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A fractional order model for obesity epidemic in a non-constant population

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Abstract

In this paper, we propose a fractional order epidemic model for obesity contagion. The population size is assumed to be nonconstant, which is more realistic. The model considers vertical transmission of obesity and also obesity-related death rate. We give local stability analysis of the model. Finally, some numerical examples are presented.

MSC: 34A08; 92B99; 34D20; 37N25

Keywords: fractional differential equations; epidemic model; stability analysis; obesity

1 Introduction

Overweight and obesity are defined as abnormal and excessive fat accumulation that presents a risk to health [1]. Obesity is measured by using a number called body mass index (BMI) which is calculated by using the formula

$$BMI = \frac{W}{H^2},$$

where W is the weight of a person in kilograms and H is the height of a person in meters. If $25 \leq BMI < 30$, then the person is said to be overweight, and if $BMI \geq 30$, then the person is said to be obese.

Obesity is one of the major risk factors for many chronic, fatal diseases including cancer, diabetes mellitus and cardiovascular disorders. According to the World Health Organization, worldwide obesity has doubled since 1980, and in 2008, 11% of adults aged 20 and over were obese [1].

Although there are some other reasons (e.g. genetic reasons, endocrine disorders), the main reason for obesity is excessive food intake and lack of physical activity. These reasons are closely related to the life-styles of the individuals within a population. Therefore, obesity can be considered as socially contagious. In [2] a detailed analysis of the obesity epidemic in the U.S. is given. Santonja et al. [3] and Ejima et al. [4] also considered obesity as an epidemic disease and gave mathematical models to explain the spread of obesity. In both of these models integer order differential equations are used and the total population size is assumed to be constant.

The epidemic models where the total population is assumed to be constant are classical models given for short-term epidemics. When the epidemic disease arises and vanishes in a short time like influenza, this kind of models give realistic results. But in the case of long-term effective diseases like hepatitis, rabies, rubella and so on, limiting the population to be constant would be a very strong assumption that affects the realism of the model. In this paper, we propose a new mathematical model in which we assume that the population is nonconstant. What is more, with a particular choice of the natural death rate function in the model, it gives a classical logistic growth for the population.

We also consider the memory dependence on the obesity contagion. The memory effect in the spread of obesity is discussed in detail in [5]. In recent years, it has frequently been observed in modeling memory-dependent processes of physical and life sciences that models based on fractional order derivatives provide better agreement between solutions and real data [6–8]. Therefore, it is reasonable to use fractional order models to understand the spread of obesity in a population. Also note that the fractional order model we give is a generalization of an integer order model, and if the order of the fractional model is one, it reduces to its integer order counterpart.

In this paper, we consider continuous fractional differential equation systems to understand the spread of obesity in a population. Discrete fractional systems are also being used to model some real life problems, and their stability results are given in recent years [9, 10]. Stability of the proposed model in this paper is examined using the method given by Matignon [11]. Some other stability results can be found in [12–15].

This paper is organized as follows. Section 2 is devoted to the model construction. In this section we propose a new fractional order epidemic model including disease-dependent death rate within a nonconstant population size. We also consider the tendency to obesity at birth as a result of bad nutritional habits during pregnancy, by means of vertical transmission. In Section 3, a detailed local stability analysis for the model is given. Finally, in Section 4, we give some numerical examples to illustrate our results.

2 Mathematical model

We first give some basic definitions of fractional calculus.

Definition 1 The fractional integral of order $\mu > 0$ of a function $f : R^+ \to R$ is defined by

$$I^\mu f(t) = \frac{1}{\Gamma(\mu)} \int_0^t (t-\tau)^{\mu-1} f(\tau)\, d\tau.$$

Here and elsewhere Γ denotes the gamma function.

Fractional derivative has several different definitions [16]. In this paper we use the Caputo definition due to its advantages in applied problems. The Caputo definition of fractional derivative allows us to use initial conditions of the classical form, avoiding solvability problems.

Definition 2 Caputo fractional derivative of order $\mu > 0$ of the function $f : R^+ \to R$ is defined by (if exists)

$$D^\mu f(t) = I^{n-\mu} D^n f(t),$$

where n is the integer part of μ and $D = \frac{d}{dt}$.

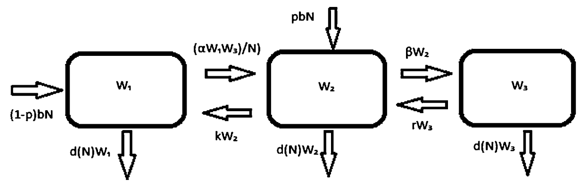

Figure 1 The schematic diagram of the model.

In our model, we assume that the total population $N(t)$ is partitioned into three classes $W_1(t)$, $W_2(t)$, $W_3(t)$ denoting normal weight, overweight and obese individuals at time t, respectively. For the model we study in this paper, we consider both horizontal and vertical transmission of the disease. We treat excess weight gain as an epidemic disease that spreads via social contact. We also take into account the fact that overweight babies may be born because of bad nutritional habits during the pregnancy periods. The parameters we use in our model are as follows:

p: probability of having an overweight baby;
b: natural birth rate;
$d(\cdot)$: natural death rate function (dependent on the total population);
α: transmission rate of the disease by social contact;
β: rate at which an overweight individual moves to the obese class;
k, r: treatment rates for overweight and obese individual, respectively;
θ: obesity-related (from the diseases that are caused by obesity) death rate.

We assume that b, α, β, k, r and θ are all nonnegative constants. Let the natural death rate function be a continuous and nondecreasing function of N. Also assume that there exists a positive constant M such that $d(M) = b$.

The schematic diagram of our model can be seen in Figure 1. The system of fractional order nonlinear ordinary differential equations for the proposed model is given by:

$$\begin{aligned} D^\mu W_1 &= (1-p)bN + kW_2 - \frac{\alpha W_1 W_3}{N} - d(N)W_1, \\ D^\mu E &= pbN + \frac{\alpha W_1 W_3}{N} + rW_3 - W_2\bigl(\beta + k + d(N)\bigr), \\ D^\mu I &= \beta W_2 - \bigl(r + d(N) + \theta\bigr) W_3, \end{aligned} \tag{1}$$

where $\mu \in (0,1]$ and $N = W_1 + W_2 + W_3$, $(W_1, W_2, W_3) \in R^3_+$. The fractional order model Equation (1) is in fact obtained by modifying the classical integer order model. Equation (1) includes a free parameter μ that may help the theoretical formulation of the solution fit better with the real data [17–19]. To the best of our knowledge, this model (also the integer order counterpart) is a new model for the obesity contagion.

Adding up the equations given in (1), we have

$$D^\mu N = \bigl(b - d(N)\bigr)N - \theta W_3. \tag{2}$$

We should note that for the disease-free case (i.e. $W_3 = 0$), if d is a linear function, then the total population has logistic growth.

Theorem 3 *There is a unique solution for the initial value problem given by* (1) *and the initial conditions*

$$W_1(0) = W_{10}, \qquad W_2(0) = W_{20}, \qquad W_3(0) = W_{30}, \tag{3}$$

and the solution remains in R^3_+.

Proof It is easy to see the existence and uniqueness of the solution of the initial value problem (1)-(3) in $(0,\infty)$. We will show that the domain R^3_+ remains positively invariant.

Since

$$\begin{aligned} D^\mu W_1|_{W_1=0} &= (1-p)b(W_2+W_3) + kW_2 \geq 0, \\ D^\mu W_2|_{W_2=0} &= pb(W_1+W_2) + \frac{\alpha W_1 W_3}{W_1+W_3} + rW_3 \geq 0, \\ D^\mu W_3|_{W_3=0} &= \beta W_2 \geq 0, \end{aligned}$$

on each hyperplane bounding the nonnegative orthant, the vector field points into R^3_+. □

3 Equilibrium points and stability

For simplicity in calculations, we will consider the system

$$\begin{aligned} D^\mu W_1 &= (1-p)bN + k(N - W_1 - W_3) - \frac{\alpha W_1 W_3}{N} - d(N)W_1, \\ D^\mu W_3 &= \beta(N - W_1 - W_3) - \big(r + d(N) + \theta\big) W_3, \\ D^\mu N &= \big(b - d(N)\big)N - \theta W_3 \end{aligned} \tag{4}$$

with the initial conditions

$$W_1(0) = W_{10}, \qquad W_3(0) = W_{30}, \qquad N(0) = N_0,$$

where $0 < \mu < 1$. The disease-free equilibrium (DFE) $F_0 = (M,0,M)$ of system (4) exists only if $p = 0$. The basic reproductive number of the given system is

$$R_0 = \frac{k + \alpha - \theta}{(k+b)(\beta + r + b + \theta)}. \tag{5}$$

Theorem 4 *The DFE of system* (4) (*if exists*) *is asymptotically stable if* $R_0 < 1$.

Proof The characteristic equation of system (4) is

$$f(\lambda) = \big(d'(M)M + \lambda\big)\big(A\lambda^2 + B\lambda + C\big) = 0, \tag{6}$$

where

$$\begin{aligned} A &= -1, \\ B &= -(k + 2b + \beta + r + \theta), \end{aligned}$$

$$C = \beta(\alpha + k - \theta) - (k + b)(\beta + r + b + \theta).$$

The DFE of system (4) is asymptotically stable if all of the roots of the characteristic equation (6) λ_i, $i = 1, 2, 3$, satisfy the following condition [11, 20]:

$$|\operatorname{Arg}\lambda_i| > \mu\frac{\pi}{2}. \tag{7}$$

From (6),

$$\lambda_1 = -d'(M)M < 0$$

and

$$\lambda_{2,3} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}. \tag{8}$$

It is easy to show that if $\frac{k+\alpha-\theta}{(k+b)(\beta+r+b+\theta)} < 1$, then the eigenvalues given with (8) satisfy condition (7). □

We now discuss the existence and stability of positive equilibrium. Positive equilibrium of system (4) is $F_1 = (W_1^*, W_3^*, N^*)$, where

$$W_1^* = N^*\left[1 - \frac{(\beta + r + d(N^*) + \theta)(b - d(N^*))}{\beta\theta}\right],$$
$$W_3^* = \frac{(b - d(N^*))N^*}{\theta}$$

and $d(N^*)$ is the positive root of the polynomial

$$\tilde{A}d^3(N^*) + \tilde{B}d^2(N^*) + \tilde{C}d(N^*) + \tilde{D} \tag{9}$$

with

$$\tilde{A} = \alpha - \theta,$$
$$\tilde{B} = \theta(b - \theta - \beta - k - r) + \alpha(\beta + r + \theta - 2b),$$
$$\tilde{C} = \theta^2(b - \beta - k) + \alpha b\bigl(b - 2(\beta + r + \theta)\bigr) + \theta\bigl(\beta(\alpha + b) + k(b - r) + br\bigr),$$
$$\tilde{D} = b\theta^2(k + \beta - p\beta) + \alpha b^2(\beta + r + \theta) + \theta b(kr - \alpha\beta).$$

We shall note that if $\tilde{A} \times \tilde{D}$ is negative, then (9) has at least one positive root. Also if $b > \theta$ then for

$$0 < d(N^*) < A_1 \quad \text{and} \quad d(N^*) > A_2,$$

there exists positive W_1^*, where

$$A_{1,2} = \frac{-(X_1 + X_2) \pm \sqrt{(X_1 + X_2)^2 - 4(X_2 + X_3)}}{2},$$

$$X_1 = r + \theta + \beta,$$
$$X_2 = -b,$$
$$X_3 = \theta\beta.$$

The Jacobian matrix $J(F_1)$ evaluated at the endemic equilibrium is given by

$$J(F_1) = \begin{pmatrix} -k - \frac{\alpha B_1}{\theta} - d(N^*) & -k - 1 + \frac{B_1 B_2}{\beta\theta} & (1-p)b + k + (\frac{\alpha B_1}{\theta} - N^*)(1 - \frac{B_1 B_2}{\beta\theta}) \\ -\beta & -B_2 & \beta - \frac{B_1 B_3}{\theta} \\ 0 & -\theta & B_1 - B_3 \end{pmatrix},$$

where

$$B_1 = b - d(N^*),$$
$$B_2 = \beta + r + d(N^*) + \theta,$$
$$B_3 = N^* d'(N^*).$$

Then the characteristic equation of the linearized system is in the form

$$a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0 \tag{10}$$

with

$$\begin{aligned} a_1 &= -1, \\ a_2 &= -\left(\left(\frac{\alpha}{\theta} - 1\right)B_1 + k + d(N^*) + B_2 + B_3\right), \\ a_3 &= \left(B_2 + \frac{\alpha}{\theta}B_1\right)(B_1 - B_3) + \left(d(N^*) + k\right)(B_1 - B_2 - B_3) \\ &\quad + B_1 B_3 + \beta(1 + k - \theta) + \frac{B_1 B_2}{\theta}(1 + \alpha), \\ a_4 &= (B_1\theta + B_3\beta)\left(1 + k - \frac{B_1 B_2}{\beta\theta}\right) + \frac{\alpha B_1}{\theta}(B_1 B_2 - B_2 B_3 + B_1 B_3) \\ &\quad + \theta\beta\left[B_1 - N^* - pb - \frac{B_1}{\theta}\left(\frac{\alpha B_1 B_2}{\beta\theta} - \alpha - \frac{B_2 N^*}{\beta}\right)\right]. \end{aligned} \tag{11}$$

Corollary 5 *Let* a_4 *be as given in* (11). *If* $a_4 > 0$, *then the positive equilibrium point of system* (4) *is unstable.*

Proof Using Descartes' rule of signs, it is clear that if $a_4 > 0$ then the characteristic equation (10) has at least one positive root. So, the positive equilibrium F_1 of system (4) is unstable. □

4 Numerical results

In this section we consider four sets of parameters to discuss different cases. We use the solution technique given in [21] to evaluate the numerical solutions of the system for μ values $1, 0.8$ and 0.6. Two main theorems about this technique are given below.

Theorem 6 ([21]) *Let* $\|\cdot\|$ *denote any convenient norm on* R^n. *Assume that* $f \in C[R_1, R^n]$, *where* $R_1 = [(t,X) : 0 \leq t \leq a$ *and* $\|X - X_0\| \leq b]$, $f = (f_1, f_2, \ldots, f_n)^T$, $X = (x_1, x_2, \ldots, x_n)^T$, *and let* $\|f(t,X)\| \leq S$ *on* R_1. *Then there exists at least one solution for the system of FDEs given by*

$$D^\alpha X(t) = f\bigl(t, X(t)\bigr) \tag{12}$$

with the initial conditions

$$X(0) = X_0 \tag{13}$$

on $0 \leq t \leq \beta$, *where* $\beta = \min(a, [\frac{b}{S}\Gamma(\alpha+1)]^{\frac{1}{\alpha}})$, $0 < \alpha < 1$.

Theorem 7 ([21]) *Consider the initial value problem given by* (12) *and* (13) *of order* α, $0 < \alpha < 1$. *Let*

$$g\bigl(\nu, X_*(\nu)\bigr) = f\bigl(t - \bigl(t^\alpha - \nu\Gamma(\alpha+1)\bigr)^{1/\alpha}, X\bigl(t - \bigl(t^\alpha - \nu\Gamma(\alpha+1)\bigr)^{1/\alpha}\bigr)\bigr)$$

and assume that the conditions of Theorem 3 *hold. Then a solution of* (12) *and* (13) *can be given by*

$$X(t) = X_*\bigl(t^\alpha / \Gamma(\alpha+1)\bigr),$$

where $X_*(\nu)$ *is a solution of the system of integer order differential equations*

$$\frac{d(X_*(\nu))}{d\nu} = g\bigl(\nu, X_*(\nu)\bigr)$$

with the initial conditions

$$X_*(0) = X_0.$$

We use the corresponding parameter values given in Table 1 for each case. Population data for Turkey are used in numerical simulations. Some of the values given in Table 1 are taken from real data. A linear, population-dependent death rate function is evaluated

Table 1 Parameter values for Case 1-4

	Case 1	Case 2	Case 3	Case 4
p =	0.2	0.2	0.2	0.0
[22] b =	0.017	0.017	0.017	0.017
M =	243.926	243.926	243.926	243.926
α =	0.3	0.3	0.3	0.01
β =	0.2	0.2	0.2	0.2
k =	0.005	0.05	0.1	0.008
r =	0.002	0.02	0.2	0.2
[23] θ =	0.009	0.009	0.009	0.009
$N(0)$ =	76	76	76	76
[24] $S(0)$ =	26.676	26.676	26.676	26.676
[24] $I(0)$ =	22.8	22.8	22.8	22.8

using the data in [25] as $d(N) = (1.72385 + 0.0626262N)10^{-4}$. M value is calculated using the function d and the value b.

This linear death rate function lets the total population N have logistic growth. For the exact solution of a fractional logistic equation, [26] used the Carleman embedding technique, but there is a controversy between the results of West and Area et al. [27]. However, in this paper we use a totally different technique [21] to find the solution of the system including the logistic equation.

Case 1: System (4) has a positive fixed point $F^*_{1,1} = (6.31442, 103.487, 120.283)$. The roots of the characteristic polynomial of the given system at $F^*_{1,1}$ are

$$\lambda_1 = -0.439302,$$
$$\lambda_2 = -0.0265552 + 0.161532i,$$
$$\lambda_3 = -0.0265552 - 0.161532i.$$

For all $\mu \in (0,1]$, $F^*_{1,1}$ is asymptotically stable.

Case 2: In Case 2, we use the same parameter values with the previous case except for k and r which are the control parameters. These parameters can be adjusted according to the disease prevention and control strategies. Using the new k and r parameter values, the positive fixed point of system (4) can be evaluated as $F^*_{1,2} = (11.8416, 102.443, 134.33)$. As k and r increase, I^* decreases. The roots of the characteristic polynomial of the given system at $F^*_{1,2}$ are

$$\lambda_1 = -0.524651,$$
$$\lambda_2 = -0.00247813 - 0.203507i,$$
$$\lambda_3 = -0.00247813 - 0.203507i.$$

So, even for this case, the positive equilibrium point is asymptotically stable for all $\mu \in (0,1]$.

Case 3: In this case, we use greater k and r values compared to the previous cases. The positive fixed point in this case is $F^*_{1,3} = (123.259, 62.5809, 256.809)$. The roots of the characteristic polynomial at $F^*_{1,3}$ are

$$\lambda_1 = -0.919697,$$
$$\lambda_2 = 0.144044 + 0.469084i,$$
$$\lambda_3 = 0.144044 - 0.469084i.$$

For $\mu = 1$, $F^*_{1,3}$ is unstable. Using (7), it can be shown that for μ value less than 0.810329, $F^*_{1,3}$ is asymptotically stable.

Case 4: We now consider the case where vertical transmission of the disease does not exist, i.e. $p = 0$. The value of p is also related to the disease control and prevention strategies. For this case, the DFE exists. For $F^*_0 = (243.926, 0, 243.926)$, the basic reproductive number $R_0 = 0.84507$, which states that F^*_0 is asymptotically stable for all $\mu \in (0,1]$.

The graphics of the solutions for each case can be seen in Figures 2-13.

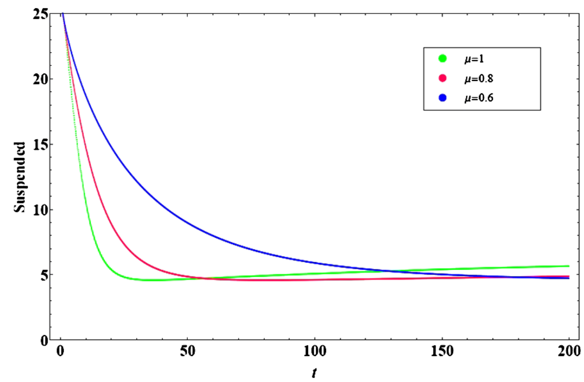

Figure 2 Normal-weight class for Case 1.

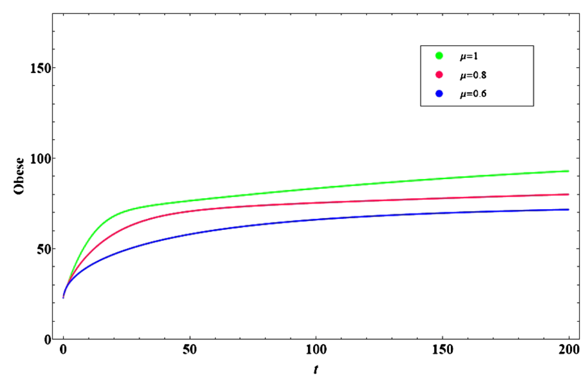

Figure 3 Obese class for Case 1.

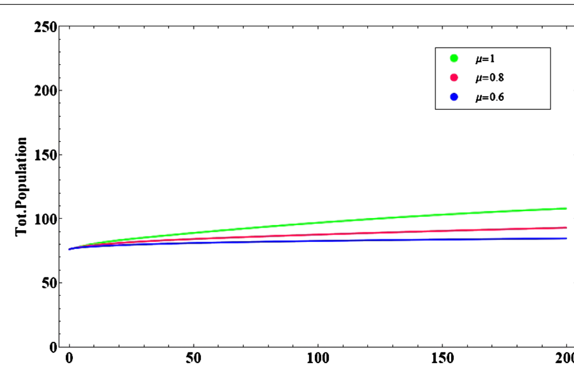

Figure 4 Total population for Case 1.

5 Conclusions

In this paper, a fractional order mathematical model of obesity epidemic, including vertical transmission within a nonconstant population size, is proposed. The order of the proposed system is a free parameter which can be used to have a better fit between the real data and a theoretical formulation of the solutions. We should also note that fractional order models give more realistic predictions in modeling procedures with short memory effect [16, 19].

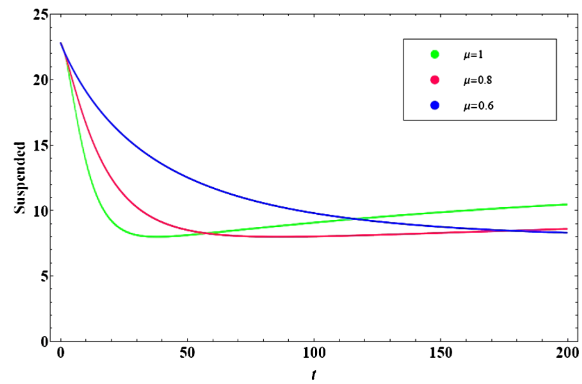

Figure 5 Normal-weight class for Case 2.

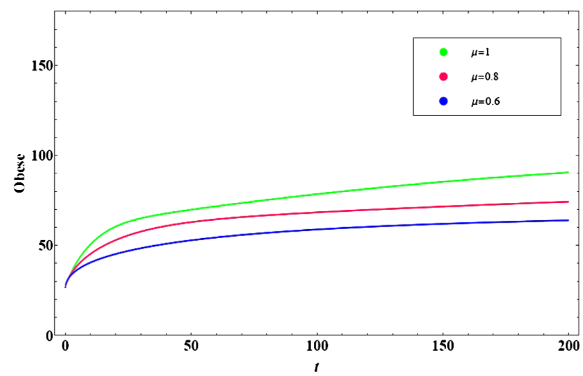

Figure 6 Obese class for Case 2.

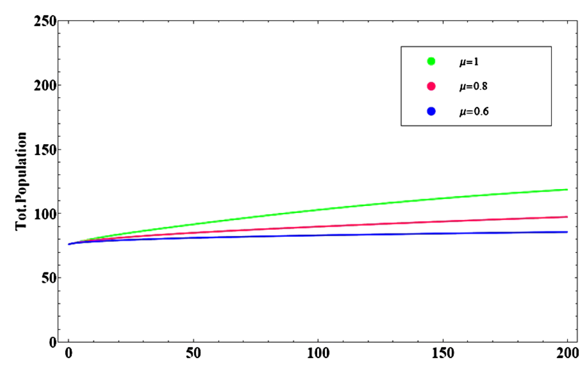

Figure 7 Total population for Case 2.

Since epidemic dynamics of obesity can be considered as a memory-dependent process, fractional order systems may be good tools for modeling the contagion of obesity. Obesity is one of the major health problems all over the world. Because of the economic impact of obesity-related diseases, the dynamics of obesity epidemic is important for countries. In the United States, nationwide excess medical costs for obesity is as much as $147 billion annually for adults and $14.3 billion annually for children [28]. Due to these economic

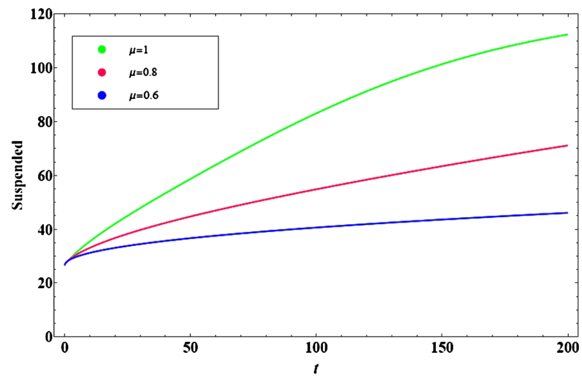

Figure 8 Normal-weight class for Case 3.

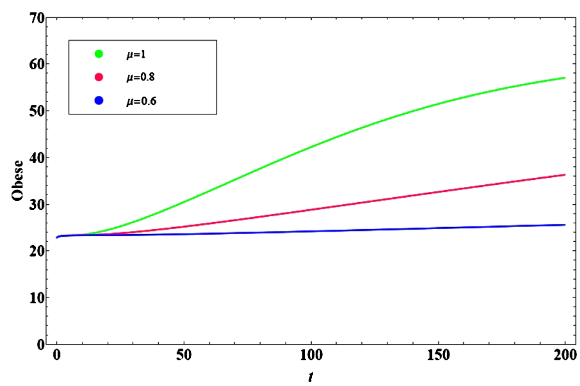

Figure 9 Obese class for Case 3.

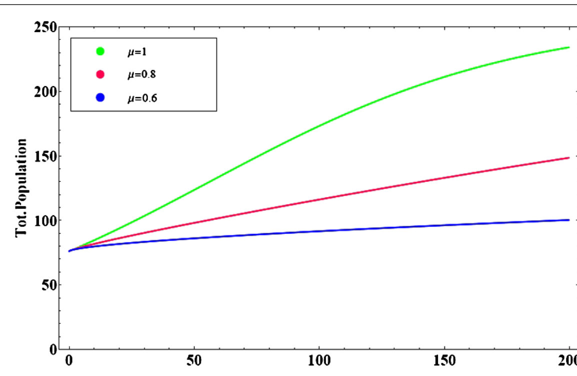

Figure 10 Total population for Case 3.

reasons, disease control is very important for countries. The parameters k and r in our model are control parameters for disease control. In the final part of the paper, we simulated the system for different parameter values given in Table 1. We should point out that by adjusting the control parameters r and k, disease can be kept under control.

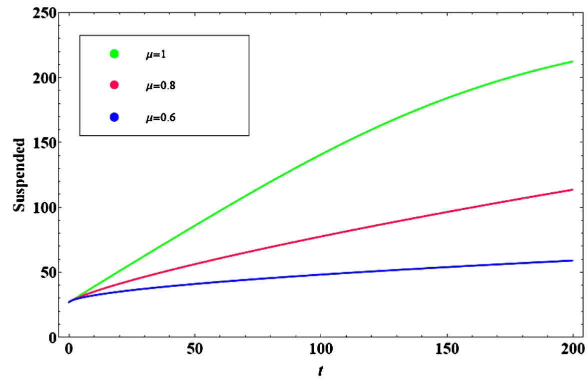

Figure 11 Normal-weight class for Case 4.

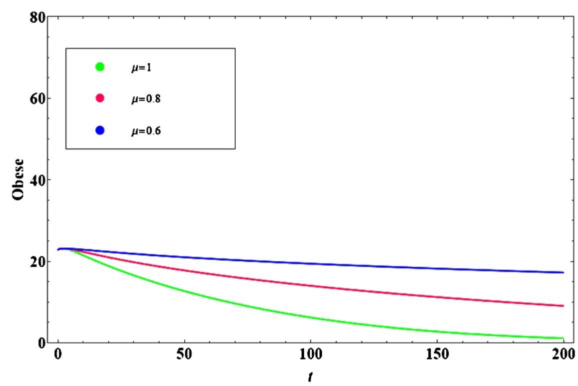

Figure 12 Obese class for Case 4.

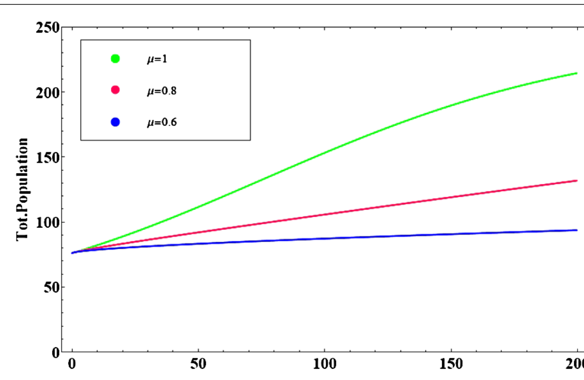

Figure 13 Total population for Case 4.

Competing interests
The author declares that she has no competing interests.

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References

1. http://www.who.int/topics/obesity/en/, 07.23.2014
2. Wang, Y, Beydoun, MA, Liang, L, Caballero, B, Kumanyika, SK: Will all americans become overweight or obese? Estimating the progression and cost of the US obesity epidemic. Obesity **16**, 2323-2330 (2008). doi:10.1038/oby.2008.351
3. Santonja, F-J, Villanueva, R-L, Jodar, L, Gonzalez-Parra, G: Mathematical modelling of social obesity epidemic in the region of Valencia, Spain. Math. Comput. Model. Dyn. Syst. **16**(1), 23-34 (2010). doi:10.1080/13873951003590149
4. Ejima, K, Aihara, K, Nishiura, H: Modeling the obesity epidemic: social contagion and its implications for control. Theor. Biol. Med. Model. **10**, 17 (2013)
5. Barry, D, Clarke, M, Petry, NM: Obesity and its relationship to addictions: is overeating a form of addictive behavior? Am. J. Addict. **18**(6), 439-451 (2009)
6. Baleanu, D, Diethelm, K, Scalas, E, Trujillo, JJ: Fractional Calculus: Models and Numerical Methods. World Scientific, Singapore (2012)
7. Diethelm, K: A fractional calculus based model for the simulation of an outbreak of dengue fever. Nonlinear Dyn. **71**(4), 613-619 (2013)
8. Arshad, S, Baleanu, D, Huang, J, Tang, Y, Al Qurashi, MM: Dynamical analysis of fractional order model of immunogenic tumors. Adv. Mech. Eng. **8**(7), 1-13 (2016)
9. Wu, GC, Baleanu, D: Chaos synchronization of the discrete fractional logistic map. Signal Process. **102**, 96-99 (2014)
10. Chen, FL: A review of existence and stability results for discrete fractional equations. J. Comput. Complex. Appl. **1**(1), 22-53 (2015)
11. Matignon, D: Stability results for fractional differential equations with applications to control processing. In: Computational Engineering in Systems Applications, vol. 2, pp. 963-968 (1996)
12. Li, Y, Chen, YQ, Podlubny, I: Stability of fractional-order nonlinear dynamic systems: Lyapunov direct method and generalized Mittag-Leffler stability. Comput. Math. Appl. **59**, 1810-1821 (2010)
13. Aguila-Camacho, N, Duarte-Mermoud, MA, Gallegos, JA: Lyapunov functions for fractional order systems. Commun. Nonlinear Sci. Numer. Simul. **19**, 2951-2957 (2014)
14. Wu, GC, Baleanu, D, Xie, HP, Chen, FL: Chaos synchronization of fractional chaotic maps based on stability results. Physica A **460**, 374-383 (2016)
15. Hu, JB, Zhao, LD: Stabilization and synchronization of fractional chaotic systems with delay via impulsive control. J. Comput. Complex. Appl. **2**, 103-111 (2016)
16. Podlubny, I: Fractional Differential Equations. Academic Press, New York (1999)
17. Demirci, E, Unal, A, Ozalp, N: A fractional order SEIR model with density dependent death rate. Hacet. J. Math. Stat. **40**(2), 287-295 (2011)
18. Ozalp, N, Demirci, E: A fractional order SEIR model with vertical transmission. Math. Comput. Model. **54**(1), 1-6 (2011)
19. Pinto, AMC, Machado, AT: Fractional dynamics of computer virus propagation. Math. Probl. Eng. **2014**, Article ID 476502 (2014)
20. Ahmed, E, El-Sayed, AMA, El-Saka, HAA: Equilibrium points, stability and numerical solutions of fractional-order predator-prey and rabies models. J. Math. Anal. Appl. **332**, 709-726 (2007)
21. Demirci, E, Ozalp, N: A method for solving differential equations of fractional order. J. Comput. Appl. Math. **236**(11), 2754-2762 (2012)
22. data.worldbank.org/indicator/SP.DYN.CBRT.IN
23. wvdhhr.org/bph/oehp/obesity/mortality.htm
24. beslenme.gov.tr/index.php?page=40
25. https://www.cia.gov/library/publications/the-world-factbook/
26. West, BJ: Exact solution to fractional logistic equation. Phys. A, Stat. Mech. Appl. **429**, 103-108 (2015)
27. Area, I, Losada, J, Nieto, JJ: A note on fractional logistic equation. Phys. A, Stat. Mech. Appl. **444**, 182-187 (2016)
28. Hammond, RA, Levine, R: The economic impact of obesity in the United States. Diabetes Metab. Syndr. Obes. **3**, 285-295 (2010)