T(s)K\Phi_2K^T\bar{x}(s)\,ds \\ &\quad - \int_{t-\tau_{ij}(t)}^{t-h_1} \zeta^T\Phi_3\zeta\,ds - \int_{t-h}^{t-\tau_{ij}(t)} \zeta^T\Phi_4\zeta\,ds - kV(t,i,j), \\ LV(t,i,j) &\leq \xi^T(t)\bar{\Upsilon}_{ij}\xi(t) - \int_{t-h_1}^{t} \zeta^T\Phi_1\zeta\,ds + \int_{t-\tau_{ij}(t)}^{t} \bar{x}^T(s)K\Phi_2K^T\bar{x}(s)\,ds \\ &\quad - \int_{t-\tau_{ij}(t)}^{t-h_1} \zeta^T\Phi_3\zeta\,ds - \int_{t-h}^{t-\tau_{ij}(t)} \zeta^T\Phi_4\zeta\,ds - kV(t,i,j). \end{aligned} \tag{23}$$

Considering $\Pi_{ij} < 0$, $\bar{\Pi}_{ij} < 0$, one can obtain $\Upsilon_{ij} < 0$, $\bar{\Upsilon}_{ij} < 0$.

Then with (15a)-(15m) it can be concluded

$$LV(t,i,j) \leq -kV(t,i,j). \tag{24}$$

Hence

$$L\big(e^{kt}V(t,i,j)\big) = e^{kt}\big(LV(t,i,j) + kV(t,i,j)\big) \leq 0. \tag{25}$$

With Dynkin's formula, one can obtain

$$\begin{aligned} \mathrm{E}V(t,i,j)e^{kt} &= \mathrm{E}V(t_0,i,j)e^{kt_0} + \mathrm{E}\int_{t_0}^{t} L\big(e^{ks}V(s,i,j)\big)\,ds \\ &\leq \mathrm{E}V(t_0,i,j)e^{kt_0}. \end{aligned} \tag{26}$$

Then

$$\lambda_{\min}(F_{ij})\mathrm{E}\|\bar{x}(t)\|^2 \leq \mathrm{E}V(t,i,j) \leq \mathrm{E}V(t_0,i,j)e^{-k(t-t_0)}.$$

According to the definition of $V(t,i,j)$, we have

$$\begin{aligned} \mathrm{E}V(t_0,i,j) &\leq \big[\lambda_{\max}(F_{ij}) + h\lambda_{\max}(M_{ij}) + h^2\lambda_{\max}(M) + h\lambda_{\max}(Q_1) + h\lambda_{\max}(Q_2) \\ &\quad + h_1^2\lambda_{\max}(R_1) + h_1^2\lambda_{\max}(W) + h_{12}^3\lambda_{\max}(R_2)\big]\mathrm{E}\|\psi\|_h^2. \end{aligned} \tag{27}$$

The following inequality can be concluded:

$$\mathrm{E}\|\bar{x}(t)\| \leq ae^{-\frac{k}{2}(t-t_0)}\mathrm{E}\|\bar{\psi}\|_h, \tag{28}$$

where

$$a = \sqrt{\frac{(\lambda_{\max}(F_{ij}) + h\lambda_{\max}(M_{ij}) + h^2\lambda_{\max}(M) + h_1\lambda_{\max}(Q_1) + h\lambda_{\max}(Q_2) + h_1^2\lambda_{\max}(R_1) + h^2\lambda_{\max}(W) + h_{12}^3\lambda_{\max}(R_2))}{\lambda_{\min}(F_{ij})}}.$$

By Definition 1, it can be derived that the fault detection system (1) without $w(t)$ is mean-square exponentially stable. Then, based on Definition 2, we can conclude that the filtering error system (5) is mean-square exponentially stable with H_∞ performance γ.

Now let us consider the case $d_n \geq 1$. Choose the Lyapunov function candidate as follows:

$$V(t,i,j) = \sum_{i=1}^{6} V_i(t,i,j).$$

Remark 3 For $d_n \geq 1$, it can be concluded that $-(1-\dot{\tau}_{ij}(t)) \geq 0$, which means $V_7(t,i,j)$ and $V_8(t,i,j)$ will increase the conservatism of theoretical results. Hence, in this case, $V_7(t,i,j)$ and $V_8(t,i,j)$ will not be included to construct the Lyapunov function.

Then the following inequality can be concluded:

$$\begin{aligned} LV(t,i,j) \leq{}& \eta^T(t)\bar{\Xi}_{ij0}\eta(t) - \int_{t-h_1}^{t} \zeta^T\Phi_1\zeta\,ds \\ & - \int_{t-\tau_{ij}(t)}^{t-h_1} \zeta^T\Phi_3\zeta\,ds - \int_{t-h}^{t-\tau_{ij}(t)} \zeta^T\Phi_4\zeta\,ds - kV(t,i,j), \end{aligned}$$

where

$$\begin{aligned} \bar{\Xi}_{ij0} &= \begin{bmatrix} \bar{\Upsilon}_{ij0} & \Xi_{2ij} \\ * & 0 \end{bmatrix}, \\ \bar{\Upsilon}_{ij0} &= \Sigma_{ij} + \Theta + \Theta^T + he^{kh}W. \end{aligned}$$

As above proof, it can be concluded that

$$\mathrm{E}\|\bar{x}(t)\| \leq ae^{-\frac{k}{2}(t-t_0)}\mathrm{E}\|\bar{\psi}\|_h, \tag{29}$$

where

$$a = \sqrt{\frac{(\lambda_{\max}(F_{ij}) + h_1\lambda_{\max}(Q_1) + h\lambda_{\max}(Q_2) + h_1^2\lambda_{\max}(R_1) + h^2\lambda_{\max}(W) + h_{12}^3\lambda_{\max}(R_2))}{\lambda_{\min}(F_{ij})}}.$$

Considering Definition 1, it can be derived that the filtering error system (5) without $w(t)$ is mean-square exponentially stable. Then, combined with Definition 2, we can conclude that the filtering error system (5) is mean-square exponentially stable with H_∞ performance γ. The proof of Theorem 1 is thus completed. □

Based on Theorem 1 and LMI techniques, the fault detection filter design problem is addressed as follows.

Theorem 2 *For $d_n < 1$, given positive scalars h, h_1 and k, if there exist R_1, R_2, S_{12}, M, Q_1, Q_2, U, U_1, U_2, W, M_{ij}, F_{ij} with appropriate dimension, such that*

$$\begin{bmatrix} T_{ij} & \Xi_{2ij} \\ * & -\gamma^2 I \end{bmatrix} < 0, \tag{30a}$$

$$\begin{bmatrix} \bar{T}_{ij} & \Xi_{2ij} \\ * & -\gamma^2 I \end{bmatrix} < 0, \tag{30b}$$

$$\begin{bmatrix} W & U \\ * & R_1 \end{bmatrix} > 0, \tag{30c}$$

$$\begin{bmatrix} W & U_2 \\ * & R_1 \end{bmatrix} > 0, \tag{30d}$$

$$\begin{bmatrix} W & U_3 \\ * & R_1 \end{bmatrix} > 0, \tag{30e}$$

$$\begin{bmatrix} R_2 & S_{12} \\ S_{12} & R_2 \end{bmatrix} > 0, \tag{30f}$$

$$-M + \sum_{l\in S_1} \pi_{il}^{(j)} M_{lj} + \sum_{k\in S_2} \lambda_{jk} M_{ik} < 0, \tag{30g}$$

where

$$\begin{aligned} T_{ij} = {} & T_{ij0} - \left\{ w_7 - w_9 - \frac{M+N}{2}(w_1 - e_5) \right\} \left\{ w_7 - w_9 - N(w_1 - w_5) \right\}^T \\ & - \left\{ w_9 - w_{10} - \frac{M+N}{2}(w_5 - w_4) \right\} \left\{ w_9 - w_{10} - N(w_5 - w_4) \right\}^T \\ & - \left\{ w_{10} - w_{11} - \frac{M+N}{2}(w_4 - w_6) \right\} \left\{ w_{10} - w_{11} - N(w_4 - w_6) \right\}^T, \end{aligned}$$

$$T_{ij0} = \Sigma^{ij} + \Theta + \Theta^T - (1-d_n) w_4^T M_{ij} w_4 + h e^{kh} W + p_{ij} p_{ij}^T,$$

$$\Theta = [U, 0, 0, U_3 - U_2, U_2 - U, -U_3, 0, 0, 0, 0, 0],$$

$$p_{ij} = [0, L_{ij}, -L_{fij}, 0, 0, 0, 0, 0, 0, 0, 0]^T, \qquad H = h e^{kh} R_1 + h_{12}^2 e^{kh_{12}} R_2,$$

$$\begin{aligned} \bar{T}_{ij} = {} & \bar{T}_{ij0} - \left\{ w_7 - w_9 - M(w_1 - w_5) \right\} \left\{ w_7 - w_9 - \frac{M+N}{2}(w_1 - w_5) \right\}^T \\ & - \left\{ w_9 - w_{10} - M(w_5 - w_4) \right\} \left\{ w_9 - w_{10} - \frac{M+N}{2}(w_5 - w_4) \right\}^T \\ & - \left\{ w_{10} - w_{11} - M(w_4 - w_6) \right\} \left\{ w_{10} - w_{11} - \frac{M+N}{2}(w_4 - w_6) \right\}^T, \end{aligned}$$

$$\Xi_{2ij} = \begin{vmatrix} E_{uij}^T F_{1ij}^T & 0 & 0 & 0 & 0 & 0 & 0 & E_{uij}^T V & 0 & 0 & 0 \\ E_{ldij}^T F_{1ij}^T & E_{ldij}^T \hat{B}_{fij}^T & 0 & 0 & 0 & 0 & 0 & E_{ldij}^T V & 0 & 0 & 0 \\ E_{lfij}^T F_{1ij}^T & E_{lfij}^T \hat{B}_{fij}^T & E_{wij}^T F_{3ij}^T & 0 & 0 & 0 & 0 & E_{lfij}^T V & 0 & 0 & 0 \end{vmatrix}^T,$$

$$\Sigma_{1,1}^{ij} = F_{1ij}A_{ij} + A_{ij}^T F_{1ij} + kF_{1ij} + \sum_{l\in S_1} \pi_{il}^{(j)} F_{1lj} + \sum_{k\in S_2} \lambda_{jk} F_{1ik} + M_{ij} + hM + Q_1 + Q_2 + l^2 e^{kl} Q_3,$$

$$\Sigma_{1,2}^{ij} = A_{lij}^T \hat{B}_{fij}^T,$$

$$\Sigma_{2,2}^{ij} = \hat{A}_{fij} + \hat{A}_{fij}^T + kF_{2ij} + \sum_{l\in S_1} \pi_{il}^{(j)} F_{2lj} + \sum_{k\in S_2} \lambda_{jk} F_{2ik},$$

$$\Sigma_{3,3}^{ij} = F_{3i}A_{wij} + A_{wij}^T F_{3i} + kF_{3ij} + \sum_{l\in S_1} \pi_{il}^{(j)} F_{3lj} + \sum_{k\in S_2} \lambda_{jk} F_{3ik},$$

$$\Sigma_{1,4}^{ij} = F_{1ij}B_{ij},$$

$$\Sigma_{2,4}^{ij} = \hat{B}_{fij}B_{lij},$$

$$\Sigma_{4,4}^{ij} = -2R_2 + S_{12} + S_{12}^T,$$

$$\Sigma_{5,5}^{ij} = -Q_1 e^{-kh_1} - R_2,$$

$$\Sigma_{4,6}^{ij} = 2R_2 - 2S_{12},$$

$$\Sigma_{5,6}^{ij} = 2S_{12},$$

$$\Sigma_{6,6}^{ij} = -Q_2 e^{-kh} - R_2,$$

$$\Sigma_{1,7}^{ij} = F_{1ij}D_{ij},$$

$$\Sigma_{2,7}^{ij} = \hat{B}_{fij}D_{lij},$$

$$\Sigma_{1,8}^{ij} = A_{ij}^T V,$$

$$\Sigma_{4,8}^{ij} = B_{ij}^T V,$$

$$\Sigma_{7,8}^{ij} = D_{ij}^T V,$$

$$\Sigma_{8,8}^{ij} = H - V.$$

For $d_n \geq 1$, given positive scalars h, h_1 and k, if there exist R_1, R_2, S_{12}, Q_1, Q_2, U, U_1, U_2, W, F_{ij} with appropriate dimension, such that

$$\begin{bmatrix} H_{ij} & \Xi_{2ij} \\ * & -\gamma^2 I \end{bmatrix} < 0, \tag{30h}$$

$$\begin{bmatrix} \bar{H}_{ij} & \Xi_{2ij} \\ * & -\gamma^2 I \end{bmatrix} < 0, \tag{30i}$$

$$\begin{bmatrix} W & U \\ * & R_1 \end{bmatrix} > 0, \tag{30j}$$

$$\begin{bmatrix} W & U_2 \\ * & R_1 \end{bmatrix} > 0, \tag{30k}$$

$$\begin{bmatrix} W & U_3 \\ * & R_1 \end{bmatrix} > 0, \tag{30l}$$

$$\begin{bmatrix} R_2 & S_{12} \\ S_{12} & R_2 \end{bmatrix} > 0, \tag{30m}$$

where

$$\begin{aligned}
H_{ij} &= H_{ij0} - \left\{w_7 - w_9 - \frac{M+N}{2}(w_1 - w_5)\right\}\left\{w_7 - w_9 - N(w_1 - w_5)\right\}^T \\
&\quad - \left\{w_9 - w_{10} - \frac{M+N}{2}(w_5 - w_4)\right\}\left\{e_9 - e_{10} - N(w_5 - w_4)\right\}^T \\
&\quad - \left\{w_{10} - w_{11} - \frac{M+N}{2}(w_4 - w_6)\right\}\left\{e_{10} - e_{11} - N(w_4 - w_6)\right\}^T, \\
H_{ij0} &= \bar{\Sigma}^{ij} + \Theta + \Theta^T + he^{kh}W + p_{ij}p_{ij}^T, \\
\bar{H}_{ij} &= H_{ij0} - \left\{w_7 - w_9 - M(w_1 - w_5)\right\}\left\{w_7 - w_9 - \frac{M+N}{2}(w_1 - w_5)\right\}^T \\
&\quad - \left\{w_9 - w_{10} - M(w_5 - w_4)\right\}\left\{w_9 - w_{10} - \frac{M+N}{2}(w_5 - w_4)\right\}^T \\
&\quad - \left\{w_{10} - w_{11} - M(w_4 - w_6)\right\}\left\{w_{10} - w_{11} - \frac{M+N}{2}(w_4 - w_6)\right\}^T, \\
\Xi_{2ij} &= \begin{vmatrix}
E_{uij}^T F_{1ij}^T & 0 & 0 & 0 & 0 & 0 & 0 & E_{uij}^T V & 0 & 0 & 0 \\
E_{ldij}^T F_{1ij}^T & E_{ldij}^T \hat{B}_{fij}^T & 0 & 0 & 0 & 0 & 0 & E_{ldij}^T V & 0 & 0 & 0 \\
E_{lfij}^T F_{1ij}^T & E_{lfij}^T \hat{B}_{fij}^T & E_{wij}^T F_{3ij}^T & 0 & 0 & 0 & 0 & E_{lfij}^T V & 0 & 0 & 0
\end{vmatrix}^T, \\
\bar{\Sigma}_{1,1}^{ij} &= F_{1ij}A_{ij} + A_{ij}^T F_{1ij} + kF_{1ij} + \sum_{l\in S_1} \pi_{il}^{(j)} F_{1lj} + \sum_{k\in S_2} \lambda_{jk} F_{1ik} + Q_1 + Q_2 + l^2 e^{kl} Q_3, \\
\bar{\Sigma}_{1,2}^{ij} &= A_{lij}^T \hat{B}_{fij}^T, \\
\bar{\Sigma}_{2,2}^{ij} &= \hat{A}_{fij} + \hat{A}_{fij}^T + kF_{2ij} + \sum_{l\in S_1} \pi_{il}^{(j)} F_{2lj} + \sum_{k\in S_2} \lambda_{jk} F_{2ik}, \\
\bar{\Sigma}_{3,3}^{ij} &= F_{3i}A_{wij} + A_{wij}^T F_{3i} + kF_{3ij} + \sum_{l\in S_1} \pi_{il}^{(j)} F_{3lj} + \sum_{k\in S_2} \lambda_{jk} F_{3ik}, \\
\bar{\Sigma}_{1,4}^{ij} &= F_{1ij}B_{ij}, \\
\bar{\Sigma}_{2,4}^{ij} &= \hat{B}_{fij}B_{lij}, \\
\bar{\Sigma}_{4,4}^{ij} &= -2R_2 + S_{12} + S_{12}^T, \\
\bar{\Sigma}_{5,5}^{ij} &= -Q_1 e^{-kh_1} - R_2, \\
\bar{\Sigma}_{4,6}^{ij} &= 2R_2 - 2S_{12}, \\
\bar{\Sigma}_{5,6}^{ij} &= 2S_{12}, \\
\bar{\Sigma}_{6,6}^{ij} &= -Q_2 e^{-kh} - R_2, \\
\bar{\Sigma}_{1,7}^{ij} &= F_{1ij}D_{ij}, \\
\bar{\Sigma}_{2,7}^{ij} &= \hat{B}_{fij}D_{lij}, \\
\bar{\Sigma}_{1,8}^{ij} &= A_{ij}^T V, \\
\bar{\Sigma}_{4,8}^{ij} &= B_{ij}^T V, \\
\bar{\Sigma}_{7,8}^{ij} &= D_{ij}^T V, \\
\bar{\Sigma}_{8,8}^{ij} &= H - V.
\end{aligned}$$

Then the filtering error system (5) *is mean-square exponentially stable with H_∞ performance γ, and the desired parameters of FDF are determined by*

$$B_{fij} = F_{2ij}^{-1}\hat{B}_{fij}, \qquad A_{fij} = F_{2ij}^{-1}\hat{A}_{fij}, \qquad L_{fij} = \hat{L}_{fij}. \tag{31}$$

Proof First define $F_{ij} = \mathrm{diag}\{F_{1ij}, F_{2ij}, F_{3ij}\}$. Based on (15a)-(15m) and (31), one can obtain (30a)-(30m). Then, combined with Theorem 1 and Definition 1, it can be concluded that the filtering error system (5) is mean-square exponentially stable with H_∞ performance γ. The proof of Theorem 2 is thus completed. □

4 Simulation results

In this section, we will verify the proposed methodology by giving an illustrative example. Consider MJNDSs with parameters, Markovian switching modes and state-space matrices as follows:

$$\begin{aligned}
&A_1 = \begin{bmatrix} -12 & 0 \\ 0.5 & -9 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} -1.1 & 0.5 \\ 1.5 & -1 \end{bmatrix}, \qquad D_1 = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.5 \end{bmatrix}, \\
&E_{u1} = \begin{bmatrix} 0.15 \\ 0.1 \end{bmatrix}, \qquad E_{d1} = \begin{bmatrix} 0.5 \\ 0.2 \end{bmatrix}, \qquad E_{f1} = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}, \\
&A_{l1} = [1.8 \quad 2.0], \qquad B_{l1} = [1.5 \quad 0], \qquad D_{l1} = [0.15 \quad 0.1], \\
&E_{dl1} = 0.15, \qquad E_{fl1} = 0.2, \\
&A_2 = \begin{bmatrix} -11 & 1.5 \\ -2 & -13 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} -1 & 1.2 \\ 0.5 & -0.9 \end{bmatrix}, \qquad D_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix}, \\
&E_{u2} = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, \qquad E_{d2} = \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix}, \qquad E_{f2} = \begin{bmatrix} 0.2 \\ 0 \end{bmatrix}, \\
&A_{l2} = [2.1 \quad 1.9], \qquad B_{l2} = [1.0 \quad 0.8], \qquad D_{l2} = [0.1 \quad 0.1], \\
&E_{dl2} = 0.2, \qquad E_{fl2} = 0.1, \\
&G(t) = 0.25\big(\big|x(t)+1\big| + \big|x(t)-1\big|\big).
\end{aligned}$$

The piecewise homogeneous TP matrices are

$$\Pi_1 = \begin{bmatrix} -0.4 & 0.4 \\ 0.5 & -0.5 \end{bmatrix}, \qquad \Pi_2 = \begin{bmatrix} -0.6 & 0.6 \\ 0.3 & -0.3 \end{bmatrix}, \qquad \Pi_3 = \begin{bmatrix} -0.8 & 0.8 \\ 0.4 & -0.4 \end{bmatrix}.$$

The HTP matrix is

$$\Lambda = \begin{bmatrix} -0.7 & 0.3 & 0.4 \\ 0.3 & -0.8 & 0.5 \\ 0.4 & 0.2 & -0.6 \end{bmatrix}.$$

Other parameters are $\tau_1(t) = 0.3 + 0.3\sin(5t)$, $\tau_2(t) = 0.4 + 0.2\cos(6t)$, $h_1 = 0.4$, $h = 0.6$, $h_{12} = 0.3$, $d_n = 1.5$, $M = 0$, $N = 0.5I$, $\gamma = 1.0$, $k = 0.1$. The weighting matrix is $W(s) = 5/(s+5)$. Then, based on (4), it can be concluded that $A_w = -5$, $E_w = 5$, $L_w = 1$. Based on

Theorem 2, the filtering parameters are determined as follows:

$$A_{f11} = \begin{bmatrix} -3.3246 & 1.0424 \\ -1.4333 & -0.5130 \end{bmatrix}, \qquad B_{f11} = \begin{bmatrix} 0.5686 \\ 0.0371 \end{bmatrix},$$

$$L_{f11} = [\,4.4313 \quad 0.4725\,],$$

$$A_{f12} = \begin{bmatrix} -2.4723 & -1.7799 \\ -3.4085 & -3.5492 \end{bmatrix}, \qquad B_{f12} = \begin{bmatrix} 0.4426 \\ 0.8135 \end{bmatrix},$$

$$L_{f12} = [\,5.4478 \quad 3.9307\,],$$

$$A_{f13} = \begin{bmatrix} -2.5576 & -1.4977 \\ -3.2110 & -3.2455 \end{bmatrix}, \qquad B_{f13} = \begin{bmatrix} 0.4552 \\ 0.7358 \end{bmatrix},$$

$$L_{f13} = [\,5.3461 \quad 3.5849\,],$$

$$A_{f21} = \begin{bmatrix} -0.3764 & 0.1019 \\ -0.2568 & -2.9246 \end{bmatrix}, \qquad B_{f21} = \begin{bmatrix} 0.0452 \\ 0.9800 \end{bmatrix},$$

$$L_{f21} = [\,-0.0492 \quad 4.3755\,],$$

$$A_{f22} = \begin{bmatrix} -3.5381 & -0.1402 \\ 0.0847 & -0.5042 \end{bmatrix}, \qquad B_{f22} = \begin{bmatrix} 0.5712 \\ 0.0383 \end{bmatrix},$$

$$L_{f22} = [\,4.8461 \quad -0.0796\,],$$

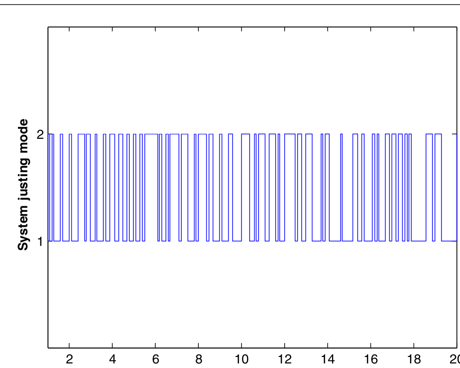
$$A_{f23} = \begin{bmatrix} -0.6926 & 0.0777 \\ -0.2226 & -2.6826 \end{bmatrix}, \qquad B_{f23} = \begin{bmatrix} 0.0978 \\ 0.8859 \end{bmatrix},$$

$$L_{f23} = [\,0.4404 \quad 3.9300\,].$$

Remark 4 It is noticed that $d_n = 1.5$, which means that our theoretical results are suitable for the case that the derivative of time delay is bigger than 1.

For numerical simulation, the initial state is $\varphi(\theta) = [0.4, -0.6]^T$, $r_0 = 1$, $\sigma_0 = 1$. The disturbance input $d(t)$ is the uniform distribution noise between $[-1, 1]$. The fault signal $f(t)$ is a square wave signal with unit amplitude. Corresponding numerical simulation results are shown in Figures 1-5.

Figure 1 Evolution of system jumping mode with time-varying TPs.



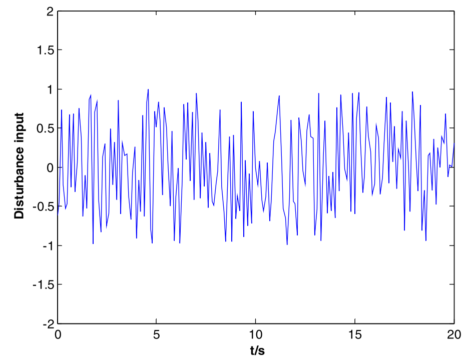


Figure 2 Time response of disturbance input $d(t)$.

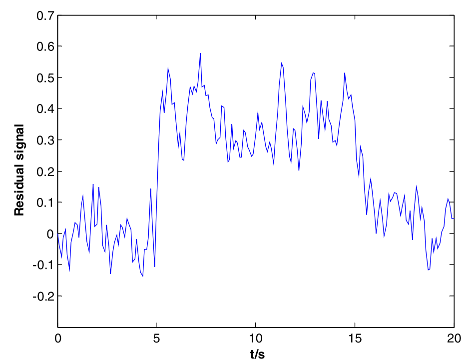


Figure 3 Time response of residual signal $r(t)$.

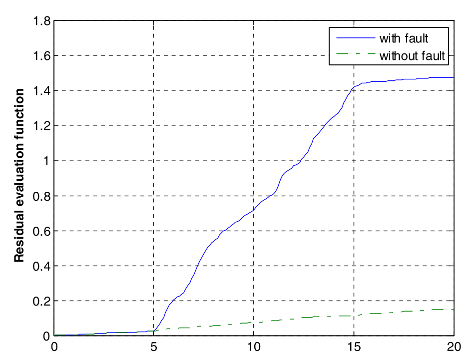
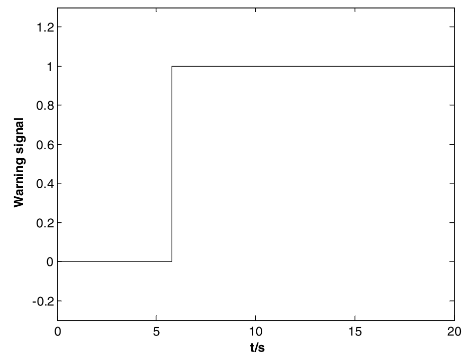


Figure 4 Evolution of residual evaluation function $J(r)$.

Remark 5 Figure 1 depicts the evolution of system jumping mode with time-varying TPs, which is more random compared with time-invariant TPs. Figure 2 depicts the time response of disturbance input $d(t)$. Figure 3 depicts the time response of the residual signal $r(t)$. Figure 4 depicts the evolution of residual evaluation function $J(r)$. Figure 5 depicts the time response of warning signal. For the case without fault, one can get $\int_0^{20} r^T(t)r(t)\,dt = 0.1760$. We can choose threshold $J_{\mathrm{th}} = 0.18$. Then, considering the case that fault exists, one can get $\int_0^{5.8} r^T(t)r(t)\,dt = 0.1863 > J_{\mathrm{th}}$. From Figure 5 it can be found that the alert is triggered at about 5.8 seconds, which means that it will take 0.8 seconds to detect the fault.

Figure 5 Time response of warning signal.



5 Conclusions

In this paper, the problem on fault detection filter design for continuous-time NMJSs with mode-dependent delay and time-varying TPs has been investigated. Based on Lyapunov-Krasovskii function approach and convex polyhedron technique, a FDF has been constructed for the possible application in fault detection such that the mean-square exponential stability and a prescribed level of disturbance attenuation are satisfied. Finally, the typical numerical example has been included to verify the correctness of theoretical results.

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Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors read and approved the final manuscript.

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