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On a power-type coupled system with mean curvature operator in Minkowski space

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Abstract

We study the Dirichlet problem for the prescribed mean curvature equation in Minkowski space

$$\begin{cases} \mathcal{M}(u) + v^{\alpha} = 0 & \text{in } B, \\ \mathcal{M}(v) + u^{\beta} = 0 & \text{in } B, \\ u|_{\partial B} = v|_{\partial B} = 0, \end{cases}$$

where $\mathcal{M}(w) = \mathrm{div}(\frac{\nabla w}{\sqrt{1-|\nabla w|^2}})$ and B is a unit ball in $\mathbb{R}^N (N \geq 2)$. We use the index theory of fixed points for completely continuous operators to obtain the existence, nonexistence and uniqueness results of positive radial solutions under some corresponding assumptions on α, β.

MSC: 34B15; 35J66

Keywords: Minkowski curvature operator; System; Positive radial solution; Uniqueness

1 Introduction and main results

Consider the Dirichlet problem of a quasilinear differential system of the type

$$\begin{cases} \mathcal{M}(u) + v^{\alpha} = 0 & \text{in } B, \\ \mathcal{M}(v) + u^{\beta} = 0 & \text{in } B, \\ u|_{\partial B} = v|_{\partial B} = 0, \end{cases} \tag{1.1}$$

where $\mathcal{M}$ stands for the mean curvature operator in Minkowski space

$$\mathcal{M}(w) := \mathrm{div}\left(\frac{\nabla w}{\sqrt{1-|\nabla w|^2}}\right),$$

$B = \{x \in \mathbb{R}^N : |x| < 1\}$, $N \geq 2$ is an integer.

Minkowski-curvature equations are quasilinear second-order PDEs, and there are important applications in differential geometry and the theory of relativity. Geometrically, these are related to maximal and constant mean curvature spacelike hypersurfaces having the property that the trace of the extrinsic curvature is zero, respectively, constant (see [12]).

It is known (see [1]) that the study of spacelike submanifolds of codimension one in the flat Minkowski space $\mathbb{L}^{N+1}$ ($\mathbb{L}^{N+1} := \{(x,t) : x \in \mathbb{R}^N, t \in \mathbb{R}\}$ endowed with the Lorentzian metric $\sum_{j=1}^{N}(dx_j)^2 - (dt)^2$, where (x,t) are the canonical coordinates in $\mathbb{R}^{N+1}$) with prescribed mean extrinsic curvature, can lead to the type

$$\mathcal{M}v = H(x,v) \quad \text{in } \Omega, \qquad v = 0 \quad \text{on } \partial\Omega, \tag{1.2}$$

where Ω is a bounded domain in $\mathbb{R}^N$ and the nonlinearity $H : \Omega \times \mathbb{R} \to \mathbb{R}$ is continuous. The existence and multiplicity of positive solutions of problem (1.2) have been discussed in the last two decades by several authors (see [1–7, 15, 16, 20, 22, 23]) in connection with various configurations of H. If Ω is a bounded domain and H is a bounded function defined on $\Omega \times \mathbb{R}$, Bartnik and Simon [1] proved that the problem (1.2) has a strictly spacelike solution. In particular, if $\Omega = B_R := \{x \in \mathbb{R}^N : |x| < R\}$ with $R > 0$, Bereanu, Jebelean and Torres [2, 3] established some existence/nonexistence and multiplicity results for positive radial solutions of problem (1.2) via a Leray–Schauder degree argument and critical point theory. In [6, 7, 15, 20], by using the bifurcation method, the authors studied the existence, multiplicity, and the global behavior of positive solutions of problem (1.2) with $H = \lambda f(x,v)$ on the unit ball. However, to the authors' best knowledge, the study of the Dirichlet problem of a quasilinear differential system with mean curvature operator $\mathcal{M}$ seems to be in its early stages, we refer the reader to [10–13, 16, 17, 21] and the references therein. For instance, Gurban et al. [11] investigated the following two-parameter problem

$$\begin{cases} \mathcal{M}(u) + \lambda_1 g_1(|x|,u,v) = 0 & \text{in } B, \\ \mathcal{M}(v) + \lambda_2 g_2(|x|,u,v) = 0 & \text{in } B, \\ u|_{\partial B} = v|_{\partial B} = 0. \end{cases} \tag{1.3}$$

By using the fixed-point index, they obtained the following results:

Theorem A *Suppose $g_i : [0,1] \times [0,\infty)^2 \to [0,\infty)$, $i = 1,2$ are continuous, quasimonotone nondecreasing with respect to both s, t and satisfy for every $r \in (0,1]$,*

$$g_i(r,s,t) > 0, \quad \forall s,t > 0, \qquad g_1(r,\xi,0) = g_2(r,0,\xi) = 0, \quad \forall \xi > 0$$

and

$$\int_0^b \tau^{N-1} g_i(\tau,\alpha,\alpha)\,d\tau > 0, \quad i = 1,2,$$

where $b \in (0,1)$, $0 < \alpha < 1 - b$ are constants. Then, there exist $\lambda_1^* > 0 < \lambda_2^*$ such that for all $\lambda_1 > \lambda_1^*$ and $\lambda_2 > \lambda_2^*$, problem (1.3) has at least one positive radial solution. Note that (1.1) is a special case of (1.3) and Theorem A does not cover the case where $\lambda_1 = \lambda_2 = 1$.

In 2015, Zhang and Qi [24] studied the following system coupled by Monge–Ampère equations:

$$\begin{cases} \det D^2 u_1 = (-u_2)^{\alpha} & \text{in } \Omega, \\ \det D^2 u_2 = (-u_1)^{\beta} & \text{in } \Omega, \\ u_1 < 0, \qquad u_2 < 0 & \text{in } \Omega, \\ u_1 = 0, \qquad u_2 = 0 & \text{on } \partial\Omega, \end{cases} \tag{1.4}$$

where Ω is a ball in $\mathbb{R}^N, N \geq 2$, $\alpha > 0$, $\beta > 0$, $\det D^2 u$ stands for the determinant of the Hessian matrix $(\frac{\partial^2 u}{\partial x_i \partial x_j})$ of u. By reducing it to a system coupled by ODEs and using the fixed-point index, they obtained the existence, uniqueness results and nonexistence of radial convex solutions under some corresponding assumptions on α, β.

Motivated by these studies, the main objective of this paper is to investigate the existence/nonexistence and uniqueness of positive radial solutions for system (1.1) on the unit ball B mainly by the fixed-point index in a cone in the same way as in [24]. Our results are completely new and complementary to the results of [11].

We obtain:

Theorem 1.1 *System* (1.1) *has a positive radial solution if* $\alpha > 0$, $\beta > 0$ *and* $\alpha\beta < 1$.

Theorem 1.2 *Let* $\alpha > 0$, $\beta > 0$ *and* $\alpha\beta < 1$, *then system* (1.1) *has a unique positive radial solution.*

Theorem 1.3 *If* $\alpha > 0$, $\beta > 0$ *and* $\alpha\beta = 1$, *then system* (1.1) *has no positive radial solution.*

This paper is organized as follows: In Sect. 2, some preliminaries are given; in Sect. 3, we obtain the main results.

2 Preliminaries

In order to present the existence results of positive radial solutions for system (1.1), setting $r = |x|$ and $u(|x|) = u(r)$, $v(|x|) = v(r)$, the system (1.1) reduces to the homogeneous mixed boundary-value problem:

$$\begin{cases} (r^{N-1}\phi(u'))' + r^{N-1}v^{\alpha} = 0, \\ (r^{N-1}\phi(v'))' + r^{N-1}u^{\beta} = 0, \\ u'(0) = u(1) = 0 = v(1) = v'(0). \end{cases} \tag{2.1}$$

By a solution of (2.1) we mean a couple of nonnegative functions $(u, v) \in C^1[0,1] \times C^1[0,1]$ with $\|u'\| < 1$, $\|v'\| < 1$ and $r \mapsto r^{N-1}\phi(u'(r))$, $r \mapsto r^{N-1}\phi(v'(r))$ of class C^1 on $[0,1]$, which satisfies problem (2.1). Here and below, $\|\cdot\|$ stands for the usual sup-norm on $C := C[0,1]$.

The following lemma is a direct consequence of [18, Lemma 2.2].

Lemma 2.1 *For any* $u \in C([0,1],[0,\infty))$ *for which* $u'(r)$ *is decreasing in* $[0,1]$ *we have*

$$\min_{r \in [\frac{1}{4},\frac{3}{4}]} u(r) \geq \frac{1}{4}\|u\|.$$

Lemma 2.2 ([14]) *Let* $\phi(s) := s/\sqrt{1-s^2}$. *Then,* $\phi^{-1}(s) = s/\sqrt{1+s^2}$ *and*

$$\phi^{-1}(s_1)\phi^{-1}(s_2) \le \phi^{-1}(s_1 s_2) \le s_1 s_2, \quad \forall s_1, s_2 \in [0,\infty).$$

In particular, for $0 < s_1 \le 1$ *we have*

$$\phi^{-1}(s_1 s_2) \ge s_1 \phi^{-1}(s_2).$$

Define P to be a cone in C by

$$P = \left\{ u \in C : u(t) > 0, t \in [0,1], \text{and } \min_{t\in[\frac{1}{4},\frac{3}{4}]} u(t) \ge \frac{1}{4}\|u\| \right\}.$$

Define $P_R = \{u \in P : \|u\| < R\}$ for $R > 0$.

For each $u \in P$, we define two solution operators $T_i : P \to P$ $(i = 1, 2)$ as follows:

$$(T_1 u)(r) = \int_r^1 \phi^{-1}\left(\frac{1}{t^{N-1}} \int_0^t s^{N-1} u^\alpha \, ds\right) dt \tag{2.2}$$

and

$$(T_2 u)(r) = \int_r^1 \phi^{-1}\left(\frac{1}{t^{N-1}} \int_0^t s^{N-1} u^\beta \, ds\right) dt. \tag{2.3}$$

From [19], we know that each operator T_i, $i = 1, 2$ is a nonnegative concave function, this combines with Lemma 2.1, we have $T_i : P \to P$ is a completely continuous operator. Define a composite operator $T = T_1 \circ T_2$, which is also completely continuous from P to itself. This implies from (2.2) and (2.3) that $(v_1, v_2) \in C^1[0,1] \times C^1[0,1]$ is the solution of (2.1) if and only if $v_1 = T_1 v_2$, $v_2 = T_2 v_1$, where $(v_1, v_2) \in P \setminus \{0\} \times P \setminus \{0\}$.

Thus, if $v_1 \in P \setminus \{0\}$ is a fixed point of T, define $v_2 = T_2 v_1$, then $v_2 \in P \setminus \{0\}$ so that $(v_1, v_2) \in C^1[0,1] \times C^1[0,1]$ solves (1.1); conversely, if $(v_1, v_2) \in C^1[0,1] \times C^1[0,1]$ solves (1.1), then v_1 must be a nonzero fixed point of T in P. Hence, our task is to search for nonzero fixed points of T.

Lemma 2.3 ([8]) *Let E be a Banach space and K a cone in E. For $r > 0$, define $K_r = K \cap B_r$. Assume that $T : \bar{K}_r \to K$ is completely continuous such that $Tx \neq x$ for $x \in \partial K_r = \{x \in K : \|x\| = r\}$.*

(i) *If $\|Tx\| \ge \|x\|$ for $x \in \partial K_r$, then $i(T, K_r, K) = 0$.*

(ii) *If $\|Tx\| \le \|x\|$ for $x \in \partial K_r$, then $i(T, K_r, K) = 1$.*

Definition 2.4 ([9, 18]) Let K be a cone in real Banach space Y. Let $A : K \to K$ and $u_0 > \theta$, where θ denotes the zero element Y.

(i) For any $x > \theta$, there exist $\theta_1, \theta_2 > 0$ such that

$$\theta_1 u_0 \le A(x) \le \theta_2 u_0.$$

(ii) For any $\alpha u_0 \le x \le \beta u_0$ and $t \in (0,1)$, there exists some $\eta > 0$ such that

$$A(tx) \ge (1+\eta) t A x.$$

Then, A is called u_0-sublinear.

Lemma 2.5 ([9, 18]) *An increasing and u_0-sublinear operator T can have at most one positive fixed point.*

3 Proof of main results

Proof of Theorem 1.1 By Lemma 2.2 and the definition of T_2, for each $u \in P$, we have

$$\begin{aligned}\|T_2u\| &= \int_0^1 \phi^{-1}\left(\frac{1}{t^{N-1}}\int_0^t s^{N-1}u^\beta\,ds\right)dt \\ &\geq \int_{\frac{1}{4}}^{\frac{3}{4}} \phi^{-1}\left(\frac{1}{t^{N-1}}\int_{\frac{1}{4}}^t s^{N-1}\left(\frac{1}{4}\|u\|\right)^\beta ds\right)dt \\ &\geq \int_{\frac{1}{4}}^{\frac{3}{4}} \phi^{-1}\left(\left(\frac{1}{4}\|u\|\right)^\beta\right)\int_{\frac{1}{4}}^t \left(\frac{s}{t}\right)^{N-1} ds\,dt \\ &= \phi^{-1}\left(\left(\frac{1}{4}\|u\|\right)^\beta\right)\int_{\frac{1}{4}}^{\frac{3}{4}}\int_{\frac{1}{4}}^t \left(\frac{s}{t}\right)^{N-1} ds\,dt \\ &\geq \Gamma_1\phi^{-1}\left(\|u\|^\beta\right),\end{aligned}$$

where Γ_1 is a positive constant given by

$$\Gamma_1 = \frac{1}{4^\beta}\int_{\frac{1}{4}}^{\frac{3}{4}}\int_{\frac{1}{4}}^t \left(\frac{s}{t}\right)^{N-1} ds\,dt.$$

By using the same method, we can obtain

$$\|T_1u\| \geq \Gamma_1\phi^{-1}\left(\|u\|^\alpha\right).$$

Hence, we have

$$\begin{aligned}\left\|T(u)\right\| &= \left\|T_1 \circ T_2(u)\right\| \\ &\geq \Gamma_1\phi^{-1}\left(\|T_2u\|^\alpha\right) \\ &\geq \Gamma_1^{1+\alpha}\phi^{-1}\left(\left(\phi^{-1}\left(\|u\|^\beta\right)\right)^\alpha\right) \\ &\geq \frac{1}{\sqrt{2}}\Gamma_1^{1+\alpha}\frac{\|u\|^{\alpha\beta}}{(1+\|u\|^{2\beta})^{\frac{\alpha}{2}}},\end{aligned}$$

which implies that

$$\|Tu\| \geq \Gamma_2\frac{\|u\|^{\alpha\beta}}{(1+\|u\|^{2\beta})^{\frac{\alpha}{2}}}, \tag{3.1}$$

where $\Gamma_2 = \frac{1}{\sqrt{2}}\Gamma_1^{1+\alpha}$.

On the other hand, for each $u \in P$, we have

$$\begin{aligned}\|T_2u\| &= \int_0^1 \phi^{-1}\left(\frac{1}{t^{N-1}}\int_0^t s^{N-1}u^\beta(s)\,ds\right)dt \\ &\leq \int_0^1 \phi^{-1}\left(\frac{1}{t^{N-1}}\int_0^t s^{N-1}\|u\|^\beta\,ds\right)dt\end{aligned}$$

$$\leq \int_0^1 \phi^{-1}\bigl(\|u\|^{\beta}\bigr)\,dt$$
$$\leq \|u\|^{\beta}.$$

In the same way, we can obtain

$$\|T_1 u\| \leq \|u\|^{\alpha}.$$

Moreover,

$$\|Tu\| \leq \bigl\|T_1 \circ T_2(u)\bigr\| \leq \|u\|^{\alpha\beta}. \tag{3.2}$$

Now, let us consider the case of $\alpha\beta < 1$. Since $\lim_{s\to 0} \frac{s^{\alpha\beta-1}}{(1+s^{2\beta})^{\frac{\alpha}{2}}} = \infty$, there exists $R_1 \in (0,1)$ small enough such that for every $u \in P$ satisfying $\|u\| = R_1$, we have

$$\Gamma_2 \frac{\|u\|^{\alpha\beta-1}}{(1+\|u\|^{2\beta})^{\frac{\alpha}{2}}} > 1.$$

Now, by (3.1), we have

$$\|Tu\| > \|u\|.$$

Moreover, by (3.2), we know that there exists $R_2 > R_1$, and for each $u \in P$ satisfying $\|u\| = R_2$ it holds that

$$\|Tu\| < \|u\|.$$

By Lemma 2.3, we have

$$i(T, P_{R_1}, P) = 0, \qquad i(T, P_{R_2}, P) = 1.$$

Therefore, $i(T, P_{R_2} \setminus \bar{P}_{R_1}, P) = 1$, which implies that T has a fixed-point $u_1 \in P_{R_2} \setminus \bar{P}_{R_1}$. □

Remark 3.1 Bereanu et al. [3] studied the existence and multiplicity of positive radial solutions for the problem

$$\mathcal{M}u + \lambda u^{q} = 0 \quad \text{in } B, \qquad u = 0 \quad \text{on } \partial B, \tag{3.3}$$

where $q > 1$. By using the Leray–Schauder degree argument and critical point theory, they obtained a sharper result: there exists $\Lambda > 0$ such that problem (3.3) has zero, at least one or at least two positive radial solutions according to $\lambda \in (0,\Lambda)$, $\lambda = \Lambda$ or $\lambda > \Lambda$. In [11], Gurban et al. investigated the existence of positive radial solutions for the Dirichlet problem of a quasilinear differential system of type

$$\begin{cases} \mathcal{M}(u) + \lambda_1 u^{p_1} v^{q_1} = 0 & \text{in } B, \\ \mathcal{M}(v) + \lambda_2 u^{p_2} v^{q_2} = 0 & \text{in } B, \\ u|_{\partial B} = v|_{\partial B} = 0, \end{cases} \tag{3.4}$$

where p_1, q_2 are nonnegative, while q_1, p_2 are positive exponents. By using the fixed-point index, they proved that there exist $\lambda_i^* > 0$, such that for all $\lambda > \lambda_i^*$, system (3.4) has a positive radial solution (u, v). We note that the relationship between Λ and 1 (λ_i^* and 1) is still uncertain. For the same reason, in this paper, if $\alpha\beta > 1$, it is difficult to obtain any results in the case of $\lambda_i = 1$, $i = 1, 2$.

Next, we can further prove that the positive radial solution obtained in Theorem 1.1 is the unique positive radial solution to problem (1.1).

Proof of Theorem 1.2 By the definitions of T_1 and T_2, it is clear that T_1 and T_2 are both increasing operators induced by K, where $K := \{u \in C[0,1] : u(t) \geq 0, t \in [0,1]\}$ is a cone in $C[0,1]$. Obviously, $P \subset K$. Let $T = T_1 \circ T_2$. Then, by Theorem 1.1, if we want to obtain the solution of system (1.1), we only need to prove that T has at most one fixed-point in K. Furthermore, by Lemma 2.5, it suffices to verify that $T : K \to K$ is u_0-sublinear for some u_0 positive in $C[0,1]$. First, let us show that T_2 satisfies the Definition 2.4(i). In fact,

$$\begin{aligned}
(T_2u)(r) &= \int_r^1 \phi^{-1}\left(\frac{1}{t^{N-1}}\int_0^t s^{N-1}u^\beta \, ds\right) dt \\
&\leq \int_r^1 \phi^{-1}\left(\frac{1}{t^{N-1}}\int_0^t s^{N-1}\|u\|^\beta \, ds\right) dt \\
&\leq \int_r^1 \phi^{-1}\left(\frac{\|u\|^\beta}{t^{N-1}}\frac{t^N}{N}\right) dt \\
&\leq \int_r^1 \phi^{-1}\left(\frac{\|u\|^\beta}{N}t\right) dt \\
&\leq \frac{1}{2}\frac{\|u\|^\beta}{N}\left(1-r^2\right) \\
&\leq \frac{\|u\|^\beta}{N}(1-r).
\end{aligned}$$

Now, let $u_0 = 1 - r$, $r \in [0,1]$ and $\theta_2 = \frac{\|u\|^\beta}{N}$, then $T_2(u)(r) \leq \theta_2 u_0$.

Next, let $c \in (0,1)$ be a fixed number and $\Gamma_3 = \phi^{-1}(\frac{1}{c^{N-1}}\int_0^c s^{N-1}u^\beta(s)\, ds)$. Note that $T_2(u)(r)$ decreases with the variable r, then we obtain that

$$T_2(u)(r) \geq T_2u(c) \geq \Gamma_3(1-c), \quad r \in [0,c].$$

By Lemma 2.2 and the fact that $r \in [c,1]$, we have

$$\begin{aligned}
(T_2u)(r) &\geq \int_r^1 \phi^{-1}\left(\frac{1}{t^{N-1}}\int_0^t s^{N-1}\left(\frac{1}{4}\|u\|\right)^\beta ds\right) dt \\
&= \int_r^1 \phi^{-1}\left(\frac{(\frac{1}{4}\|u\|)^\beta}{N}t\right) dt \\
&\geq \phi^{-1}\left(\frac{(\frac{1}{4}\|u\|)^\beta}{N}\right)\int_r^1 t\, dt
\end{aligned}$$

$$\geq \phi^{-1}\left(\frac{(\frac{1}{4}\|u\|)^\beta}{N}\right)\int_r^1 c\,dt$$
$$\geq c\phi^{-1}\left(\frac{(\frac{1}{4}\|u\|)^\beta}{N}\right)(1-r)\,dt.$$

Choose $\Gamma_3' = \min\{\Gamma_3, c\phi^{-1}(\frac{(\frac{1}{4}\|u\|)^\beta}{N})\}$. Then, we have $T_2(u)(r) \geq \Gamma_3'(1-c)(1-r)$. Now, if we take $\theta_1 = \Gamma_3'(1-c)$, then $\theta_1 u_0 \leq T_2(u) \leq \theta_2 u_0$, which satisfies Definition 2.4(i). Similarly, T_1 also satisfies Definition 2.4(i). This implies that the operator T satisfies Definition 2.4(i).

Secondly, we prove that for any $\theta_1 u_0 \leq x \leq \theta_2 u_0$ and $\xi \in (0,1)$, there exists some $\eta > 0$ such that

$$T(\xi u) \geq (1+\eta)\xi Tu.$$

From the definition of T_1 and T_2, it is easy to obtain

$$T_2(\xi x) \geq \xi^\beta (T_2 x), \qquad T_1(\xi x) \geq \xi^\alpha (T_1 x).$$

Moreover, for $0 < \alpha\beta < 1$, there exists $\eta > 0$ such that

$$T(\xi x) \geq T_1 \circ \left(\xi^\beta T_2(x)\right) \geq \xi^{\alpha\beta} T_1 \circ T_2(x) \geq (1+\eta)\xi Tx,$$

which implies that T satisfies Definition 2.4(ii). Then, T is u_0-sublinear and T has at most one fixed-point in K by Lemma 2.5. Therefore, the system (1.1) has a unique positive radial solution. □

Finally, we prove the nonexistence results.

Proof of Theorem 1.3 Suppose on the contrary that T has a positive fixed point $v_0 \in P$, then $\|v_0\| = \|Tv_0\|$ and v_0 is a concave function satisfying

$$v_0(1) = 0, \qquad v_0(t) > 0, \quad t \in [0,1).$$

From the proof of Theorem 1.1, we know that for any $u \in P$, $\|T_1(u)\| \leq \|u\|^\alpha$ and $\|T_2(u)\| \leq \|u\|^\beta$. Let $u = v_0$, then combining this with the concavity property of v_0, we obtain that $\|T_1(v_0)\| < \|v_0\|^\alpha$ and $\|T_2(v_0)\| < \|v_0\|^\beta$. Moreover,

$$\left\|T(v_0)\right\| < \|v_0\|^{\alpha\beta} = \|v_0\|,$$

which is a contradiction. Therefore, the system (1.1) has no positive radial solution. □

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Competing interests
The authors declare that they have no competing interests.

Authors' contributions
ZQH completed the main study and wrote the manuscript, YZZ and LYM checked the proofs process and verified the calculations. Moreover, all the authors read and approved the last version of the manuscript.

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