Ren et al. *Boundary Value Problems* (2017) 2017:118
DOI 10.1186/s13661-017-0849-y

Boundary Value Problems
a SpringerOpen Journal

RESEARCH Open Access

Maximum and minimum solutions for a nonlocal p-Laplacian fractional differential system from eco-economical processes

Teng Ren[1*], Sidi Li[2], Xinguang Zhang[3,4] and Lishan Liu[4,5]

*Correspondence: hunantengren@163.com
[1]School of Logistics and Transportation, Central South University of Forestry and Technology, Changsha, Hunan 410004, China
Full list of author information is available at the end of the article

Abstract

This paper focuses on the maximum and minimum solutions for a fractional order differential system, involving a p-Laplacian operator and nonlocal boundary conditions, which arises from many complex processes such as ecological economy phenomena and diffusive interaction. By introducing new type growth conditions and using the monotone iterative technique, some new results about the existence of maximal and minimal solutions for a fractional order differential system is established, and the estimation of the lower and upper bounds of the maximum and minimum solutions is also derived. In addition, the iterative schemes starting from some explicit initial values and converging to the exact maximum and minimum solutions are also constructed.

Keywords: maximum and minimum solutions; fractional differential system; p-Laplacian operator; nonlocal boundary value problem

1 Introduction

The interest in using fractional differential equations in modeling ecological economy and diffusive processes has wide literature. Especially when one wants to model long-range ecological economy phenomena and diffusive interaction, fractional differential operator has higher accuracy than integer order differential model in depicting the co-evolution process of economic, social and ecological subsystems and the transport of solute in highly heterogeneous porous media. Recently this interest has also been activated by recent progress in the mathematical theory and psycho-socio-economical dynamics, see [1, 2]. On the other hand, since fractional order derivative, which exhibits a long time memory behavior, is nonlocal, thus except for diffusive processes in porous medium flow and ecological economy phenomena, the differential equation with fractional derivative can also describe many other physical phenomena in natural sciences and engineering, such as earthquake, traffic flow, measurement of viscoelastic material properties, polymer rheology and various material processes [3–37].

In this paper, we study the existence of maximum and minimum solutions for the following nonlocal fractional differential system:

$$\begin{cases} -\mathscr{D}_t^{\beta_1}(\varphi_{p_1}(-\mathscr{D}_t^{\alpha_1}x_1))(t) = f_1(x_1(t),x_2(t)), \\ -\mathscr{D}_t^{\beta_2}(\varphi_{p_2}(-\mathscr{D}_t^{\alpha_2}x_2))(t) = f_2(x_1(t),x_2(t)) \\ x_1(0) = 0, \qquad \mathscr{D}_t^{\alpha_1}x_1(0) = \mathscr{D}_t^{\alpha_1}x_1(1) = 0, \qquad x_1(1) = \int_0^1 x_1(t)\,dA_1(t), \\ x_2(0) = 0, \qquad \mathscr{D}_t^{\alpha_2}x_2(0) = \mathscr{D}_t^{\alpha_2}x_2(1) = 0, \qquad x_2(1) = \int_0^1 x_2(t)\,dA_2(t), \end{cases} \tag{1.1}$$

where $\mathscr{D}_t^{\alpha_i}$, $\mathscr{D}_t^{\beta_i}$ are the standard Riemann-Liouville derivatives satisfying $1 < \alpha_i, \beta_i \leq 2$, $\int_0^1 x(s)\,dA_i(s)$ denotes a Riemann-Stieltjes integral and A_i is a function of bounded variation, φ_{p_i} is the p-Laplacian operator defined by $\varphi_{p_i}(s) = |s|^{p_i-2}s$, $p_i > 1$, where $i = 1,2$. Obviously, $\varphi_{p_i}(s)$ is invertible and its inverse operator is $\varphi_{q_i}(s)$, where $q_i = \frac{p_i}{p_i-1}$, $i = 1,2$, are conjugate indices of p_i.

Recently, some interesting results about the existence of positive solutions for nonlinear fractional equation with p-Laplacian operator have been reported [21–26, 38]. In [23], Chen and Liu investigated the existence of solutions for the anti-periodic fractional order p-Laplacian boundary value problem with the following form:

$$\begin{cases} \mathscr{D}_t^{\beta}(\varphi_p(\mathscr{D}_t^{\alpha}x))(t) = f(t,x(t)), \quad t \in (0,1), \\ x(0) = -x(1), \qquad \mathscr{D}_t^{\alpha}x(0) = -\mathscr{D}_t^{\alpha}x(1), \end{cases} \tag{1.2}$$

where $0 < \alpha, \beta \leq 1, 1 < \alpha + \beta \leq 2$, $\mathscr{D}_t^{\beta}$ is a Caputo fractional derivative, and $f : [0,1] \times R^2 \to R$ is continuous. Under certain nonlinear growth conditions of the nonlinearity, a new existence result is obtained by using Schaefer's fixed point theorem. By means of upper and lower solutions method, Wang et al. [26] studied the existence of positive solutions for the following nonlocal fractional p-Laplacian equation:

$$\begin{cases} \mathscr{D}_t^{\beta}(\varphi_p(\mathscr{D}_t^{\alpha}x))(t) + f(t,x(t)) = 0, \quad t \in (0,1), \\ x(0) = 0, \qquad x(1) = ax(\xi), \qquad \mathscr{D}_t^{\alpha}x(0) = 0, \qquad \mathscr{D}_t^{\alpha}x(1) = b\mathscr{D}_t^{\alpha}x(\eta), \end{cases} \tag{1.3}$$

where $1 < \alpha, \beta \leq 2$, $0 \leq a, b \leq 1$, $0 < \xi, \eta < 1$. More recently, Zhang et al. [38] considered the uniqueness of positive solution for the following fractional order differential equation:

$$\begin{cases} \mathscr{D}_t^{\beta}(\varphi_p(-\mathscr{D}_t^{\alpha}x))(t) = -f(x(t), \mathscr{D}_t^{\gamma}x(t)), \quad t \in (0,1), \\ \mathscr{D}_t^{\alpha}x(0) = \mathscr{D}_t^{\alpha+1}x(0) = \mathscr{D}_t^{\alpha}x(1) = 0, \\ \mathscr{D}_t^{\gamma}x(0) = 0, \qquad \mathscr{D}_t^{\gamma}x(1) = \int_0^1 \mathscr{D}_t^{\gamma}x(s)\,dA(s), \end{cases} \tag{1.4}$$

where $\mathscr{D}_t^{\alpha}$, $\mathscr{D}_t^{\beta}$, $\mathscr{D}_t^{\gamma}$ are the standard Riemann-Liouville derivatives, $\int_0^1 x(s)\,dA(s)$ denotes a Riemann-Stieltjes integral and $0 < \gamma \leq 1 < \alpha \leq 2 < \beta < 3$, $\alpha - \gamma > 1$, A is a function of bounded variation and dA can be a signed measure. Under the case where the nonlinearity $f(u,v)$ may be singular at both $u = 0$ and $v = 0$, the uniqueness of positive solution for equation (1.4) was established via the fixed point theorem of the mixed monotone operator.

However, to the best of our knowledge, there are relatively few results on a fractional order differential system involving the p-Laplacian operator and nonlocal Riemann-Stieltjes integral boundary conditions, and no work has been done concerning the maximal and minimal solutions of system (1.1). Thus, motivated by the above work, in this paper, we consider the maximum and minimum solutions for a fractional order p-Laplacian system subject to a nonlocal Riemann-Stieltjes integral boundary condition. Difference from the above mentioned work is that in this paper we introduce new type growth condition of nonlinearity which covers a large number of nonlinear functions; at the same time, the existence, estimation of the lower and upper bounds and the convergent iterative scheme of minimal and maximal solutions for system (1.1) are also established.

2 Preliminaries and lemmas

A number of definitions for the fractional derivative have emerged over the years, and in this paper, we carry out our work base on the sense of Riemann-Liouville fractional derivatives; for details, see [15, 39, 40]. Here we only recall a famous semigroup property for Riemann-Liouville fractional calculus.

Proposition 2.1 (see [15, 39, 40]) *Let $\alpha > 0$, and $f(t)$ is integrable, then*

$$I^{\alpha}\mathscr{D}_t^{\alpha}f(t) = f(t) + c_1t^{\alpha-1} + c_2t^{\alpha-2} + \cdots + c_nt^{\alpha-n},$$

where $c_i \in \mathbb{R}$ $(i = 1, 2, \ldots, n)$, n is the smallest integer greater than or equal to α.

Let $i = 1, 2$, we firstly focus on the following linear fractional differential equation subject to the nonlocal Riemann-Stieltjes integral boundary condition:

$$\begin{cases} -\mathscr{D}_t^{\alpha_i}x_i(t) = h_i(t), & t \in (0,1), \\ x_i(0) = 0, & x_i(1) = \int_0^1 x_i(t)\,dA_i(t). \end{cases} \tag{2.1}$$

In order to establish the existence of positive solutions for system (1.1), it is necessary to find Green's function of BVP (2.1). The following result has been given in [38].

Lemma 2.1 (see [38]) *Given $h_i \in L^1(0,1)$ and $1 < \alpha_i \leq 2$, then the following boundary value problem*

$$\begin{cases} -\mathscr{D}_t^{\alpha_i}x_i(t) = h_i(t), & 0 < t < 1, \\ x_i(0) = x_i(1) = 0, \end{cases} \tag{2.2}$$

has the unique solution

$$x_i(t) = \int_0^1 G_{\alpha_i}(t,s)h_i(s)\,ds,$$

where

$$G_{\alpha_i}(t,s) = \frac{1}{\Gamma(\alpha_i)} \begin{cases} [t(1-s)]^{\alpha_i-1}, & 0 \leq t \leq s \leq 1, \\ [t(1-s)]^{\alpha_i-1} - (t-s)^{\alpha_i-1}, & 0 \leq s \leq t \leq 1. \end{cases} \tag{2.3}$$

By Proposition 2.1, it is easy to get that the unique solution of the following boundary value problem

$$\begin{cases} -\mathscr{D}_t^{\alpha_i} x_i(t) = 0, \quad 0 < t < 1, \\ x_i(0) = 0, \qquad x_i(1) = 1, \end{cases} \tag{2.4}$$

is $t^{\alpha_i - 1}$. Thus, by [16] and [17], we have the following lemma.

Lemma 2.2 (see [16]) *If* $1 < \alpha_i \leq 2$ *and* $h_i \in L^1[0,1]$, *then the boundary value problem* (2.1) *has the unique solution*

$$x_i(t) = \int_0^1 H_i(t,s) h_i(s)\, ds, \tag{2.5}$$

where

$$\begin{aligned} &H_i(t,s) = \frac{t^{\alpha_i - 1}}{1 - \mathcal{A}_i} \mathcal{G}_{A_i}(s) + G_{\alpha_i}(t,s), \quad \mathcal{A}_i = \int_0^1 t^{\alpha_i - 1}\, dA_i(t), \\ &\mathcal{G}_{A_i}(s) = \int_0^1 G_{\alpha_i}(t,s)\, dA_i(t), \end{aligned} \tag{2.6}$$

and $G_{\alpha_i}(t,s)$ *is defined by* (2.3).

Now we introduce the following necessary condition to ensure the nonnegativity of Green's function.

(H0) A_i is functions of bounded variation satisfying $\mathcal{G}_{A_i}(s) \geq 0$ for $s \in [0,1]$ and $0 \leq \mathcal{A}_i < 1$.

Lemma 2.3 (see [16]) *Assume* (H0) *holds, then* $G_{\alpha_i}(t,s)$ *and* $H_i(t,s)$ *have the following properties*:

(1) $G_{\alpha_i}(t,s)$ *and* $H_i(t,s)$ *are nonnegative and continuous for* $(t,s) \in [0,1] \times [0,1]$.

(2) *For any* $t,s \in [0,1]$, $G_{\alpha_i}(t,s)$ *satisfies*

$$\frac{t^{\alpha_i - 1}(1-t)s(1-s)^{\alpha_i - 1}}{\Gamma(\alpha_i)} \leq G_{\alpha_i}(t,s) \leq \frac{\alpha_i - 1}{\Gamma(\alpha_i)} s(1-s)^{\alpha_i - 1}. \tag{2.7}$$

(3) *There exist two constants* a, b *such that*

$$a t^{\alpha_i - 1} \mathcal{G}_{A_i}(s) \leq H_i(t,s) \leq b t^{\alpha_i - 1}, \quad s,t \in [0,1]. \tag{2.8}$$

For convenience of writing, for $h_i \in L^1[0,1]$ and $p_i > 1$, $h_i \geq 0$, $i = 1,2$, we rewrite the following linear boundary value problems:

$$\begin{cases} -\mathscr{D}_t^{\beta_1}(\varphi_{p_1}(-\mathscr{D}_t^{\alpha_1} x_1))(t) = h_1(t), \quad t \in (0,1), \\ x_1(0) = 0, \qquad \mathscr{D}_t^{\alpha_1} x(0) = \mathscr{D}_t^{\alpha_1} x_1(1) = 0, \qquad x_1(1) = \int_0^1 x_1(t)\, dA_1(t), \end{cases}$$

and

$$\begin{cases} -\mathscr{D}_t^{\beta_2}(\varphi_{p_2}(-\mathscr{D}_t^{\alpha_2}x_2))(t) = h_2(t), \quad t \in (0,1), \\ x_2(0) = 0, \qquad \mathscr{D}_t^{\alpha_2}x(0) = \mathscr{D}_t^{\alpha_2}x_2(1) = 0, \qquad x_2(1) = \int_0^1 x_2(t)\,dA_2(t), \end{cases}$$

as

$$\begin{cases} -\mathscr{D}_t^{\beta_i}(\varphi_{p_i}(-\mathscr{D}_t^{\alpha_i}x_i))(t) = h_i(t), \quad t \in (0,1), \\ x_i(0) = 0, \qquad \mathscr{D}_t^{\alpha_i}x(0) = \mathscr{D}_t^{\alpha_i}x_i(1) = 0, \qquad x_i(1) = \int_0^1 x_i(t)\,dA_i(t). \end{cases} \tag{2.9}$$

Lemma 2.4 *The linear boundary value problem* (2.9) *has a unique positive solution*

$$x_i(t) = \int_0^1 H_i(t,s)\left(\int_0^1 G_{\beta_i}(s,\tau)h_i(\tau)\,d\tau\right)^{q_i-1} ds,$$

where q_i is conjugate indices of p_i.

Proof Let $w_i = -\mathscr{D}_t^{\alpha_i}x_i$, $v_i = \varphi_{p_i}(w_i)$, by Lemma 2.1, the unique solution of the boundary value problem

$$\begin{cases} -\mathscr{D}_t^{\beta_i}v_i(t) = h_i(t), \quad t \in (0,1), \\ v_i(0) = v_i(1) = 0, \end{cases}$$

is

$$v_i(t) = \int_0^1 G_{\beta_i}(t,s)h_i(s)\,ds, \quad t \in [0,1]. \tag{2.10}$$

Noting that $-\mathscr{D}_t^{\alpha_i}x = w_i$, $w_i = \varphi_{p_i}^{-1}(v_i)$ as well as (2.10), we get that the solution of (2.9) satisfies

$$\begin{cases} -\mathscr{D}_t^{\alpha_i}x_i(t) = \varphi_{p_i}^{-1}(\int_0^1 G_{\beta_i}(t,s)h_i(s)\,ds), \quad t \in (0,1), \\ x_i(0) = 0, \qquad x_i(1) = \int_0^1 x_i(t)\,dA_i(t), \end{cases}$$

which implies that

$$x_i(t) = \int_0^1 H_i(t,s)\varphi_{p_i}^{-1}\left(\int_0^1 G_{\beta_i}(s,\tau)h_i(\tau)\,d\tau\right) ds, \quad t \in [0,1],$$

that is,

$$x_i(t) = \int_0^1 H_i(t,s)\left(\int_0^1 G_{\beta_i}(s,\tau)h_i(\tau)\,d\tau\right)^{q_i-1} ds, \quad t \in [0,1]. \qquad \square$$

To establish the existence of positive solution of system (1.1), the following new growth condition for nonlinearity will be used in the rest of the paper.

(H1) $f_1, f_2 : [0,+\infty) \times [0,+\infty) \to (0,+\infty)$ are continuous and nondecreasing in the first variable and second variable, and there exist positive constants $\epsilon_1 > \frac{1}{q_1-1}$, $\epsilon_2 > \frac{1}{q_2-1}$ and M such that

$$\max\left\{ \sup_{\substack{s,t\geq 0\\ s+t\neq 0}} \frac{f_1(s,t)}{(s+t)^{\epsilon_1}}, \sup_{\substack{s,t\geq 0\\ s+t\neq 0}} \frac{f_2(s,t)}{(s+t)^{\epsilon_2}} \right\} \leq M. \tag{2.11}$$

Remark 2.1 In this work, we introduce the growth condition (2.11) for the first time which differs from previous work [10, 11, 16–26, 38]. In particular, it includes a large number of nonlinear functions, some basic examples of f_1, f_2 satisfying (H1) are

(1) $f(s,t) = a_0 + \sum_{i=1}^m a_i(s+t)^{\gamma_i}$, where $a_i, \gamma_i > 0$, $i = 0,1,2,\dots,m$.

(2) $f(s,t) = [a + \sum_{i=1}^m a_i(s+t)^{\mu_i}]^{\frac{1}{\mu}}$, where a, μ, a_i, μ_i $(i = 1,2,\dots,m)$ are positive constants.

(3) $f(s,t) = (s+t+1)^{\mu+1}\ln(1+\frac{1}{1+s+t}) + (s+t)^{\mu} + a$, $a, \mu > 0$.

(4) $f(s,t) = \ln(2+s+t)$.

Proof (1)-(4) are obvious, we omit the proof. □

Remark 2.2 If $f_i(s,t) = g_i(s+t)$, then the following interesting cases are also included by (2.11):

Case 1. There exists a constant $\epsilon_i > \frac{1}{q_i-1}$ such that $\frac{g_i(x)}{x^{\epsilon_i}}$ is increasing on x and

$$\lim_{x\to+\infty} \frac{g_i(x)}{x^{\epsilon_i}} = M > 0.$$

Case 2. There exists a constant $\epsilon_i > \frac{1}{q_i-1}$ such that $\frac{g_i(x)}{x^{\epsilon_i}}$ is nonincreasing on x.

Case 1 and Case 2 indicate that g_i can be superlinear or sublinear or mixed cases of them; moreover, this shows that assumption (2.11) is very easy to be satisfied.

Let $E = C[0,1] \times C[0,1]$ be the Banach space of all continuous functions with the norm

$$\|(x_1,x_2)\| = \|x_1\| + \|x_2\|, \qquad \|x_i\| = \max\{x_i(t) : t \in [0,1]\}.$$

Define a cone P in E

$$\begin{aligned} P = \{&(x_1,x_2) \in E : \text{there exist nonnegative numbers } l_{x_i} < L_{x_i} \text{ such that} \\ & l_{x_i} t^{\alpha_i-1} \leq x_i(t) \leq L_{x_i} t^{\alpha_i-1}, t \in [0,1], i = 1,2\}, \end{aligned}$$

and operators T_1, T_2, T,

$$\begin{aligned} T_1(x_1,x_2)(t) &= \int_0^1 H_1(t,s)\left(\int_0^1 G_{\beta_1}(s,\tau) f_1(x_1(\tau),x_2(\tau))\,d\tau\right)^{q_1-1} ds, \\ T_2(x_1,x_2)(t) &= \int_0^1 H_2(t,s)\left(\int_0^1 G_{\beta_2}(s,\tau) f_2(x_1(\tau),x_2(\tau))\,d\tau\right)^{q_2-1} ds, \\ T(x_1,x_2)(t) &=: (T_1(x_1,x_2)(t), T_2(x_1,x_2)(t)), \end{aligned}$$

then the fixed point of operator T in E is the solution of system (1.1).

Lemma 2.5 *Assume that* (H0)-(H1) *hold. Then* $T : P \to P$ *is a continuous, compact operator.*

Proof For any $(x_1, x_2) \in P$, we can find four nonnegative numbers $L_{x_i} > l_{x_i} \geq 0$ such that

$$l_{x_i} t^{\alpha_i - 1} \leq x_i(t) \leq L_{x_i} t^{\alpha_i - 1}, \quad t \in [0,1], i = 1, 2. \tag{2.12}$$

If $(x_1, x_2) = (0, 0)$, notice that $f_i(0,0) \not\equiv 0$, then it follows from (2.8) that $T(x_1, x_2) \in P$. Otherwise, by (H1), we know that T_1 and T_2 are increasing with respect to x_1, x_2. Now we divide into two cases to prove the right-hand side of inequality of (2.12) is valid for T.

Case 1. If $x_1(t) + x_2(t) \geq 1$, by (2.8), we get

$$\begin{aligned} T_1(x_1,x_2)(t) &\leq bt^{\alpha_1-1} \int_0^1 \left(\int_0^1 G_{\beta_1}(s,\tau) f_1\big(x_1(\tau), x_2(\tau)\big)\, d\tau \right)^{q_1-1} ds \\ &\leq bt^{\alpha_1-1} \int_0^1 \left(\int_0^1 G_{\beta_1}(s,\tau) \frac{f_1(x_1(\tau), x_2(\tau))}{(x_1(\tau)+x_2(\tau))^{\epsilon_1}} \big(x_1(\tau)+x_2(\tau)\big)^{\epsilon_1}\, d\tau \right)^{q_1-1} ds \\ &\leq bM^{\epsilon_1(q_1-1)} t^{\alpha_1-1} \int_0^1 \left(\int_0^1 G_{\beta_1}(s,\tau) \big(x_1(\tau)+x_2(\tau)\big)^{\epsilon_1}\, d\tau \right)^{q_1-1} ds \\ &\leq bM^{\epsilon_1(q_1-1)} t^{\alpha_1-1} \left(\int_0^1 \frac{\beta_1-1}{\Gamma(\beta_1)} \tau(1-\tau)^{\beta_1-1} \big(L_{x_1}\tau^{\alpha_1-1} + L_{x_2}\tau^{\alpha_2-1}\big)^{\epsilon_1}\, d\tau \right)^{q_1-1} \\ &\leq bM^{\epsilon_1(q_1-1)} \left(\frac{\beta_1-1}{\Gamma(\beta_1)} (L_{x_1}+L_{x_2})^{\epsilon_1} \right)^{q_1-1} t^{\alpha_1-1} \leq L_{x_1}^* t^{\alpha_1-1}. \end{aligned} \tag{2.13}$$

Case 2. If $x_1(t) + x_2(t) < 1$, we have

$$\begin{aligned} T_1(x_1,x_2)(t) &\leq bt^{\alpha_1-1} \int_0^1 \left(\int_0^1 G_{\beta_1}(s,\tau) f_1\big(x_1(\tau), x_2(\tau)\big)\, d\tau \right)^{q_1-1} ds \\ &\leq bt^{\alpha_1-1} \int_0^1 \left(\int_0^1 G_{\beta_1}(s,\tau) f_1(1,1)\, d\tau \right)^{q_1-1} ds \\ &\leq bt^{\alpha_1-1} \left(\int_0^1 \frac{\beta_1-1}{\Gamma(\beta_1)} \tau(1-\tau)^{\beta_1-1} f_1(1,1)\, d\tau \right)^{q_1-1} \\ &\leq b \left(\frac{\beta_1-1}{\Gamma(\beta_1)} f_1(1,1) \right)^{q_1-1} t^{\alpha_1-1} \leq L_{x_1}^* t^{\alpha_1-1}, \end{aligned} \tag{2.14}$$

where

$$L_{x_1}^* = \max\left\{ bM^{\epsilon_1(q_1-1)} \left(\frac{\beta_1-1}{\Gamma(\beta_1)} (L_{x_1}+L_{x_2})^{\epsilon_1} \right)^{q_1-1}, b\left(\frac{\beta_1-1}{\Gamma(\beta_1)} f_1(1,1) \right)^{q_1-1} \right\}.$$

On the other hand,

$$\begin{aligned} T_1(x_1,x_2)(t) &\geq at^{\alpha_1-1} \int_0^1 \mathcal{G}_{A_1}(s) \left(\int_0^1 G_{\beta_1}(s,\tau) f_2\big(x_1(\tau), x_2(\tau)\big)\, d\tau \right)^{q_1-1} ds \\ &\geq at^{\alpha_1-1} \int_0^1 \mathcal{G}_{A_1}(s) \left(\int_0^1 \frac{s^{\beta_1-1}(1-s)\tau(1-\tau)^{\beta_1-1}}{\Gamma(\beta_1)} f_1(0,0) \right)^{q_1-1} ds \end{aligned}$$

$$\begin{aligned}
&= a\left(\frac{f_1(0,0)}{\Gamma(\beta_1)}\right)^{q_1-1}\int_0^1 \mathcal{G}_{A_1}(s)s^{(\beta_1-1)(q_1-1)}(1-s)^{q_1-1}\,ds \\
&\quad \times\left(\int_0^1 \tau(1-\tau)^{\beta_1-1}\,d\tau\right)^{q_1-1} t^{\alpha_1-1} \\
&= a\left(\frac{f_1(0,0)}{\Gamma(\beta_1+2)}\right)^{q_1-1}\int_0^1 \mathcal{G}_{A_1}(s)s^{(\beta_1-1)(q_1-1)}(1-s)^{q_1-1}\,ds t^{\alpha_1-1} \\
&= l_{x_1}^* t^{\alpha_1-1},
\end{aligned} \tag{2.15}$$

where

$$l_{x_1}^* = a\left(\frac{f_1(0,0)}{\Gamma(\beta_1+2)}\right)^{q_1-1}\int_0^1 \mathcal{G}_{A_1}(s)s^{(\beta_1-1)(q_1-1)}(1-s)^{q_1-1}\,ds.$$

Thus (2.13)-(2.15) yield that

$$l_{x_1}^* t^{\alpha_1-1} \le T_1(x_1,x_2)(t) \le L_{x_1}^* t^{\alpha_1-1}.$$

In the same way, there exist two constants $L_{x_2}^* > l_{x_2}^* \ge 0$ such that

$$l_{x_2}^* t^{\alpha_2-1} \le T_2(x_1,x_2)(t) \le L_{x_2}^* t^{\alpha_2-1}.$$

Therefore T is well defined and uniformly bounded and $T(P) \subset P$.

On the other hand, according to the Arzela-Ascoli theorem and the Lebesgue dominated convergence theorem, we know that $T: P \to P$ is completely continuous. □

3 Main results

Lemma 3.1 *Suppose $\epsilon_i(q_i-1) > 1$, $i = 1,2$, then the equation*

$$\left(\frac{\beta_1-1}{\Gamma(\beta_1+2)}2^{\epsilon_1}M\right)^{q_1-1} x^{\epsilon_1(q_1-1)-1} + \left(\frac{\beta_2-1}{\Gamma(\beta_2+2)}2^{\epsilon_2}M\right)^{q_2-1} x^{\epsilon_2(q_2-1)-1} = b^{-1} \tag{3.1}$$

has a unique positive solution r^* in $[0,\infty)$.

Proof Let

$$\begin{aligned}
\varphi(x) = b^{-1} &- \left(\frac{\beta_1-1}{\Gamma(\beta_1+2)}2^{\epsilon_1}M\right)^{q_1-1} x^{\epsilon_1(q_1-1)-1} \\
&- \left(\frac{\beta_2-1}{\Gamma(\beta_2+2)}2^{\epsilon_2}M\right)^{q_2-1} x^{\epsilon_2(q_2-1)-1},
\end{aligned} \tag{3.2}$$

then $\varphi(x)$ is continuous in $[0,\infty)$, and

$$\varphi(0) = b^{-1} > 0, \tag{3.3}$$

$$\begin{aligned}
&\varphi\left(\left(\frac{(\beta_1-1)2^{\epsilon_1}Mb^{\frac{1}{q_1-1}}}{\Gamma(\beta_1+2)}\right)^{-\frac{q_1-1}{\epsilon_1(q_1-1)-1}}\right) \\
&\quad = -\left(\frac{\beta_2-1}{\Gamma(\beta_2+2)}2^{\epsilon_2}M\right)^{q_2-1}\left(\frac{(\beta_1-1)2^{\epsilon_1}Mb^{\frac{1}{q_1-1}}}{\Gamma(\beta_1+2)}\right)^{-\frac{(q_1-1)(\epsilon_2(q_2-1)-1)}{\epsilon_1(q_1-1)-1}} < 0.
\end{aligned} \tag{3.4}$$

On the other hand, we also have

$$\begin{aligned}\varphi'(x) &= -\big[\epsilon_1(q_1-1)-1\big]\left(\frac{\beta_1-1}{\Gamma(\beta_1+2)}2^{\epsilon_1}M\right)^{q_1-1}x^{\epsilon_1(q_1-1)-2}\\ &\quad -\big[\epsilon_2(q_2-1)-1\big]\left(\frac{\beta_2-1}{\Gamma(\beta_2+2)}2^{\epsilon_2}M\right)^{q_2-1}x^{\epsilon_2(q_2-1)-2}\\ &< 0, \quad x\in(0,\infty). \end{aligned} \tag{3.5}$$

Thus by (3.2)-(3.5), equation (3.1) has a unique positive solution r^* in $[0,\infty)$. □

Theorem 3.1 *Suppose conditions* (H0) *and* (H1) *hold, and*

$$\frac{(\beta_i-1)f_i(0,0)}{\Gamma(\beta_i+2)} \le \left(\frac{r^*}{b}\right)^{\frac{1}{q_i-1}}, \quad i=1,2. \tag{3.6}$$

Then system (1.1) *has the minimal and maximal solutions,* $x^*=(x_1^*,x_2^*)$ *and* $y^*=(y_1^*,y_2^*)$, *which are positive; and there exist some nonnegative numbers* $m_i<n_i$, $i=1,2,3,4$, *such that*

$$\begin{aligned} &m_1t^{\alpha-1}\le x_1^*(t)\le n_1t^{\alpha-1}, \qquad m_2t^{\alpha-1}\le y_1^*(t)\le n_2t^{\alpha-1}, \quad t\in[0,1],\\ &m_3t^{\alpha-1}\le x_2^*(t)\le n_3t^{\alpha-1}, \qquad m_4t^{\alpha-1}\le y_2^*(t)\le n_4t^{\alpha-1}, \quad t\in[0,1]. \end{aligned} \tag{3.7}$$

Moreover, for initial values $u^{(0)}(t)=(0,0)$, $w^{(0)}(t)=(r^*,r^*)$, *let* $\{u^{(n)}\}$ *and* $\{w^{(n)}\}$ *be the iterative sequences generated by*

$$u^{(n)}(t)=Tu^{(n-1)}(t)=T^nu^{(0)}(t), \qquad w^{(n)}(t)=Tw^{(n-1)}(t)=T^nw^{(0)}(t), \tag{3.8}$$

then

$$\lim_{n\to+\infty}u^{(n)}(t)=x^*(t), \qquad \lim_{n\to+\infty}w^{(n)}(t)=y^*(t)$$

uniformly for $t\in[0,1]$.

Proof Take $P[0,r^*]=\{(x_1,x_2)\in P: 0\le\|x_1\|\le r^*, 0\le\|x_2\|\le r^*\}$, we firstly prove $T(P[0,r^*])\subset P[0,r^*]$.

In fact, for any $(x_1,x_2)\in P[0,r^*]$, if $(x_1,x_2)\equiv(0,0)$, it follows from (2.8) that

$$\begin{aligned}\big\|T_1(x_1,x_2)\big\| &= \max_{t\in[0,1]}\left\{\int_0^1 H_1(t,s)\left(\int_0^1 G_{\beta_1}(s,\tau)f_1\big(x_1(\tau),x_2(\tau)\big)\,d\tau\right)^{q_1-1}ds\right\}\\ &\le b\int_0^1\left(\int_0^1\frac{\beta_1-1}{\Gamma(\beta_1)}\tau(1-\tau)^{\beta_1-1}f_1(0,0)\,d\tau\right)^{q_1-1}ds\\ &= b\left(\frac{\beta_1-1}{\Gamma(\beta_1+2)}f_1(0,0)\right)^{q_1-1}\le r^*, \end{aligned} \tag{3.9}$$

and

$$\begin{aligned} \|T_2(x_1,x_2)\| &= \max_{t\in[0,1]}\left\{\int_0^1 H_2(t,s)\left(\int_0^1 G_{\beta_2}(s,\tau)f_2\big(x_1(\tau),x_2(\tau)\big)\,d\tau\right)^{q_2-1}ds\right\} \\ &\le b\int_0^1\left(\int_0^1 \frac{\beta_2-1}{\Gamma(\beta_2)}\tau(1-\tau)^{\beta_2-1}f_2(0,0)\,d\tau\right)^{q_2-1}ds \\ &= b\left(\frac{\beta_2-1}{\Gamma(\beta_2+2)}f_2(0,0)\right)^{q_2-1} \le r^*. \end{aligned} \tag{3.10}$$

Otherwise, for any $t\in(0,1)$, we have

$$0 < x_1(t)+x_2(t) \le \max_{t\in[0,1]} x_1(t) + \max_{t\in[0,1]} x_2(t) \le 2r^*. \tag{3.11}$$

So, by (H1), we get

$$\begin{aligned} \|T_1(x_1,x_2)\| &= \max_{t\in[0,1]}\left\{\int_0^1 H_1(t,s)\left(\int_0^1 G_{\beta_1}(s,\tau)f_1\big(x_1(\tau),x_2(\tau)\big)\,d\tau\right)^{q_1-1}ds\right\} \\ &\le b\int_0^1\left(\int_0^1 \frac{\beta_1-1}{\Gamma(\beta_1)}\tau(1-\tau)^{\beta_1-1}M\big[x_1(\tau)+x_2(\tau)\big]^{\epsilon_1}\,d\tau\right)^{q_1-1}ds \\ &\le b\int_0^1\left(\int_0^1 \frac{\beta_1-1}{\Gamma(\beta_1)}\tau(1-\tau)^{\beta_1-1}M2^{\epsilon_1}\big(r^*\big)^{\epsilon_1}\,d\tau\right)^{q_1-1}ds \\ &\le b\left(\frac{\beta_1-1}{\Gamma(\beta_1+2)}2^{\epsilon_1}M\right)^{q_1-1}\big(r^*\big)^{\epsilon_1(q_1-1)} \\ &\le b\left(\frac{\beta_1-1}{\Gamma(\beta_1+2)}2^{\epsilon_1}M\right)^{q_1-1}\big(r^*\big)^{\epsilon_1(q_1-1)} + b\left(\frac{\beta_2-1}{\Gamma(\beta_2+2)}2^{\epsilon_2}M\right)^{q_2-1}\big(r^*\big)^{\epsilon_1(q_2-1)} \\ &= r^*. \end{aligned} \tag{3.12}$$

Similar to (3.12), we have

$$\begin{aligned} \|T_2(x_1,x_2)\| &\le b\left(\frac{\beta_1-1}{\Gamma(\beta_1+2)}2^{\epsilon_1}M\right)^{q_1-1}\big(r^*\big)^{\epsilon_1(q_1-1)} \\ &\quad + b\left(\frac{\beta_2-1}{\Gamma(\beta_2+2)}2^{\epsilon_2}M\right)^{q_2-1}\big(r^*\big)^{\epsilon_1(q_2-1)} = r^*, \end{aligned} \tag{3.13}$$

which implies that $T(P[0,r^*])\subset P[0,r^*]$.

Let $u^{(0)}(t) = (u_1^{(0)}(t), u_2^{(0)}(t)) = (0,0)$ and

$$\begin{aligned} u^{(1)}(t) &=: \big(u_1^{(1)}(t), u_2^{(1)}(t)\big) = \big(\big(T_1\big(u_1^{(0)},u_2^{(0)}\big)\big)(t), \big(T_2\big(u_1^{(0)},u_2^{(0)}\big)\big)(t)\big) \\ &= \big(\big(T_1(0,0)\big)(t), \big(T_2(0,0)\big)(t)\big), \quad t\in[0,1], \end{aligned}$$

it follows from $u^{(0)}(t)\in P([0,r^*])$ that $u^{(1)}(t)\in T(P[0,r^*])$.

Denote

$$u^{(n+1)}(t) = Tu^{(n)}(t) = T^{n+1}u^{(0)}(t), \quad n = 1,2,\ldots.$$

It follows from $T(P[0,r^*]) \subset P[0,r^*]$ that $u_n(t) \in P[0,r^*]$ for $n \geq 1$. Noticing that T is compact, we get that $\{u^{(n)}\}$ is a sequentially compact set.

On the other hand, since $u^{(1)}(t) \geq 0 = u^{(0)}(t)$, we have

$$u^{(2)}(t) = \big(Tu^{(1)}\big)(t) \geq \big(Tu^{(0)}\big)(t) = u^{(1)}(t), \quad t \in [0,1].$$

By induction, we get

$$u^{(n+1)} \geq u^{(n)}, \quad n = 1,2,\ldots.$$

Consequently, there exists $x^* = (x_1^*, x_2^*) \in P[0,r]$ such that $u^{(n)} \to x^*$. Letting $n \to +\infty$, from the continuity of T and $Tu^{(n)} = u^{(n-1)}$, we obtain $Tx^* = x^*$, which implies that $x^* = (x_1^*, x_2^*)$ is a nonnegative solution of the nonlinear integral equation (1.1). Since $x^* \in P$, there exist constants $0 < m_1 < n_1$, $0 < m_2 < n_2$ such that

$$0 < m_1 t^{\alpha_1 - 1} \leq x_1^*(t) \leq n_1 t^{\alpha_1 - 1}, 0 < m_2 t^{\alpha_2 - 1} \leq x_2^*(t) \leq n_2 t^{\alpha_2 - 1}, \quad t \in (0,1),$$

and consequently x^* is a positive solution of system (1.1).

Now let $w^{(0)}(t) = (w_1^{(0)}(t), w_2^{(0)}(t)) = (r^*, r^*)$ and

$$w^{(1)}(t) =: \big(w_1^{(1)}(t), w_2^{(1)}(t)\big) = \big(\big(T_1\big(w_1^{(0)}, w_2^{(0)}\big)\big)(t), \big(T_2\big(w_1^{(0)}, w_2^{(0)}\big)\big)(t)\big), \quad t \in [0,1].$$

Since $w^{(0)}(t) = (r^*, r^*) \in P[0,r^*]$, and then $w^{(1)}(t) \in P[0,r^*]$. Thus denote

$$w^{(n+1)}(t) = Tw^{(n)}(t) = T^{n+1} w^{(0)}(t), \quad n = 1,2,\ldots.$$

It follows from $T(P[0,r^*]) \subset P[0,r^*]$ that

$$w^{(n)}(t) \in P\big[0,r^*\big], \quad n = 0,1,2,\ldots.$$

From Lemma 2.5, T is compact, consequently $\{w^{(n)}\}$ is a sequentially compact set.

Now, since $w^{(1)}(t) \in P[0,r^*]$, we get

$$0 \leq w_1^{(1)}(t) \leq \big\|w_1^{(1)}\big\| \leq r = w_1^{(0)}(t), \qquad 0 \leq w_2^{(1)}(t) \leq \big\|w_2^{(1)}\big\| \leq r^* = w_2^{(0)}(t).$$

It follows from (H1) that $w^{(2)}(t) = Tw^{(1)}(t) \leq Tw^{(0)}(t) = w^{(1)}(t)$. By induction, we obtain

$$w^{(n+1)}(t) \leq w^{(n)}(t), \quad n = 0,1,2,\ldots.$$

Consequently, there exists $y^*(t) \in P[0,r^*]$ such that $w^{(n)}(t) \to y^* = (y_1^*, y_2^*)$. Letting $n \to +\infty$, from the continuity of T and $Tw^{(n)}(t) = w^{(n-1)}(t)$, we have $Ty^* = y^*$, which implies that y^* is another nonnegative solution of the boundary value problem (1.1) and y^* also satisfies (3.7) since $y^* \in P$.

In the end, we prove that x^* and y^* are maximum and minimum solutions for system (1.1). Let $\tilde{x}$ be any positive solution of system (1.1), then $u^{(0)} = 0 \leq \tilde{x} \leq r^* = w^{(0)}$, and $u^{(1)} = Tu^{(0)} \leq T\tilde{x} = \tilde{x} \leq T(w^{(0)}) = w^{(1)}$. By induction, we have $u^{(n)} \leq \tilde{x} \leq w^{(n)}$, $n = 1,2,3,\ldots$. Taking limit, we have $x^* \leq \tilde{x} \leq y^*$. This implies that x^* and y^* are the maximal and minimal solutions of system (1.1), respectively. The proof is completed. □

Remark In particular, if $\alpha_i = \beta_i = p_i = 2$, then the nonlocal fractional system (1.1) will reduce to a fourth order classical beam system of ordinary differential equation,

$$\begin{cases} -x_1^{(4)}(t) = f_1(x_1(t), x_2(t)), \\ -x_2^{(4)}(t) = f_2(x_1(t), x_2(t)) \\ x_1(0) = 0, \qquad x_1''(0) = x_1''(1) = 0, \qquad x_1(1) = \int_0^1 x_1(t)\,dA_1(t), \\ x_2(0) = 0, \qquad x_2''(0) = x_2''(1) = 0, \qquad x_2(1) = \int_0^1 x_2(t)\,dA_2(t), \end{cases} \tag{3.14}$$

we have the following good result.

Corollary 3.1 *Suppose conditions* (H0) *and* (H1) *hold, and*

$$f_i(0,0) \le \frac{6}{b} r^*, \quad i = 1, 2, \tag{3.15}$$

then system (3.14) *has the minimal and maximal solutions*, $x^* = (x_1^*, x_2^*)$ *and* $y^* = (y_1^*, y_2^*)$, *which are positive; and there exist some nonnegative numbers* $m_i < n_i$, $i = 1, 2, 3, 4$, *such that*

$$\begin{aligned} &m_1 t \le x_1^*(t) \le n_1 t, m_2 t \le y_1^*(t) \le n_2 t, \quad t \in [0,1], \\ &m_3 t \le x_2^*(t) \le n_3 t, m_4 t \le y_2^*(t) \le n_4 t, \quad t \in [0,1]. \end{aligned} \tag{3.16}$$

Moreover, for initial values $u^{(0)}(t) = (0,0)$, $w^{(0)}(t) = (r^*, r^*)$, *let* $\{u^{(n)}\}$ *and* $\{w^{(n)}\}$ *be the iterative sequences generated by*

$$u^{(n)}(t) = Tu^{(n-1)}(t) = T^n u^{(0)}(t), \qquad w^{(n)}(t) = Tw^{(n-1)}(t) = T^n w^{(0)}(t), \tag{3.17}$$

then

$$\lim_{n\to+\infty} u^{(n)}(t) = x^*(t), \qquad \lim_{n\to+\infty} w^{(n)}(t) = y^*(t)$$

uniformly for $t \in [0,1]$.

In the end, we know that fractional order integral and derivative operators can describe an important characteristics exhibiting long-memory in time in many complex processes and systems. With this advantage, in many eco-economical systems and diffusive processes with long time memory behavior [2, 3, 15, 39], fractional calculus provides an excellent tool to describe the hereditary properties of them. Here we give a specific example arising from the above complex processes.

Example Consider the following nonlocal boundary value problem of the fractional p-Laplacian equation:

$$\begin{cases} -\mathscr{D}_t^{\frac{4}{3}}(\varphi_{\frac{5}{2}}(-\mathscr{D}_t^{\frac{3}{2}} x_1))(t) \\ \quad = (x_1 + x_2)^2 + (x_1 + x_2)^2(x_1 + x_2 + 1)\ln(1 + \frac{1}{1+x_1+x_2}) + 1, \\ -\mathscr{D}_t^{\frac{6}{5}}(\varphi_{\frac{3}{2}}(-\mathscr{D}_t^{\frac{7}{6}} x_2))(t) = \ln(2 + x_1 + x_2), \quad t \in (0,1), \\ x_1(0) = 0, \qquad \mathscr{D}_t^{\frac{3}{2}} x_1(0) = \mathscr{D}_t^{\frac{3}{2}} x_1(1) = 0, \qquad x_1(1) = \int_0^1 x_1(s)\,dA_1(s), \\ x_2(0) = 0, \qquad \mathscr{D}_t^{\frac{7}{6}} x_2(0) = \mathscr{D}_t^{\frac{7}{6}} x_2(1) = 0, \qquad x_2(1) = \int_0^1 x_2(s)\,dA_2(s), \end{cases} \tag{3.18}$$

where

$$A_1(t) = \begin{cases} 0, & t \in [0, \frac{1}{2}), \\ 2, & t \in [\frac{1}{2}, \frac{3}{4}), \\ 1, & t \in [\frac{3}{4}, 1], \end{cases} \qquad A_2(t) = \begin{cases} 0, & t \in [0, \frac{1}{3}), \\ \frac{3}{2}, & t \in [\frac{1}{3}, \frac{2}{3}), \\ \frac{1}{2}, & t \in [\frac{2}{3}, 1]. \end{cases}$$

Then system (3.18) has the positive minimal and maximal solutions, $x^* = (x_1^*, x_2^*)$ and $y^* = (y_1^*, y_2^*)$; and there exist some nonnegative numbers $m_i \le n_i$, $i = 1, 2, 3, 4$, such that

$$\begin{aligned} &m_1 t^{\frac{1}{2}} \le x_1^*(t) \le n_1 t^{\frac{1}{2}}, \qquad m_2 t^{\frac{1}{2}} \le y_1^*(t) \le n_2 t^{\frac{1}{2}}, \quad t \in [0,1], \\ &m_3 t^{\frac{1}{6}} \le x_2^*(t) \le n_3 t^{\frac{1}{6}}, \qquad m_4 t^{\frac{1}{6}} \le y_2^*(t) \le n_4 t^{\frac{1}{6}}, \quad t \in [0,1]. \end{aligned} \tag{3.19}$$

By simple computation, problem (3.18) is equivalent to the following multipoint boundary value problem:

$$\begin{cases} -\mathscr{D}_t^{\frac{4}{3}} (\varphi_{\frac{5}{2}} (-\mathscr{D}_t^{\frac{3}{2}} x_1))(t) \\ \quad = (x_1 + x_2)^2 + (x_1 + x_2)^2 (x_1 + x_2 + 1) \ln(1 + \frac{1}{1 + x_1 + x_2}) + 1, \\ -\mathscr{D}_t^{\frac{6}{5}} (\varphi_{\frac{3}{2}} (-\mathscr{D}_t^{\frac{7}{6}} x_2))(t) = \ln(2 + x_1 + x_2), \quad t \in (0,1), \\ x_1(0) = 0, \qquad \mathscr{D}_t^{\frac{3}{2}} x_1(0) = \mathscr{D}_t^{\frac{3}{2}} x_1(1) = 0, \qquad x_1(1) = 2x_1(\frac{1}{2}) - x_1(\frac{3}{4}), \\ x_2(0) = 0, \qquad \mathscr{D}_t^{\frac{7}{6}} x_2(0) = \mathscr{D}_t^{\frac{7}{6}} x_2(1) = 0, \qquad x_2(1) = \frac{3}{2} x(\frac{1}{3}) - x(\frac{2}{3}). \end{cases}$$

Let

$$\alpha_1 = \frac{3}{2}, \qquad \alpha_2 = \frac{7}{6}, \qquad \beta_1 = \frac{4}{3}, \qquad \beta_2 = \frac{6}{5}, \qquad p_1 = \frac{5}{2}, \qquad p_2 = \frac{3}{2},$$

and

$$f_1(s,t) = (s+t)^2 (s+t+1) \ln\left(1 + \frac{1}{1+s+t}\right) + (s+t)^2 + 1, \qquad f_2(s,t) = \ln(2+s+t).$$

Firstly, we have

$$\begin{aligned} \mathcal{A}_1 &= \int_0^1 t^{\alpha_1 - 1}\, dA_1(t) = 2 \times \left(\frac{1}{2}\right)^{\frac{1}{2}} - \left(\frac{3}{4}\right)^{\frac{1}{2}} = 0.5482 < 1, \\ \mathcal{A}_2 &= \int_0^1 t^{\alpha_2 - 1}\, dA_2(t) = \frac{3}{2} \times \left(\frac{1}{3}\right)^{\frac{1}{6}} - \left(\frac{2}{3}\right)^{\frac{1}{6}} = 0.3053 < 1, \end{aligned}$$

and by simple computation, we have $\mathcal{G}_{A_i}(s) \ge 0$, $i = 1, 2$, and so (H0) holds.

Obviously, $f_1, f_2 : [0, +\infty) \times [0, +\infty) \to [0, +\infty)$ are continuous and nondecreasing in the first variable and second variable and $f_i(0,0) \ne 0$, $i = 1, 2$. Noticing that

$$\begin{aligned} &\sup_{s+t \ge 1} \frac{f_1(s,t)}{(s+t)^2} \le \sup_{s+t \ge 1} \left\{ \ln\left(1 + \frac{1}{1+s+t}\right)^{1+s+t} + 2 \right\} \le 3, \\ &\sup_{s+t \ge 1} \frac{f_2(s,t)}{(s+t)} = \sup_{s+t \ge 1} \frac{\ln(2+s+t)}{(s+t)} \le \ln 3 \le 3, \end{aligned}$$

and then (2.11) is satisfied with $\epsilon_1 = 2 > \frac{1}{q_1-1} = \frac{3}{2}$, $\epsilon_2 = 1 > \frac{1}{q_2-1} = \frac{1}{2}$, $M = 3$. Thus, by Theorem 3.1, system (3.18) has maximal and minimal solutions which satisfy (3.19).

4 Conclusion

In this work, we have established an existence result on the maximum and minimum solutions for a class of fractional order differential systems involving a p-Laplacian operator and nonlocal boundary conditions. This type of differential systems actually arise from some complex natural processes such as ecological economy phenomena and diffusive interaction, moreover fractional differential operator can more accurately depict the coevolution process of economic, social and ecological subsystems and the transport of solute in highly heterogeneous porous media. The main contribution is that we introduced some new type growth conditions for nonlinearity and adopted the monotone iterative technique to establish the existence and estimation of the lower and upper bounds of the maximum and minimum solutions. Furthermore, the iterative schemes converging to the exact maximum and minimum solutions, which start from some explicit initial values, are also constructed.

Acknowledgements
We are thankful to the editor and the anonymous reviewers for many valuable suggestions to improve this paper.

Funding
The first author and second author are partially supported by the Youth Project of MOE (Ministry of Education in China) Humanity and Social Science Foundation (16YJC630101), the Youth Project of Hunan Provincial Philosophy and Social Science Foundation (15YBA407), the Foundation of the Excellent Youth Scholars of Hunan Provincial Department of Education (14B188) and the Project of Hunan Provincial Situation Decision-making Consultation (16JCC009). The third author and fourth author are partially supported financially by the National Natural Science Foundation of China (11571296,11371221) and the Natural Science Foundation of Shandong Province of China (ZR2014AM009).

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
The study was carried out in collaboration among all authors. TR, SL and XZ completed the main part of this article and gave one example; LL corrected the main theorems and polished the manuscript. All authors read and approved the final manuscript.

Author details
[1]School of Logistics and Transportation, Central South University of Forestry and Technology, Changsha, Hunan 410004, China. [2]School of Tourism Management, Central South University of Forestry and Technology, Changsha, Hunan 410004, China. [3]School of Mathematical and Informational Sciences, Yantai University, Yantai, Shandong 264005, China. [4]Department of Mathematics and Statistics, Curtin University of Technology, Perth, WA 6845, Australia. [5]School of Mathematical Sciences, Qufu Normal University, Qufu, Shandong 273165, China.

Publisher's Note

Received: 4 February 2017 Accepted: 3 August 2017 Published online: 18 August 2017

References

1. Caffarelli, L, Vazquez, J: Nonlinear porous medium flow with fractional potential pressure. Arch. Ration. Mech. Anal. **202**, 537-565 (2011)
2. Ivancevic, VG, Ivancevic, TT: Geometrical Dynamics of Complex Systems: A Unified Modelling Approach to Physics, Control, Biomechanics, Neurodynamics and Psycho-Socio-Economical Dynamics. Springer, Berlin (2006)
3. Tarasov, V: Lattice model of fractional gradient and integral elasticity: long-range interaction of Grünwald-Letnikov-Riesz type. Mech. Mater. **70**, 106-114 (2014)
4. Yin, D, Duan, X, Zhou, X: Fractional time-dependent deformation component models for characterizing viscoelastic Poisson's ratio. Eur. J. Mech. A, Solids **42**, 422-429 (2013)
5. Paola, M, Pinnola, F, Zingales, M: Fractional differential equations and related exact mechanical models. Comput. Math. Appl. **66**, 608-620 (2013)
6. Das, S, Maharatna, K: Fractional dynamical model for the generation of ECG like signals from filtered coupled Van-der Pol oscillators. Comput. Methods Programs Biomed. **122**, 490-507 (2013)

7. Sumelka, W: Fractional viscoplasticity. Mech. Res. Commun. **56**, 31-36 (2014)
8. Hartley, T, Lorenzo, C, Qammer, H: Chaos in fractional order Chua's system. IEEE Trans. Circuits Syst. I, Fundam. Theory Appl. **42**(8), 485-490 (1995)
9. Caputo, M: Free modes splitting and alterations of electro chemically polarizable media. Rend Fis. Accad. Lincei **4**, 89-98 (1993)
10. Zhang, X, Liu, L, Wu, Y: Multiple positive solutions of a singular fractional differential equation with negatively perturbed term. Math. Comput. Model. **55**, 1263-1274 (2012)
11. Goodrich, C: Existence and uniqueness of solutions to a fractional difference equation with nonlocal conditions. Comput. Math. Appl. **61**, 191-202 (2011)
12. Paola, M, Zingales, M: The multiscale stochastic model of fractional hereditary materials (FHM). Proc. IUTAM **6**, 50-59 (2013)
13. El-Saka, H: The fractional-order SIS epidemic model with variable population size. J. Egypt. Math. Soc. **22**, 50-54 (2014)
14. Mongiovi, M, Zingales, M: A non-local model of thermal energy transport: the fractional temperature equation. Int. J. Heat Mass Transf. **67**, 593-601 (2013)
15. Miller, K, Ross, B: An Introduction to the Fractional Calculus and Fractional Differential Equations. Wiley, New York (1993)
16. Zhang, X, Han, Y: Existence and uniqueness of positive solutions for higher order nonlocal fractional differential equations. Appl. Math. Lett. **25**, 555-560 (2012)
17. Webb, J: Nonlocal conjugate type boundary value problems of higher order. Nonlinear Anal. **71**, 1933-1940 (2009)
18. Zhang, X, Liu, L, Wu, Y: The uniqueness of positive solution for a singular fractional differential system involving derivatives. Commun. Nonlinear Sci. Numer. Simul. **18**, 1400-1409 (2013)
19. Zhang, X, Liu, L, Wu, Y: The eigenvalue problem for a singular higher fractional differential equation involving fractional derivatives. Appl. Math. Comput. **218**, 8526-8536 (2012)
20. Zhang, X, Liu, L, Wu, Y: Existence results for multiple positive solutions of nonlinear higher order perturbed fractional differential equations with derivatives. Appl. Math. Comput. **219**, 1420-1433 (2012)
21. Liu, X, Jia, M, Xiang, X: On the solvability of a fractional differential equation model involving the *p*-Laplacian operator. Comput. Math. Appl. **64**, 3267-3275 (2012)
22. Zhang, X, Liu, L, Wu, Y, Wiwatanapataphee, B: Nontrivial solutions for a fractional advection dispersion equation in anomalous diffusion. Appl. Math. Lett. **66**, 1-8 (2017)
23. Chen, T, Liu, W: An anti-periodic boundary value problem for the fractional differential equation with a *p*-Laplacian operator. Appl. Math. Lett. **25**, 1671-1675 (2012)
24. Jiang, W: Solvability of fractional differential equations with *p*-Laplacian at resonance. Appl. Math. Comput. **260**, 48-56 (2015)
25. Chen, T, Liu, W, Liu, J: Solvability of periodic boundary value problem for fractional *p*-Laplacian equation. Appl. Math. Comput. **244**, 422-431 (2014)
26. Wang, J, Xiang, H: Upper and lower solutions method for a class of singular fractional boundary value problems with *p*-Laplacian operator. Abstr. Appl. Anal. **2010**, Article ID 971824 (2010)
27. Zhang, X, Mao, C, Liu, L, Wu, Y: Exact iterative solution for an abstract fractional dynamic system model for bioprocess. Qual. Theory Dyn. Syst. **16**, 205-222 (2017)
28. Zhang, X, Wu, Y, Caccetta, L: Nonlocal fractional order differential equations with changing-sign singular perturbation. Appl. Math. Model. **39**, 6543-6552 (2015)
29. Zhang, X, Liu, L, Wu, Y: Variational structure and multiple solutions for a fractional advection-dispersion equation. Comput. Math. Appl. **68**, 1794-1805 (2014)
30. Zhang, X, Liu, L, Wiwatanapataphee, B, Wu, Y: The eigenvalue for a class of singular *p*-Laplacian fractional differential equations involving the Riemann-Stieltjes integral boundary condition. Appl. Math. Comput. **235**, 412-422 (2014)
31. Guo, Z, Yuan, H: Pricing European option under the time-changed mixed Brownian-fractional Brownian model. Physica A **406**, 73-79 (2014)
32. Zhang, X, Liu, L, Wu, Y, Wiwatanapataphee, B: The spectral analysis for a singular fractional differential equation with a signed measure. Appl. Math. Comput. **257**, 252-263 (2015)
33. Sun, F, Liu, L, Zhang, X, Wu, Y: Spectral analysis for a singular differential system with integral boundary conditions. Mediterr. J. Math. **13**, 4763-4782 (2016)
34. Liu, L, Sun, F, Zhang, X, Wu, Y: Bifurcation analysis for a singular differential system with two parameters via to degree theory. Nonlinear Anal., Model. Control **22**(1), 31-50 (2017)
35. Liu, L, Li, H, Liu, C, Wu, Y: Existence and uniqueness of positive solutions for singular fractional differential systems with coupled integral boundary value problems. J. Nonlinear Sci. Appl. **10**, 243-262 (2017)
36. Guo, L, Liu, L, Wu, Y: Existence of positive solutions for singular fractional differential equations with infinite-point boundary conditions. Nonlinear Anal., Model. Control **21**(5), 635-650 (2016)
37. Liu, L, Zhang, X, Jiang, J, Wu, Y: The unique solution of a class of sum mixed monotone operator equations and its application to fractional boundary value problems. J. Nonlinear Sci. Appl. **9**(5), 2943-2958 (2016)
38. Zhang, X, Liu, L, Wu, Y: The uniqueness of positive solution for a fractional order model of turbulent flow in a porous medium. Appl. Math. Lett. **37**, 26-33 (2014)
39. Podlubny, I: Fractional Differential Equations. Mathematics in Science and Engineering. Academic Press, New York (1999)
40. Kilbas, A, Srivastava, H, Trujillo, J: Theory and Applications of Fractional Differential Equations. North-Holland Mathematics Studies, vol. 204. Elsevier, Amsterdam (2006)