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Actuarial approach in a mixed fractional Brownian motion with jumps environment for pricing currency option

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Abstract

This research aims to investigate the strategy of fair insurance premium actuarial approach for pricing currency option, when the value of foreign currency option follows the mixed fractional Brownian motion with jumps and the European call and put currency option are presented. It has certain reference significance to avoiding foreign exchange risk.

Keywords: currency option; actuarial approach; mixed fractional Brownian motion; jump process

1 Introduction

A currency option is a contract that gives the holder the right to buy or sell a certain amount of foreign currency at a fixed exchange rate (exercise price) upon exercise of the option. American options are options that can be exercised at any time before they expire. European options can be exercised only during a specified period immediately before expiration.

Option pricing was introduced by Black-Scholes in 1973 [1]. In a work by Garman and Kohlhagen (*GK*) [2], the Black-Scholes model was developed in order to evaluate European currency option. However, some researchers (see [3]) pointed to the evidences, which reflect the mispriced currency options by the *GK* model. The significant causes of why this model is not suitable for stock markets are due to the fact that the currencies are different from the stocks in main respects, and geometric Brownian motion is unable to resolve the conduct of the currency return [4, 5]. Since then, in order to tackle these problems, many systems for pricing currency options have been proposed using the extensions of the *GK* model [6–9]. Since fractional Brownian motion (*FBM*) includes two prominent properties: long-range correlation and self-similarity, it can get the typical tail behavior from stock markets.

Unfortunately, owing to the fact that *FBM* is neither a Markov process nor a semimartingale, we are unable to employ the prevalent stochastic calculus to analyze it [10]. To resolve these problems, with respect to the long memory feature and to capture the fluctuations from stock markets, the mixed fractional Brownian motion (*MFBM*) has been introduced [11, 12]. Cheridito [11] had proved that, for $H \in (\frac{3}{4}, 1)$, the mixed model with dependent Brownian motion (*BM*) and *FBM* was equivalent to one with *BM*. Therefore, we assume that $H \in (\frac{3}{4}, 1)$. Moreover, the empirical studies demonstrate that discontinuous or jumps are vital components for analyzing financial data (see [13–19]). Then, we present the combination of Poisson jump process and *MFBM* in order to highlight all these properties. Actuarial approach to option pricing was put forward in 1998 by Bladt and Rydberg [20]. In this study, we assess the actuarial approach for pricing currency options, whose price is governed by jump process and *MFBM*. In this model, we propose the actuarial approach to pricing currency options into a problem of equivalent of fair insurance premium. No economic assumptions are considered in the actuarial approach, and it is not only valid in complete, arbitrage-free and equilibrium markets but also reliable in incomplete, arbitrage and non-equilibrium markets.

Definition 1.1 A *MFBM* of parameters ϵ, α and H is a linear compound of different *FBM*s under probability space (Ω, F, P) for any $t \in R_+$ by

$$M_t^H = \epsilon B(t) + \alpha B_H(t), \tag{1}$$

where $B(t)$ is a *BM*, $B_H(t)$ is an independent *FBM* with Hurst parameter $H \in (0,1)$, ϵ and α are two real constants such that $(\epsilon, \alpha) \neq (0,0)$; to get more information about *MFBM* you can see [21, 22].

The *MFBM* has the following properties:

1. M_t^H is a centered Gaussian process and not a Markovian one for $H \in (0,1) \setminus \frac{1}{2}$;
2. $M_0^H = 0$ P-almost surely;
3. The covariation function of $M_t^H(\alpha,\beta)$ and $M_t^H(a,b)$ for any $t, s \in R_+$ is given by

$$\mathrm{Cov}\big(M_t^H, M_s^H\big) = \alpha^2 (t \wedge s) + \frac{\beta}{2}\big(t^{2H} + s^{2H} - |t-s|^{2H}\big), \tag{2}$$

where $\wedge$ denotes the minimum of two numbers;

4. The increments of $M_t^H(\alpha,\beta)$ are stationary and mixed-self similar for any $h > 0$

$$M_{ht}^H(\alpha,\beta) \triangleq M_t^H\big(\alpha h^{\frac{1}{2}}, \beta h^H\big), \tag{3}$$

where $\triangleq$ means 'to same law';

5. The increments of M_t^H are positively correlated if $\frac{1}{2} < H < 1$, uncorrelated if $H = \frac{1}{2}$ and are negatively correlated if $0 < H < \frac{1}{2}$;
6. The increments of M_t^H are long range dependent if and only if $H > \frac{1}{2}$;
7. For all $t \in R_+$, we have

$$E\big[\big(M_t^H(\alpha,\beta)\big)^n\big] = \begin{cases} 0, & n = 2l+1, \\ \frac{(2l)!}{2^l l!}(\alpha^2 t + \beta^2 t^{2H})^l, & n = 2l. \end{cases} \tag{4}$$

To derive a *MFBM* with jumps model for pricing currency options, the greater attention should be paid to the following conditions:

(i) No transaction expenses or taxes should be determined and all securities are perfectly divisible;

(ii) Safety trading is continuous;
(iii) The domestics interest rate r_d and foreign interest rate r_f in the short-term are defined and stable over time;
(iv) There are no risk-free arbitrage opportunities.

The spot exchange rate in the *MFBM* with jumps model is given by

$$\begin{aligned} dS(t) &= S(t)(\mu - \lambda\mu_{J(t)})\,dt + \sigma S(t)\,d\widehat{B}(t) + \sigma S(t)\,d\widehat{B}_H(t) \\ &\quad + S(t)\big(e^{J(t)} - 1\big)\,dN_t, \quad 0 < t \leq T, S(0) = S > 0. \end{aligned} \tag{5}$$

Suppose that B_t^d and B_t^f show the domestic and foreign price of risk-free bond, respectively. Thus, B_t^d and B_t^f satisfy in Equations (6) and (7):

$$dB_t^d = B_t^d r_d\,dt, \qquad B_T^d = 1B_t^d = e^{-r_d(T-t)}, \tag{6}$$

$$dB_t^f = B_t^f r_f\,dt, \qquad B_T^f = 1B_t^f = e^{-r_f(T-t)}, \tag{7}$$

where $S(t)$ denotes the spot exchange rate at time t of one unit of the foreign currency measured in the domestic currency; the drift μ and volatility σ are supposed to be constants; $\widehat{B}(t)$ and $\widehat{B}_H(t)$ are a *BM* and a *FBM*, respectively; N_t is a Poisson process with rate λ; $(e^{J(t)} - 1)$ is jump size at t which is a sequence of independent identically distributed and $J(t) \sim N(-\frac{\sigma_J^2}{2}, \sigma_J^2)$. Moreover, all three sources of randomness, the *FBM*, $\widehat{B}_H(t)$, the Poisson process N_t and the jump size $e^{J(t)} - 1$, are supposed to be independent.

By using the fractional Girsanov equation and the following variables change

$$B(t) + B_H(t) = \frac{\mu - \lambda\mu_{J(t)} + r_f - r_d}{\sigma}t + \widehat{B}(t) + \widehat{B}_H(t), \tag{8}$$

Equation (5) is transformed to the following equation:

$$\begin{aligned} dS(t) &= S_t(r_d - r_f)\,dt + \sigma S(t)\,dB(t) + \sigma S(t)\,dB_H(t) \\ &\quad + S(t)\big(e^{J(t)} - 1\big)\,dN_t, \quad 0 < t \leq T, S(0) = S > 0. \end{aligned} \tag{9}$$

Lemma 1.2 *By applying the Ito formula, the solution for the stochastic differential Equation* (9) (*see* [23]) *is given by*

$$S(t) = S\exp\left[(r_d - r_f)t + \sigma B(t) + \sigma B_H(t) - \frac{1}{2}\sigma^2 t - \frac{1}{2}\sigma^2 t^{2H} + \sum_{i=1}^{N_t} J(t_i)\right] \tag{10}$$

and the mean

$$\begin{aligned} &E\big(S(t)\big) \\ &\quad = E\left[S\exp\left((r_d - r_f)t + \sigma B(t) + \sigma B_H(t) - \frac{1}{2}\sigma^2 t - \frac{1}{2}\sigma^2 t^{2H} + \sum_{i=1}^{N_t} J(t_i)\right)\right] \\ &\quad = S\exp\left((r_d - r_f)t - \frac{1}{2}\sigma^2 t - \frac{1}{2}\sigma^2 t^{2H}\right)E\big[\exp\big(\sigma B(t) + \sigma B_H(t)\big)\big]E\left[\exp\left(\sum_{i=1}^{N_t} J(t_i)\right)\right] \end{aligned}$$

$$\begin{aligned}&= S\exp\left((r_d - r_f)t - \frac{1}{2}\sigma^2 t - \frac{1}{2}\sigma^2 t^{2H}\right)\exp\left(\frac{1}{2}\sigma^2 t - \frac{1}{2}\sigma^2 t^{2H}\right)\exp\left(\frac{1}{2}n\sigma_J^2 t\right)\\ &= S\exp\left[\left((r_d - r_f) + \frac{1}{2}n\sigma_J^2\right)t\right].\end{aligned} \tag{11}$$

2 Actuarial approach for pricing currency option

This section deals with the new pricing model for the currency options using the actuarial approach, when the spot exchange rate follows the *MFBM* with jumps process. This model can be applied to different financial markets, for example, in the arbitrage-free, equilibrium and complete markets and also in the arbitrage, non-equilibrium and incomplete markets.

Definition 2.1 ([20]) The expectation return rate $\theta(t)$ of S_t on $t \in [0, T]$ is defined $\int_0^T \theta(s)\,ds$ as follows:

$$\frac{E(S(t))}{S(0)} = \exp\left(\int_0^T \theta(s)\,ds\right). \tag{12}$$

Definition 2.2 Suppose that $C(K, T)$ and $P(K, T)$ show the European call and put currency options, respectively, whose spot exchange rate is $S(t)$, the exercise price is K and the time maturity is T. Thus, the value of European option by actuarial approach can be written as follows:

$$C(K, T) = E\left[\left(\exp\left(-\int_0^T \theta(t)\,dt\right)S(T)B_0^f - KB_0^d\right)I_A\right], \tag{13}$$

$$P(K, T) = E\left[\left(KB_0^d - \exp\left(-\int_0^T \theta(t)\,dt\right)S(T)B_0^f\right)I_B\right]. \tag{14}$$

The essential condition for performing the European call and put currency options on the expiry date are, respectively,

$$\begin{aligned}&\exp\left(-\int_0^T \theta(t)\,dt\right)S(T)B_0^f > KB_0^d,\\ &KB_0^d > \exp\left(-\int_0^T \theta(t)\,dt\right)S(T)B_0^f.\end{aligned} \tag{15}$$

Theorem 2.3 *Let the spot exchange rate $S(t)$ satisfy Equation* (5). *Thus, the value of the European call and put currency option at time $t = 0$ is as follows, respectively*:

$$\begin{aligned}C(K, T) &= E\left[\left(\exp\left(-\int_0^T \theta(t)\,dt\right)S(T)B_0^f - KB_0^d\right)I_A\right]\\ &= E\left[\left(\exp\left(-(r_d - r_f)T - \frac{N_T\sigma_J^2}{2}T\right)S(T)B_0^f - KB_0^d\right)I_A\right],\end{aligned} \tag{16}$$

$$\begin{aligned}C(K, T) = {}& SB_0^f \sum_{n=0}^{\infty}\left[\frac{(\lambda T)^n}{n!}\exp\left(\sum_{i=1}^{n} J(t_i) - \lambda T - \frac{n\sigma_J^2}{2}T\right)\right]\Phi(b_n)\\ &- KB_0^d \sum_{n=0}^{\infty} e^{-\lambda T}\frac{(\lambda T)^n}{n!}\Phi(b'_n),\end{aligned} \tag{17}$$

$$\begin{aligned}
P(K,T) &= E\left[\left(KB_0^d - \exp\left(-\int_0^T \theta(t)\,dt\right)S(T)B_0^f\right)I_B\right] \\
&= E\left[\left(KB_0^d - \exp\left(-(r_d - r_f)T - \frac{N_t\sigma_J^2}{2}T\right)S(T)B_0^f\right)I_B\right] \\
&= KB_0^d \sum_{n=0}^{\infty} e^{-\lambda T}\frac{(\lambda T)^n}{n!}\Phi\left(-b_n'\right) - SB_0^f \\
&\quad \times \sum_{n=0}^{\infty}\left[\frac{(\lambda T)^n}{n!}\exp\left(\sum_{i=1}^{n} J(t_i) - \lambda T - \frac{n\sigma_J^2}{2}T\right)\right]\Phi(-b_n),
\end{aligned} \tag{18}$$

here

$$y_n = \frac{m - \sum_{i=1}^{n} J(t_i) + \frac{n\sigma_J^2}{2}}{\sigma}, \qquad m = \ln\frac{KB_0^d}{SB_0^f} + \frac{1}{2}\sigma^2 T + \frac{1}{2}\sigma^2 T^{2H}, \tag{19}$$

$$b_n = \frac{\sigma T + \sigma T^{2H} - y_n}{\sqrt{T + T^{2H}}}, \qquad b_n' = \frac{-y_n}{\sqrt{T + T^{2H}}}. \tag{20}$$

Proof From Lemma 1.2 we have

$$S(T) = S\exp\left[(r_d - r_f)T + \sigma B(T) + \sigma B_H(T) - \frac{1}{2}\sigma^2 T - \frac{1}{2}\sigma^2 T^{2H} + \sum_{i=1}^{N_T} J(t_i)\right]. \tag{21}$$

The $\exp(-\int_0^T \theta(t)\,dt)S(T)B_0^f > KB_0^d$ is equivalent to the following equation:

$$\begin{aligned}
&\exp\left(-(r_d - r_f)T - \frac{N_T\sigma_J^2}{2}T\right) \times S\exp\Biggl[(r_d - r_f)T + \sigma B(T) + \sigma B_H(T) \\
&\quad - \frac{1}{2}\sigma^2 T - \frac{1}{2}\sigma^2 T^{2H} + \sum_{i=1}^{N_T} J(t_i)\Biggr] \times B_0^f > KB_0^d.
\end{aligned} \tag{22}$$

Then we have $\sigma B(T) + \sigma B_H(T) + \sum_{i=1}^{N_T} J(t_i) - \frac{N_T\sigma_J^2}{2}T > m$.

$$\begin{aligned}
C_1(K,T) &= E\left[\exp\left(-\int_0^T \theta(t)\,dt\right)S(T)B_0^f I_{\exp(-\int_0^T \theta(t)\,dt)S(T)B_0^f > KB_0^d}\right] \\
&= E\Biggl\{\exp\left(-(r_d - r_f)T - \frac{N_T\sigma_J^2}{2}T\right) \\
&\quad \times S\exp\left[(r_d - r_f)T + \sigma B(T) + \sigma B_H(T) - \frac{1}{2}\sigma^2 T - \frac{1}{2}\sigma^2 T^{2H} + \sum_{i=1}^{N_T} J(t_i)\right] \\
&\quad \times B_0^f I_{\sigma B(T) + \sigma B_H(T) + \sum_{i=1}^{N_T} J(t_i) - \frac{N_T\sigma_J^2}{2}T > m}\Biggr\} \\
&= SB_0^f \exp\left[-\frac{1}{2}\sigma^2 T - \frac{1}{2}\sigma^2 T^{2H}\right] \\
&\quad \times E\Biggl[\exp\left(\sigma B(T) + \sigma B_H(T) + \sum_{i=1}^{N_T} J(t_i) - \frac{N_T\sigma_J^2}{2}T\right)
\end{aligned}$$

$$\begin{aligned}
&\quad \times I_{\sigma B(T)+\sigma B_H(T)+\sum_{i=1}^{N_T} J(t_i)-\frac{N_T\sigma_J^2}{2}T>m}\Big] \\
&= SB_0^f \exp\left[-\frac{1}{2}\sigma^2 T - \frac{1}{2}\sigma^2 T^{2H}\right] \\
&\quad \times E\left[E\left[\exp\left(\sigma B(T) + \sigma B_H(T) + \sum_{i=1}^{N_T} J(t_i) - \frac{N_T\sigma_J^2}{2}T\right)\right.\right. \\
&\quad \times I_{\sigma B(T)+\sigma B_H(T)+\sum_{i=1}^{N_T} J(t_i)-\frac{N_T\sigma_J^2}{2}T>m}\bigg| N_T\bigg]\bigg] \\
&= SB_0^f \exp\left[-\frac{1}{2}\sigma^2 T - \frac{1}{2}\sigma^2 T^{2H}\right]\sum_{n=0}^{\infty} P(N_T = n) \\
&\quad \times E\left[\exp\left(\sigma B(T) + \sigma B_H(T) + \sum_{i=1}^{N_T} J(t_i) - \frac{N_T\sigma_J^2}{2}T\right)\right. \\
&\quad \times I_{\sigma B(T)+\sigma B_H(T)+\sum_{i=1}^{N_T} J(t_i)-\frac{N_T\sigma_J^2}{2}T>m}\bigg| n\bigg] \\
&= SB_0^f \exp\left[-\frac{1}{2}\sigma^2 T - \frac{1}{2}\sigma^2 T^{2H}\right]\sum_{n=0}^{\infty}\left[\frac{(\lambda T)^n}{n!}\exp\left(\sum_{i=1}^{n} J(t_i) - \lambda T - \frac{n\sigma_J^2}{2}T\right)\right] \\
&\quad \times E\big[\exp\big(\sigma B(T) + \sigma B_H(T)\big) I_{\sigma B(T)+\sigma B_H(T)+\sum_{i=1}^{N_T} J(t_i)-\frac{N_T\sigma_J^2}{2}T>m}\big] \\
&= SB_0^f \sum_{n=0}^{\infty}\left[\frac{(\lambda T)^n}{n!}\exp\left(\sum_{i=1}^{n} J(t_i) - \lambda T - \frac{n\sigma_J^2}{2}T\right)\right]\frac{1}{\sqrt{2\pi(T+T^{2H})}} \\
&\quad \times \int_{y_n}^{+\infty} e^{-\frac{(x-\sigma T-\sigma T^{2H})^2}{2(T+T^{2H})}}\,dx \\
&= SB_0^f \sum_{n=0}^{\infty}\left[\frac{(\lambda T)^n}{n!}\exp\left(\sum_{i=1}^{n} J(t_i) - \lambda T - \frac{n\sigma_J^2}{2}T\right)\right] P(Z > y_n), \qquad (23)
\end{aligned}$$

$$\begin{aligned}
C_1(K,T) &= SB_0^f \sum_{n=0}^{\infty}\left[\frac{(\lambda T)^n}{n!}\exp\left(\sum_{i=1}^{n} J(t_i) - \lambda T - \frac{n\sigma_J^2}{2}T\right)\right] \\
&\quad \times P\left(\frac{Z-\sigma T-\sigma T^{2H}}{\sqrt{T+T^{2H}}} > \frac{y_n-\sigma T-\sigma T^{2H}}{\sqrt{T+T^{2H}}}\right) \\
&= SB_0^f \sum_{n=0}^{\infty}\left[\frac{(\lambda T)^n}{n!}\exp\left(\sum_{i=1}^{n} J(t_i) - \lambda T - \frac{n\sigma_J^2}{2}T\right)\right]\Phi(b_n). \qquad (24)
\end{aligned}$$

Moreover,

$$\begin{aligned}
C_2(K,T) &= E\big[KB_0^d I_{\exp(-\int_0^T \theta(t)\,dt)S(T)B_0^f > KB_0^d}\big] \\
&= KB_0^d P\left[\sigma B(T) + \sigma B_H(T) + \sum_{i=1}^{N_T} J(t_i) - \frac{N_T\sigma_J^2}{2}T > m\right] \\
&= KB_0^d \sum_{n=0}^{\infty} P(N_T = n) P\left[\sigma B(T) + \sigma B_H(T) + \sum_{i=1}^{N_T} J(t_i) - \frac{n\sigma_J^2}{2}T > m\right]
\end{aligned}$$

$$
\begin{aligned}
&= KB_0^d \sum_{n=0}^{\infty} \frac{e^{-\lambda T}(\lambda T)^n}{n!} P\left[\frac{B(T) + B_H(T)}{\sqrt{T + T^{2H}}} > \frac{y_n}{\sqrt{T + T^{2H}}}\right] \\
&= KB_0^d \sum_{n=0}^{\infty} \frac{e^{-\lambda T}(\lambda T)^n}{n!} \Phi\left(-\frac{y_n}{\sqrt{T + T^{2H}}}\right) \\
&= KB_0^d \sum_{n=0}^{\infty} \frac{e^{-\lambda T}(\lambda T)^n}{n!} \Phi\bigl(b_n'\bigr),
\end{aligned} \tag{25}
$$

where

$$
y_n = \frac{m - \sum_{i=1}^n J(t_i) + \frac{n\sigma_J^2}{2}}{\sigma}, \qquad m = \ln \frac{KB_0^d}{SB_0^f} + \frac{1}{2}\sigma^2 T + \frac{1}{2}\sigma^2 T^{2H}, \tag{26}
$$

$$
b_n = \frac{\sigma T + \sigma T^{2H} - y_n}{\sqrt{T + T^{2H}}}, \qquad b_n' = \frac{-y_n}{\sqrt{T + T^{2H}}}, \tag{27}
$$

and $\Phi(\cdot)$ is the cumulative normal distribution. From Equation (17) we can get

$$
\begin{aligned}
C(K,T) &= E\left[\left(\exp\left(-\int_0^T \theta(t)\,dt\right)S(T)B_0^f - KB_0^d\right)I_A\right] \\
&= C_1(K,T) - C_2(K,T) \\
&= SB_0^f \sum_{n=0}^{\infty}\left[\frac{(\lambda T)^n}{n!}\exp\left(\sum_{i=1}^n J(t_i) - \lambda T - \frac{n\sigma_J^2}{2}T\right)\right]\Phi(b_n) \\
&\quad - KB_0^d \sum_{n=0}^{\infty} e^{-\lambda T}\frac{(\lambda T)^n}{n!}\Phi\bigl(b_n'\bigr).
\end{aligned} \tag{28}
$$

The proof of Equation (18) is the same way. □

3 Conclusion

In the actuarial approach, we do not need the economic knowledge of financial data in which the outcome is accurate in all kinds of markets. It is important to note that our model in this study is easy to use against the Black-Scholes model because there is no need to investigate an equivalent martingale measure. In addition, in this paper, we supposed that the spot price follows the *MFBM* with jumps is a clear reference, which is important to eschewing foreign exchange risk.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors jointly worked on deriving the results and approved the final manuscript.

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