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Fault detection for switched T-S fuzzy systems in finite frequency domain

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Abstract

This paper is concerned with the problem of the fault detection for switched T-S fuzzy systems with actuator faults. The system faults and unknown disturbances are considered to be in a finite frequency domain. An effective fault detection filter is designed to measure the fault sensitivity and disturbance robustness. By using Parseval's theorem and S-procedure, a new finite frequency method of fault detection for switched T-S fuzzy systems is formulated. Sufficient linear matrix inequalities (LMIs) conditions are proposed to design the fault detection filter, which can guarantee the finite frequency H_- and H_∞ performance. The effectiveness of the given finite frequency method for switched T-S fuzzy systems is illustrated through two numerical simulation examples.

Keywords: fault detection; filter; finite frequency; switched T-S fuzzy systems; Parseval's theorem

1 Introduction

Due to the more increasing demand for reliability and safety in industrial control processes, the issue of fault detection is required in various kinds of practical complex systems. The controlled problems are relative to sensors, actuators, controlled hardware, of course, also including the controlled process and so on. However, for the malfunctioning of the sensors and actuators, deterioration of plant equipment or the ageing of controller hardware, faults are developed. In order to ensure the safety and reliability of the controlled systems, the fault detection problem was proposed.

Nowadays, for the increasing demand for reliability and safety in industrial processes, fault detection problem has attracted more and more attention. Many more fault detection approaches have been proposed for different systems, designing fault detection filters to monitor systems; see *e.g.* [1–4]. The objectives of fault detection filter designing can be defined as (i) detecting faults as soon as possible, (ii) avoiding false alarms for uncontrolled inputs signals like disturbance. Among these results, various optimization methods have been proposed, such as the mixed H_2/H_∞ [5] optimization method and the H_-/H_∞ [6, 7] optimization method, where the H_- index can measure the maximum influence of a fault on the residual signal and the H_∞ index [8] can measure the minimum of the disturbance on the residual signal. Besides, there is a method to deal with the first one index in [2]: a weighting matrix is introduced to transform the H_- constraint into a H_∞ constraint. How-

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ever, it is not easy to select appropriate weights and the additional weights will increase the complexity of fault detection systems.

The Takagi-Sugeno (T-S) fuzzy model has witnessed great progress since it is an effective tool in approximating most complex nonlinear systems [9–11]. In [12], a new indirect adaptive switching fuzzy control method is proposed for fuzzy dynamical systems based on T-S multiple models. Further, the method in [13] is extended to a class of unknown nonlinear systems with a new robust adaptive multiple model-based fuzzy control scheme. In [14, 15], the problem of fault detection are investigated for T-S fuzzy systems by translating the FD performance requirements over a finite frequency range into an H_∞ tracking problem. References [16, 17] proposed the FD method for uncertain fuzzy systems based on the delta operator approach. In [2, 18], the fault detection problems are considered for T-S fuzzy systems in finite frequency domain. The fault detection and isolation problem is investigated in [19, 20].

In another study, by using generalized the Kalman-Yakubovic-Popov (GKYP) lemma [21] and Parselval's theorem [22], some fault detection methods with less conservatism compared to full frequency domain approaches have been developed for linear time-invariant systems [23, 24], singular systems [25, 26], T-S fuzzy systems [2, 18, 26, 27], switched systems [28–30], Markovian jump systems [31], and so on. In [32], a fault detection filter design method is presented for switched fuzzy systems with average dwell time. However, there is no result on fault detection in a finite frequency domain for switched T-S fuzzy systems [33–35].

In this paper, the problem of fault detection for a class of switched T-S fuzzy systems in finite frequency domain is addressed. We consider the finite frequency H_- performance and H_∞ performance, which can better detect the fault of the control systems and reduce the effect of disturbance by using the finite frequency domain approach. Then a new bounded real lemma (BRL) is obtained. Based on the new BRL, the sufficient conditions for designing the filter which can guarantee the H_- and H_∞ performances are given in terms of solving a set of LMIs. Compared to the existing FD method in the full frequency domain [32], the designed filter is more sensitive to the fault signal and more robust against disturbances due to the introduced additional slack matrix variable. Finally, numerical simulation examples are given to show the effectiveness of the proposed finite frequency fault detection.

The sections of this paper are organized as follows. In Section 2, the preliminary and problem statement of fault detection filter are presented. The fault detection filter performance analysis and design conditions are proposed in Section 3. The fault detection threshold is given in Section 4, and in Section 5 numerical simulation examples are given.

Notation The symmetric terms in a symmetric matrix are denoted by $*$. For a matrix P, its complex conjugate transpose is denoted by P^*, and $He(P) = P + P^*$.

2 Preliminaries and problem statement

2.1 System description

The switched T-S fuzzy model is described by fuzzy IF-THEN rules, which locally represent linear input-output relations of nonlinear systems. Let us consider the following

switched T-S fuzzy systems model:

$$\dot{x}(t) = A_\sigma(h)x(t) + B_{d\sigma}(h)d(t) + B_\sigma(h)f(t),$$

$$y(t) = C_\sigma(h)x(t) + D_{d\sigma}(h)d(t),\tag{1}$$

where $x(t) \in R^n$ is the state, $y(t) \in R^q$ is the measured output, $d(t) \in R^d$ is the disturbance input, $f(t) \in R^f$ represents the actuator fault signal, the piecewise constant function $\sigma(t)$ is the switch signal, and $\sigma(t) = i \in N = \{1, 2, \ldots, r\}$ means the ith subsystem is active, $A_\sigma(h) = \sum_{j=1}^r h_j A_{\sigma j}(t)$, $B_{d\sigma}(h)$, $C_{d\sigma}(h)$, $D_{d\sigma}(h)$ are similar to $A_{d\sigma}(h)$. The $z_1(t), \ldots, z_s(t)$ are premise variables and μ_{jk} $(j = 1, \ldots, s, k = 1, \ldots, q)$ are fuzzy sets, s is the number of IF-THEN rules, and q is the number of premise variables and

$$h_j\big(z(t)\big) = \frac{\mu_j(z(t))}{\sum_{j=1}^s \mu_j(z(t))}, \qquad \mu_j\big(z(t)\big) = \prod_{k=1}^q \mu_{jk}\big(z_k(t)\big);$$

$\mu_{jk}(z_k(t))$ is the grade of the membership of $z_k(t)$ in μ_{jk}. For all t we have

$$h_j\big(z(t)\big) \le 0, \quad j = 1, \ldots, s, \sum_{j=1}^s h_j\big(z(t)\big) = 1.$$

The corresponding switched fuzzy subsystem matrices are denoted by given fuzzy matrices $A_i(h)$, $B_{di}(h)$, $B_{fi}(h)$, $C_i(h)$, $D_{di}(h)$, $D_{fi}(h)$ $(i \in N)$. The frequencies of $d(t)$ and $f(t)$ reside in a known finite frequency set Θ, which is defined as

$$\Theta := \begin{cases} \{\omega \in R : |\omega| \le \varpi_l, \varpi_l \ge 0\} & \text{(LF)}, \\ \{\omega \in R : \varpi_1 \le \omega \le \varpi_2, \varpi_1 \le \varpi_2\} & \text{(MF)}, \\ \{\omega \in R : |\omega| \ge \varpi_h, \varpi_h \ge 0\} & \text{(HF)}. \end{cases}\tag{2}$$

Remark 1 Switched nonlinear systems can be found in various domains [36], such as mobile robots, network control systems, automotive, dc converters, and so on. Recently, some results have been proposed for switched nonlinear systems based on the T-S fuzzy model [32, 37].

Assumption 1 There exists a switching function $\sigma(t)$ such that the system (1) with $d(t) = 0$ and $f(t) = 0$ is admissible.

Definition 1 For any $t_2 > t_1 \ge 0$, let $N(t_1, t_2)$ denote the number of switchings of $\sigma(t)$ over (t_1, t_2). If $N(t_1, t_2) \le N_0 + (t_2 - t_1)/T$ holds for $T > 0$, $N_0 \ge 0$, then T is called the average dwell time.

2.2 Fault detection filter
In order to detect the actuator fault, the following fault detection filter is introduced:

$$\dot{x}_f(t) = A_{f\sigma}(h)x_f(t) + B_{f\sigma}(h)y(t),$$

$$r(t) = C_{f\sigma}(h)x_f(t),\tag{3}$$

where $x_f(t) \in R^n$ is the filter state, $y(t) \in R^q$ is the measured output, $r(t) \in R^d$ is the filter output, the piecewise continuous function $\sigma(t)$ is the switching signal, and $\sigma(t) = i \in N = \{1, 2, \ldots, r\}$ means the ith subsystem is active, and the switched fuzzy system matrices $A_{f\sigma}(h) = \sum_{j=1}^{r} h_j A_{f\sigma j}(t)$, and $B_{f\sigma}(h)$, $C_{f\sigma}(h)$ are similar to be $A_{f\sigma}(h)$.

Augmenting the model of system (1) to include the states of (3), we obtain the following system:

$$\dot{\eta}(t) = \bar{A}_\sigma(h)\eta(t) + \bar{B}_{d\sigma}(h)d(t) + \bar{B}_\sigma(h)f(t),$$
$$r(t) = \bar{C}_\sigma(h)\eta(t), \tag{4}$$

where

$$\bar{A}_\sigma(h) = \begin{bmatrix} A_\sigma(h) & 0 \\ B_{f\sigma}(h)C_\sigma(h) & A_{f\sigma}(h) \end{bmatrix}, \qquad \bar{B}_{d\sigma}(h) = \begin{bmatrix} B_{d\sigma}(h) \\ B_{f\sigma}(h)D_{d\sigma}(h) \end{bmatrix},$$

$$\bar{B}_\sigma(h) = \begin{bmatrix} B_\sigma(h) \\ 0 \end{bmatrix}, \qquad \bar{C}_\sigma = \begin{bmatrix} 0 \\ C_{f\sigma}^*(h) \end{bmatrix}^*, \qquad \eta(t) = \begin{bmatrix} x(t) \\ x_f(t) \end{bmatrix}.$$

2.3 Problem statement

In practice, for some systems, the main effects of disturbances and faults occupy different frequency domains. Particularly, fault signals usually emerge in the low frequency domain, for example, the constant struck fault just belongs to the low frequency domain [38]. Meanwhile, the disturbances are always in the high frequency such as high frequency noise.

Remark 2 For an incipient signal, the fault information is always contained within a low frequency band as the fault development is slow. For the type fault signal, a FD filter design typically requires a high fault sensitivity in a low frequency range.

As mentioned earlier, in this paper, we assume that faults are low frequency signals, while disturbances reside in the high frequency domain. Consider the following two finite frequency ranges for frequency ω in disturbance $d(t)$ and fault $f(t)$:

$$\Theta_d := \{\omega \in R : |\omega| \geq \varpi_h, \varpi_h \geq 0\}, \tag{5}$$
$$\Theta_f := \{\omega \in R : |\omega| \leq \varpi_l, \varpi_l \geq 0\}. \tag{6}$$

Denote by $F(f(t))$ the Fourier transformation of $f(t)$.

Definition 2 The system (4) has the finite frequency H_∞ index bound γ, if the inequality

$$\int_0^\infty e^{-\alpha t} r^*(t) r(t) \, dt \leq \gamma^2 \int_0^\infty d^*(t) d(t) \, dt \tag{7}$$

holds for all solutions of (4) with $d(t) \in L_2$ such that

$$\int_0^\infty \tau(w + \varpi_h)(w - \varpi_h)\eta(\omega)\dot{\eta}(\omega) \, d\omega > 0 \tag{8}$$

under the zero initial condition.

Definition 3 The system (4) has the finite frequency H_- index bound β, if the inequality

$$\int_0^\infty r^*(t)r(t)\,dt \geq \beta^2 \int_0^\infty e^{-\alpha t}f^*(t)f(t)\,dt \tag{9}$$

holds for all solutions of (4) with $f(t) \in L_2$ such that

$$\int_0^\infty -\tau(w + \varpi_l)(w - \varpi_l)\eta(\omega)\dot{\eta}(\omega)\,d\omega > 0 \tag{10}$$

under the zero initial condition.

In this paper, an H_-/H_∞ fault detection filter is designed to be sensitive to both fault and disturbance, and to guarantee that the augmented model (4) is asymptotically stable. To this end, considering the frequency characterizations of $d(t)$ and $f(t)$, the finite frequency FD problem can be formulated as follows: Given three scalars α, γ, and β, under the condition that the filter (4) is stable, solve the following optimization problem:

$$\begin{aligned} &\max \beta \\ &\text{s.t. (7), (9).} \end{aligned} \tag{11}$$

Remark 3 In the optimization problem (11), condition (7) is a H_∞ performance constraint where γ denotes the worst case criterion for the effects of the disturbance $d(t)$ on the residual $r(t)$ and condition (9) is a H_- performance constraint where β is a measurement of the fault sensitivity in the worst case from faults $f(t)$ to the residual $r(t)$.

2.4 Preliminaries

Lemma 1 [39] (Finsler's lemma) *Let $x \in R^n$, $Q \in R^{n \times n}$, and $U \in R^{n \times m}$. The following statements are equivalent*:

(i) $x^*Qx < 0$, $\forall U^*x = 0$, $x \neq 0$,

(ii) $\exists Y \in R^{m \times n} : Q + UY + Y^*U^* < 0$.

Lemma 2 [40] *The switched T-S fuzzy system (1) is said to be globally uniformly asymptotically stable with average dwell time $T > T^* = \frac{\ln \mu}{\alpha}$, if there exist Lyapunov functions $V_i(x(t))$, $\forall i \in 1, 2, \ldots, r$, such that*

$$V_i\big(x(t_k)\big) \leq \mu V_j\big(x(t_k)\big), \quad t_k \text{ is the switching point,}$$
$$\dot{V}_i\big(x(t)\big) \leq -\alpha V_i\big(x(t)\big).$$

Lemma 3 [41] *If there exist functions $f(t)$ and $g(t)$ satisfying*

$$\dot{f}(t) \leq -\alpha f(t) + mg(t),$$

where α and m are constant, then

$$f(t) \leq e^{-\alpha(t-t_0)}f(t_0) + m \int_{t_0}^t e^{-\alpha(t-s)}g(s)\,ds.$$

3 Main results

In this section, the fault detection filter in the finite frequency design for switched T-S fuzzy systems problem which is formulated in the previous section based on an LMI approach will be solved. Four subsections will be given to illustrate this problem. The first two subsections give the sufficient conditions for (7) and (9), the third one gives the sufficient conditions for (S.3). Finally, we give the sufficient condition to guarantee the stability of the developed filter.

3.1 Finite frequency performance analysis

In this subsection, we assume that the fault detection filter parameters in (3) are known, and the proposed the sufficient condition such that the filter error system (4) satisfies these two specifications (7) and (9). Then we propose the theorems to obtain the filter which guarantee (7) and (9) based on the previous theorems.

Lemma 4 *For any signal $\eta(t)$ residing in a known finite frequency set Θ defined in* (2), *if there exist a symmetric matrix P and a symmetric positive definite matrix Q, then*

$$\int_0^\infty \vartheta^*(t)\Xi_d\vartheta(t)\,dt = \int_0^\infty \vartheta^*(t)(\Phi \otimes P + \Psi \otimes Q)\vartheta(t)\,dt \geq 0, \tag{12}$$

where $\vartheta(t) = \begin{bmatrix} \dot{\eta}(t) \\ \eta(t) \end{bmatrix}$, $\Xi_d = (\Phi \otimes P + \Psi \otimes Q)$, $\Phi = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and Ψ is defined in Table 1.

Proof Assume that $\eta(t)$ resides in the high frequency range, $\{\omega \in R : (\omega + \varpi_h)(\omega - \varpi_h) \geq 0\}$, then from Table 1, $\Psi = \begin{bmatrix} 1 & 0 \\ 0 & -\varpi_h^2 \end{bmatrix}$. We obtain

$$\int_0^\infty \vartheta^*(t)(\Phi \otimes P + \Psi \otimes Q)\vartheta(t)\,dt$$

$$= \int_0^\infty \left[He(\dot{\eta}^*(t)P\eta(t)) + \dot{\eta}^*(t)Q\dot{\eta}(t) - \varpi_h^2 \eta^*(t)Q\eta(t) \right] dt. \tag{13}$$

By Parseval's theorem [22], we have

$$\int_0^\infty \dot{\eta}^*(t)Q\dot{\eta}(t)\,dt$$

$$= \int_0^\infty \text{tr}\left[\dot{\eta}(t)\dot{\eta}^*(t)Q \right] dt = \text{tr}\left[\int_0^\infty \dot{\eta}(t)\dot{\eta}^*(t)\,dt \right] Q$$

$$= \text{tr}\left[\frac{1}{2\pi} \int_{-\infty}^\infty \left(j\omega\eta(\omega) \right)\left(-j\omega\eta^*(\omega) \right) d\omega \right] Q = \text{tr}\left[\frac{1}{2\pi} \int_{-\infty}^\infty \omega^2 \eta(\omega)\eta^*(\omega)\,d\omega \right] Q, \tag{14}$$

where $\eta(\omega)$ is the Fourier transform of $\eta(t)$. In a similar way,

$$\int_0^\infty He(\dot{\eta}^*(t)P\eta(t))\,dt = 0$$

Table 1 Values Ψ for different frequency ranges [21]

	$\omega \leq \varpi_l$ (LF)	$\varpi_1 \leq \omega \leq \varpi_2$ (MF)	$\omega \geq \varpi_h$ (HF)
Ψ	$\begin{bmatrix} -1 & 0 \\ 0 & \varpi_l^2 \end{bmatrix}$	$\begin{bmatrix} -1 & j\frac{\varpi_1+\varpi_2}{2} \\ -j\frac{\varpi_1+\varpi_2}{2} & -\varpi_1\varpi_2 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & -\varpi_h^2 \end{bmatrix}$

and

$$\int_0^\infty -\varpi_h^2 \eta^*(t) Q \eta(t)\, dt = \mathrm{tr}\left[\frac{1}{2\pi}\int_{-\infty}^\infty -\varpi_h^2 \eta(\omega)\eta^*(\omega)\, d\omega\right]Q.$$

Thus, considering the fact that $(\omega + \varpi_h)(\omega - \varpi_h) \geq 0$ and $Q > 0$, (13) yields

$$\int_0^\infty \vartheta^*(t)(\Phi \otimes P + \Psi \otimes Q)\vartheta(t)\, dt$$

$$= \mathrm{tr}\left[\frac{1}{2\pi}\int_{-\infty}^\infty (\omega + \varpi_h)(\omega - \varpi_h)\eta(\omega)\eta^*(\omega)\, d\omega\right]Q \geq 0. \tag{15}$$

For other frequency range cases, similar conclusions can be obtained. The proof is completed. □

3.1.1 Inequality condition for finite frequency H_∞ performance

As is well known the disturbances usually are high frequency signals in practice. So, in this paper the disturbances are assumed to belong to the following high frequency domain:

$$\Theta = \left\{\omega \in R : |\omega| \geq \varpi_h, \varpi_h \geq 0\right\}. \tag{16}$$

Set $f(t) = 0$, the filter error system (4) becomes

$$\dot{\eta}(t) = \bar{A}_\sigma(h)\eta(t) + \bar{B}_{d\sigma}(h)d(t),$$

$$r(t) = \bar{C}_\sigma(h)\eta(t). \tag{17}$$

Theorem 1 *For a given scalar $\alpha > 0$, $\gamma > 0$, the high-frequency H_∞ performance (7) is guaranteed for system (17), if there exist symmetric positive definite matrices P_i^d, Q_d, symmetric matrices P_d, a matrix V_{di}, $i = 1, \ldots, r$, and a scalar μ such that for a switching signal with average dwell time satisfying $T > T^* = \frac{\ln \mu}{\alpha}$*

$$\Lambda_{di}(h) + \Lambda_{d1i}(h) + \Lambda_{d2i}(h) < 0, \tag{18}$$

$$\Lambda_{di}(h) \geq 0, \tag{19}$$

$$P_i^d \leq \mu P_j^d, \quad i \neq j, \tag{20}$$

where

$$\Lambda_{di}(h) = \begin{bmatrix} \bar{A}_i(h) & \bar{B}_{di}(h) \\ I & 0 \end{bmatrix}^* (\Phi \otimes P_d + \Psi \otimes Q_d)\begin{bmatrix} \bar{A}_i(h) & \bar{B}_{di}(h) \\ I & 0 \end{bmatrix},$$

$$\Lambda_{d1i}(h) = \begin{bmatrix} \bar{C}_i(h) & 0 \\ 0 & I \end{bmatrix}^* \begin{bmatrix} I & 0 \\ 0 & -\gamma^2 I \end{bmatrix}\begin{bmatrix} \bar{C}_i(h) & 0 \\ 0 & I \end{bmatrix}, \qquad \Phi = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},$$

$$\Lambda_{d2i}(h) = \begin{bmatrix} \bar{A}_i(h) & \bar{B}_{di}(h) \\ I & 0 \end{bmatrix}^* \begin{bmatrix} 0 & P_i^d \\ P_i^d & \alpha P_i^d \end{bmatrix}\begin{bmatrix} \bar{A}_i(h) & \bar{B}_{di}(h) \\ I & 0 \end{bmatrix},$$

with the full column rank matrix U satisfying $\bar{E}^* U = 0$.

Proof First, choosing a Lyapunov functional candidate of the form

$$V(t) = \eta^*(t)P^d_{\sigma(t)}\eta(t) = \xi^*(t)\begin{bmatrix} P^d_\sigma & 0 \\ 0 & 0 \end{bmatrix}\xi(t)$$

$$= \xi^*(t)\begin{bmatrix} \bar{A}_\sigma(h) & \bar{B}_{d\sigma}(h) \\ I & 0 \end{bmatrix}^* \begin{bmatrix} 0 & 0 \\ 0 & P^d_\sigma \end{bmatrix}\begin{bmatrix} \bar{A}_\sigma(h) & \bar{B}_{d\sigma}(h) \\ I & 0 \end{bmatrix}\xi(t), \tag{21}$$

then the derivative of $V(t)$ along (17) satisfies

$$\dot{V}(t) = \dot{\eta}^*(t)P^d_\sigma\eta(t) + \eta^*(t)P^d_\sigma\dot{\eta}(t)$$

$$= \eta^*(t)\big(\bar{A}^*_\sigma(h)\big(P^d_\sigma + P^d_\sigma\bar{A}\sigma(t)\big)\eta(t)$$

$$\quad + d^*(t)\bar{B}^*_{d\sigma}(P^d_\sigma\eta(t) + \eta^*(t)P^d_\sigma(h)\bar{B}_{d\sigma}(h)d(t)$$

$$= \xi^*(t)\begin{bmatrix} \bar{A}_\sigma(h) & \bar{B}_{d\sigma}(h) \\ I & 0 \end{bmatrix}^* \begin{bmatrix} 0 & P^d_\sigma \\ P^d_\sigma & 0 \end{bmatrix}\begin{bmatrix} \bar{A}_\sigma(h) & \bar{B}_{d\sigma}(h) \\ I & 0 \end{bmatrix}\xi(t). \tag{22}$$

Second, from system (17), we have

$$r^*(t)r(t) - \gamma^2 d^*(t)d(t)$$

$$= \xi^*(t)\begin{bmatrix} C_\sigma(h)^*C_\sigma(h) & 0 \\ 0 & -\gamma^2 I \end{bmatrix}\xi(t)$$

$$= \xi^*(t)\begin{bmatrix} \bar{C}_\sigma(h) & 0 \\ 0 & I \end{bmatrix}^* \begin{bmatrix} I & 0 \\ 0 & -\gamma^2 I \end{bmatrix}\begin{bmatrix} \bar{C}_\sigma(h) & 0 \\ 0 & I \end{bmatrix}\xi(t). \tag{23}$$

Third, we let $0 = t_0 < t_1 < \cdots < t_k < \cdots$, $k = 1, \ldots$, denote the switching point, and we suppose that the i_kth subsystem is active when $i \in [t_k, t_{k+1})$. From (19) we have

$$\xi^*(t)\big(\Lambda_{di_k}(h) + \Lambda_{d1i_k}(h) + \Lambda_{d2i_k}(h)\big)\xi(t)$$

$$= \xi^*(t)\Bigg(\begin{bmatrix} \bar{A}_i(h) & \bar{B}_{di}(h) \\ I & 0 \end{bmatrix}^* \begin{bmatrix} 0 & P^d_i \\ P^d_i & 0 \end{bmatrix}\begin{bmatrix} \bar{A}_i(h) & \bar{B}_{di}(h) \\ I & 0 \end{bmatrix}$$

$$\quad + \begin{bmatrix} \bar{A}_i(h) & \bar{B}_{di}(h) \\ I & 0 \end{bmatrix}^* \begin{bmatrix} 0 & 0 \\ 0 & \alpha P^d_i \end{bmatrix}\begin{bmatrix} \bar{A}_i(h) & \bar{B}_{di}(h) \\ I & 0 \end{bmatrix}$$

$$\quad + \begin{bmatrix} \bar{C}_i(h) & 0 \\ 0 & I \end{bmatrix}^* \begin{bmatrix} I & 0 \\ 0 & -\gamma^2 I \end{bmatrix}\begin{bmatrix} \bar{C}_i(h) & 0 \\ 0 & I \end{bmatrix}\Bigg)\xi(t)$$

$$\quad + \xi^*(t)\Bigg(\begin{bmatrix} \bar{A}_i(h) & \bar{B}_{di}(h) \\ I & 0 \end{bmatrix}^* (\Phi \otimes P_d + \Psi \otimes Q_d)\begin{bmatrix} \bar{A}_i(h) & \bar{B}_{di}(h) \\ I & 0 \end{bmatrix}\Bigg)\xi(t)$$

$$= \dot{V}_{i_k}(t) + \alpha V_{i_k}(t) + \Sigma_{i_k} + e^{N(0,t)In\mu}\vartheta^*_{i_k}\Xi_d\vartheta_{i_k} < 0, \tag{24}$$

where

$$\vartheta_{i_k} = e^{-N(0,1)In\mu/2}\begin{bmatrix} \dot{\eta}(t) \\ \eta(t) \end{bmatrix} = \begin{bmatrix} \dot{x}_{i_k}(t) \\ x_{i_k}(t) \end{bmatrix} \tag{25}$$

and

$$\Lambda_{d2ik}(h) = r_{i_k}^*(t)r_{i_k}(t) - \gamma^2 d^*(t)d(t), \tag{26}$$

$$\Lambda_{dik}(h) = \eta^*(t)(\Phi \otimes P_{di} + \Psi \otimes Q_d)\eta(t), \tag{27}$$

$$\Xi_d = (\Phi \otimes P_{di} + \Psi \otimes Q_d). \tag{28}$$

Set $\eta(t_{k+1}) = \eta_{i_{k+1}}(t_{k+1})$, the state of the switched system can be expressed as

$$\eta(t) = \sum_{k=0}^{\infty}\big(\eta_{i_k}(t)\big(u(t - t_k) - u(t - t_{k+1})\big)\big), \quad t \in [0, \infty). \tag{29}$$

We have $\vartheta(t) = [(\dot{\chi}(t))^*, (\chi(t))^*]^*$. Considering (29),

$$\chi(t) = \sum_{k=0}^{\infty}\big(e^{-\frac{k\ln\mu}{2}}\eta_{i_k}(t)\big(u(t - t_k) - u(t - t_{k+1})\big)\big), \quad t \in [0, \infty). \tag{30}$$

Obviously, $\chi(t)$ is also in the frequency range Θ_d and asymptotically stable. In addition, from (30)

$$\dot{\chi}(t) = \sum_{k=0}^{\infty}\Big\{e^{-\frac{k\ln\mu}{2}}\dot{\eta}_{i_k}(t)\big(u(t - t_k) - u(t - t_{k+1})\big)$$

$$+ e^{-\frac{k\ln\mu}{2}}\eta_{i_k}(t)\big(\delta(t - t_k) - \delta(t - t_{k+1})\big)\Big\}, \quad t \in [0, \infty), \tag{31}$$

where $\delta(t)$ is the unit pulse signal. Therefore, by Lemma 4, we have

$$\int_0^{\infty} \vartheta^*(s)\Xi_d\vartheta(s)\,ds \geq 0. \tag{32}$$

From Lemma 3, we have

$$V_{i_k}(t) - e^{-\alpha(t-t_k)}V_{i_k}(t_k) + \int_{t_k}^t e^{-\alpha(t-s)}\big(\Sigma_{i_k}(s) + e^{N(0,t)In\mu}\vartheta_{i_k}^*\Xi_d\vartheta_{i_k}\big)\,ds < 0. \tag{33}$$

Besides, from (21), we have

$$V_{i_k}(t_k) \leq \mu V_{i_{k-1}}\big(t_k^-\big). \tag{34}$$

Then we can obtain the following formula with (33), (34):

$$V_{i_k}(t) \leq \mu e^{-\alpha(t-t_k)}V_{i_{k-1}}\big(t_k^-\big) - \int_{t_k}^t e^{-\alpha(t-s)}\big(\Sigma_{i_k}(s)$$

$$+ e^{N(0,t)In\mu}\vartheta_{i_k}^*\Xi_d\vartheta_{i_k}\big)\,ds$$

$$\leq \mu^{N(0,t)}e^{-\alpha t}V_{i_0}(0) - \mu^{N(0,t)}\int_0^{t_1} e^{-\alpha(t-s)}\big(\Sigma_{i_k}(s)$$

$$+ e^{N(0,t)In\mu}\vartheta_{i_k}^*\Xi_d\vartheta_{i_k}\big)\,ds$$

$$- \cdots - \mu^0 \int_{t_k}^{t} e^{-\alpha(t-s)} \left(\Sigma_{i_k}(s) + e^{N(0,t)In\mu} \vartheta_{i_k}^* \Xi_d \vartheta_{i_k} \right) ds$$

$$= e^{-\alpha(t-t_k)+N(0,t)\ln\mu} V_{i_0}(0) - \int_0^t e^{-\alpha(t-s)+N(s,t)\ln\mu} \left(\Sigma_{\sigma(s)}(s) \right.$$

$$\left. + e^{N(0,t)In\mu} \vartheta_{i_k}^* \Xi_d \vartheta_{i_k} \right) ds. \tag{35}$$

Considering the zero initial condition and $V_{i_k}(t) > 0$, (36) implies

$$\int_0^t e^{-\alpha(t-s)+N(s,t)\ln\mu} \left(\Sigma_{\sigma(s)}(s) + e^{N(0,t)In\mu} \vartheta_{i_k}^* \Xi_d \vartheta_{i_k} \right) ds \le 0, \tag{36}$$

and multiplying both sides of (37) by $e^{-N(0,t)\ln\mu}$ we have

$$\int_0^t e^{-\alpha(t-s)-N(0,s)\ln\mu} r_{i_k}^*(s) r_{i_k}(s)\, ds \le \int_0^t e^{-\alpha(t-s)-N(0,s)\ln\mu} \left(\gamma^2 d^*(s)d(s) \right.$$

$$\left. - e^{N(0,t)In\mu} \vartheta_{i_k}^* \Xi_d \vartheta_{i_k} \right) ds. \tag{37}$$

Because of the fact that $N(0,s) \le \frac{s}{T} < \alpha = \frac{s}{T^*}$ and $T^* = \frac{\ln\mu}{\alpha}$, we have $N(0,s)\ln\mu < \alpha s$ and (37) implies

$$\int_0^t e^{-\alpha(t-s)-\alpha s} r_{i_k}^*(s) r_{i_k}(s)\, ds \le \int_0^t e^{-\alpha(t-s)} \left(\gamma^2 d^*(s)d(s) - \vartheta_{i_k}^* \Xi_d \vartheta_{i_k} \right) ds. \tag{38}$$

Integrating (39) from $t = 0$ to $t = \infty$

$$\int_0^\infty \left(\int_0^t e^{-\alpha(t-s)-\alpha s} r_{i_k}^*(s) r_{i_k}(s)\, ds \right) dt$$

$$\le \int_0^\infty \left(\int_0^t e^{-\alpha(t-s)} \left(\gamma^2 d^*(s)d(s) + \vartheta_{i_k}^* \Xi_d \vartheta_{i_k} \right) ds \right) dt, \tag{39}$$

$$\left(\int_0^\infty e^{-\alpha s} r_{i_k}^*(s) r_{i_k}(s)\, ds \right) \left(\int_s^\infty e^{-\alpha(t-s)}\, dt \right)$$

$$\le \left(\int_0^\infty \left(\gamma^2 d^*(s)d(s) - \vartheta_{i_k}^* \Xi_d \vartheta_{i_k} \right) ds \right) \left(\int_s^\infty e^{-\alpha(t-s)}\, dt \right); \tag{40}$$

(41) implies

$$\int_0^\infty e^{-\alpha t} r^*(t) r(t)\, dt \le \gamma^2 \int_0^\infty d^*(t)d(t)\, dt - \int_0^\infty \vartheta_{i_k}^* \Xi_d \vartheta_{i_k}\, dt. \tag{41}$$

Because of the premise lemma, Lemma 4, we have

$$\int_0^\infty \vartheta_{i_k}^* \Xi_d \vartheta_{i_k}\, dt \ge 0. \tag{42}$$

So, we obtain

$$\int_0^\infty e^{-\alpha t} r^*(t) r(t)\, dt \le \gamma^2 \int_0^\infty d^*(t)d(t)\, dt. \tag{43}$$

The proof is completed.　　　　　　　　　　　　　　　　　　　　　　　　　　　□

3.1.2 Inequality condition for low-frequency H_- performance

Just as mentioned previously, on the contrary, the fault signals usually emerge in the low frequency domain where disturbances usually are high frequency signals. Therefore, in this paper, we assume that the fault signals belong to the following low frequency range:

$$\mathbf{\Theta} = \left\{ \omega \in R : |\omega| \le \varpi_l, \varpi_l \ge 0 \right\}. \tag{44}$$

Set $d(t) = 0$, the filter error system (4) becomes

$$\dot{\eta}(t) = \bar{A}_\sigma(h)\eta(t) + \bar{B}_\sigma(h)f(t),$$
$$r(t) = \bar{C}_\sigma(h)\eta(t). \tag{45}$$

Theorem 2 *For a given scalar $\alpha > 0$ $\beta > 0$, the finite frequency weighted H_- performance* (9) *is satisfied for system* (45), *if there exist symmetric positive definite matrices P_i^f, Q_{fi}, symmetric matrices P_{fi}, matrices V_{fi}, $i = 1, \ldots, N$, and scalar μ such that switching signal with average dwell time satisfying $T > T^* = \frac{\ln \mu}{\alpha}$*

$$\Lambda_{fi}(h) + \Lambda_{f1i}(h) + \Lambda_{f2i}(h) < 0, \tag{46}$$

$$\Lambda_{fi}(h) \ge 0, \tag{47}$$

$$P_i^d \le \mu P_j^d, \quad i \ne j, \tag{48}$$

where

$$\Lambda_{fi}(h) = \begin{bmatrix} \bar{A}_i(h) & \bar{B}_i(h) \\ I & 0 \end{bmatrix}^* (\Phi \otimes P_d + \Psi \otimes Q_d) \begin{bmatrix} \bar{A}_i(h) & \bar{B}_i(h) \\ I & 0 \end{bmatrix}, \qquad \Phi = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},$$

$$\Lambda_{f1i}(h) = \begin{bmatrix} \bar{C}_i(h) & 0 \\ 0 & I \end{bmatrix}^* \begin{bmatrix} I & 0 \\ 0 & -\gamma^2 I \end{bmatrix} \begin{bmatrix} \bar{C}_i(h) & 0 \\ 0 & I \end{bmatrix},$$

$$\Lambda_{f2i}(h) = \begin{bmatrix} \bar{A}_i(h) & \bar{B}_i(h) \\ I & 0 \end{bmatrix}^* \begin{bmatrix} 0 & P_i^d \\ P_i^d & \alpha P_i^d \end{bmatrix} \begin{bmatrix} \bar{A}_i(h) & \bar{B}_i(h) \\ I & 0 \end{bmatrix}.$$

Proof Theorem 2 is proposed by using a similar method to Theorem 1.　　　　　□

3.2 Fault detection filter design conditions

In the preliminaries, Theorems 1, 2 provide finite frequency performance analysis conditions when the filter parameters are known. However, the parameters of the filter (3) are unknown. In the following, we will develop some decoupling techniques to transform the conditions in Theorems 1, 2 into LMIs conditions to obtain the fault detection filter gains.

3.2.1 Fault detection filter for low-frequency H_- performance

The next theorem provides fault detection filter design conditions under which the finite frequency performance (9) can be guaranteed based on Theorem 2.

Theorem 3 *Considering the switched T-S fuzzy system* (1). *For a given positive scalar α, β, there exists a fault detection filter in the form of* (3), *such that the filter error system* (4)

with $d(t) = 0$ *satisfies the finite frequency performance* (9) *when* ω *belongs to* (44) *if there exist matrices*

$$P_i^f = \begin{bmatrix} P_i^{f11} & 0 \\ 0 & P_i^{f22} \end{bmatrix} > 0, \qquad Q_f = \begin{bmatrix} Q_f^{11} & Q_f^{12} \\ Q_f^{12*} & Q_f^{22} \end{bmatrix} > 0, \tag{49}$$

$$P_f = \begin{bmatrix} P_f^{11} & 0 \\ 0 & P_f^{22} \end{bmatrix}, \quad i = 1, \dots, r, \tag{50}$$

the matrices $\hat{A}_{fi}(h)$, $\hat{B}_{fi}(h)$, $C_{fi}(h)$, M_{f1i}, M_{f2i}, M_{f3}, Z_{f1i}, Z_{f2i}, Z_{f3}, *and* N, $i = 1, \dots, r$ *and scalar* $\mu > 1$, ς *such that the following inequalities hold:*

$$\varsigma_i^2 - C_{fi} T < 0, \quad i = 1, \dots, r, \tag{51}$$

$$P_i^f \le \mu P_j^f, \quad i \ne j, \tag{52}$$

$$\begin{bmatrix} \Xi_{11ij} & \Xi_{12ij} & \Xi_{13ij} \\ * & \Xi_{22ij} & \Xi_{23ij} \\ * & * & \Xi_{33ij} \end{bmatrix} < 0, \tag{53}$$

where

$$T \in \Re^n, \qquad \|T\|_2 = \varsigma,$$

$$\Xi_{11ij} = \begin{bmatrix} -Q_f^{11} - He(Z_{f1i}) & -Q_f^{12} - N_i - M_{f1i}{}^* \\ * & -Q_f^{22} - He(N_i) \end{bmatrix},$$

$$\Xi_{12ij} = \begin{bmatrix} P_i^{f11} + P_f^{11} + Z_{f1i}A_{ij} + \hat{B}_{fij}C_{ij} - Z_{f2i}^* & \hat{A}_{fij} - M_{f2i}^* \\ M_{f1i}A_{ij} + \hat{B}_{fij}C_{ij} - N_i^* & P_i^{f22} + P_f^{22} + \hat{A}_{fij} - N_i^* \end{bmatrix},$$

$$\Xi_{13ij} = \begin{bmatrix} Z_{f1i}B_{ij} + Z_{f3i}^*B_{ij} \\ M_{f1i}B_{ij} + \hat{B}_{ij} \end{bmatrix},$$

$$\Xi_{22ij} = \begin{bmatrix} \varpi_l^2 Q_f^{11} + He(Z_{f2i}A_{ij} + \hat{B}_{fij}C_{ij}) & (\varpi_l^2 Q_f^{12} + \hat{A}_{fij} + A_{ij}^*M_{f2i}^* + C_{ij}^*\hat{B}_{fij}^*) \\ * & \varpi_l^2 Q_f^{22} + He(\hat{A}_{fij}) - \varsigma_i^2 I \end{bmatrix},$$

$$\Xi_{23ij} = \begin{bmatrix} Z_{f2i}B_{ij} - A_{ij}^*Z_{f3i}^*B_{ij} - C_{ij}^*\hat{B}_{fij}^*B_{ij} \\ M_{f2i}B_{ij} - \hat{A}_{fij}^*B_{ij} \end{bmatrix},$$

$$\Xi_{33ij} = \beta^2 I - He(B_{ij}Z_{f3i}B_{ij}).$$

Then $A_{fij} = N_i^{-1}\hat{A}_{fij}$, $B_{fij} = N_i^{-1}\hat{B}_{fij}$, *and* C_{fij} *are the parameters of the filter* (3).

Proof Considering Theorem 2, from (47), we have

$$\begin{bmatrix} \bar{A}_i(h) & \bar{B}_i(h) \\ I & 0 \end{bmatrix}^* \begin{bmatrix} -Q_f & P_i^f + P_f \\ * & \alpha P_i^f + \varpi_l^2 Q_f \end{bmatrix} \begin{bmatrix} \bar{A}_i(h) & \bar{B}_i(h) \\ I & 0 \end{bmatrix}$$

$$+ \begin{bmatrix} \bar{C}_i(h) & 0 \\ 0 & I \end{bmatrix}^* \begin{bmatrix} -I & 0 \\ 0 & \beta^2 I \end{bmatrix} \begin{bmatrix} \bar{C}_i(h) & 0 \\ 0 & I \end{bmatrix} < 0. \tag{54}$$

By setting

$$\Pi = \begin{bmatrix} -Q_f & P_i^f + P_f & 0 \\ * & (\alpha P_i^f + \varpi_l^2 Q_f) - \bar{C}_{fi}^*(h)\bar{C}_{fi}(h) & 0 \\ * & * & \beta^2 I \end{bmatrix} \tag{55}$$

(54) can be rewritten as

$$S_{1i}^* \Pi S_{1i} < 0, \tag{56}$$

where $S_{1i} = \begin{bmatrix} \bar{A}_i(h) & \bar{B}_i(h) \\ I & 0 \\ 0 & I \end{bmatrix}$. Then applying Lemma 1, (57) is equivalent to

$$\Pi + He(Y_{fi} S_{2i}) < 0, \tag{57}$$

where $S_{2i} = [-I\ \bar{A}_i(h)\ \bar{B}_i(h)]$, $S_{2i} S_{1i} = 0$, and Y_{fi} can be set as $[Y_{f1i}^*\ Y_{f2i}^*\ -Y_{f3i}^*\bar{B}_i(h)]^*$, which is an additional matrix as introduced by Lemma 1.

The following inequality is obtained from (58):

$$\begin{bmatrix} -Q_f - He(Y_{f1i}) & P_i^f + P_f + Y_{f1i}\bar{A}_i(h) - Y_{f2i}^* \\ * & \alpha P_i^f + \varpi_l^2 Q_f + He(Y_{f2i}\bar{A}_i)(h) - \bar{C}_i^*(h)\bar{C}_i(h) \\ * & * \end{bmatrix}$$
$$\begin{matrix} Y_{f1i}\bar{B}_i(h) + Y_{f3i}^*\bar{B}_i(h) \\ Y_{f2i}\bar{B}_i(h) - \bar{A}_i^*(h)Y_{f3i}^*\bar{B}_i(h) \\ \beta^2 I - He(\bar{B}_i(h)^* Y_{f3i}\bar{B}_i(h)) \end{matrix} \Bigg] < 0. \tag{58}$$

We can see that the block $(2,2)$ in (59) is nonlinear for $\bar{C}_i^*(h)\bar{C}_i(h) = \text{diag}\{0, C_{fi}^*(h)C_{fi}(h)\}$, so we refer to [42] and introduce another constraint (51) which can guarantee

$$C_{fi}^* C_{fi} > \varsigma^2 I. \tag{59}$$

So, the block $(2,2)$ in (59) can be replaced by $\alpha P_i^f + \varpi_l^2 Q_f + He(Y_{f2i}\bar{A}_i(h)) - \text{diag}\{0, \varsigma^2 I\}$.

Next, we suppose Y_{1i}, Y_{2i}, Y_{3i} have the following forms:

$$Y_{f1i} = \begin{bmatrix} Z_{f1i} & N_i \\ M_{f1i} & N_i \end{bmatrix}, \qquad Y_{f2i} = \begin{bmatrix} Z_{f2i} & N_i \\ M_{f2i} & N_i \end{bmatrix}, \qquad Y_{f3i} = \begin{bmatrix} Z_{f3i} & N_i \\ M_{f3i} & N_i \end{bmatrix}. \tag{60}$$

Further,

$$Y_{fji}\bar{A}_i(h) = \begin{bmatrix} Z_{fji}A_i(h) + \hat{B}_{fi}(h)C_i(h) & \hat{A}_{fi}(h) \\ M_{fji}A_i(h) + \hat{B}_{fi}(h)C_i(h) & \hat{A}_{fi}(h) \end{bmatrix}, \tag{61}$$

$$Y_{fji}\bar{B}_i(h) = \begin{bmatrix} Z_{fji}B_i(h) \\ M_{fji}B_i(h) \end{bmatrix}, \qquad i = 1, \dots, r, j = 1, 2, 3, \tag{62}$$

$$\bar{B}_i^*(h)Y_{f3i}\bar{B}_i(h) = B_i^*(h)Z_{f3i}B_i(h). \tag{63}$$

After the defuzzification, we obtain

$$
\begin{bmatrix}
\Xi_{11ij} & \Xi_{12ij} & \Xi_{13ij} \\
* & \Xi_{22ij} & \Xi_{23ij} \\
* & * & \Xi_{33ij}
\end{bmatrix} < 0,
\tag{64}
$$

where

$$
T \in \Re^n, \qquad \|T\|_2 = \varsigma,
$$

$$
\Xi_{11ij} = \begin{bmatrix}
-Q_f^{11} - He(Z_{f1i}) & -Q_f^{12} - N_i - M_{f1i}^* \\
* & -Q_f^{22} - He(N_i)
\end{bmatrix},
$$

$$
\Xi_{12ij} = \begin{bmatrix}
P_i^{f11} + P_f^{11} + Z_{f1i}A_{ij} + \hat{B}_{fij}C_{ij} - Z_{f2i}^* & \hat{A}_{fij} - M_{f2i}^* \\
M_{f1i}A_{ij} + \hat{B}_{fij}C_{ij} - N_i^* & P_i^{f22} + P_f^{22} + \hat{A}_{fij} - N_i^*
\end{bmatrix},
$$

$$
\Xi_{13ij} = \begin{bmatrix}
Z_{f1i}B_{ij} + Z_{f3i}^* B_{ij} \\
M_{f1i}B_{ij} + \hat{B}_{ij}
\end{bmatrix},
$$

$$
\Xi_{22ij} = \begin{bmatrix}
\varpi_l^2 Q_f^{11} + He(Z_{f2i}A_{ij} + \hat{B}_{fij}C_{ij}) & \varpi_l^2 Q_f^{12} + \hat{A}_{fij} + A_{ij}^* M_{f2i}^* + C_{ij}^* \hat{B}_{fij}^* \\
* & \varpi_l^2 Q_f^{22} + He(\hat{A}_{fij}) - \varsigma_i^2 I
\end{bmatrix},
$$

$$
\Xi_{23ij} = \begin{bmatrix}
Z_{f2i}B_{ij} - A_{ij}^* Z_{f3i}^* B_{ij} - C_{ij}^* \hat{B}_{fij}^* B_{ij} \\
M_{f2i}B_{ij} - \hat{A}_{fij}^* B_{ij}
\end{bmatrix},
$$

$$
\Xi_{33ij} = \beta^2 I - He(B_{ij}Z_{f3i}B_{ij}).
$$

Then $A_{fij} = N_i^{-1}\hat{A}_{fij}$, $B_{fij} = N_i^{-1}\hat{B}_{fij}$, and C_{fij} are the parameters of the filter (3). Then, applying (60), (61), (61), (63), and (59) to (59), we get (64).

The proof is completed. $\qquad\square$

3.2.2 Fault detection filter for high-frequency H_∞ performance
This subsection provides fault detection filter design conditions under which the finite frequency performance (7) can be guaranteed.

Theorem 4 *Consider the switched T-S fuzzy system* (1). *For a given positive scalar* α, γ, *there exists a fault detection filter in the form of* (3), *such that the filter error system* (4) *with* $f(t) = 0$ *satisfies the finite frequency performance* (7) *when* ω *belongs to* (16) *if there exist matrices*

$$
P_i^d = \begin{bmatrix} P_i^{d11} & 0 \\ 0 & P_i^{d22} \end{bmatrix} > 0, \qquad
Q_d = \begin{bmatrix} Q_d^{11} & Q_d^{12} \\ Q_d^{12*} & Q_d^{22} \end{bmatrix} > 0,
\tag{65}
$$

$$
P_d = \begin{bmatrix} P_d^{11} & 0 \\ 0 & P_d^{22} \end{bmatrix}, \quad i = 1, \dots, r,
\tag{66}
$$

the matrices $\hat{A}_{fi}(h)$, $\hat{B}_{fi}(h)$, $C_{fi}(h)$, M_{d1i}, M_{d2i}, Z_{d1i}, Z_{d2i}, *and* N, $i = 1, \dots, r$, *and a scalar* $\mu > 1$ *such that the following inequalities hold*:

$$
P_i^d \le \mu P_j^d, \quad i \ne j,
\tag{67}
$$

$$\begin{bmatrix} \Xi_{11ij} & \Xi_{12i} & \Xi_{13ij} & 0 \\ * & \Xi_{22ij} & \Xi_{23ij} & \Xi_{24ij} \\ * & * & \Xi_{33ij} & 0 \\ * & * & * & \Xi_{44ij} \end{bmatrix} < 0, \tag{68}$$

where

$$\Xi_{11ij} = \begin{bmatrix} Q_{di}^{11} - He(Z_{d1i}) & Q_{di}^{12} - N_i - M_{d1i}{}^* \\ * & Q_{di}^{22} - He(N_i) \end{bmatrix},$$

$$\Xi_{13ij} = \begin{bmatrix} Z_{d1i}B_{dij} + \hat{B}_{fij}D_{dij} \\ M_{d1i}B_{dij} + \hat{B}_{fij}D_{dij} \end{bmatrix},$$

$$\Xi_{12ij} = \begin{bmatrix} P_i^{d11} + P_{di}^{11} + Z_{d1i}A_{ij} + \hat{B}_{fij}C_{ij} - Z_{d2i}^* & \hat{A}_{fij} - M_{d2i}^* \\ M_{d1i}A_{ij} + \hat{B}_{fij}C_{ij} - N_i^* & P_i^{d22} + P_{di}^{22} + \hat{A}_{fij} - N_i^* \end{bmatrix},$$

$$\Xi_{22ij} = \begin{bmatrix} \alpha P_i^{d11} - \varpi_h^2 Q_{di}^{11} + He(Z_{d2i}A_{ij} + \hat{B}_{fij}C_{ij}) & -\varpi_h^2 Q_{di}^{12} + \hat{A}_{fij} + A_{ij}^* M_{d2i}^* + C_{ij}^* \hat{B}_{fij}^* \\ * & \alpha P_i^{d22} - \varpi_h^2 Q_{di}^{22} + He(\hat{A}_{fij}) \end{bmatrix},$$

$$\Xi_{23ij} = \begin{bmatrix} Z_{d2i}B_{dij} + \hat{B}_{fij}D_{dij} \\ M_{d2i}B_{dij} + \hat{B}_{fij}D_{dij} \end{bmatrix},$$

$$\Xi_{24ij} = [0 \quad C_{fij}]^*,$$

$$\Xi_{33ij} = -\gamma^2 I,$$

$$\Xi_{44ij} = -I.$$

Then $A_{fij} = N_i^{-1}\hat{A}_{fij}$, $B_{fij} = N_i^{-1}\hat{B}_{fij}$, and C_{fij} are the parameters of the filter (3).

Proof Theorem 3 is proved by using a similar method to Theorem 4. □

3.3 Stability condition (S.3)

Considering the performance (S.3), because of Assumption 1, we only have to provide some conditions to guarantee the stability of fault detection filter (3).

Theorem 5 *For a given positive scalar α, the fault detection filter in the form of* (3) *is stable with average dwell time $T > T^* = \frac{\ln \mu}{\alpha}$, if there exist matrices $P_i = \begin{bmatrix} P_i^{11} & 0 \\ 0 & P_i^{22} \end{bmatrix} > 0$, $i = 1, \dots, r$, matrices $\hat{A}_{fij}$, $\hat{B}_{fij}$, C_{fij}, M_{1i}, M_{2i}, Z_{1i}, Z_{2i}, and N, $i = 1, \dots, r$, and a scalar $\mu > 1$ such that the following inequalities hold*:

$$P_i \leq \mu P_j, \quad i \neq j, \tag{69}$$

$$\begin{bmatrix} \Xi_{11ij} & \Xi_{12ij} & \Xi_{13ij} \\ * & \Xi_{22ij} & \Xi_{23ij} \\ * & * & \Xi_{33ij} \end{bmatrix} < 0, \tag{70}$$

where

$$\Xi_{11ij} = \begin{bmatrix} -He(Z_{1i}) & -N_i - M_{1i}{}^* \\ * & -He(N_i) \end{bmatrix},$$

$$\Xi_{12ij} = \begin{bmatrix} P_i^{11} + Z_{1i}A_{ij} + \hat{B}_{fij}C_{ij} - Z_{2i}^* & \hat{A}_{fij} - M_{2i}^* \\ M_{1i}A_{ij} + \hat{B}_{fij}C_{ij} - N_i^* & P_i^{22} + \hat{A}_{fij} - N_i^* \end{bmatrix},$$

$$\Xi_{22ij} = \begin{bmatrix} \alpha P_i^{11} + He(Z_{2i}A_{ij} + \hat{B}_{fij}C_{ij}) & \hat{A}_{fij} + A_{ij}^* M_{2i}^* + C_{ij}^* \hat{B}_{fij}^* \\ * & \alpha P_i^{22} + He(\hat{A}_{fij}) \end{bmatrix}.$$

So $A_{fij} = N_i^{-1}\hat{A}_{fij}$, $B_{fij} = N_i^{-1}\hat{B}_{fij}$, and C_{fij} are the parameters of the filter (3).

Proof First, with a similar method to the one used in proving Theorem 3, we have

$$P_i^d \le \mu P_j^d, \quad i \ne j, \tag{71}$$

$$\begin{bmatrix} \Delta_{11i} & \Delta_{12i} & \Delta_{13i} \\ * & \Delta_{22i} & \Delta_{23i} \\ * & * & \Delta_{33i} \end{bmatrix} \le 0, \tag{72}$$

where

$$\Delta_{11i} = \begin{bmatrix} Q_{di}^{11} - He(Z_{d1i}) & Q_{di}^{12} - N_i - M_{d1i}^* \\ * & Q_{di}^{22} - He(N_i) \end{bmatrix},$$

$$\Delta_{13i} = \begin{bmatrix} Z_{d1i}B_{di}(h) + \hat{B}_{fi}(h)D_{di}(h) \\ M_{d1i}B_{di}(h) + \hat{B}_{fi}(h)D_{di}(h) \end{bmatrix},$$

$$\Delta_{12i} = \begin{bmatrix} P_{di}^{11} + Z_{d1i}A_i(h) + \hat{B}_{fi}(h)C_i(h) - Z_{d2i}^* & \hat{A}_{fi}(h) - M_{d2i}^* \\ M_{d1i}A_i(h) + \hat{B}_{fi}(h)C_i(h) - N_i^* & P_{di}^{22} + \hat{A}_{fi}(h) - N_i^* \end{bmatrix},$$

$$\Delta_{22i} = \begin{bmatrix} -\varpi_h^2 Q_{di}^{11} + He(Z_{d2i}A_i(h) + \hat{B}_{fi}(h)C_i(h)) \\ * \\ \end{bmatrix.$$

$$\left.\begin{matrix} \varpi_h^2 Q_{di}^{12} + \hat{A}_{fi}(h) + A_i^*(h)M_{d2i}^* + C_i^*(h)\hat{B}_{fi}^*(h) \\ -\varpi_h^2 Q_{di}^{22} + He(\hat{A}_{fi}(h)) \end{matrix}\right],$$

$$\Delta_{23i} = \begin{bmatrix} Z_{d2i}B_{di}(h) + \hat{B}_{fi}(h)D_{di}(h) \\ M_{d2i}B_{di}(h) + \hat{B}_{fi}(h)D_{di}(h) \end{bmatrix},$$

$$\Delta_{33i} = -\gamma^2 I.$$

Equation (71) is equivalent to

$$\begin{aligned} \dot{V}(t) &= \dot{\eta}^*(t)P_\sigma \eta(t) + \eta^*(t)P_\sigma \dot{\eta}(t) \\ &= \eta^*(t)\big(\bar{A}_\sigma^*(h)P_\sigma + P_\sigma \bar{A}_\sigma(h)\big)\eta(t) \\ &= \eta^*(t)\begin{bmatrix} \bar{A}_\sigma(h) \\ I \end{bmatrix}^* \begin{bmatrix} 0 & P_\sigma \\ P_\sigma & 0 \end{bmatrix}\begin{bmatrix} \bar{A}_\sigma(h) \\ I \end{bmatrix}\eta(t), \end{aligned} \tag{73}$$

$$V_i \le \mu V_j, \quad i \ne j, \tag{74}$$

$$\begin{bmatrix} \bar{A}_i(h) \\ I \end{bmatrix}^* \begin{bmatrix} 0 & P_i \\ * & \alpha P_i \end{bmatrix}\begin{bmatrix} \bar{A}_i(h) \\ I \end{bmatrix} < 0. \tag{75}$$

Next, from (75) and (76) we can obtain

$$\dot{V}_i(t) + \alpha V_i(t) = \eta(t)^* \begin{bmatrix} \bar{A}_i(h) \\ I \end{bmatrix}^* \begin{bmatrix} 0 & P_i \\ * & \alpha P_i \end{bmatrix} \begin{bmatrix} \bar{A}_i(h) \\ I \end{bmatrix} \eta(t) < 0 \tag{76}$$

and

$$V_{i_k}(t) \le \mu e^{-\alpha(t-t_k)} V_{i_{k-1}}\left(t_k^-\right)$$
$$\le e^{-(\alpha - \ln \mu/T)} V_{i_0}(t_0). \tag{77}$$

Obviously, we have

$$V(t) \ge a\left\|\widetilde{\eta}_1(t)\right\|^2, \qquad V(0) \le b\left\|\widetilde{\eta}_1(0)\right\|^2. \tag{78}$$

So we obtain

$$\left\|\widetilde{\eta}_1(t)\right\|^2 \le 1/a V(t) \le 1/a e^{-(\alpha - In\mu/T)} V(t_0) \le B/a e^{-(\alpha - In\mu/T)} \left\|\widetilde{\eta}_1(0)\right\|^2. \tag{79}$$

After the defuzzification, $A_{fij} = N_i^{-1}\hat{A}_{fij}$, $B_{fij} = N_i^{-1}\hat{B}_{fij}$, and C_{fij} are the parameters of the filter (3).

Therefore, with Lemma 2 the filter (3) is stable. □

3.4 FD filter parameters

Now, based on Theorems 3, 4, and 5, when γ, α, and μ are given, we can derive the FD filter parameters by solving the optimization problem (11):

$$\max \beta$$
$$\text{s.t. } (50), (51), (52), (53), (54), (66), (67), (68), (69), (70), (71). \tag{80}$$

We have $A_{fij} = N_i^{-1}\hat{A}_{fij}$, $B_{fij} = N_i^{-1}\hat{B}_{fij}$.

Then the filters which we need are defined by the gain matrices A_{fij}, B_{fij}, C_{fij}.

4 Detection threshold design

The next step is to evaluate the residual signal and compare it with some threshold value to detect the fault in the system. In this section, the threshold for detecting faults is designed and the detection logic unit is based on the results proposed in [43].

The evaluation function based on the RMS energy of the residual signal is used in this paper. So we have

$$J_r(t) = \|r\|_{rms} := \sqrt{\frac{1}{t} \int_0^t r^T(\tau) r(\tau)\, d\tau}. \tag{81}$$

The threshold J_{th} is obtained by

$$J_{th} = \sup_{f(t)=0, d(t)\in L_2,} J_r(t). \tag{82}$$

Finally the occurrence of a fault can be detected by the following logic rule:

$$J_r > J_{th} \quad \Rightarrow \quad \text{alarm},\tag{83}$$

$$J_r \leq J_{th} \quad \Rightarrow \quad \text{no faults}.\tag{84}$$

Remark 4 A new fault detection scheme has been developed for switched T-S fuzzy systems such that the generated residual is designed to be sensitive to fault signals for the faulty cases, while it is robust against the disturbances for the fault-free case. Since the fault signals and disturbance inputs are considered to be in a finite frequency domain, an additional slack matrix variable Q in Lemma 4 has been introduced to reduce the conservatism. Therefore, compared to the method [32] in the full frequency domain, it is shown that the generated residuals are more sensitive to fault signals and more robust against the disturbances, and hence, the faults are easier to detect. However, it should be pointed out that the LMIs solved in (81) could be very complex, which may make the computation very costly. The degree of complexity depends on the dimensions of the considered systems, the number of switching modes, and the number of IF-THEN rules in T-S fuzzy systems.

5 Example

In this section, two examples are presented to illustrate the effectiveness of the fault detection filter design method proposed in this paper.

Example 1 Consider the switched T-S fuzzy system (1) with the following parameters:

Region 1:

Subsystems 1:

$$A_{11} = \begin{bmatrix} -11.5 & 4 \\ -0.843 & -1.0493 \end{bmatrix}, \qquad B_{11} = \begin{bmatrix} -10.01 \\ -0.2946 \end{bmatrix}, \qquad C_{11} = \begin{bmatrix} 2 \\ -1 \end{bmatrix},$$

$$B_{d11} = \begin{bmatrix} 0.4 \\ 0.1 \end{bmatrix}, \qquad D_{d11} = [1].$$

Subsystems 2:

$$A_{12} = \begin{bmatrix} -4.8 & 1 \\ -0.232 & -1.4312 \end{bmatrix}, \qquad B_{12} = \begin{bmatrix} -10.2 \\ -0.1516 \end{bmatrix}, \qquad C_{12} = \begin{bmatrix} -0.8 \\ 0.1 \end{bmatrix},$$

$$B_{d12} = \begin{bmatrix} -0.5 \\ -0.1 \end{bmatrix}, \qquad D_{d12} = [1].$$

Region 2:

Subsystems 1:

$$A_{21} = \begin{bmatrix} -10 & 0.6 \\ -0.6 & -0.8 \end{bmatrix}, \qquad B_{21} = \begin{bmatrix} -3.4 \\ -1.7 \end{bmatrix}, \qquad C_{21} = \begin{bmatrix} 5 \\ -1 \end{bmatrix},$$

$$B_{d21} = \begin{bmatrix} 0.2 \\ -0.2 \end{bmatrix}, \qquad D_{d21} = [1].$$

Subsystems 2:

$$A_{22} = \begin{bmatrix} -12 & 1.2 \\ -2 & -4.529 \end{bmatrix}, \qquad B_{22} = \begin{bmatrix} -1.2 \\ -0.1765 \end{bmatrix}, \qquad C_{22} = \begin{bmatrix} 1.6 \\ 4 \end{bmatrix},$$

$$B_{d22} = \begin{bmatrix} 0.2 \\ 0.2 \end{bmatrix}, \qquad D_{d22} = [1].$$

Choose $\mu = 1.2, \alpha = 0.2$. Then using the method proposed in this paper, the gain matrices of the fault detection filter are as follows.

Region 1:

Subsystems 1:

$$A_{f11} = \begin{bmatrix} -0.8642 & 0.7609 \\ 0.1323 & -1.1915 \end{bmatrix}, \qquad B_{f11} = \begin{bmatrix} 0.5108 \\ 0.0338 \end{bmatrix}, \qquad C_{f11} = \begin{bmatrix} 54.3567 \\ 0.7044 \end{bmatrix}.$$

Subsystems 2:

$$A_{f12} = \begin{bmatrix} -1.0693 & -0.8440 \\ -0.0042 & -1.9275 \end{bmatrix}, \qquad B_{f12} = \begin{bmatrix} -0.6602 \\ -0.0464 \end{bmatrix}, \qquad C_{f12} = \begin{bmatrix} 54.5732 \\ 1.6529 \end{bmatrix}.$$
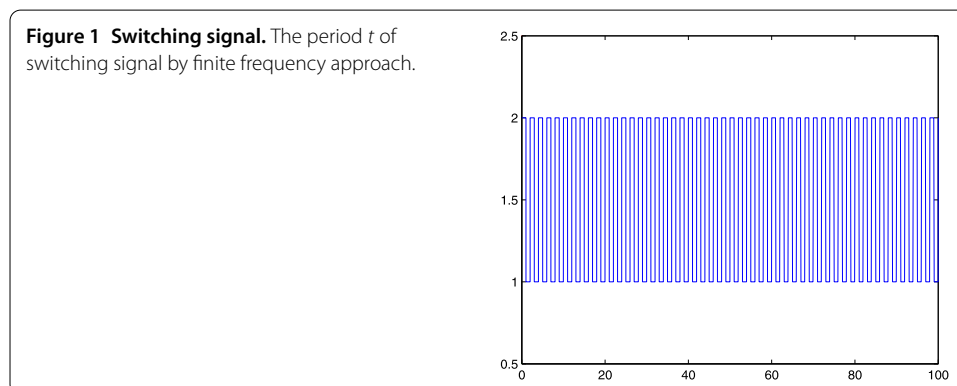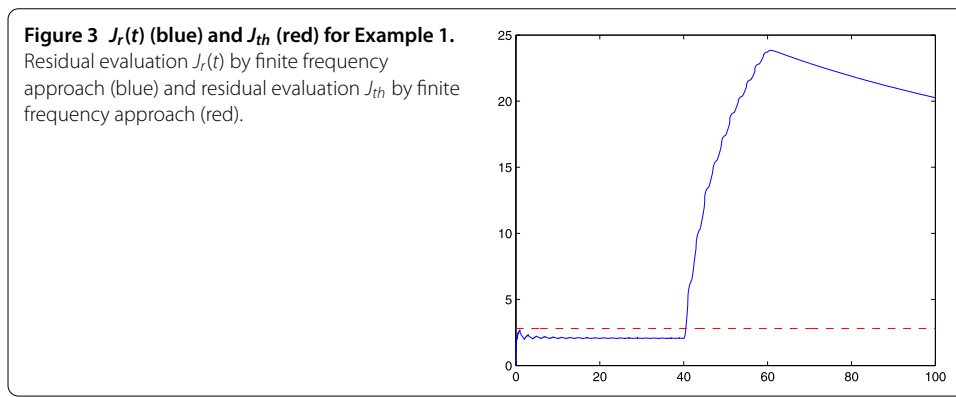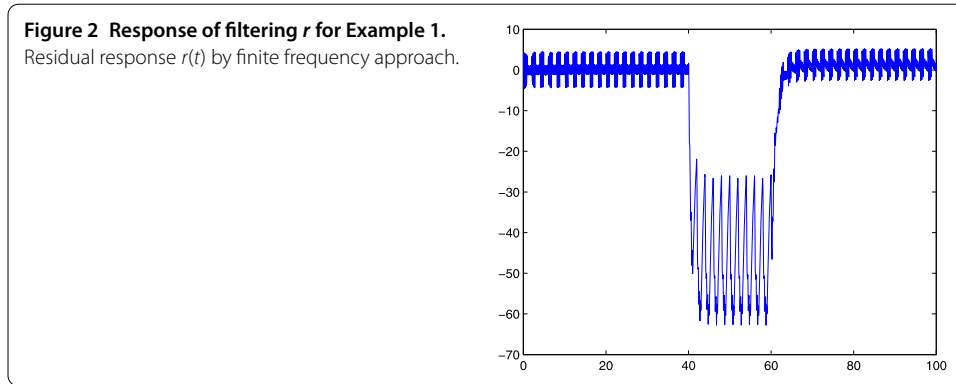
Region 2:

Subsystems 1:

$$A_{f21} = \begin{bmatrix} -1.5063 & 0.7571 \\ 0.0175 & -1.3025 \end{bmatrix}, \qquad B_{f21} = \begin{bmatrix} 1.7794 \\ 0.3428 \end{bmatrix}, \qquad C_{f21} = \begin{bmatrix} 54.2551 \\ -1.7895 \end{bmatrix}.$$

Subsystems 2:

$$A_{f22} = \begin{bmatrix} -4.4844 & -1.0169 \\ -0.5034 & -1.6580 \end{bmatrix}, \qquad B_{f22} = \begin{bmatrix} -0.4740 \\ -0.1913 \end{bmatrix}, \qquad C_{f22} = \begin{bmatrix} 55.1978 \\ 1.1419 \end{bmatrix}.$$

In a simulation, for $t \in [0, 100]$, the disturbance is chosen as $d(t) = 3\cos(7\pi t)$, and the fault occurs from 40 s to 60 s and satisfies $f(t) = 1$ for $40 < t < 60$. The switch signal with average dell time $T^* = \frac{\ln \mu}{\alpha} = 0.9116$ is shown in Figure 1.

Figure 1 Switching signal. The period t of switching signal by finite frequency approach.

Figure 2 Response of filtering *r* for Example 1.
Residual response *r(t)* by finite frequency approach.

Figure 3 $J_r(t)$ (blue) and J_{th} (red) for Example 1.
Residual evaluation $J_r(t)$ by finite frequency approach (blue) and residual evaluation J_{th} by finite frequency approach (red).

Then we apply the proposed finite frequency method to a switched T-S fuzzy system (1). The residual $r(t)$ of the FD filter and the corresponding residual evaluations $J_r(t)$ are shown in Figures 2 and 3. The fault threshold is set as $J_{th} = 2.8$. It is clear that the residual $r(t)$ in Figure 2 generated by proposed method is sensitive to the fault and robust against the disturbance, and according to (83) and (84), the residual evaluation function $J_r(t)$ in Figure 3 indicates that the fault is detected at approximately $t = 40.51$ s.

Example 2 To further illustrate the proposed results, we apply the proposed FD method to a chemical process example [44]. We consider a continuous stirred tank reactor where an irreversible process occurs. For each type of operation, the mathematical model for the process takes the form

$$\dot{C}_A = \frac{F_\sigma}{V}(C_{A_\sigma} - C_A) - k_0 e^{-E/RT_R} C_A,$$

$$\dot{T}_R = \frac{F_\sigma}{V}(T_{A_\sigma} - T_A) + \frac{-\Delta H}{\rho c_p} k_0 e^{-E/RT_R} C_A + \frac{Q_\sigma}{\rho c_p V}, \tag{85}$$

where CA denotes the concentration of the species A, T_R denoted the temperature of the reactor, Q_σ is the heat removed from the reactor, V is the volume of the reactor, k_0, E, ΔH are the pre-exponential constant, the activation energy, and enthalpy of the reaction, c_p and ρ are the heat capacity and fluid density in the reactor and $\sigma(t) \in \{1, 2\}$ is a discrete variable. The values of all process parameters can be found in [44].

By using the method in [44], we can rewrite (85) in the form of system (1) with following parameters:

$$A_{f11} = \begin{bmatrix} -4.5803 \times 10^{-2} & 6.6748 \times 10^{-5} \\ 2.4807 & -3.61 \times 10^{-3} \end{bmatrix}, \qquad A_{f12} = \begin{bmatrix} -3.5728 & 5.1826 \times 10^{-5} \\ 707.89 & -0.010286 \end{bmatrix},$$

$$A_{f21} = \begin{bmatrix} -0.029103 & 5.1833 \times 10^{-5} \\ 2.4807 & -0.0036045 \end{bmatrix},$$

$$A_{f22} = \begin{bmatrix} -4.5803 * 1e{-}2 & 6.6748 \times 10^{-5} \\ 706.14 & -0.010265 \end{bmatrix},$$

$$B_{11} = B_{12} = B_{21} = B_{22} = B_{d11} = B_{d12} = B_{d21} = B_{d22} = B = \begin{bmatrix} 4.1841 \times 10^{-2} \\ 0 \end{bmatrix},$$

$$C_{11} = C_{12} = C_{21} = C_{22} = C = [1 \quad 0], \qquad D_{11} = D_{12} = D_{21} = D_{22} = 0.$$

To ensure the system stability, it is assumed that the output feedback controller $u = Ky(t)$ has been designed beforehand. By using the common Lyapunov function method and solving LMIs, we can obtain the controller parameters as $K = -71.0535$. Therefore, the closed-loop system with the fault is in the following form:

$$\dot{x}(t) = \big(A_\sigma(h) + BKC\big)x(t) + B_{d\sigma}(h)d(t) + B_\sigma(h)f(t),$$
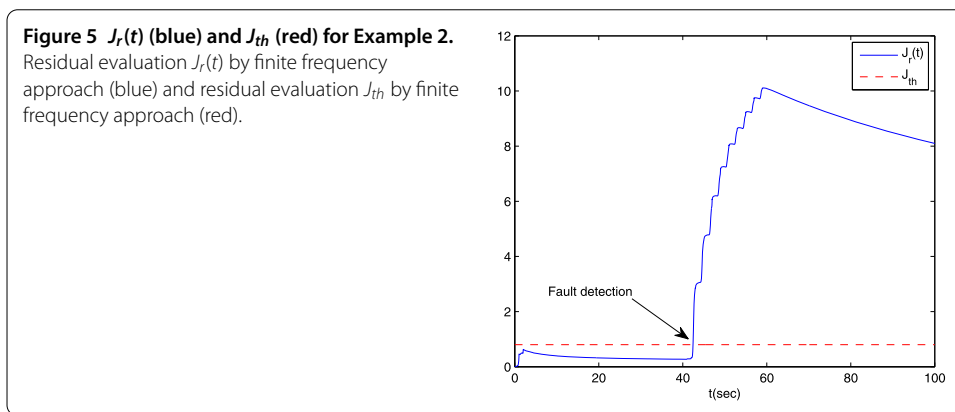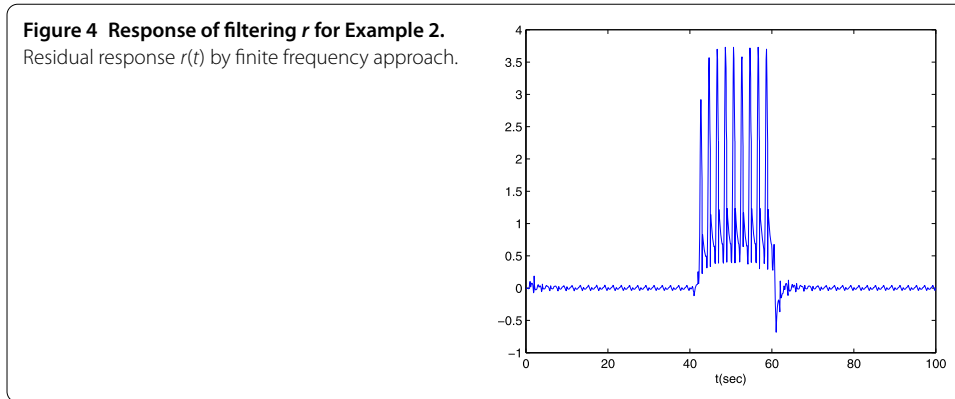
$$y(t) = Cx(t). \tag{86}$$

Then using the method proposed in this paper, for given $\gamma = 0.2$ and choosing the same parameters μ and T^* as Example 1, we obtain the gain matrices of the fault detection filter:

$$A_{f11} = \begin{bmatrix} -0.9490 & -0.0227 \\ 0.5654 & -2.0963 \end{bmatrix}, \qquad A_{f12} = \begin{bmatrix} 0.0329 & -1.0361 \\ 81.3220 & -85.0437 \end{bmatrix},$$

$$A_{f21} = \begin{bmatrix} -7.1351 & 2.8578 \\ 3.4565 & -4.8024 \end{bmatrix}, \qquad A_{f22} = \begin{bmatrix} -13.5811 & 7.6217 \\ 9.3433 & -10.2626 \end{bmatrix},$$

$$B_{f11} = \begin{bmatrix} -0.5392 \\ 14.6825 \end{bmatrix}, \qquad B_{f12} = \begin{bmatrix} 0.4506 \\ -19.8973 \end{bmatrix},$$

$$B_{f21} = \begin{bmatrix} -2.3287 \\ -2.0118 \end{bmatrix}, \qquad B_{f22} = \begin{bmatrix} 11.8345 \\ -11.4497 \end{bmatrix},$$

$$C_{f11} = [0.3391 \quad -0.0428], \qquad C_{f12} = [0.2573 \quad 0.0974],$$

$$C_{f21} = [0.2824 \quad -0.0223], \qquad C_{f22} = [0.2985 \quad -0.0299],$$

and the optimal value on fault sensitivity performance index is $\beta = 1.1$.

In simulation, the switching signal is chosen the same as Example 1. The fault is set as $f(t) = 1$ occurring after $t = 40$ s, and the disturbance is set as $d(t) = 5\sin(t)$.

The simulations are shown in Figures 4 and 5. It is clear that the residual $r(t)$ in Figure 4 generated by proposed method is sensitive to a fault and robust against the disturbance,

Figure 4 Response of filtering *r* for Example 2.
Residual response $r(t)$ by finite frequency approach.

Figure 5 $J_r(t)$ (blue) and J_{th} (red) for Example 2.
Residual evaluation $J_r(t)$ by finite frequency approach (blue) and residual evaluation J_{th} by finite frequency approach (red).

and according to (83) and (84), the residual evaluation function $J_r(t)$ in Figure 5 indicates that the fault is detected at approximately $t = 42.44$ s.

6 Conclusions

In this paper, we have studied the fault detection problem for switched T-S fuzzy systems in finite frequency. Based on Parseval's lemma and S-procedure, we have obtained some sufficient conditions which ensure that the augmented filter system has the H_- fault affection level, the H_∞ disturbance attention level, and we have stability. The fault detection filter design conditions have been derived in terms of solving a set of LMIs. Two numerical examples have been provided to demonstrate the effectiveness of the proposed method.

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References

1. Hwang, I, Kim, S, Kim, Y, Seah, CE: A survey of fault detection, isolation, and reconfiguration methods. IEEE Trans. Control Syst. Technol. **18**, 636-653 (2010)
2. Li, XJ, Yang, GH: Fault detection for T-S fuzzy systems with unknown membership functions. IEEE Trans. Fuzzy Syst. **22**, 139-152 (2014)
3. Lsennann, R: Model-based fault detection and diagnosis status and applications. Annu. Rev. Control **29**, 71-85 (2005)
4. Nguang, SK, Shi, P, Ding, S: Fault detection for uncertain fuzzy systems: an LMI approach. IEEE Trans. Fuzzy Syst. **15**, 1251-1262 (2007)
5. Khosrowjerdi, MJ, Nikoukhah, R, Safari-Shad, N: A mixed H_2/H_∞ approach to simultaneous fault detection and control. Automatica **40**, 261-267 (2004)
6. Gertler, J: A survey of model-based failure detection and isolation in complex plants. IEEE Control Syst. Mag. **8**, 3-11 (1988)
7. Xiao, N, Gao, ZW: Robust fault detection for descriptor Markovian jump systems. In: Proceedings of IEEE International Conference on Control and Automation, pp. 835-840 (2007)
8. Wu, L, Lam, J: Weighted H_∞ filtering of switched systems with time-varying delay: average dwell time approach. Circuits Syst. Signal Process. **28**, 1017-1036 (2009)
9. Takagi, T, Sugeno, M: Fuzzy identification of systems and its applications to modelling and control. IEEE Trans. Syst. Man Cybern. **15**, 116-132 (1985)
10. De Souza, WA, Teixeira, MCM, Cardim, R, Assuncao, E: On switched regulator design of uncertain nonlinear systems using Takagi-Sugeno fuzzy models. IEEE Trans. Fuzzy Syst. **22**, 1720-1727 (2014)
11. Zhao, XD, Zhang, LX, Shi, P, Karimi, HR: Novel stability criteria for T-S fuzzy systems. IEEE Trans. Fuzzy Syst. **22**, 313-323 (2014)
12. Sofianos, NA, Boutalis, YS: Stable indirect adaptive switching control for fuzzy dynamical systems based on T-S multiple models. Int. J. Syst. Sci. **44**, 1546-1565 (2013)
13. Sofianos, NA, Boutalis, YS: Robust adaptive multiple models based fuzzy control of nonlinear systems. Neurocomputing **173**, 1733-1742 (2016)
14. Zhao, Y, Lam, J, Gao, HJ: Fault detection for fuzzy systems with intermittent measurements. IEEE Trans. Fuzzy Syst. **17**, 398-410 (2009)
15. Ding, WG, Mao, ZH, Jiang, B, Shi, P, He, X: H_∞ fault detection for a class of T-S fuzzy model-based nonlinear networked control systems. In: Proceedings of the 19th IFAC World Congress, pp. 11647-11652 (2014)
16. Yang, HJ, Li, X, Liu, ZX, Hua, CC: Fault detection for uncertain fuzzy systems based on the delta operator approach. Circuits Syst. Signal Process. **33**, 733-759 (2014)
17. Li, HY, Gao, YB, Wu, LG, Lam, HK: Fault detection for T-S fuzzy time-delay systems: delta operator and input-output methods. IEEE Trans. Cybern. **45**, 229-241 (2015)
18. Li, XJ, Yang, GH: Fault detection for switched linear parameter-varying systems: an average dwell-time approach. IET Control Theory Appl. **7**, 1120-1130 (2013)
19. Ichtev, A, Hellendoorn, J, BabuSka, R: Fault detection and isolation using multiple Takagi-Sugeno fuzzy models. In: The 10th IEEE International Conference on Fuzzy Systems, pp. 1498-1502 (2001)
20. Yang, GH, Wang, HM: Fault detection and isolation for a class of uncertain state-feedback fuzzy control systems. IEEE Trans. Fuzzy Syst. **23**, 139-151 (2015)
21. Iwasaki, T, Hara, S: Generalized KYP lemma: united frequency domain inequalities with design applications. IEEE Trans. Autom. Control **50**, 41-59 (2005)
22. Zhou, K, Doyle, J: Essentials of Robust Control. Prentice Hall, Englewood Cliffs (1998)
23. Yang, GH, Wang, H: Fault detection for a class of uncertain state-feedback control systems. IEEE Trans. Control Syst. Technol. **18**, 201-212 (2010)
24. Yang, HJ, Xia, YQ, Zhang, JH: Generalised finite-frequency KYP lemma in delta domain and applications to fault detection. Int. J. Control **84**, 511-525 (2011)
25. Zhai, D, Li, JH, Zhang, QL: Fault detection for singular multiple time-delay systems with application to electrical circuit. J. Franklin Inst. **351**, 5411-5436 (2014)
26. Zhai, D, Lu, AY, Li, JH, Zhang, QL: Robust H_∞ control for switched singular linear systems with uncertainties in the derivative matrices: an improved average dwell time approach. Optim. Control Appl. Methods (2015). doi:10.1002/oca.2179
27. Yang, H, Liu, B: Fault detection for T-S fuzzy discrete systems in finite-frequency domain. IEEE Trans. Syst. Man Cybern., Part B, Cybern. **41**, 911-920 (2011)
28. Zhao, XG, Li, J, Ye, D: Fault detection for switched systems with finite-frequency specifications. Nonlinear Dyn. **70**, 409-420 (2012)
29. Zhai, D, Lu, AY, Li, JH, Zhang, QL: Fault detection for singular switched linear systems with multiple time-varying delay in finite frequency domain. Int. J. Syst. Sci. (2015). doi:10.1080/00207721.2015.1112932
30. Zhai, D, Lu, AY, Li, JH, Zhang, QL: Simultaneous fault detection and control for switched linear systems with mode-dependent averaged dwell-time. Appl. Math. Comput. **273**, 767-792 (2016)
31. Zhai, D, An, LW, Li, JH, Zhang, QL: Fault detection for stochastic parameter-varying Markovian jump systems with application to networked control systems. Appl. Math. Model. **40**, 2368-2383 (2016)
32. Li, J, Su, QY, Wang, HM: Fault detection filter design for switched fuzzy systems with average dwell time. In: Proceeding of 25th Chinese Control and Decision Conference (CCDC), pp. 1054-1059 (2013)
33. Tanaka, K, Iwasaki, M, Wang, HO: Stability and smoothness conditions for switching fuzzy systems. In: 43rd IEEE Conference on Decision and Control, pp. 2474-2478 (2000)
34. Choi, DJ, Park, P: Guaranteed cost controller design for discrete-time switching fuzzy systems. IEEE Trans. Syst. Man Cybern., Part B, Cybern. **34**, 110-119 (2004)
35. Zhai, D, Lu, AY, Dong, JX, Zhang, QL: Dynamic output feedback H_∞ control for switched T-S fuzzy systems via discretized Lyapunov function technique. Neurocomputing **177**, 651-669 (2016)
36. Venkataramanan, V, Chen, BM, Lee, TH, Guo, G: A new approach to the design of mode switching control in hard disk drive servo systems. Control Eng. Pract. **10**, 925-939 (2002)

37. Zhao, XD, Yin, YF, Niu, B, Zheng, XL: Stabilization for a class of switched nonlinear systems with novel average dwell time switching by T-S fuzzy modeling. IEEE Trans. Cybern. (2015). doi:10.1109/TCYB.2015.2458896
38. Iwasaki, T, Hara, S, Fradkov, AL, LoConte, L, Sillitoe, I: Restricted frequency inequality is equivalent to restricted dissipativity. In: 43rd IEEE Conference on Decision and Control, pp. 426-431 (2004)
39. Gahinet, P, Apkarian, P: An linear matrix inequality approach to H_∞ control. Int. J. Robust Nonlinear Control **4**, 422-448 (1994)
40. Zhou, L, Ho, DWC, Zhai, GS: Stability analysis of switched linear singular systems. Automatica **49**, 1481-1487 (2013)
41. Tseng, CS, Chen, BS: $L_\infty Z$ gain fuzzy control for nonlinear dynamic systems with persistent bounded disturbance. In: Proceedings of the 13th IEEE International Conference on Fuzzy Systems, Budapest, Hungary, Luly, pp. 783-788 (2004)
42. Casavola, A, Famularo, D, Franze, G: Robust fault detection of uncertain linear systems via quasi-LMIs. Automatica **44**, 289-295 (2008)
43. Frank, M, Ding, X: Survey of robust residual generation and evaluation methods in observer-based fault detection systems. J. Process Control **7**, 402-424 (1997)
44. Chiou, JS, Wang, CJ, Chen, CM, Wang, CC: Analysis and synthesis of switched nonlinear systems using the T-S fuzzy model. Appl. Math. Model. **34**, 1467-1481 (2010)