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Numerical solution of Korteweg-de Vries-Burgers equation by the compact-type CIP method

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Abstract

In this paper, a hybrid compact-CIP scheme is proposed to solve Korteweg-de Vries-Burgers equation. The nonlinear advective terms are computed based on the classical constrained interpolation profile (CIP) method, which is coupled with a high-order compact scheme for third-order derivatives in Korteweg-de Vries-Burgers equation. The strong stability preserving third-order Runge-Kutta time discretizations is adopted in this work. A test case is presented to demonstrate the high-resolution properties of the proposed compact-CIP scheme.

Keywords: high-order compact schemes; CIP schemes; Korteweg-de Vries-Burgers equation

1 Introduction

In 1895, Korteweg and de Vries [1] developed the Korteweg de Vries (KdV) equation to model weakly nonlinear waves. It has been used in several different fields to describe various physical phenomena of interest. The KdV-Burgers (KdVB) equation which is derived by Su and Gardner [2] appears in the study of the weak effects of dispersion, dissipation, and nonlinearity in waves propagating in a liquid-filled elastic tube. Recently, the nonlinear fractional partial differential equations, such as fractional KdV-Burgers equation [3], fractional Schrödinger-Korteweg-de Vries equations [4] and fractional Burgers' equations [5], were also presented to describe many important phenomena and dynamic processes in physics. Some theoretical issues concerning the KdVB equation, such as the traveling wave solution, have received considerable attention [6]. A number of exact solitary wave solutions to KdVB equations have been found in the past few years. The exact solutions of a compound KdVB equation were obtained by using a homogeneous balance method in [7]. By using the special truncated expansion method, Hassan [8] constructed solitary wave solutions for the compound KdVB equation and discussed the generalized two-dimensional KdVB equation. The Exp-function method is applied to obtain generalized solitary solutions and periodic solutions for the KdVB equation in [9]. In the past several decades, many authors have paid attention to studying the numerical methods for solving KdVB equations. Soliman extended the variational iterations method to solve the KdVB equations [10]. A new decomposition method was presented to find the explicit and numerical solutions of the KdVB equations without any transformations, linearization or weak nonlinearity assumptions in [11]. The element-free Galerkin (EFG) method for numerically solving the compound KdVB equation was discussed by Rong-Jun and Yu-Min in [12]. The explicit restrictive Taylor approximation (RTA) was implemented to find numerical solution of KdV-Burgers in [13]. Nonlinear dispersive wave propagation problems that described the KdVB equations in [14] were simulated by high-order compact finite difference schemes coupled with high-order low-pass filter and the classical fourth-order Runge-Kutta scheme.

In 1992, based on implicit interpolations, high-order compact (HOC) difference schemes for different derivatives were developed by Lele [15]. These implicit schemes were very accurate in smooth regions, and they have spectral-like resolution properties by using the global grid. Li and Visbal applied the compact schemes coupled with high-order low-pass filter for solving KdV-Burgers equations in [14]. In the past few years, it has been popular for using the less diffusive and less oscillating CIP scheme which was developed by Takewaki *et al.* [16] to solve hyperbolic equation. The classical CIP schemes which were essentially written as the semi-Lagrangian formulation were low-diffusion and stable. The scheme can solve hyperbolic equations with third-order accuracy in space [17]. However, the original CIP method [16, 18–20] utilizes auxiliary boundary conditions for the spatial gradient information. Usually, in order to get the values of derivation on the node, it has to differentiate the equation with spatial variable. The procedure is easy while the velocity is constant, but it is difficult for complex equations. By using the compact scheme for the values of derivation on the nodes, we present a new compact scheme based on the characteristic method for solving KdV-Burgers equations.

In this paper, a new numerical method named compact-type CIP schemes based on combination of CIP and high-order compact schemes is advanced to solve the KdV-Burgers equations. The present scheme is mainly based on the idea of characteristic method; as a new ingredient, the high-order compact scheme is employed to obtain the derivatives rather than differentiate the equation with spatial variable to construct a CIP scheme, and then resolution properties can also be obtained. By comparing with the classical compact scheme for solving KdV-Burgers equations, no filter is used to overcome non-physical oscillations.

The remainder of the paper is organized as follows. In Section 2, CIP is described in brief, then high-order compact schemes are given. The numerical algorithm of the present schemes is described in this section. The merit of our present method for KdVB equation is displayed in Section 3, a comparison of numerical solutions with exact solutions is carried out to illustrate the capability of the method for nonlinear dispersive equations. At last, a short discussion of the present method is given in Section 4.

2 Descriptions of methods

In this paper, we consider the following generalized KdV-Burgers equation [8]:

$$u_t + (\alpha + \beta u)uu_x + \gamma u_{xx} - \delta u_{xxx} = 0, \tag{2.1}$$

where α, β, γ and δ are real constants. The equation can be split into two parts

$$u_t + a(u)u_x = 0, \tag{2.2}$$

$$u_t = -\gamma u_{xx} + \delta u_{xxx}, \tag{2.3}$$

where $a(u) = (\alpha + \beta u)u$.

2.1 The CIP method

In this section, we review the CIP method briefly. The CIP method in [19] uses cubic-polynomial interpolation to get the values of function on nodes. The primary goal of the numerical algorithm will be to retrieve the lost information inside the grid cell between these digitized points. We differentiate the advective phase of equation (2.2) with the spatial variable x, then we get [21]

$$\frac{\partial g}{\partial t} + a(u)\frac{\partial g}{\partial x} = -g\frac{\partial a(u)}{\partial x}, \tag{2.4}$$

where $g = \partial u/\partial x$ stands for the spatial derivatives of u. For the computational domain $[a, b]$, we only consider a uniform grid with a space step $\Delta x = \frac{b-a}{N}$. If both the values of u and g are given at two grid points, the cubic polynomial at the nth step can be written as

$$U_i^n(x) = a_i X^3 + b_i X^2 + g_i^n X + u_i^n, \tag{2.5}$$

where $X = x - x_i$, and coefficients a_i, b_i, c_i and d_i will be obtained with the following constrains:

$$U_i^n(x_i) = u_i^n, \qquad U_i^n(x_{iup}) = u_{iup}^n, \tag{2.6}$$

where $iup = i - \operatorname{sgn}(a(u_i))$, the sign $\operatorname{sgn}(a(u_i))$ stands for the sign of $a(u_i)$. Then the coefficients of the cubic polynomial are given

$$\begin{aligned} a_i &= \frac{g_i^n + g_{iup}^n}{\Delta x_i^2} + \frac{2(u_i^n - u_{iup}^n)}{\Delta x_i^3}, \\ b_i &= \frac{3(u_{iup}^n - u_i^n)}{\Delta x_i^2} - \frac{2g_i^n + g_{iup}^n}{\Delta x_i}, \end{aligned} \tag{2.7}$$

where $\Delta x_i = x_{iup} - x_i$. Thus, the profile u and g at the $(n + 1)$th step can be obtained by shifting the profile by $a(u_i)\Delta t$

$$\begin{aligned} u_i^{n+1} &= U_i^n\bigl(x_i - a(u_i)\Delta t\bigr), \\ g_i^{n+1} &= U_i'^n\bigl(x_i - a(u_i)\Delta t\bigr). \end{aligned} \tag{2.8}$$

We define $\xi_i = -a(u_i)\Delta t$, then the formulates are rewritten as

$$u_i^{n+1} = a_i\xi_i^3 + b_i\xi_i^2 + g_i^n\xi_i + u_i^n, \qquad g_i^{n+1} = 3a_i\xi_i^n + 2b_i\xi_i^n + g_i^n. \tag{2.9}$$

It can be seen that we only use two points in CIP schemes to get u_i^{n+1}. Then we display the implementation of this method, while the computational boundary is complex and less boundary points need to be handled. The CIP method uses only two neighboring stencils, but keeps third-order precision. In this sense, high-order precision is gained though less computational stencils are used. For more details about CIP schemes, readers can refer to [21].

2.2 High-order compact scheme

Lele developed high-order linear compact difference schemes based on implicit interpolations in [15]. These implicit schemes are very accurate in smooth regions and have spectral-like resolution properties by using the global grid. The finite difference approximation to the derivative of the function is expressed as a linear combination of the given function values, then, by solving a tridiagonal or pentadiagonal system, the derivatives of the function can be obtained. In this section, a review of formulas for first-order, second-order and third-order derivatives is presented. For more details about the high-order compact schemes, readers can refer to [15, 22].

2.2.1 The derivatives at interior nodes

In this paper, the KdVB equation on a uniform mesh is considered, the point values and the derivatives are indicated by u_i, u_i', $i = 1, \dots, N$. For the first-order derivatives at interior nodes, we have the formula [15]

$$u_i' + \alpha\left(u_{i-1}' + u_{i+1}'\right) + \beta\left(u_{i-2}' + u_{i+2}'\right) = c\frac{u_{i+3} - u_{i-3}}{6h} + b\frac{u_{i+2} - u_{i-2}}{4h} + a\frac{u_{i+1} - u_{i-1}}{2h}. \quad (2.10)$$

If the schemes are restricted to $\beta \geq 0$ and $c = 0$, this provides a one-parameter α-family of fourth-order tridiagonal scheme with

$$\beta = 0, \qquad c = 0, \qquad a = \frac{2}{3}(\alpha + 2), \qquad b = \frac{1}{3}(4\alpha - 1). \quad (2.11)$$

A simple sixth-order tridiagonal scheme is given by the coefficients

$$\alpha = \frac{1}{3}, \qquad \beta = 0, \qquad c = 0, \qquad a = \frac{14}{9}, \qquad b = \frac{1}{9}. \quad (2.12)$$

The scheme can be rewritten as follows:

$$u_i' + \frac{1}{3}\left(u_{i-1}' + u_{i+1}'\right) = \frac{14}{9}\frac{u_{i+1} - u_{i-1}}{2h} + \frac{1}{9}\frac{u_{i+2} - u_{i-2}}{2h}. \quad (2.13)$$

For the second-order derivatives at interior nodes, we have the formula [15]

$$\begin{aligned} &u_i'' + \alpha\left(u_{i-1}'' + u_{i+1}''\right) + \beta\left(u_{i-2}'' + u_{i+2}''\right) \\ &\quad = c\frac{u_{i+3} - 2u_i + u_{i-3}}{9h^2} + b\frac{u_{i+2} - 2u_i + u_{i-2}}{4h^2} + a\frac{u_{i+1} - 2u_i + u_{i-1}}{h^2}, \end{aligned} \quad (2.14)$$

which provides a one-parameter α-family of fourth-order tridiagonal schemes with

$$\beta = 0, \qquad c = 0, \qquad a = \frac{4}{3}(1 - \alpha), \qquad b = \frac{1}{3}(-1 + 10\alpha). \quad (2.15)$$

A sixth-order tridiagonal scheme is also given with

$$\alpha = \frac{2}{11}, \qquad \beta = 0, \qquad c = 0, \qquad a = \frac{12}{11}, \qquad b = \frac{3}{11}, \quad (2.16)$$

then the sixth-order tridiagonal scheme for (2.14) can be rewritten as follows

$$u_i'' + \frac{2}{11}\left(u_{i-1}'' + u_{i+1}''\right) = \frac{3}{11}\frac{u_{i+2} - 2u_i + u_{i-2}}{4h^2} + \frac{12}{11}\frac{u_{i+1} - 2u_i + u_{i-1}}{h^2}. \quad (2.17)$$

For the third-order derivatives at interior nodes, the following formula is given in [15]:

$$\alpha\left(u'''_{i-1} + u'''_{i+1}\right) + u'''_i = b\frac{u_{i+3} - 3u_{i+1} + 3u_{i-1} - u_{i-3}}{8h^3} + a\frac{u_{i+2} - 2u_{i+1} + 2u_{i-1} - u_{i-2}}{2h^3}, \quad (2.18)$$

which provides an α-family of fourth-order tridiagonal schemes with $a = 2$, $b = 2\alpha - 1$. The simple sixth-order tridiagonal scheme is given with $\alpha = \frac{7}{16}$, $a = 2$, $b = -\frac{1}{8}$.

2.2.2 Non-periodic boundaries

For those near boundary nodes, approximation formulas for the first-order derivatives of non-periodic boundary problems are given by one-side formulation as follows [15]:

$$\begin{aligned} &u'_1 + \alpha u'_2 = \frac{1}{h}(au_1 + bu_2 + cu_3 + du_4),\\ &u'_N + \alpha u'_{N-1} = -\frac{1}{h}(au_N + bu_{N-1} + cu_{N-2} + du_{N-3}). \end{aligned} \quad (2.19)$$

The coefficients for the schemes of third- and fourth-order derivatives are given by

$$\begin{aligned} &a = -\frac{11 + 2\alpha}{6}, \quad b = \frac{6-\alpha}{2}, \quad c = \frac{2\alpha - 3}{2}, \quad d = \frac{2-\alpha}{6} \quad \text{(third order)},\\ &\alpha = 3, \quad a = -\frac{17}{6}, \quad b = \frac{3}{2}, \quad c = \frac{3}{2}, \quad d = -\frac{1}{6} \quad \text{(fourth order)}. \end{aligned} \quad (2.20)$$

The sixth-order scheme is also given, where the first and second points need to be handled. For the first point, the formula is

$$u'_1 + \alpha u'_2 = \frac{1}{h}(a_1u_1 + a_2u_2 + a_3u_3 + a_4u_4 + a_5u_5 + a_6u_6), \quad (2.21)$$

where

$$\begin{aligned} &\alpha = 5, \quad a_1 = -\frac{197}{60}, \quad a_2 = -\frac{12}{5}, \quad a_3 = 5,\\ &a_4 = -\frac{5}{3}, \quad a_5 = \frac{5}{12}, \quad a_6 = -\frac{1}{20}. \end{aligned} \quad (2.22)$$

For the second point, the formula is

$$\alpha u'_1 + u'_2 + \alpha u'_3 = \frac{1}{h}(b_1u_1 + b_2u_2 + b_3u_3 + b_4u_4 + b_5u_5 + b_6u_6), \quad (2.23)$$

where

$$\begin{aligned} &\alpha = \frac{2}{11}, \quad b_1 = -\frac{20}{33}, \quad b_2 = -\frac{35}{132}, \quad b_3 = \frac{34}{33},\\ &b_4 = -\frac{7}{33}, \quad b_5 = \frac{2}{33}, \quad b_6 = -\frac{1}{132}. \end{aligned} \quad (2.24)$$

The dissymmetry condition is used for the Nth and $(N-1)$th points.

The boundary formulations for the second-order derivatives also were constructed in [15].

$$\begin{aligned} u_1'' + \alpha u_2'' &= \frac{1}{h^2}(au_1 + bu_2 + cu_3 + du_4 + eu_5), \\ u_N'' + \alpha u_{N-1}'' &= -\frac{1}{h^2}(au_N + bu_{N-1} + cu_{N-2} + du_{N-3} + eu_{N-4}). \end{aligned} \tag{2.25}$$

For the third-order accuracy, the coefficients are given as follows:

$$\begin{aligned} &a = \frac{11\alpha + 35}{12}, \qquad b = -\frac{5\alpha + 26}{3}, \qquad c = \frac{\alpha + 19}{2}, \\ &d = \frac{\alpha - 14}{3}, \qquad e = \frac{11 - \alpha}{12}. \end{aligned} \tag{2.26}$$

2.3 The proposed compact-type CIP method

In this section, a new compact-type CIP scheme is proposed for equation (2.1). The present scheme is mainly based on the idea of characteristic method; as a new ingredient, the high-order compact scheme is employed to obtain the derivatives rather than differentiate the equation with spatial variable to construct a CIP scheme. To explain the present scheme, we consider the KdVB equations as follows:

$$u_t + \alpha u u_x + \gamma u_{xx} - \delta u_{xxx} = 0, \tag{2.27}$$

where α and δ are constants. We split the solution of equation into two phases

$$\frac{\partial u}{\partial t} + \alpha u \frac{\partial u}{\partial x} = 0, \tag{2.28}$$

$$\frac{\partial u}{\partial t} = \delta u_{xxx} - \gamma u_{xx}. \tag{2.29}$$

We consider a 1-D grid with $x_0, x_1, x_2, \ldots, x_{N-1}, x_N$. At the nth step, the point values of u are denoted by $u_0^n, u_1^n, \ldots, u_{N-1}^n, u_N^n$. At first, CIP method is applied to the advective equation (2.28). If both the values of u_i and $u_i'^n$ are given at two grid points, the cubic polynomial at the nth time stage can be written as follows:

$$U_i^n(X) = a_i^n X^3 + b_i^n X^2 + c_i^n X + u_i^n, \tag{2.30}$$

where $X = x_i - x$, the coefficients a_i^n, b_i^n, c_i^n are given by (2.7). The predictor-corrector scheme is employed to calculate the value u^*.

To formulate the classical CIP scheme, equation (2.4) was used to get the values of $u_i'^n$. In the present method, the high-order compact scheme (2.10) is employed to evaluate the derivatives $u_i'^n$, $0 \le i \le N$. In this paper, we use a simple sixth-order tridiagonal scheme for interior points and boundary points, then the coefficients a_i^n, b_i^n, c_i^n in (2.7) can be obtained. Temporal discretization for equation (2.29) can be solved by using a third-order Runge-Kutta method as follows:

$$\begin{aligned} u^{(1)} &= u^n + \Delta t L(u^*), \\ u^{(2)} &= \frac{3}{4}u^n + \frac{1}{4}u^{(1)} + \frac{1}{4}\Delta t L(u^{(1)}), \\ u^{n+1} &= \frac{1}{3}u^n + \frac{2}{3}u^{(2)} + \frac{2}{3}\Delta t L(u^{(2)}), \end{aligned} \tag{2.31}$$

where $L(u) = -\gamma u_{xx} + \delta u_{xxx}$. The high-order compact formulas (2.17) and (2.18) are used to solve the second- and third-order derivatives in equation (2.31). In this paper, we use the sixth-order tridiagonal scheme with the periodic boundary condition.

Supposing the values u_i^n have been obtained, the essential ingredients of the computational algorithm for equation (2.27) consist of the following steps:

1. CIP method is used to obtain u^*
   a. The values of the first-order derivative on all the nodes are obtained by using the HOC scheme (2.13).
   b. Predictor-corrector CIP scheme:
      (a) Predictor step

$$u_i^{**} = U_i^n\left(x_i - \alpha u_i^n \Delta t\right) = a_i^n {\xi_i}^3 + b_i^n {\xi_i}^2 + c_i^n \xi_i + u_i^n,$$

      where $\xi_i = -\alpha u_i^n \Delta t$. We also get u^{***} at the (n+$\frac{1}{2}$)th time stage by using linear interpolation or QUICK scheme based on the value u_i^n.
      (b) Corrector step (CIP method)

$$\hat{u}_i^* = U_i^n\left(x_i - \alpha u_i^\diamond \Delta t\right) = a_i^n {\xi_i}^3 + b_i^n {\xi_i}^2 + c_i^n \xi_i + u_i^n,$$

      where $u^\diamond = \frac{1}{2}(u^{**} + u^{***})$.
      (c) The predictor and corrector steps are employed again to get u^*.
2. High-order compact schemes and Runge-Kutta method for solving equation (2.29)
   (a) High-order formulas (2.17) and (2.18) are used to get second- and third-order derivatives, respectively.
   (b) The third-order SSP Runge-Kutta method (2.31) is used to get the value u_i^{n+1}.

3 Numerical results

In this section, we provide a numerical example with two different initial conditions for the present compact-CIP scheme with the third-order SSP Runge-Kutta time discretization. The non-periodic boundary formulation is applied to (2.28) (HOC approximation formulas for first- and second-order derivatives are used) and periodic boundary conditions for third-order derivatives in the following example.

Example 3.1 We consider the KdVB equation

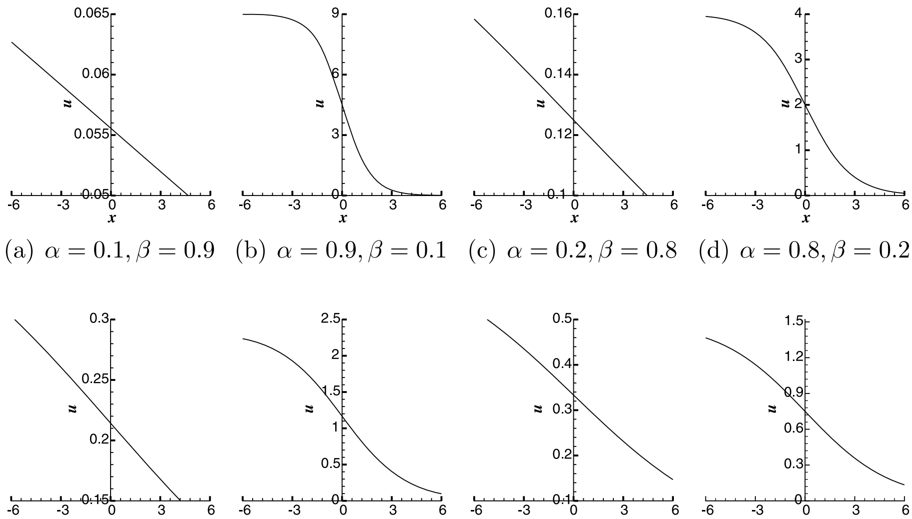
$$u_t + (\alpha + \beta u)uu_x + \gamma u_{xx} - \delta u_{xxx} = 0 \tag{3.1}$$

with the initial solution for $\gamma = 0$, $\delta = -1$

$$u(x,0) = -\frac{\alpha}{2\beta}\left(1 + \tanh\left(\frac{\alpha}{2\sqrt{(-6\beta)}}(x)\right)\right). \tag{3.2}$$

We show the numerical solutions for different values of α and β in Figure 1. If we let $\beta = 0$, $\alpha = 2$, $\gamma = -5$, $\delta = -3$. The exact solution for this case is [14]

$$u(x,t) = \frac{1}{3}\left(\mathrm{sech}^2(\theta/2) + 2\tanh(\theta/2) + 2\right), \tag{3.3}$$



(a) $\alpha = 0.1, \beta = 0.9$ (b) $\alpha = 0.9, \beta = 0.1$ (c) $\alpha = 0.2, \beta = 0.8$ (d) $\alpha = 0.8, \beta = 0.2$

(e) $\alpha = 0.3, \beta = 0.7$ (f) $\alpha = 0.7, \beta = 0.3$ (g) $\alpha = 0.4, \beta = 0.6$ (h) $\alpha = 0.6, \beta = 0.4$

Figure 1 Numerical solutions of Example 3.1 for the equation $u_t + (\alpha - \beta u)uu_x + u_{xxx} = 0$ with different α and β.

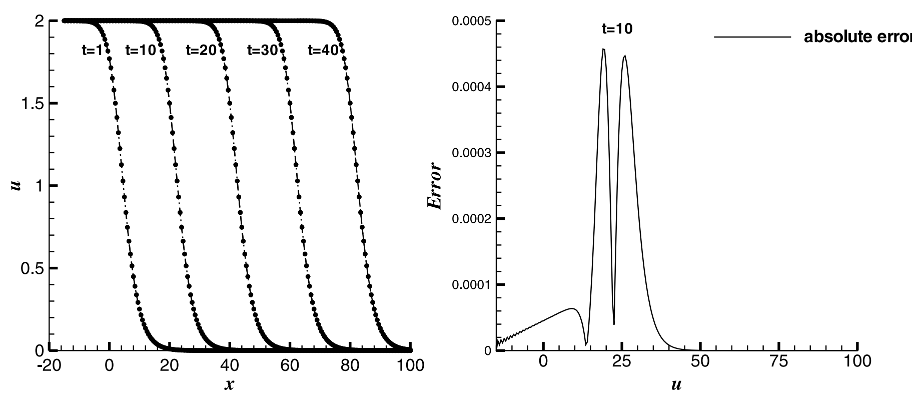


Figure 2 Numerical and analytical solutions of Example 3.1 at various time stages.

where $\theta = -\frac{1}{3}x + \frac{2}{3}t$. The numerical and analytical solutions are shown in Figure 2. The numerical solutions are identical to the exact solution.

4 Conclusions

In this paper, a high-order compact-CIP scheme is applied to simulate Korteweg-de Vries Burgers equations. The proposed scheme is mainly based on the idea of characteristic method; as a new ingredient, the high-order compact scheme is employed to obtain the derivatives rather than differentiate the equation with spatial variable to construct a CIP scheme, and then resolution properties can also be obtained. The numerical results show the good performance and high resolution property of the proposed scheme.

Competing interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Authors' contributions

All authors contributed equally and significantly in writing this article. All authors read and approved the final manuscript.

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