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Application of the enhanced modified simple equation method for Burger-Fisher and modified Volterra equations

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Abstract

The enhanced modified simple equation method plays a vital role in finding an exact traveling wave solution of nonlinear evolution equations (NLEEs) in engineering and mathematical physics. In this article, we use the enhanced modified simple equation method to find the exact solutions of NLEEs via the Burger-Fisher equation and the modified Volterra equations and achieve exact solutions involving parameters. When the parameters receive special values, the solitary wave solutions are derived from the exact solutions. It is established that the enhanced modified simple equation method offers a further influential mathematical tool for constructing exact solutions of NLEEs in mathematical physics.

Keywords: Burger-Fisher equation; modified Volterra equations; exact solutions; enhanced modified simple equation method; sample

1 Introduction

It is well known that nonlinear phenomena occur in various areas of science and engineering, such as fluid mechanics, plasma, solid-state physics, biophysics, etc., and could be modeled by nonlinear evolution equations (NLEEs). So, many NLEEs are widely employed to describe these complex physical phenomena. Thus, the issue is to look for exact solutions of NLEEs which can help understand that the internal mechanism of intricate physical phenomena plays a vital role. Consequently, many powerful and efficient methods and techniques, such as Darboux transformations method [1], Bäcklund transformation method [2], Hirota's bilinear method [3], Painlevé expansions method [4], symmetry method [5], the tanh method [6], the homogeneous balance method [7], the Jacobi-elliptic function method [8], the (G'/G)-expansion method [9–15], F-expansion method [16], the exp-expansion method [17, 18], Exp-function method [18–20], the modified simple equation method [20–25], the generalized and improved (G'/G)-expansion method [26], and so on, were established to obtain exact traveling wave solutions of nonlinear physical phenomena.

$$u_t + uu_x + u_{xx} + u(1-u) = 0 \tag{1}$$

and the modified Volterra equations

$$\begin{cases} u_t + \alpha u_x - u + uv = 0, \\ v_t + \beta v_x + v - uv = 0. \end{cases} \tag{2}$$

Eq. (1) not only arises in genetics, biology, heat and mass transfer, but also acts as a prototype model for describing the interaction between the reaction mechanism, convection effect and diffusion transport [27]. Eq. (2), also known as the predator-prey equations, is a pair of nonlinear evolution equations frequently used to describe the dynamics of biological systems with competition, disease and mutualism [28].

The article is organized as follows. In Section 2, the enhanced modified simple equation method is discussed. In Section 3, we apply this method to the nonlinear evolution equations pointed out above. In Section 4 and 5 discussions and conclusions are given, respectively.

2 The enhanced modified simple equation method

The proposed method can be described as follows. Suppose that we have a nonlinear evolution equation in the form

$$F(u, u_t, u_x, u_{xx}, \ldots) = 0, \tag{3}$$

where F is a polynomial of u and its partial derivatives, in which the highest order derivatives and nonlinear terms are involved. In the following, we give the main steps of this method.

Step 1. Using the generalized wave transformation

$$u(x,t) = u(\xi), \qquad \xi = p(t)x + q(t), \tag{4}$$

where $p(t)$ and $q(t)$ are differentiable functions of t, from (3) and (4) we have the following ODE:

$$F\bigl[u, \bigl(\dot{p}(t)x + \dot{q}(t)\bigr)u', p(t)u', \ldots\bigr] = 0, \tag{5}$$

where $\cdot \equiv d/dt, ' \equiv d/d\xi$.

Step 2. Suppose that Eq. (5) has the formal solution

$$u(\xi) = \sum_{k=0}^{N} A_k(t)\left[\frac{\psi'(\xi)}{\psi(\xi)}\right]^k, \tag{6}$$

where $A_k(t)$ are functions of t, $A_k(t)$ and $\psi(\xi)$ are unknown functions to be determined later such that $A_N \neq 0$.

Step 3. Determine the positive integer N in Eq. (6) by considering the homogeneous balance between the highest order derivatives and the nonlinear terms in Eq. (5).

Step 4. Substitute Eq. (6) into Eq. (5), calculate all the necessary derivatives $u', u'', \ldots$ of the unknown function $u(\xi)$ and obtain the function $\psi(\xi)$. As a result, a polynomial of $\frac{\psi'(\xi)}{\psi(\xi)}$ and its derivatives can be obtained. Then we gather all the terms in this polynomial of the same power of $\psi(\xi)^{-j}$, where $j \geq 0$, and equate all the coefficients of these terms to zero. This operation yields a system of equations which can be solved to find $A_k(t)$ and $\psi(\xi)$. Consequently, we can get the exact solutions of Eq. (3).

3 Applications

In this section, we will employ the enhanced modified simple equation method to obtain the exact solutions and then the solitary wave solutions of the Burger-Fisher equation and the modified Volterra equations.

3.1 The Burger-Fisher equation

The exact solutions of Eq. (1) have been investigated by using different methods, e.g., the exp-function method in Ref. [29] and the (G'/G)-expansion method in Ref. [30]. In this subsection, we will solve Eq. (1) by using the enhanced modified simple equation method. First, we use the generalized wave transformation (4) to reduce Eq. (1) to the following ODE:

$$\left[\dot{p}(t)x + \dot{q}(t)\right]u' + p(t)uu' + p^2(t)u'' + u(1-u) = 0. \tag{7}$$

Taking the homogenous balance between the highest-order derivative u'' and the nonlinear term of the highest order u^2, we obtain $N = 1$. Therefore, the solution of Eq. (7) has the formal solution

$$u(\xi) = A_0(t) + A_1(t)\left[\frac{\psi'(\xi)}{\psi(\xi)}\right], \tag{8}$$

where $A_0(t)$ and $A_1(t)$ are functions of t to be determined later such that $A_1(t) \neq 0$. It is easy to see that

$$u' = A_1(t)\left(\frac{\psi''}{\psi} - \frac{\psi'^2}{\psi^2}\right), \tag{9}$$

$$u'' = A_1(t)\left(\frac{\psi'''}{\psi} - 3\frac{\psi'\psi''}{\psi^2} + 2\frac{\psi'3}{\psi^3}\right). \tag{10}$$

Substituting Eqs. (8)-(10) into Eq. (7) and equating all the coefficients of ψ^0, ψ^{-1}, $\psi^{-1}x$, ψ^{-2}, $\psi^{-2}x$ and ψ^{-3} to zero, we respectively obtain

$$A_0^2(t) - A_0(t) = 0, \tag{11}$$

$$p^2(t)\psi''' + \left[\dot{q}(t) + p(t)A_0(t)\right]\psi'' + \left[1 - 2A_0(t)\right]\psi' = 0, \tag{12}$$

$$\dot{p}(t)A_1(t)\psi'' = 0, \tag{13}$$

$$\left[p(t)A_1(t) - 3p^2(t)\right]\psi'' - \left[\dot{q}(t) + p(t)A_0(t) - A_1(t)\right]\psi' = 0, \tag{14}$$

$$\dot{p}(t)A_1(t)\psi'^2 = 0, \tag{15}$$

$$2p^2(t)A_1(t)\psi'3 - p(t)A_1^2(t)\psi'3 = 0. \tag{16}$$

Eqs. (11), (13), (15) and (16) give the results

$$p(t) = k, \qquad A_1(t) = 2k, \qquad A_0(t) = 0, \qquad A_0(t) = 1,$$

where k is a constant of integration. Let us now discuss the following cases.

Case 1. If $A_0(t) = 0$, Eq. (12) and Eq. (14) reduce to

$$k^2\psi''' + \dot{q}(t)\psi'' + \psi' = 0, \tag{17}$$

$$\psi' = \frac{k^2}{2k - \dot{q}(t)}\psi''. \tag{18}$$

Substituting Eq. (18) into Eq. (17), we conclude that

$$\frac{\psi'''}{\psi''} = \frac{2k\dot{q}(t) - \dot{q}^2(t) + k^2}{\dot{q}(t)k^2 - 2k^3}. \tag{19}$$

Integrating Eq. (19) with respect to ξ yields

$$\psi'' = c_1(t)\exp\left[\frac{2k\dot{q}(t) - \dot{q}^2(t) + k^2}{\dot{q}(t)k^2 - 2k^3}\xi\right]. \tag{20}$$

Substituting Eq. (20) into Eq. (18), we conclude that

$$\psi' = \frac{k^2c_1(t)}{2k - \dot{q}(t)}\exp\left[\frac{2k\dot{q}(t) - \dot{q}^2(t) + k^2}{\dot{q}(t)k^2 - 2k^3}\xi\right], \tag{21}$$

and then

$$\psi = c_2(t) + \frac{k^4c_1(t)}{\dot{q}^2(t) - 2k\dot{q}(t) - k^2}\exp\left[\frac{2k\dot{q}(t) - \dot{q}^2(t) + k^2}{\dot{q}(t)k^2 - 2k^3}\xi\right], \tag{22}$$

where $c_1(t), c_2(t)$ and $q(t)$ are arbitrary functions of t. Now the exact solution of Eq. (1) has the form

$$u_1(x,t) = \frac{\frac{2k^3c_1(t)}{2k-\dot{q}(t)}\exp[\frac{2k\dot{q}(t)-\dot{q}^2(t)+k^2}{\dot{q}(t)k^2-2k^3}\xi]}{c_2(t) + \frac{k^4c_1(t)}{\dot{q}^2(t)-2k\dot{q}(t)-k^2}\exp[\frac{2k\dot{q}(t)-\dot{q}^2(t)+k^2}{\dot{q}(t)k^2-2k^3}\xi]}. \tag{23}$$

If we set $c_2(t) = \pm 1, c_1(t) = \frac{\dot{q}^2(t)-2k\dot{q}(t)-k^2}{k^4}$ in Eq. (23), then we have the following solitary-like wave solutions:

$$u_{11}(x,t) = \frac{\dot{q}^2(t) - 2k\dot{q}(t) - k^2}{4k^2 - 2k\dot{q}(t)}\left\{1 + \tanh\left[\frac{2k\dot{q}(t) - \dot{q}^2(t) + k^2}{2\dot{q}(t)k^2 - 4k^3}\bigl(kx + q(t)\bigr)\right]\right\}, \tag{24}$$

$$u_{12}(x,t) = \frac{\dot{q}^2(t) - 2k\dot{q}(t) - k^2}{4k^2 - 2k\dot{q}(t)}\left\{1 + \coth\left[\frac{2k\dot{q}(t) - \dot{q}^2(t) + k^2}{2\dot{q}(t)k^2 - 4k^3}\bigl(kx + q(t)\bigr)\right]\right\}. \tag{25}$$

Case 2. If $A_0(t) = 1$, Eqs. (12) and (14) reduce to

$$\frac{\psi'''}{\psi''} = \frac{\dot{q}^2(t)}{k^3 - k^2\dot{q}(t)}. \tag{26}$$

Similar to case 1, we conclude that

$$\psi = c_2(t) + \frac{k^4c_1(t)}{\dot{q}^2(t)}\exp\left[\frac{\dot{q}^2(t)}{k^3 - k^2\dot{q}(t)}\xi\right], \tag{27}$$

where $c_1(t), c_2(t)$ and $q(t)$ are arbitrary functions of t. Now the exact solution of Eq. (1) has the form

$$u_2(x,t) = 1 + \frac{\frac{2k^3 c_1(t)}{k-\dot{q}(t)} \exp[\frac{\dot{q}^2(t)}{k^3-k^2\dot{q}(t)}\xi]}{c_2(t) + \frac{k^4 c_1(t)}{\dot{q}^2(t)} \exp[\frac{\dot{q}^2(t)}{k^3-k^2\dot{q}(t)}\xi]}. \tag{28}$$

If we set $c_2(t) = \pm 1, c_1(t) = \frac{\dot{q}^2(t)}{k^4}$ in Eq. (28), then we have the following solitary-like wave solutions:

$$u_{21}(x,t) = 1 + \frac{\dot{q}^2(t)}{k^2 - k\dot{q}(t)} \left\{1 + \tanh\left[\frac{\dot{q}^2(t)}{2k^3 - 2k^2\dot{q}(t)}\left(kx + q(t)\right)\right]\right\}, \tag{29}$$

$$u_{22}(x,t) = 1 + \frac{\dot{q}^2(t)}{k^2 - k\dot{q}(t)} \left\{1 + \coth\left[\frac{\dot{q}^2(t)}{2k^3 - 2k^2\dot{q}(t)}\left(kx + q(t)\right)\right]\right\}. \tag{30}$$

3.2 The modified Volterra equations

Now we will study the enhanced modified simple equation method to solve Eq. (1) in the following way.

Using the generalized wave transformation

$$\begin{cases} u(x,t) = u(\xi), & \xi = p(t)x + q(t), \\ v(x,t) = v(\xi), & \xi = p(t)x + q(t), \end{cases} \tag{31}$$

where $p(t)$ and $q(t)$ are differentiable functions of t, we have the following ODE:

$$\begin{cases} [\dot{p}(t)x + \dot{q}(t) + \alpha p(t)]u' - u + uv = 0, \\ [\dot{p}(t)x + \dot{q}(t) + \beta p(t)]v' + v - uv = 0. \end{cases} \tag{32}$$

Balancing the highest order derivative u', v', and the nonlinear term of the highest order uv, yields $N = 1$. Through the enhanced modified simple equation method, for $N = 1$ Eq. (32) become

$$\begin{cases} u(\xi) = A_0(t) + A_1(t)[\frac{\psi'(\xi)}{\psi(\xi)}], \\ v(\xi) = B_0(t) + B_1(t)[\frac{\psi'(\xi)}{\psi(\xi)}], \end{cases} \tag{33}$$

where $A_0(t), B_0(t), A_1(t)$ and $B_1(t)$ are functions of t to be determined later such that $A_1(t) \neq 0$ and $B_1(t) \neq 0$. It is easy to see that

$$\begin{cases} u' = A_1(t)(\frac{\psi''}{\psi} - \frac{\psi'^2}{\psi^2}), \\ v' = B_1(t)(\frac{\psi''}{\psi} - \frac{\psi'^2}{\psi^2}). \end{cases} \tag{34}$$

Substituting Eqs. (33)-(34) into (32) and equating all the coefficients of ψ^0, ψ^{-1}, $\psi^{-1}x$, ψ^{-2}, and $\psi^{-2}x$ to zero yield a set of over-determined differential equations with respect to $\{\xi, A_0(t), A_1(t), B_0(t), B_1(t), p(t), q(t)\}$. Solving these over-determined differential equations, we obtain the following results: $p(t) = k$, $A_0(t) = B_0(t) = 0$ or $A_0(t) = B_0(t) = 1$, $A_1(t) = -\dot{q}(t) - \beta k$, $B_1(t) = \dot{q}(t) + \alpha k$, $q(t)$ are arbitrary functions of t.

Case 1. If $A_0(t) = B_0(t) = 0$, therefore, the general form of solutions of Eqs. (2) can be expressed by

$$\begin{cases} u_1(x,t) = [-\dot{q}(t) - \beta k]c_1(t)\frac{\exp[\frac{1}{\dot{q}(t)+\alpha k}\xi]}{c_2(t)+c_1(t)[\dot{q}(t)+\alpha k]\exp[\frac{1}{\dot{q}(t)+\alpha k}\xi]}, \\ v_1(x,t) = [\dot{q}(t) + \alpha k]c_3(t)\frac{\exp[\frac{-1}{\dot{q}(t)+\beta k}\xi]}{c_4(t)-c_3(t)[\dot{q}(t)+\beta k]\exp[\frac{-1}{\dot{q}(t)+\beta k}\xi]}. \end{cases} \tag{35}$$

If we set $c_2(t) = c_4(t) = \pm 1$, $c_1(t) = \frac{1}{\dot{q}(t)+\alpha k}$ and $c_3(t) = \frac{-1}{\dot{q}(t)+\beta k}$ in Eqs. (35), then we have the following solitary-like wave solutions:

$$\begin{cases} u_{11}(x,t) = -\frac{\dot{q}(t)+\beta k}{2[\dot{q}(t)+\alpha k]}\{1 + \tanh[\frac{1}{2[\dot{q}(t)+\alpha k]}(kx + q(t))]\}, \\ v_{11}(x,t) = -\frac{\dot{q}(t)+\alpha k}{2[\dot{q}(t)+\beta k]}\{1 + \tanh[\frac{-1}{2[\dot{q}(t)+\beta k]}(kx + q(t))]\}, \end{cases} \tag{36}$$

and

$$\begin{cases} u_{12}(x,t) = -\frac{\dot{q}(t)+\beta k}{2[\dot{q}(t)+\alpha k]}\{1 + \coth[\frac{1}{2[\dot{q}(t)+\alpha k]}(kx + q(t))]\}, \\ v_{12}(x,t) = -\frac{\dot{q}(t)+\alpha k}{2[\dot{q}(t)+\beta k]}\{1 + \coth[\frac{-1}{2[\dot{q}(t)+\beta k]}(kx + q(t))]\}. \end{cases} \tag{37}$$

Case 2. If $A_0(t) = B_0(t) = 1$, the exact solutions of Eqs. (2) can be expressed by

$$\begin{cases} u_2(x,t) = 1 - [\dot{q}(t) + \beta k]c_1(t)\frac{\exp[\frac{1}{\dot{q}(t)+\beta k}\xi]}{c_2(t)+c_1(t)[\dot{q}(t)+\beta k]\exp[\frac{1}{\dot{q}(t)+\beta k}\xi]}, \\ v_2(x,t) = 1 + [\dot{q}(t) + \alpha k]c_3(t)\frac{\exp[\frac{-1}{\dot{q}(t)+\alpha k}\xi]}{c_4(t)-c_3(t)[\dot{q}(t)+\alpha k]\exp[\frac{-1}{\dot{q}(t)+\alpha k}\xi]}. \end{cases} \tag{38}$$

If we set $c_2(t) = c_4(t) = \pm 1$, $c_1(t) = \frac{1}{\dot{q}(t)+\beta k}$ and $c_3(t) = \frac{-1}{\dot{q}(t)+\alpha k}$ in Eqs. (38), then we have the following solitary-like wave solutions:

$$\begin{cases} u_{21}(x,t) = \frac{1}{2}\{1 - \tanh[\frac{1}{2[\dot{q}(t)+\beta k]}(kx + q(t))]\}, \\ v_{21}(x,t) = \frac{1}{2}\{1 - \tanh[\frac{-1}{2[\dot{q}(t)+\alpha k]}(kx + q(t))]\}, \end{cases} \tag{39}$$

and

$$\begin{cases} u_{22}(x,t) = \frac{1}{2}\{1 - \coth[\frac{1}{2[\dot{q}(t)+\beta k]}(kx + q(t))]\}, \\ v_{22}(x,t) = \frac{1}{2}\{1 - \coth[\frac{-1}{2[\dot{q}(t)+\alpha k]}(kx + q(t))]\}. \end{cases} \tag{40}$$

Remark Solutions (24), (25), (29), (30), (36), (37), (39), (40) can be calculated and checked by hand. And if $q(t)$ is a linear function, the solitary wave solutions are derived from Eqs. (24), (25), (29), (30), (36), (37), (39) and (40). When $q(t)$ is not a linear function, we can find a rich variety of exact and fresh solutions of the Burger-Fisher and the modified Volterra equations.

4 Discussions

It should be mentioned that, to the authors' knowledge, this is the first attempt to solve the Burger-Fisher and the modified Volterra equations with the enhanced modified simple equation method. The advantages of the method over the modified simple equation (MSE) is that the method provides more general and larger amount of new exact traveling wave solutions with several free parameters. Besides, by means of the enhanced modified simple equation method, the exact solutions to these equations have been gained in this article without using the symbolic computing software, such as Mathematica and Maple, since the computations are very simple.

5 Conclusions

In this letter, we considered the Burger-Fisher and the modified Volterra equations. We put forth the enhanced modified simple equation method for the solitary-like wave solutions and the solitary solutions of these nonlinear equations. This study reveals that the enhanced modified simple equation method is quite efficient and practically well suited to be used in finding exact solutions of NLEEs. Also, we observe that the promising and powerful method can be applied to many other NLEEs in mathematical physics.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
Both the authors, ZC and ZZ, with the consultation of each other carried out this work and drafted the manuscript together. Both the authors approved the final manuscript.

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