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Permanence of a Gilpin-Ayala predator-prey system with time-dependent delay

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Abstract
In this paper, a Gilpin-Ayala predator-prey model with time-dependent delay with m preys and $(n-m)$ predators is studied, which can be seen as a modification of the traditional Lotka-Volterra competition predator-prey system model. Two sets of sufficient conditions on the permanence of the system are obtained. One set is delay dependent, while the other set is delay independent.

Keywords: permanence; non-autonomous system; Gilpin-Ayala predator-prey model; time-dependent delay

1 Introduction

Predator-prey phenomena are always interesting topics in both ecology and mathematical ecology, which attract a lot of attention due to its universal existence and importance [1]. Since Volterra [2] proposed a differential equation model to successfully explain the ecological phenomenon of Finme Fish Harbor in 1926, and Lotka [3] also derived the model to describe a hypothetical chemical reaction in which the chemical concentrations oscillate in 1925, the famous Lotka-Volterra equations were accepted by many experts. The classical Lotka-Volterra predator-prey model is a rudimentary model of mathematical ecology which can be expressed as follows:

$$\begin{aligned} \dot{x}(t) &= x(t)\bigl(b - ay(t)\bigr), \\ \dot{y}(t) &= y(t)\bigl(-d + cx(t)\bigr), \end{aligned} \tag{1.1}$$

where $x(t)$ is the density of the prey species at time t, $y(t)$ is the density of the predator species at time t. b is the intrinsic growth rate of the prey, a is the per-capita rate of predation of the predator, d is the death rate of the predator, c denotes the product of the per-capita rate of predation and the rate of conversing prey into predator.

Since then the Lotka-Volterra equation and its various generalized forms have been frequently used to describe the population dynamics with predator-prey relations and a lot of extensive research results have already been obtained and one has seen great progress [4–25]. However, regardless of this fact, the Lotka-Volterra equation has a property which is considered as a disadvantage and that is the linearity of this model (*i.e.* the rate of change in the size of each species is a linear function of the sizes of the interacting species). Particularly, in 1973, Gilpin and Ayala [26] claimed that a little more complicated model was needed in order to obtain more realistic solutions and proposed the following model:

$$\dot{x}_i(t) = r_i x_i(t)\left[1 - \left(\frac{x_i(t)}{K_i}\right)^{\theta_i} - \sum_{j=1, j\neq i}^{n} a_{ij}(t)\frac{x_j(t)}{K_j}\right], \quad i = 1, 2, \ldots, n, \tag{1.2}$$

where $x_i(t)$ is the population density of species i at time t, r_i is the intrinsic exponential growth rate of species i, K_i is the environment carrying capacity of species i in the absence of competition, θ_i provides a nonlinear measure of intraspecific interference, and a_{ij} $(i \neq j)$ provides a measure of interspecific competition. So, Li and Lu [5] introduced the following more complicated non-autonomous prey-competition model:

$$\begin{aligned} \dot{x}_i(t) &= x_i(t)\left[r_i(t) - \sum_{j=1}^{n} a_{ij}(t)x_j^{\alpha_{ij}}(t)\right], \quad i = 1, 2, \ldots, m, \\ \dot{x}_i(t) &= x_i(t)\left[-r_i(t) + \sum_{j=1}^{m} a_{ij}(t)x_j^{\alpha_{ij}}(t) - \sum_{j=m+1}^{n} a_{ij}(t)x_j^{\alpha_{ij}}(t)\right], \quad i = m+1, \ldots, n, \end{aligned} \tag{1.3}$$

they obtained sufficient conditions for the existence of a unique globally attractive periodic solution of system (1.3). For more work in this direction, one could refer to [5, 14–16] and the references cited therein.

Furthermore, delay due to negative feedback is a common example, because the process of a reproduction of a population is not instantaneous. The effect of these kinds of delays on the asymptotic behavior of populations has been studied by a number of authors (see, for example, [11, 15, 20, 22–25]). Chen *et al.* [6] further incorporated time delays in the model (1.3) and they proposed the following model:

$$\begin{cases} \dot{x}_i(t) = x_i(t)[r_i(t) - \sum_{j=1}^{n} a_{ij}(t)x_j^{\alpha_{ij}}(t) - \sum_{j=1}^{n} b_{ij}(t)x_j^{\beta_{ij}}(t - \tau_{ij}(t))], \quad i = 1, 2, \ldots, m, \\ \dot{x}_i(t) = x_i(t)[-r_i(t) + \sum_{j=1}^{m} a_{ij}(t)x_j^{\alpha_{ij}}(t) + \sum_{j=1}^{m} b_{ij}(t)x_j^{\beta_{ij}}(t - \tau_{ij}(t)) \\ \qquad - \sum_{j=m+1}^{n} a_{ij}(t)x_j^{\alpha_{ij}}(t) - \sum_{j=m+1}^{n} b_{ij}(t)x_j^{\beta_{ij}}(t - \tau_{ij}(t))], \quad i = m+1, \ldots, n, \end{cases} \tag{1.4}$$

with the initial conditions

$$x_i(s) = \Phi_i(s) \geq 0, \quad s \in [-\tau, 0]; \qquad \Phi_i(0) > 0; \tag{1.5}$$

where $\tau = \max_{1\leq i,j\leq n}\{\sup_{t\in[0,+\infty)}\{\tau_{ij}(t)\}\}$. $x_i(t)$, $i = 1, 2, \ldots, m$, are the densities of the prey species i at time t, $x_i(t)$, $i = m+1, \ldots, n$ are the densities of the predator species X_i at time t; α_{ij}, β_{ij} are all positive constants and represent nonlinear measures of interspecific or intraspecific interference. $r_i(t)$, $i = 1, \ldots, m$, and $r_i(t)$, $i = m+1, \ldots, n$, are the intrinsic and death rates at time t, $a_{ij}(t)$ and $b_{ij}(t)$ represent the effects of the interspecific $(i \neq j)$ and the intraspecific $(i = j)$ interaction at time t; the terms $b_{ij}(t)x_j^{\beta_{ij}}(t - \tau_{ij}(t))$ represent the negative feedback crowding. By using the Gaines and Mawhins continuation theorem of coincidence degree theory and by constructing an appropriate Lyapunov functional, they obtained a set of sufficient conditions which guarantee the existence and global attractivity of positive periodic solutions of the system (1.4).

As we know, a more important theme that interested mathematicians as well as biologists is whether all species in a multi-species community would survive in the long run, that is, whether the ecosystems are permanent (see, for example, [1, 5, 7–13, 15, 18–25]). Huang [7] studied the permanence of the system (1.4). By using comparison theory and a differential inequality, two sets of sufficient conditions which guarantee the permanence of the system (1.4) are obtained. Their results supplement the main results of Chen *et al.* [6]. In this paper, we shall also study the permanence of the system (1.4) by using comparison theory, and get the same results as [7] do, under weaker conditions.

In this paper, we also explore the system (1.4). Throughout this paper, we always assume that for all $i, j = 1, 2, \dots, n$:

(H_1) The bounded functions $r_i(t)$, $a_{ij}(t)$, $b_{ij}(t)$, $\tau_{ij}(t)$ are all nonnegative and continuous for all $t \in R$ and $a_{ii}^l \geq 0$, $b_{ii}^l \geq 0$, $a_{ii}^l + b_{ii}^l > 0$. Here, for any bounded function $f(t)$, $f^u = \lim_{t\to\infty} \sup f(t), f^l = \lim_{t\to\infty} \inf f(t)$;
(H_2) α_{ij}, β_{ij} are all positive constants.

This paper is aimed at obtaining, by developing the analytical technique of [9–11], two sets of sufficient conditions which guarantee the permanence of the system (1.4). One set is delay dependent, while the other set is delay independent. Our results improve on Theorems 2.1 and 2.2 of [7]. Moreover, we state and prove the main results in the next section and present a brief conclusion. For more background and biological adjustments of system (1.4), see [4, 6, 7, 14–17, 19–21] and the references cited therein.

2 Main results

In this section, we present two sets of sufficient conditions for the permanence of system (1.4). We denote by $R_+^n = \{(x_1, \dots, x_n) \in R^n | x_i \geq 0, i = 1, 2, \dots, n\}$ the nonnegative cone and by $\operatorname{Int} R_+^n = \{(x_1, \dots, x_n) \in R^n | x_i > 0, i = 1, 2, \dots, n\}$ the positive cone. For ecological reasons, we consider system (1.4), only in $\operatorname{Int} R_+^n$.

Definition 2.1 System (1.4) is said to be permanent, if there are positive constants m and M, such that for each positive solution $(x_1(t), \dots, x_n(t))^T$ of system (1.4) satisfies

$$m \leq \lim_{t\to+\infty} \inf x_i(t) \leq \lim_{t\to+\infty} \sup x_i(t) \leq M, \quad i = 1, 2, \dots, n.$$

It is easy to prove the following lemma.

Lemma 2.1 *The positive cone is invariant with respect to system* (1.4).

For system (1.4), we will consider two cases, $a_{ii}^l > 0$, $b_{ii}^l \geq 0$ and $a_{ii}^l \geq 0$, $b_{ii}^l > 0$, respectively, then we obtain Theorems 2.1 and 2.2.

For convenience, we introduce the following notations.

For $i = 1, \dots, m$

$$M_i = \left(\frac{r_i^u}{a_{ii}^l}\right)^{\frac{1}{\alpha_{ii}}};$$

$$m_i = \left(\frac{r_i^l - \sum_{j=1, j\neq i}^n a_{ij}^u M_j^{\alpha_{ij}} - \sum_{j=1}^n b_{ij}^u M_j^{\beta_{ij}}}{a_{ii}^u}\right)^{\frac{1}{\alpha_{ii}}};$$

for $i = m+1,\dots,n$

$$M_i = \left(\frac{-r_i^l + \sum_{j=1}^m a_{ij}^u M_j^{\alpha_{ij}} + \sum_{j=1}^m b_{ij}^u M_j^{\beta_{ij}}}{a_{ii}^l}\right)^{\frac{1}{\alpha_{ii}}};$$

$$m_i = \left(\frac{\Delta_i}{a_{ii}^u}\right)^{\frac{1}{\alpha_{ii}}},$$

where

$$\Delta_i = -r_i^u + \sum_{j=1}^m a_{ij}^l m_j^{\alpha_{ij}} + \sum_{j=1}^n b_{ij}^l m_j^{\beta_{ij}} - \sum_{j=m+1, j\neq i}^n a_{ij}^u M_j^{\alpha_{ij}} - \sum_{j=m+1}^n b_{ij}^u M_j^{\beta_{ij}}.$$

Theorem 2.1 *Assume that system* (1.4) *satisfies* $a_{ii}^l > 0$ *and the following.*

(H_3) $r_i^l > \sum_{j=1,j\neq i}^n a_{ij}^u M_j^{\alpha_{ij}} + \sum_{j=1}^n b_{ij}^u M_j^{\beta_{ij}}$, $i = 1,2,\dots,m$,
(H_4) $\Delta_i > 0$, $i = m+1,\dots,n$.

Then system (1.4) *is permanent.*

Proof Let $x(t) = (x_1(t),\dots,x_n(t))^T$ be any positive solution of system (1.4) with initial condition (1.5). For $i = 1,2,\dots,m$, it follows from system (1.4) that

$$\dot{x}_i(t) \le x_i(t)\big(r_i(t) - a_{ii}(t)x_i^{\alpha_{ii}}(t)\big), \tag{2.1}$$

thus

$$\frac{d(x_i^{\alpha_{ii}}(t))}{dt} \le \alpha_{ii}x_i^{\alpha_{ii}}(t)\big(r_i(t) - a_{ii}(t)x_i^{\alpha_{ii}}(t)\big). \tag{2.2}$$

Let $u_i(t) = x_i^{\alpha_{ii}}(t)$, we have

$$\dot{u}_i(t) \le \alpha_{ii}u_i(t)\big(r_i(t) - a_{ii}(t)u_i(t)\big) \le \alpha_{ii}u_i(t)\big(r_i^u - a_{ii}^l u_i(t)\big). \tag{2.3}$$

By using the comparability theorem, we obtain

$$\lim_{t\to+\infty}\sup u_i(t) \le \frac{r_i^u}{a_{ii}^l}, \tag{2.4}$$

so it immediately follows that

$$\lim_{t\to+\infty}\sup x_i(t) \le \left(\frac{r_i^u}{a_{ii}^l}\right)^{\frac{1}{\alpha_{ii}}} := M_i, \quad i = 1,2,\dots,m. \tag{2.5}$$

For any $\varepsilon > 0$ small enough, it follows from (2.5) that there exists large enough T_1 such that for all $i = 1,2,\dots,m$ and $t \ge T_1$

$$x_i(t) \le M_i + \varepsilon. \tag{2.6}$$

For $i = m+1,\dots,n$ and $t \ge T_1 + \tau$, (2.6) combining with the ith equation of system (1.4) leads to

$$\dot{x}_i(t) \le x_i(t)\left(-r_i(t) + \sum_{j=1}^{m} a_{ij}(t)(M_j+\varepsilon)^{\alpha_{ij}} + \sum_{j=1}^{m} b_{ij}(t)(M_j+\varepsilon)^{\beta_{ij}} - a_{ii}(t)x_i^{\alpha_{ii}}(t)\right), \tag{2.7}$$

thus by a similar argument, we can verify that

$$\lim_{t\to+\infty}\sup x_i(t) \le \left(\frac{-r_i^l + \sum_{j=1}^m a_{ij}^u M_j^{\alpha_{ij}} + \sum_{j=1}^m b_{ij}^u M_j^{\beta_{ij}}}{a_{ii}^l}\right)^{\frac{1}{\alpha_{ii}}} := M_i, \quad i = m+1,\dots,n. \tag{2.8}$$

From the condition (H_3) of Theorem 2.1, we could choose $\varepsilon > 0$ small enough such that

$$r_i^l > \sum_{j=1,j\ne i}^{n} a_{ij}^u(M_j+\varepsilon)^{\alpha_{ij}} + \sum_{j=1}^{n} b_{ij}^u(M_j+\varepsilon)^{\beta_{ij}}, \tag{2.9}$$

thus, for $\varepsilon > 0$ satisfies (2.9), from (2.5) and (2.8), we know that there exists $T_2 > T_1 + \tau$ such that for all $i = 1,2,\dots,n$ and $t \ge T_2$

$$x_i(t) \le M_i + \varepsilon. \tag{2.10}$$

For $i = 1,\dots,m$ and $t \ge T_2 + \tau$, by applying (2.10), from the ith equation of system (1.4), we have

$$\dot{x}_i(t) \ge x_i(t)\left(r_i(t) - \sum_{j=1,j\ne i}^{n} a_{ij}(t)(M_j+\varepsilon)^{\alpha_{ij}} - \sum_{j=1}^{n} b_{ij}(t)(M_j+\varepsilon)^{\beta_{ij}} - a_{ii}(t)x_i^{\alpha_{ii}}(t)\right), \tag{2.11}$$

thus

$$\begin{aligned}\frac{d(x_i^{\alpha_{ii}}(t))}{dt} \ge{}& \alpha_{ii}x_i^{\alpha_{ii}}(t)\left(r_i(t) - \sum_{j=1,j\ne i}^{n} a_{ij}(t)(M_j+\varepsilon)^{\alpha_{ij}}\right.\\ &\left. - \sum_{j=1}^{n} b_{ij}(t)(M_j+\varepsilon)^{\beta_{ij}} - a_{ii}(t)x_i^{\alpha_{ii}}(t)\right),\end{aligned} \tag{2.12}$$

let $u_i(t) = x_i^{\alpha_{ii}}(t)$, we get

$$\begin{aligned}\dot{u}_i(t) &\ge \alpha_{ii}u_i(t)\left(r_i(t) - \sum_{j=1,j\ne i}^{n} a_{ij}(t)(M_j+\varepsilon)^{\alpha_{ij}} - \sum_{j=1}^{n} b_{ij}(t)(M_j+\varepsilon)^{\beta_{ij}} - a_{ii}(t)u_i(t)\right)\\ &\ge \alpha_{ii}u_i(t)\left(r_i^l - \sum_{j=1,j\ne i}^{n} a_{ij}^u(M_j+\varepsilon)^{\alpha_{ij}} - \sum_{j=1}^{n} b_{ij}^u(M_j+\varepsilon)^{\beta_{ij}} - a_{ii}^u u_i(t)\right).\end{aligned} \tag{2.13}$$

According to the comparability theorem, we have

$$\lim_{t\to+\infty}\inf u_i(t) \ge \frac{r_i^l - \sum_{j=1,j\ne i}^n a_{ij}^u(M_j+\varepsilon)^{\alpha_{ij}} - \sum_{j=1}^n b_{ij}^u(M_j+\varepsilon)^{\beta_{ij}}}{a_{ii}^u},$$

setting $\varepsilon \to 0$ in above inequality, we have

$$\lim_{t\to+\infty}\inf u_i(t) \geq \frac{r_i^l - \sum_{j=1,j\neq i}^n a_{ij}^u M_j^{\alpha_{ij}} - \sum_{j=1}^n b_{ij}^u M_j^{\beta_{ij}}}{a_{ii}^u}, \tag{2.14}$$

therefore

$$\lim_{t\to+\infty}\inf x_i(t) \geq \left(\frac{r_i^l - \sum_{j=1,j\neq i}^n a_{ij}^u M_j^{\alpha_{ij}} - \sum_{j=1}^n b_{ij}^u M_j^{\beta_{ij}}}{a_{ii}^u}\right)^{\frac{1}{\alpha_{ii}}} := m_i, \quad i = 1,2,\ldots,m. \tag{2.15}$$

From the condition (H_4) of Theorem 2.1, we could choose $\varepsilon > 0$ small enough such that

$$\begin{aligned} &-r_i^u + \sum_{j=1}^m a_{ij}^l (m_j - \varepsilon)^{\alpha_{ij}} + \sum_{j=1}^m b_{ij}^l (m_j - \varepsilon)^{\beta_{ij}} \\ &\quad - \sum_{j=m+1,j\neq i}^n a_{ij}^u (M_j + \varepsilon)^{\alpha_{ij}} - \sum_{j=m+1}^n b_{ij}^u (M_j + \varepsilon)^{\beta_{ij}} \geq 0, \end{aligned} \tag{2.16}$$

thus, for $\varepsilon > 0$ it satisfies (2.16), and it follows from (2.15) that there exists large enough $T_3 > T_2 + \tau$ such that for all $i = 1,\ldots,m$ and $t \geq T_3$

$$x_i(t) \geq m_i - \varepsilon, \tag{2.17}$$

thus, for $i = m+1,\ldots,n$ and $t \geq T_3 + \tau$, (2.10) and (2.17) combining with the ith equation of system (1.4) leads to

$$\begin{aligned} \dot{x}_i(t) \geq x_i(t)\Bigg(&-r_i(t) + \sum_{j=1}^m a_{ij}(t)(m_j - \varepsilon)^{\alpha_{ij}} + \sum_{j=1}^m b_{ij}(t)(m_j - \varepsilon)^{\beta_{ij}} \\ &- \sum_{j=m+1,j\neq i}^n a_{ij}(t)(M_j + \varepsilon)^{\alpha_{ij}} - \sum_{j=m+1}^n b_{ij}(t)(M_j + \varepsilon)^{\beta_{ij}} - a_{ii}(t)x_i^{\alpha_{ii}}(t)\Bigg), \end{aligned} \tag{2.18}$$

by using (2.18), similarly, we obtain

$$\lim_{t\to+\infty}\inf x_i(t) \geq \left(\frac{\Delta_i}{a_{ii}^u}\right)^{\frac{1}{\alpha_{ii}}} := m_i, \quad i = m+1,\ldots,n, \tag{2.19}$$

where

$$\Delta_i = -r_i^u + \sum_{j=1}^m a_{ij}^l m_j^{\alpha_{ij}} + \sum_{j=1}^n b_{ij}^l m_j^{\beta_{ij}} - \sum_{j=m+1,j\neq i}^n a_{ij}^u M_j^{\alpha_{ij}} - \sum_{j=m+1}^n b_{ij}^u M_j^{\beta_{ij}}.$$

Take $M = \max_{1\leq i\leq n}\{M_i\}$, $m = \min_{1\leq i\leq n}\{m_i\}$, we have

$$m \leq \liminf_{t\to+\infty} x_i(t) \leq \limsup_{t\to+\infty} x_i(t) \leq M, \quad i = 1,2,\ldots,n.$$

This ends the proof of Theorem 2.1. □

Denote for $i = 1, \dots, m$

$$\bar{M}_i = \left(\frac{r_i^u}{b_{ii}^l}\right)^{\frac{1}{\beta_{ii}}} \exp\left[r_i^u \tau\right];$$

$$\bar{m}_i = \min_{1\leq i\leq n}\left\{\left(\frac{r_i^l}{2b_i^u}\right)^{\frac{1}{\beta_{ii}}}, l_i^{\frac{1}{\beta_{ii}}}\right\},$$

where

$$\tau = \max_{1\leq i,j\leq n}\left\{\sup_{t\in[0,+\infty)}\left\{\tau_{ij}(t)\right\}\right\};$$

$$l_i \leq \left\{\left(r_i^l - \sum_{j=1}^{n} a_{ij}^u \bar{M}_j^{\alpha_{ij}} - \sum_{j=1,j\neq i}^{n} b_{ij}^u \bar{M}_j^{\beta_{ij}}\right)\Big/ b_{ii}^u, i = 1, \dots, m\right\};$$

for $i = m+1, \dots, n$

$$\bar{M}_i = \left(\frac{\lambda_i}{b_{ii}^l}\right)^{\frac{1}{\beta_{ii}}} \exp[\lambda_i \tau];$$

$$\bar{m}_i = \min_{1\leq i\leq n}\left\{\left(\frac{-r_i^u + \sum_{j=1}^{m} a_{ij}^l m_j^{\alpha_{ij}} + \sum_{j=1}^{m} b_{ij}^l m_j^{\beta_{ij}}}{2b_i^u}\right)^{\frac{1}{\beta_{ii}}}, l_i^{\frac{1}{\beta_{ii}}}\right\},$$

where

$$\lambda_i = -r_i^l + \sum_{j=1}^{m} a_{ij}^u \bar{M}_j^{\alpha_{ij}} + \sum_{j=1}^{m} b_{ij}^u \bar{M}_j^{\beta_{ij}};$$

$$l_i \leq \left\{\nabla_i / b_{ii}^u, i = m+1, \dots, n\right\};$$

$$\nabla_i = -r_i^u + \sum_{j=1}^{m} a_{ij}^l m_j^{\alpha_{ij}} + \sum_{j=1}^{n} b_{ij}^l m_j^{\beta_{ij}} - \sum_{j=m+1}^{n} a_{ij}^u M_j^{\alpha_{ij}} - \sum_{j=m+1,j\neq i}^{n} b_{ij}^u M_j^{\beta_{ij}}.$$

Theorem 2.2 *Assume that system* (1.4) *satisfies* $b_{ii}^l > 0$ *and the following.*

(H_5) $r_i^l > \sum_{j=1}^{n} a_{ij}^u \bar{M}_j^{\alpha_{ij}} + \sum_{j=1,j\neq i}^{n} b_{ij}^u \bar{M}_j^{\beta_{ij}}$, $i = 1, 2, \dots, m$,
(H_6) $\nabla_i > 0$, $i = m+1, \dots, n$.

Then system (1.4) *is permanent.*

Proof Let $x(t) = (x_1(t), \dots, x_n(t))^T$ be any positive solution of system (1.4) with initial condition (1.5), for $i = 1, 2, \dots, m$, it follows from system (1.4) that

$$\dot{x}_i(t) \leq x_i(t)\bigl(r_i(t) - b_{ii}(t)x_i^{\beta_{ii}}\bigl(t - \tau_{ii}(t)\bigr)\bigr), \tag{2.20}$$

thus

$$\frac{d(x_i^{\beta_{ii}}(t))}{dt} \leq \beta_{ii} x_i^{\beta_{ii}}(t)\bigl(r_i(t) - b_{ii}(t)x_i^{\beta_{ii}}\bigl(t - \tau_{ii}(t)\bigr)\bigr). \tag{2.21}$$

Let $u_i(t) = x_i^{\beta_{ii}}(t)$, we have

$$\begin{aligned}\dot{u}_i(t) &\le \beta_{ii}u_i(t)\big(r_i(t) - b_{ii}(t)u_i\big(t - \tau_{ii}(t)\big)\big) \\ &\le \beta_{ii}u_i(t)\big(r_i^u - b_{ii}^l u_i\big(t - \tau_{ii}(t)\big)\big).\end{aligned} \tag{2.22}$$

Take $\tilde{M}_i = \frac{r_i^u}{b_{ii}^l}(1+h_i)$, where $0 < h_i < \exp[r_i^u\tau]-1$, $\tau = \max_{1\le i,j\le n}\{\sup_{t\in[0,+\infty)}\{\tau_{ij}(t)\}\}$. Firstly, suppose $u_i(t)$ is not oscillatory about $\tilde{M}_i$. That is, there exists a $T_1^* > 0$, for $t > T_1^*$ such that

$$u_i(t) < \tilde{M}_i, \tag{2.23}$$

or

$$u_i(t) > \tilde{M}_i. \tag{2.24}$$

If (2.23) holds, then our aim is reached. Suppose (2.24) holds, then for $t \ge T_1^* + \tau$, we obtain

$$\dot{u}_i(t) < -\beta_{ii}h_i r_i^u u_i(t),$$

thus $u_i(t) < u_i(0)\exp[-\beta_{ii}h_i r_i^u t] \to 0$, as $t \to +\infty$, which is in contradiction with (2.24). Hence there must exist $\tilde{T}_1 > T_1^* + \tau$ such that $u_i(t) < \tilde{M}_i$ for $t > \tilde{T}_1$. Secondly now assume that $u_i(t)$ is oscillatory about $\tilde{M}_i$ for $t \ge T_1^*$, that is, there exists a time sequence $\{t_n\}$ such that $\tau < t_i < t_2 < \cdots < t_n < \cdots$ is a sequence of zeros of $u_i(t_n) - \tilde{M}_i$ with $\lim_{n\to\infty} t_n = +\infty$ and $u_i(t_n) = \tilde{M}_i$. Set $\bar{t}_n$ to be a point where $u_i(t)$ attends its maximum in (t_n, t_{n+1}). Thus we get $u_i(\bar{t}_n) \ge u_i(t_n) = \tilde{M}_i$. Then it is easy to see from (2.22) that

$$0 = \dot{u}_i(t)|_{t=\bar{t}_n} \le \beta_{ii}u_i(\bar{t}_n)\big(r_i^u - b_{ii}^l u_i\big(\bar{t}_n - \tau_{ii}(\bar{t}_n)\big)\big), \tag{2.25}$$

which implies that

$$u_i\big(\bar{t}_n - \tau_{ii}(\bar{t}_n)\big) \le \frac{r_i^u}{b_{ii}^l}. \tag{2.26}$$

Integrating the both sides of (2.22) from $\bar{t}_n - \tau_{ii}(\bar{t}_n)$ to $\bar{t}_n$, it follows that

$$\ln\frac{u_i(\bar{t}_n)}{u_i(\bar{t}_n - \tau_{ii}(\bar{t}_n))} \le \int_{\bar{t}_n-\tau_{ii}(\bar{t}_n)}^{\bar{t}_n} \beta_{ii}\big(r_i^u - b_{ii}^l u_i\big(t - \tau_{ii}(t)\big)\big)\,dt \le \beta_{ii}r_i^u\tau_{ii}(\bar{t}_n). \tag{2.27}$$

From (2.26) and (2.27) we get

$$u_i(\bar{t}_n) \le \frac{r_i^u}{b_{ii}^l}\exp\big[\beta_{ii}r_i^u\tau\big] := L_i.$$

Since $u_i(\bar{t}_n)$ is an arbitrary local maximum of $u_i(t)$, we can conclude that there exists a $T_2^* > 0$ such that $u_i(t) \le L_i$ for all $t \ge T_2^*$. Thus

$$\lim_{t\to+\infty}\sup x_i(t) \le \left(\frac{r_i^u}{b_{ii}^l}\right)^{\frac{1}{\beta_{ii}}}\exp\big[r_i^u\tau\big] := \bar{M}_i. \tag{2.28}$$

For any $\varepsilon > 0$ small enough, it follows from (2.28) that there exists large enough $T_3^* > T_2^*$ such that for all $i = 1, 2, \ldots, m$ and $t \geq T_3^*$

$$x_i(t) \leq \bar{M}_i + \varepsilon. \tag{2.29}$$

For $i = m+1, \ldots, n$ and $t \geq T_3^* + \tau$, (2.29) combining with the ith equation of system (1.4) leads to

$$\begin{aligned} \dot{x}_i(t) \leq x_i(t)\Biggl(&-r_i(t) + \sum_{j=1}^{m} a_{ij}(t)(\bar{M}_j + \varepsilon)^{\alpha_{ij}} \\ &+ \sum_{j=1}^{m} b_{ij}(t)(\bar{M}_j + \varepsilon)^{\beta_{ij}} - b_{ii}(t)x_i^{\beta_{ii}}\bigl(t - \tau_{ii}(t)\bigr)\Biggr), \end{aligned} \tag{2.30}$$

from (2.30), by a procedure similar to the discussion above, we can verify that

$$\lim_{t\to+\infty} \sup x_i(t) \leq \left(\frac{\lambda_i}{b_{ii}^l}\right)^{\frac{1}{\beta_{ii}}} \exp[\lambda_i \tau] := \bar{M}_i, \quad i = m+1, \ldots, n, \tag{2.31}$$

where

$$\lambda_i = -r_i^l + \sum_{j=1}^{m} a_{ij}^u \bar{M}_j^{\alpha_{ij}} + \sum_{j=1}^{m} b_{ij}^u \bar{M}_j^{\beta_{ij}}.$$

From the condition (H_5) of Theorem 2.2, we could choose $\varepsilon > 0$ small enough such that

$$r_i^l > \sum_{j=1}^{n} a_{ij}^u (M_j + \varepsilon)^{\alpha_{ij}} + \sum_{j=1, j\neq i}^{n} b_{ij}^u (M_j + \varepsilon)^{\beta_{ij}}, \tag{2.32}$$

thus, for $\varepsilon > 0$ it satisfies (2.32), from (2.28) and (2.31), we know that there exists $T_4^* > T_3^* + \tau$ such that for all $i = 1, 2, \ldots, n$ and $t \geq T_4^*$

$$x_i(t) \leq \bar{M}_i + \varepsilon. \tag{2.33}$$

And so, for $i = 1, \ldots, m$ and $t \geq T_4^* + \tau$, by applying (2.33), from the ith equation of system (1.4), one has

$$\dot{x}_i(t) \geq x_i(t)\left(r_i(t) - \sum_{j=1}^{n} a_{ij}(t)(\bar{M}_j + \varepsilon)^{\alpha_{ij}} - \sum_{j=1}^{n} b_{ij}(t)(\bar{M}_j + \varepsilon)^{\beta_{ij}}\right) \tag{2.34}$$

and

$$\begin{aligned} \dot{x}_i(t) \geq x_i(t)\Biggl(&r_i(t) - \sum_{j=1}^{n} a_{ij}(t)(\bar{M}_j + \varepsilon)^{\alpha_{ij}} \\ &- \sum_{j=1, j\neq i}^{n} b_{ij}(t)(\bar{M}_j + \varepsilon)^{\beta_{ij}} - b_{ii}(t)x_i^{\beta_{ii}}\bigl(t - \tau_{ii}(t)\bigr)\Biggr), \end{aligned} \tag{2.35}$$

thus

$$\frac{d(x_i^{\beta_{ii}}(t))}{dt} \geq \beta_{ii} x_i^{\beta_{ii}}(t)\left(r_i(t) - \sum_{j=1}^{n} a_{ij}(t)(\bar{M}_j + \varepsilon)^{\alpha_{ij}} - \sum_{j=1}^{n} b_{ij}(t)(\bar{M}_j + \varepsilon)^{\beta_{ij}}\right) \tag{2.36}$$

and

$$\begin{aligned}\frac{d(x_i^{\beta_{ii}}(t))}{dt} \geq{}& \beta_{ii} x_i^{\beta_{ii}}(t)\Biggl(r_i(t) - \sum_{j=1}^{n} a_{ij}(t)(\bar{M}_j + \varepsilon)^{\alpha_{ij}} \\ &- \sum_{j=1,j\neq i}^{n} b_{ij}(t)(\bar{M}_j + \varepsilon)^{\beta_{ij}} - b_{ii}(t)x_i^{\beta_{ii}}\bigl(t - \tau_{ii}(t)\bigr)\Biggr).\end{aligned} \tag{2.37}$$

Let $u_i(t) = x_i^{\beta_{ii}}(t)$, we have

$$\begin{aligned}\dot{u}_i(t) &\geq \beta_{ii} u_i(t)\left(r_i(t) - \sum_{j=1}^{n} a_{ij}(t)(\bar{M}_j + \varepsilon)^{\alpha_{ij}} - \sum_{j=1}^{n} b_{ij}(t)(\bar{M}_j + \varepsilon)^{\beta_{ij}}\right) \\ &\geq \beta_{ii} u_i(t)\left(r_i^l - \sum_{j=1}^{n} a_{ij}^u(\bar{M}_j + \varepsilon)^{\alpha_{ij}} - \sum_{j=1}^{n} b_{ij}^u(\bar{M}_j + \varepsilon)^{\beta_{ij}}\right) \\ &= \beta_{ii} u_i(t)\Gamma_{i\varepsilon}\end{aligned} \tag{2.38}$$

and

$$\begin{aligned}\dot{u}_i(t) \geq{}& \beta_{ii} u_i(t)\Biggl(r_i(t) - \sum_{j=1}^{n} a_{ij}(t)(\bar{M}_j + \varepsilon)^{\alpha_{ij}} \\ &- \sum_{j=1,j\neq i}^{n} b_{ij}(t)(\bar{M}_j + \varepsilon)^{\beta_{ij}} - b_{ii}(t)u_i\bigl(t - \tau_{ii}(t)\bigr)\Biggr) \\ \geq{}& \beta_{ii} u_i(t)\Biggl(r_i^l - \sum_{j=1}^{n} a_{ij}^u(\bar{M}_j + \varepsilon)^{\alpha_{ij}} \\ &- \sum_{j=1,j\neq i}^{n} b_{ij}^u(\bar{M}_j + \varepsilon)^{\beta_{ij}} - b_{ii}^u(t)u_i\bigl(t - \tau_{ii}(t)\bigr)\Biggr).\end{aligned} \tag{2.39}$$

Note that $\frac{r_i^u}{b_{ii}^l} \leq \bar{M}_i^{\beta_{ii}}$ implies that

$$\begin{aligned}\Gamma_{i\varepsilon} &= r_i^l - \sum_{j=1}^{n} a_{ij}^u(\bar{M}_j + \varepsilon)^{\alpha_{ij}} - \sum_{j=1}^{n} b_{ij}^u(\bar{M}_j + \varepsilon)^{\beta_{ij}} \\ &\leq r_i^l - b_{ii}^u(\bar{M}_i + \varepsilon)^{\beta_{ii}} \\ &\leq r_i^u - b_{ii}^l(\bar{M}_i + \varepsilon)^{\beta_{ii}} \leq 0.\end{aligned}$$

Now we consider the following two cases.

Case (i). If $\Gamma_{i\varepsilon} = 0$, then for $t \geq T_4^* + \tau$, from Lemma 2.1 and (2.38) it follows that

$$\dot{u}_i(t) = 0,$$

this implies that $\lim_{t\to+\infty} u_i(t) =: \rho_i < \frac{r_i^l}{b_{ii}^u}$, then there exists $T_5^* > T_4^*$ such that for $t \geq T_5^*$

$$u_i(t) \leq \rho_i + \frac{r_i^l/b_{ii}^u - \rho_i}{2} < \frac{r_i^l}{b_{ii}^u} < \bar{M}_i^{\beta_{ii}}. \tag{2.40}$$

From the ith equation of system (1.4) and (2.40) if follows that

$$\begin{aligned}\dot{u}_i(t) &\geq \beta_{ii}u_i(t)\left(r_i^l - \sum_{j=1}^n a_{ij}^u\left(\frac{r_j^l/b_{jj}^u + \rho_j}{2}\right)^{\alpha_{ij}} - \sum_{j=1}^n b_{ij}^u\left(\frac{r_j^l/b_{jj}^u + \rho_j}{2}\right)^{\beta_{ij}}\right)\\ &> \beta_{ii}u_i(t)\Gamma_{i\varepsilon} = 0, \quad t \geq T_5^* + \tau.\end{aligned} \tag{2.41}$$

Thus

$$\begin{aligned}u_i(t) \geq u_i\left(T_5^* + \tau\right)\exp\Bigg[\beta_{ii}\Bigg(r_i^l - \sum_{j=1}^n a_{ij}^u\left(\frac{r_j^l/b_{jj}^u + \rho_j}{2}\right)^{\alpha_{ij}}\\ - \sum_{j=1}^n b_{ij}^u\left(\frac{r_j^l/b_{jj}^u + \rho_j}{2}\right)^{\beta_{ij}}\Bigg)\left(t - \left(T_5^* + \tau\right)\right)\Bigg],\end{aligned} \tag{2.42}$$

then we have $u_i(t) \to +\infty$ as $t \to +\infty$, which is in contradiction with $u_i(t) \leq (\bar{M}_i + \varepsilon)^{\beta_{ii}}$. Hence we have $\lim_{t\to+\infty} u_i(t) \geq \frac{r_i^l}{b_{ii}^u}$, which implies that there exists T_5' such that $u_i(t) \geq \frac{r_i^l}{2b_{ii}^u}$ for $t \geq T_5'$, that is, $x_i(t) \geq (\frac{r_i^l}{2b_{ii}^u})^{\frac{1}{\beta_{ii}}}$ for $t \geq T_5'$.

Case (ii). If $\Gamma_{i\varepsilon} < 0$, from (2.39), for $t \geq T_4^* + \tau$, it follows that

$$\begin{aligned}\dot{u}_i(t) \geq \beta_{ii}u_i(t)\Bigg(r_i^l - \sum_{j=1}^n a_{ij}^u(\bar{M}_j + \varepsilon)^{\alpha_{ij}}\\ - \sum_{j=1,j\neq i}^n b_{ij}^u(\bar{M}_j + \varepsilon)^{\beta_{ij}} - b_{ii}^u(t)u_i\left(t - \tau_{ii}(t)\right)\Bigg).\end{aligned} \tag{2.43}$$

Set

$$\tilde{m}_i = \frac{r_i^l - \sum_{j=1}^n a_{ij}^u(\bar{M}_j + \varepsilon)^{\alpha_{ij}} - \sum_{j=1,j\neq i}^n b_{ij}^u(\bar{M}_j + \varepsilon)^{\beta_{ij}}}{b_{ii}^u}(1 - \sigma_i),$$

where $0 < \sigma_i < 1 - \exp[\beta_{ii}\Gamma_{i\varepsilon}\tau]$.

Firstly, suppose $u_i(t)$ is not oscillatory about $\tilde{m}_i$. That is, there exists a $T_6^* > 0$, for $t > T_6^*$ such that

$$u_i(t) > \tilde{m}_i, \tag{2.44}$$

or

$$u_i(t) < \tilde{m}_i. \tag{2.45}$$

If (2.44) holds, then our aim is obtained. Suppose (2.45) holds, then for $t \geq T_6^* + \tau$, we obtain

$$\dot{u}_i(t) \geq \beta_{ii}\sigma_i\left(r_i^l - \sum_{j=1}^{n} a_{ij}^u(\bar{M}_j + \varepsilon)^{\alpha_{ij}} - \sum_{j=1,j\neq i}^{n} b_{ij}^u(\bar{M}_j + \varepsilon)^{\beta_{ij}}\right)u_i(t),$$

thus there must exist $T_6' > T_6^* + \tau$ such that $u_i(t) > \tilde{m}_i$ for $t > T_6'$, which is a contradiction. Hence, (2.45) could not hold. Secondly now assume that $u_i(t)$ is oscillatory about $\tilde{m}_i$ for $t \geq T_4^* + \tau$, that is, there exists a time sequence $\{t_n\}$ such that $\tau < t_i < t_2 < \cdots < t_n < \cdots$ is a sequence of zeros of $u_i(t_n) - \tilde{m}_i$ with $\lim_{n\to\infty} t_n = +\infty$ and $u_i(t_n) = \tilde{m}_i$. Set $\hat{t}_n$ to be a point where $u_i(t)$ attends its minimum in (t_n, t_{n+1}). Thus, we get $u_i(\hat{t}_n) \leq u_i(t_n) = \tilde{m}_i$. Then it follows from (2.43) that

$$\begin{aligned} 0 = \dot{u}_i(t)|_{t=\hat{t}_n} \geq \beta_{ii}u_i(\hat{t}_n)\Bigg(& r_i^l - \sum_{j=1}^{n} a_{ij}^u(\bar{M}_j + \varepsilon)^{\alpha_{ij}} \\ & - \sum_{j=1,j\neq i}^{n} b_{ij}^u(\bar{M}_j + \varepsilon)^{\beta_{ij}} - b_{ii}^u u_i\big(\hat{t}_n - \tau_{ii}(\hat{t}_n)\big)\Bigg), \end{aligned} \tag{2.46}$$

which implies that

$$u_i\big(\hat{t}_n - \tau_{ii}(\hat{t}_n)\big) \geq \frac{r_i^l - \sum_{j=1}^{n} a_{ij}^u(\bar{M}_j + \varepsilon)^{\alpha_{ij}} - \sum_{j=1,j\neq i}^{n} b_{ij}^u(\bar{M}_j + \varepsilon)^{\beta_{ij}}}{b_{ii}^u}. \tag{2.47}$$

Integrating (2.43) on the interval $[\hat{t}_n - \tau_{ii}(\hat{t}_n), \hat{t}_n]$, we have

$$\begin{aligned} \ln\frac{u_i(\hat{t}_n)}{u_i(\hat{t}_n - \tau_{ii}(\hat{t}_n))} &\geq \int_{\hat{t}_n-\tau_{ii}(\hat{t}_n)}^{\hat{t}_n} \beta_{ii}\Bigg(r_i^l - \sum_{j=1}^{n} a_{ij}^u(\bar{M}_j + \varepsilon)^{\alpha_{ij}} \\ &\qquad - \sum_{j=1,j\neq i}^{n} b_{ij}^u(\bar{M}_j + \varepsilon)^{\beta_{ij}} - b_{ii}^u(t)u_i\big(t - \tau_{ii}(t)\big)\Bigg)\,dt \\ &\geq \int_{\hat{t}_n-\tau_{ii}(\hat{t}_n)}^{\hat{t}_n} \beta_{ii}\Gamma_{i\varepsilon}\,dt \\ &= \beta_{ii}\Gamma_{i\varepsilon}\tau_{ii}(\hat{t}_n). \end{aligned} \tag{2.48}$$

From (2.47) and (2.48) we get

$$u_i(\hat{t}_n) \geq \frac{r_i^l - \sum_{j=1}^{n} a_{ij}^u(\bar{M}_j + \varepsilon)^{\alpha_{ij}} - \sum_{j=1,j\neq i}^{n} b_{ij}^u(\bar{M}_j + \varepsilon)^{\beta_{ij}}}{b_{ii}^u}\exp[\beta_{ii}\Gamma_{i\varepsilon}\tau].$$

Since $u_i(\hat{t}_n)$ is an arbitrary local minimum of $u_i(t)$, we might find there exists a $l_i \leq \tilde{m}_i$ such that $u_i(t) \geq l_i$ for all $t \geq T_7^*$. Thus, we have

$$\lim_{t\to+\infty}\inf u_i(t) \geq l_i,$$

where

$$l_i \leq \left\{ \left(r_i^l - \sum_{j=1}^{n} a_{ij}^u \bar{M}_j^{\alpha_{ij}} - \sum_{j=1, j\neq i}^{n} b_{ij}^u \bar{M}_j^{\beta_{ij}} \right) \Big/ b_{ii}^u, i = 1, 2, \ldots, m \right\}.$$

Take $\bar{m}_i = \min_{1\leq i\leq n}\{(\frac{r_i^l}{2b_i^u})^{\frac{1}{\beta_{ii}}}, l_i^{\frac{1}{\beta_{ii}}}\}$, we obtain

$$\lim_{t\to+\infty} \inf x_i(t) \geq \bar{m}_i. \tag{2.49}$$

From the condition (H_6) of Theorem 2.2, we could choose $\varepsilon > 0$ small enough such that

$$\begin{aligned} &-r_i^u + \sum_{j=1}^{m} a_{ij}^l (m_j - \varepsilon)^{\alpha_{ij}} + \sum_{j=1}^{m} b_{ij}^l (m_j - \varepsilon)^{\beta_{ij}} \\ &\quad - \sum_{j=m+1, j\neq i}^{n} a_{ij}^u (M_j + \varepsilon)^{\alpha_{ij}} - \sum_{j=m+1}^{n} b_{ij}^u (M_j + \varepsilon)^{\beta_{ij}} \geq 0, \end{aligned} \tag{2.50}$$

thus, for $\varepsilon > 0$ it satisfies (2.50), and it follows from (2.49) that there exists large enough $T_8^* > T_7^*$ such that for all $i = 1, 2, \ldots, m$ and $t \geq T_8^*$

$$x_i(t) \geq \bar{m}_i - \varepsilon. \tag{2.51}$$

For $i = m+1, \ldots, n$ and $t \geq T_8^* + \tau$, by using (2.51), from the ith equation of system (1.4), one has

$$\begin{aligned} \dot{x}_i(t) \geq x_i(t) \Bigg(&-r_i^u + \sum_{j=1}^{m} a_{ij}^l (m_j - \varepsilon)^{\alpha_{ij}} + \sum_{j=1}^{m} b_{ij}^l (m_j - \varepsilon)^{\beta_{ij}} \\ &- \sum_{j=m+1}^{n} a_{ij}^u (M_j + \varepsilon)^{\alpha_{ij}} - \sum_{j=m+1}^{n} b_{ij}^u (M_j + \varepsilon)^{\beta_{ij}} \Bigg) \end{aligned} \tag{2.52}$$

and

$$\begin{aligned} \dot{x}_i(t) \geq x_i(t) \Bigg(&-r_i(t) + \sum_{j=1}^{m} a_{ij}(t) (m_j - \varepsilon)^{\alpha_{ij}} + \sum_{j=1}^{m} b_{ij}(t) (m_j - \varepsilon)^{\beta_{ij}} \\ &- \sum_{j=m+1}^{n} a_{ij}(t) (M_j + \varepsilon)^{\alpha_{ij}} - \sum_{j=m+1, j\neq i}^{n} b_{ij}(t) (M_j + \varepsilon)^{\beta_{ij}} - b_{ii}(t) x_i^{\beta_{ii}} \big(k - \tau_{ii}(t)\big) \Bigg). \end{aligned} \tag{2.53}$$

From (2.52) and (2.53), similar to the argument of (2.34) and (2.35), we also have

$$\lim_{t\to+\infty} \inf x_i(t) \geq \bar{m}_i, \quad i = m+1, \ldots, n, \tag{2.54}$$

where

$$\bar{m}_i = \min_{1\leq i\leq n} \left\{ \left(\frac{-r_i^u + \sum_{j=1}^{m} a_{ij}^l m_j^{\alpha_{ij}} + \sum_{j=1}^{m} b_{ij}^l m_j^{\beta_{ij}}}{2b_i^u} \right)^{\frac{1}{\beta_{ii}}}, l_i^{\frac{1}{\beta_{ii}}} \right\},$$

$$l_i \leq \left\{\nabla_i / b_{ii}^u, i = m+1, \ldots, n\right\},$$

$$\nabla_i = -r_i^u + \sum_{j=1}^{m} a_{ij}^l m_j^{\alpha_{ij}} + \sum_{j=1}^{n} b_{ij}^l m_j^{\beta_{ij}} - \sum_{j=m+1}^{n} a_{ij}^u M_j^{\alpha_{ij}} - \sum_{j=m+1, j\neq i}^{n} b_{ij}^u M_j^{\beta_{ij}}.$$

Take $\bar{M} = \max_{1\leq i\leq n}\{\bar{M}_i\}$, $\bar{m} = \min_{1\leq i\leq n}\{\bar{m}_i\}$, we have

$$\bar{m} \leq \liminf_{t\to+\infty} x_i(t) \leq \limsup_{t\to+\infty} x_i(t) \leq \bar{M}, \quad i = 1, 2, \ldots, n.$$

This completes the proof of Theorem 2.2. □

3 Concluding remarks

In this paper, we study a Gilpin-Ayala predator-prey model with time-dependent delay with m preys and $(n-m)$ predators. In this system, the competition among the predator species and among the prey species are simultaneously considered. The system (1.4) can be seen as the modification of the traditional Lotka-Volterra prey-competition system. Some new and interesting sufficient conditions are obtained for the permanence of the system (1.4). In [7], under the assumption that the coefficient of the density-dependent term $a_{ii}(t)$ must be positive, by using a new differential inequality, two sets of sufficient conditions on the permanence of the system (1.4) are obtained. However, in this paper we allow the coefficient to be zero; therefore the study of the permanence of the population becomes technically more difficult. Our results are different from the existing ones such as those of Huang [7]. In some sense, our results supplement those obtained by Chen *et al.* [6], generalize the results in [7], and have further application on the population dynamics.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
HL carried out the main part of this article, GY corrected the main theorems. All authors read and approved the final manuscript.

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