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RESEARCH

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Some new and explicit identities related with the Appell-type degenerate q-Changhee polynomials

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Abstract

Recently, Kim, Kwon, and Seo (J. Nonlinear Sci. Appl. 9:2380-2392, 2016) studied the degenerate q-Changhee polynomials and numbers. In this paper, we consider the Appell-type degenerate q-Changhee polynomials and give some new and explicit identities related to these polynomials

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1 Introduction

Let p be a fixed odd prime number. In this paper, we denote the ring of p-adic integers and the field of p-adic numbers by $\mathbb{Z}_p$ and $\mathbb{Q}_p$, respectively. The p-adic norm $|\cdot|_p$ is normalized as $|p|_p = \frac{1}{p}$. Let q be an indeterminate with $|1-q|_p < p^{-\frac{1}{p-1}}$. We recall that $\mathrm{UD}(\mathbb{Z}_p)$ is the set of uniformly differentiable functions on $\mathbb{Z}_p$. For each $f \in \mathrm{UD}(\mathbb{Z}_p)$, the p-adic q-Volkenborn integral on $\mathbb{Z}_p$ is defined by Kim to be

$$I_{-q}(f) = \int_{\mathbb{Z}_p} f(x)\,d\mu_{-q}(x) = \lim_{N\to\infty} \frac{1}{[p^N]_{-q}} \sum_{x=0}^{p^N-1} f(x)q^x(-1)^x, \tag{1.1}$$

where $[x]_q = \frac{1-q^x}{1-q}$ (see [1–4]). From (1.1), we have

$$q^n I_{-q}(f_n) + (-1)^{n-1} I_{-q}(f) = [2]_q \sum_{i=0}^{n-1} (-1)^{n-1-i} q^i f(i) \tag{1.2}$$

(see [5–7]). Kwon-Kim-Seo [8] derived some identities of the degenerate Changhee polynomials which are given by the generating function

$$\frac{2\lambda}{2\lambda + \log(1+\lambda t)} \bigl(1 + \log(1+\lambda t)^{\frac{1}{\lambda}}\bigr)^x = \sum_{n=0}^{\infty} \mathrm{Ch}_{n,\lambda}(x) \frac{t^n}{n!} \tag{1.3}$$

(see [1, 3, 4, 7–14]). We note that if $x = 0$, then $\mathrm{Ch}_{n,\lambda} = \mathrm{Ch}_{n,\lambda}(0)$ are called the degenerate Changhee numbers. From (1.3), we note that

$$\lim_{\lambda\to 0} \mathrm{Ch}_{n,\lambda}(x) = \mathrm{Ch}_n(x) \quad (n \geq 0).$$

We recall that the gamma and beta functions are defined by the following definite integrals: for $\alpha > 0$, $\beta > 0$,

$$\Gamma(\alpha) = \int_0^\infty e^{-t} t^{\alpha-1}\, dt \tag{1.4}$$

and

$$\begin{aligned} B(\alpha,\beta) &= \int_0^1 t^{\alpha-1}(1-t)^{\beta-1}\, dt \\ &= \int_0^\infty \frac{t^{\alpha-1}}{(1+t)^{\alpha+\beta}}\, dt \end{aligned} \tag{1.5}$$

(see [5, 15, 16]). From (1.4) and (1.5), we show that

$$\Gamma(\alpha+1) = \alpha\Gamma(\alpha), \qquad B(\alpha,\beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}. \tag{1.6}$$

The Bell polynomials are defined by the generating function

$$e^{x(e^t-1)} = \sum_{n=0}^\infty \mathrm{Bel}_n(x)\frac{t^n}{n!} \tag{1.7}$$

(see [6]).

Recently, Kim, Kwon, and Seo [1] defined the degenerate q-Changhee polynomials, a q-extension of (1.3), by

$$\frac{q\lambda+\lambda}{q\log(1+\lambda t)+q\lambda+\lambda}\bigl(1+\log(1+\lambda t)^{\frac{1}{\lambda}}\bigr)^x = \sum_{n=0}^\infty \mathrm{Ch}_{n,\lambda,q}(x)\frac{t^n}{n!}. \tag{1.8}$$

We note that if $x = 0$, then $\mathrm{Ch}_{n,\lambda,q} = \mathrm{Ch}_{n,\lambda,q}(0)$ are called the degenerate q-Changhee numbers.

In this paper, we consider the Appell-type degenerate q-Changhee polynomials and give some explicit and new formulas for these polynomials.

2 The Appell-type degenerate q-Changhee polynomials

In this section, we define the Appell-type degenerate q-Changhee polynomials which are given by

$$\frac{q\lambda+\lambda}{q\log(1+\lambda t)+q\lambda+\lambda}e^{xt} = \sum_{n=0}^\infty \widetilde{\mathrm{Ch}}_{n,\lambda,q}(x)\frac{t^n}{n!}. \tag{2.1}$$

If $x = 0$, then $\widetilde{\mathrm{Ch}}_{n,\lambda,q} = \widetilde{\mathrm{Ch}}_{n,\lambda,q}(0)$ are called the Appell-type degenerate q-Changhee numbers. From (2.1), we note that

$$\widetilde{\mathrm{Ch}}_{n,\lambda,q}(x) = \sum_{m=0}^{n} \binom{n}{m} \widetilde{\mathrm{Ch}}_{m,\lambda,q}\, x^{n-m}. \tag{2.2}$$

By (2.2), we obtain

$$\frac{d}{dx}\widetilde{\mathrm{Ch}}_{n,\lambda,q}(x) = n\widetilde{\mathrm{Ch}}_{n-1,\lambda,q}(x) \quad (n \geq 1). \tag{2.3}$$

From (2.3), we show that

$$\begin{aligned}\int_0^1 \widetilde{\mathrm{Ch}}_{n,\lambda,q}(x)\,dx &= \frac{1}{n+1}\int_0^1 \frac{d}{dx}\widetilde{\mathrm{Ch}}_{n+1,\lambda,q}(x)\,dx \\ &= \frac{1}{n+1}\bigl(\widetilde{\mathrm{Ch}}_{n+1,\lambda,q}(1) - \widetilde{\mathrm{Ch}}_{n+1,\lambda,q}\bigr). \end{aligned} \tag{2.4}$$

We observe that

$$\begin{aligned}\int_0^1 y^n \widetilde{\mathrm{Ch}}_{n,\lambda,q}(x+y)\,dy &= \sum_{m=0}^{n}\binom{n}{m}\widetilde{\mathrm{Ch}}_{n-m,\lambda,q}(x)\int_0^1 y^{n+m}\,dy \\ &= \sum_{m=0}^{n}\binom{n}{m}\frac{\widetilde{\mathrm{Ch}}_{n-m,\lambda,q}(x)}{n+m+1}. \end{aligned} \tag{2.5}$$

On the other hand, we derive

$$\begin{aligned}&\int_0^1 y^n \widetilde{\mathrm{Ch}}_{n,\lambda,q}(x+y)\,dy \\ &\quad= \sum_{m=0}^{n}\binom{n}{m}\widetilde{\mathrm{Ch}}_{n-m,\lambda,q}(x+1)(-1)^m\int_0^1 y^n(1-y)^m\,dy \\ &\quad= \sum_{m=0}^{n}\binom{n}{m}(-1)^m\widetilde{\mathrm{Ch}}_{n-m,\lambda,q}(x+1)\frac{\Gamma(n+1)\Gamma(m+1)}{\Gamma(n+m+2)}. \end{aligned} \tag{2.6}$$

Thus, by (2.5) and (2.6), we give the first result.

Theorem 1 *For $n \in \mathbb{N} \cup \{0\}$, we have*

$$\sum_{m=0}^{n}\frac{\binom{n}{m}\widetilde{\mathrm{Ch}}_{n-m,\lambda,q}(x)}{n+m+1} = \sum_{m=0}^{n}\binom{n}{m}(-1)^m\widetilde{\mathrm{Ch}}_{n-m,\lambda,q}(x+1)\frac{\Gamma(n+1)\Gamma(m+1)}{\Gamma(n+m+2)}.$$

In particular, $x = 0$;

$$\sum_{m=0}^{n}\frac{\binom{n}{m}\widetilde{\mathrm{Ch}}_{n-m,\lambda,q}}{n+m+1} = \sum_{m=0}^{n}\binom{n}{m}(-1)^m\widetilde{\mathrm{Ch}}_{n-m,\lambda,q}(1)\frac{\Gamma(n+1)\Gamma(m+1)}{\Gamma(n+m+2)}.$$

We also observe that

$$\begin{aligned}
&\int_0^1 y^n \widetilde{\mathrm{Ch}}_{n,\lambda,q}(x+y)\,dy \\
&= \frac{\widetilde{\mathrm{Ch}}_{n,\lambda,q}(x+1)}{n+1} - \frac{n}{n+1}\int_0^1 y^{n+1}\widetilde{\mathrm{Ch}}_{n-1,\lambda,q}(x+y)\,dy \\
&= \frac{\widetilde{\mathrm{Ch}}_{n,\lambda,q}(x+1)}{n+1} - \frac{\widetilde{\mathrm{Ch}}_{n-1,\lambda,q}(x+1)}{n+1}\frac{n}{n+2} \\
&\quad + (-1)^2\frac{n(n-1)}{(n+1)(n+2)}\int_0^1 y^{n+2}\widetilde{\mathrm{Ch}}_{n-2,\lambda,q}(x+y)\,dy \\
&= \frac{\widetilde{\mathrm{Ch}}_{n,\lambda,q}(x+1)}{n+1} - \frac{n\widetilde{\mathrm{Ch}}_{n-1,\lambda,q}(x+1)}{(n+1)(n+2)} + (-1)^2\frac{n(n-1)\widetilde{\mathrm{Ch}}_{n-2,\lambda,q}(x+1)}{(n+1)(n+2)(n+3)} \\
&\quad + (-1)^3\frac{n(n-1)(n-2)}{(n+1)(n+2)(n+3)}\int_0^1 y^{n+3}\widetilde{\mathrm{Ch}}_{n-3,\lambda,q}(x+y)\,dy.
\end{aligned} \tag{2.7}$$

Continuing this process consecutively yields

$$\begin{aligned}
&\int_0^1 y^n \widetilde{\mathrm{Ch}}_{n,\lambda,q}(x+y)\,dy \\
&= \frac{\widetilde{\mathrm{Ch}}_{n,\lambda,q}(x+1)}{n+1} + \sum_{m=2}^{n-1}\frac{n(n-1)\cdots(n-m+2)(-1)^{m-1}}{(n+1)(n+2)\cdots(n+m)}\widetilde{\mathrm{Ch}}_{n-m+1,\lambda,q}(x+1) \\
&\quad + (-1)^{n-1}\frac{n(n-1)(n-2)\cdots 2}{(n+1)(n+2)\cdots(2n-1)}\int_0^1 y^{2n-1}\widetilde{\mathrm{Ch}}_{1,\lambda,q}(x+y)\,dy \\
&= \frac{\widetilde{\mathrm{Ch}}_{n,\lambda,q}(x+1)}{n+1} + \sum_{m=2}^{n-1}\frac{n(n-1)\cdots(n-m+2)(-1)^{m-1}}{(n+1)(n+2)\cdots(n+m)}\widetilde{\mathrm{Ch}}_{n-m+1,\lambda,q}(x+1) \\
&\quad + (-1)^{n-1}\frac{n!}{(n+1)(n+2)\cdots(2n-1)2n}\left(\widetilde{\mathrm{Ch}}_{1,\lambda,q}(x+1) - \frac{1}{2n+1}\right) \\
&= \sum_{m=1}^{n+1}\frac{(n)_{m-1}}{\langle n+1\rangle_m}(-1)^{m-1}\widetilde{\mathrm{Ch}}_{n-m+1,\lambda,q}(x+1),
\end{aligned} \tag{2.8}$$

where $(n)_{m-1} = n(n-1)\cdots(n-m+2)$ and $< n+1 >_m = (n+1)(n+2)\cdots(n+m)$.

Thus, by (2.5) and (2.8), we give the second result.

Theorem 2 *For $n \in \mathbb{N}$ with $n \geq 3$, we have*

$$\sum_{m=0}^{n}\frac{\binom{n}{m}\widetilde{\mathrm{Ch}}_{n-m,\lambda,q}(x)}{n+m+1} = \sum_{m=0}^{n}\frac{(n)_m}{\langle n+1\rangle_{m+1}}(-1)^m\mathrm{Ch}_{n-m,\lambda,q}(x+1).$$

For $n \in \mathbb{N}$, we have

$$\begin{aligned}
&\int_0^1 y^n \widetilde{\mathrm{Ch}}_{n,\lambda,q}(x+y)\,dy \\
&= \frac{\widetilde{\mathrm{Ch}}_{n+1,\lambda,q}(x+1)}{n+1} - \frac{n}{n+1}\int_0^1 y^{n-1}\widetilde{\mathrm{Ch}}_{n+1,\lambda,q}(x+y)\,dy
\end{aligned}$$

$$= \frac{\widetilde{\mathrm{Ch}}_{n+1,\lambda,q}(x+1)}{n+1} - \frac{n}{n+1}\sum_{m=0}^{n+1}\binom{n+1}{m}\widetilde{\mathrm{Ch}}_{n+1-m,\lambda,q}(x+1)(-1)^m \int_0^1 (1-y)^m y^{n-1}\,dy$$

$$= \frac{\widetilde{\mathrm{Ch}}_{n+1,\lambda,q}(x+1)}{n+1} - \frac{n}{n+1}\sum_{m=0}^{n+1}\binom{n+1}{m}\widetilde{\mathrm{Ch}}_{n+1-m,\lambda,q}(x+1)(-1)^m B(n,m+1). \tag{2.9}$$

Therefore, by (2.5) and (2.9), we obtain the third result.

Theorem 3 *For $n \in \mathbb{N}$, we have*

$$\sum_{m=0}^{n}\frac{\binom{n}{m}\widetilde{\mathrm{Ch}}_{n-m,\lambda,q}(x)}{n+m+1}$$
$$= \frac{\widetilde{\mathrm{Ch}}_{n+1,\lambda,q}(x+1)}{n+1} - \frac{n}{n+1}\sum_{m=0}^{n+1}\binom{n+1}{m}\widetilde{\mathrm{Ch}}_{n+1-m,\lambda,q}(x+1)(-1)^m B(n,m+1),$$

where $B(n,m+1)$ is a beta function.

Now, we observe that, for $n \in \mathbb{N}\cup\{0\}$, $m \in \mathbb{N}$,

$$\int_0^1 \widetilde{\mathrm{Ch}}_{m,\lambda,q}(x)\widetilde{\mathrm{Ch}}_{n,\lambda,q}(x)\,dx$$
$$= \sum_{l=0}^{n}\binom{n}{l}\widetilde{\mathrm{Ch}}_{l,\lambda,q}\sum_{k=0}^{m}\binom{m}{k}\widetilde{\mathrm{Ch}}_{k,\lambda,q}(1)(-1)^{m-k}\int_0^1 x^{n-l}(1-x)^{m-k}\,dx$$
$$= \sum_{l=0}^{n}\sum_{k=0}^{m}\binom{n}{l}\binom{m}{k}(-1)^{m-k}\widetilde{\mathrm{Ch}}_{k,\lambda,q}(1)\widetilde{\mathrm{Ch}}_{l,\lambda,q}B(n-l+1,m-k+1)$$
$$= \sum_{l=0}^{n}\sum_{k=0}^{m}\binom{n}{l}\binom{m}{k}(-1)^{m-k}\widetilde{\mathrm{Ch}}_{k,\lambda,q}(1)\widetilde{\mathrm{Ch}}_{l,\lambda,q}\frac{\Gamma(n-l+1)\Gamma(m-k+1)}{\Gamma(n+m-l-k+2)}$$
$$= \sum_{l=0}^{n}\sum_{k=0}^{m}\frac{\binom{n}{l}\binom{m}{k}}{\binom{n+m-l-k}{n-l}}(-1)^{m-k}\frac{\widetilde{\mathrm{Ch}}_{k,\lambda,q}(1)\widetilde{\mathrm{Ch}}_{l,\lambda,q}}{n+m-l-k+1}. \tag{2.10}$$

On the other hand,

$$\int_0^1 \widetilde{\mathrm{Ch}}_{m,\lambda,q}(x)\widetilde{\mathrm{Ch}}_{n,\lambda,q}(x)\,dx = \sum_{l=0}^{n}\sum_{k=0}^{m}\binom{n}{l}\binom{m}{k}\frac{\widetilde{\mathrm{Ch}}_{m-k,\lambda,q}\widetilde{\mathrm{Ch}}_{n-l,\lambda,q}}{k+l+1}. \tag{2.11}$$

Thus, by (2.10) and (2.11), we give the fourth result.

Theorem 4 *For $n \in \mathbb{N}\cup\{0\}$, $m \in \mathbb{N}$, we have*

$$\sum_{l=0}^{n}\sum_{k=0}^{m}\frac{\binom{n}{l}\binom{m}{k}}{\binom{n+m-l-k}{n-l}}(-1)^{m-k}\frac{\widetilde{\mathrm{Ch}}_{k,\lambda,q}(1)\widetilde{\mathrm{Ch}}_{l,\lambda,q}}{n+m-l-k+1}$$
$$= \sum_{l=0}^{n}\sum_{k=0}^{m}\binom{n}{l}\binom{m}{k}\frac{\widetilde{\mathrm{Ch}}_{m-k,\lambda,q}\widetilde{\mathrm{Ch}}_{n-l,\lambda,q}}{k+l+1}.$$

By replacing t to $\frac{1}{\lambda}(e^{\lambda t}-1)$ in (2.1), we get

$$\begin{aligned}\frac{1+q}{q(1+t)+1}e^{\frac{x}{\lambda}(e^{\lambda t}-1)} &= \left(\sum_{m=0}^{\infty}\mathrm{Ch}_{m,q}\frac{t^m}{m!}\right)\left(\sum_{l=0}^{\infty}\mathrm{Bel}_l\left(\frac{x}{\lambda}\right)\frac{(\lambda t)^l}{l!}\right)\\ &= \sum_{n=0}^{\infty}\left(\sum_{m=0}^{n}\binom{n}{m}\mathrm{Ch}_{m,q}\,\mathrm{Bel}_{n-m}\left(\frac{x}{\lambda}\right)\lambda^{n-m}\right)\frac{t^n}{n!} \end{aligned}\tag{2.12}$$

(see [6]). On the other hand,

$$\begin{aligned}&\sum_{m=0}^{\infty}\widetilde{\mathrm{Ch}}_{m,\lambda,q}(x)\frac{1}{m!}\frac{1}{\lambda^m}\left(e^{\lambda t}-1\right)^m\\ &\quad=\sum_{m=0}^{\infty}\widetilde{\mathrm{Ch}}_{m,\lambda,q}(x)\frac{1}{\lambda^m}\sum_{n=m}^{\infty}S_2(n,m)\lambda^n\frac{t^n}{n!}\\ &\quad=\sum_{n=0}^{\infty}\left(\sum_{m=0}^{n}\widetilde{\mathrm{Ch}}_{m,\lambda,q}(x)S_2(n,m)\lambda^{n-m}\right)\frac{t^n}{n!},\end{aligned}\tag{2.13}$$

where $S_2(n,m)$ is for the Stirling numbers of the second kind, given by

$$\left(e^t-1\right)^m = m!\sum_{n=m}^{\infty}S_2(n,m)\frac{t^n}{n!}.$$

By (2.12) and (2.13), we give the fifth result.

Theorem 5 *For $n\in\mathbb{N}\cup\{0\}$, we have*

$$\sum_{m=0}^{n}\widetilde{\mathrm{Ch}}_{m,\lambda,q}(x)S_2(n,m)\lambda^{-m} = \sum_{m=0}^{n}\binom{n}{m}\mathrm{Ch}_{m,q}\,\mathrm{Bel}_l\left(\frac{x}{\lambda}\right)\lambda^l.$$

3 Remarks

In this section, we derive an explicit identity related to the Appell-type degenerate q-Changhee polynomials as follows. By (1.2), we get

$$\int_{\mathbb{Z}_p}e^{y\log(1+\frac{1}{\lambda}\log(1+\lambda t))+xt}\,d\mu_{-q}(y) = \frac{q\lambda+\lambda}{q\log(1+\lambda t)+q\lambda+\lambda}e^{xt} = \sum_{n=0}^{\infty}\widetilde{\mathrm{Ch}}_{n,\lambda,q}(x)\frac{t^n}{n!}.\tag{3.1}$$

On the other hand,

$$\begin{aligned}&\int_{\mathbb{Z}_p}e^{y\log(1+\frac{1}{\lambda}\log(1+\lambda t))+xt}\,d\mu_{-q}(y)\\ &\quad=\int_{\mathbb{Z}_p}\left(1+\log(1+\lambda t)^{\frac{1}{\lambda}}\right)^y e^{xt}\,d\mu_{-q}(y)\\ &\quad=\int_{\mathbb{Z}_p}\sum_{k=0}^{\infty}\binom{y}{k}\left(\frac{1}{\lambda}\log(1+\lambda t)\right)^k e^{xt}\,d\mu_{-q}(y)\\ &\quad=\int_{\mathbb{Z}_p}\sum_{k=0}^{\infty}(y)_k\frac{1}{\lambda^k}\sum_{m=k}^{\infty}S_1(m,k)\frac{(\lambda t)^m}{m!}e^{xt}\,d\mu_{-q}(y)\end{aligned}$$

$$
\begin{aligned}
&= \int_{\mathbb{Z}_p} \sum_{m=0}^{\infty} \sum_{k=0}^{m} (y)_k \lambda^{m-k} S_1(m,k) \frac{t^m}{m!} \sum_{s=0}^{\infty} x^s \frac{t^s}{s!}\, d\mu_{-q}(y) \\
&= \int_{\mathbb{Z}_p} \sum_{n=0}^{\infty} \sum_{m=0}^{n} \sum_{k=0}^{m} \binom{n}{m} (y)_k \lambda^{m-k} S_1(m,k) x^{n-m} \frac{t^n}{n!} d\mu_{-q}(y) \\
&= \sum_{n=0}^{\infty} \left(\sum_{m=0}^{n} \sum_{k=0}^{m} \binom{n}{m} \lambda^{m-k} S_1(m,k) x^{n-m} \int_{\mathbb{Z}_p} (y)_k \, d\mu_{-q}(y) \right) \frac{t^n}{n!} \\
&= \sum_{n=0}^{\infty} \left(\sum_{m=0}^{n} \sum_{k=0}^{m} \binom{n}{m} \lambda^{m-k} S_1(m,k) x^{n-m} \operatorname{Ch}_{k,q} \right) \frac{t^n}{n!},
\end{aligned} \tag{3.2}
$$

where $S_1(m,k)$ is the Stirling numbers of the first kind which is given by

$$
\bigl(\log(1+t)\bigr)^k = k! \sum_{n=k}^{\infty} S_1(m,k) \frac{t^m}{m!}.
$$

By (3.1) and (3.2), we give the final result.

Theorem 6 *For $n \geq 0$, we have*

$$
\widetilde{\operatorname{Ch}}_{n,\lambda,q}(x) = \sum_{m=0}^{n} \sum_{k=0}^{m} \binom{n}{m} \lambda^{m-k} S_1(m,k) x^{n-m} \operatorname{Ch}_{k,q}.
$$

4 Conclusions

We consider special numbers and polynomials such as Appell polynomials over the years: Bernoulli, Euler, Genocchi polynomials, and also Changhee polynomials and numbers have many applications in all most all branches of the mathematics and mathematical physics.

In Theorems 1, 2, 3, and 4, by using p-adic q-Volkenborn integral and generating functions, we derived many new and novel identities and relations related to the Appell-type degenerate q-Changhee polynomials and also q-Changhee numbers. In Theorem 5, we also gave some relations between q-Changhee type polynomials and the Stirling numbers of the first kind and Changhee numbers.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally to this work. All authors read and approved the final manuscript.

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