Zhang *Boundary Value Problems* (2015) 2015:174
DOI 10.1186/s13661-015-0435-0

Boundary Value Problems
a SpringerOpen Journal

RESEARCH Open Access

Passivity analysis of spatially and temporally BAM neural networks with the Neumann boundary conditions

Weiyuan Zhang*

*Correspondence: ahzwy@163.com
Institute of Nonlinear Science, Xianyang Normal University, Shaanxi, Xianyang 712000, P.R. China

Abstract

In the paper, a delay-differential equation modeling a bidirectional associative memory neural networks (BAM NNs) with reaction-diffusion terms is investigated, for which the input and output variables are varied with the time and space variables. By taking advantage of the inequality techniques, some Lyapunov-Krasovskii functional candidates are introduced to arrive at the novel sufficient conditions that warrant the passivity of spatially and temporally BAM NNs with mixed time delays. Moreover, when the parameter uncertainties appear in spatially and temporally BAM NNs, the criterion for robust passivity is also given. Novel passivity criteria are proposed in terms of inequalities, which can be checked easily. A numerical example is provided to demonstrate the effectiveness of the proposed results.

Keywords: BAM NNs; reaction-diffusion; delays; passivity; Lyapunov functional

1 Introduction

Since NNs related to BAM had been proposed by Kosko [1], the BAM NNs have been one of the most interesting research and extensively studied topics due to their potential applications in pattern recognition, *etc.* This class of NNs have been successfully applied to pattern recognition, signal and image processing, and artificial intelligence due to its generalization of the single-layer auto-associative Hebbian correlation to a two-layer pattern matched hetero-associative circuits. Some of these applications require that the designed network has a unique stable equilibrium point [2]. Recently, these two-layer hetero associative networks called BAM networks with axonal signal transmission delays have been studied in [2, 3], which have been used to obtain important advances in many fields such as pattern recognition and automatic control. Recently, studies on neural dynamical systems not only involved a discussion of stability properties, but they also involved many dynamic behaviors such as periodic oscillatory behavior, bifurcation, and chaos [2–10].

Stability problems are often linked to the theory of dissipative systems, which postulate that the energy dissipated inside a dynamic system is less than that supplied from an external source. Passivity is part of a broader and a general theory of dissipativeness [11, 12]. The passivity theory originated from circuit theory and plays an important role in the analysis and design of linear and nonlinear systems. It should be pointed out that the essence of the passivity theory is that the passive properties of a system can keep the system's internal stability. The passivity theory was first proposed in the circuit analysis [13], and since then it has found successful applications in diverse areas such as stability [14, 15], signal processing [16], complexity [17], fuzzy control [18], and synchronization control [19]. These are the main reasons why passivity theory has become a very relevant topic across many fields, and much investigative attention has been focused on this topic. It is noted that research on passivity has attracted so much attention, little of that had been devoted to the passivity properties of delayed NNs until [20] derived the conditions for passivity of delayed NNs. Recently, considerable attention has also been paid to the passivity analysis of NNs with time delay [20–24]. For example, some passivity conditions have been proposed for continuous-time NNs with time delay in [20]. Considering this, some delay-dependent conditions guaranteeing the passivity of NNs with time-varying delays have been proposed in [22] by use of an LMI approach. The results on passivity analysis of discrete-time NNs with time-varying delays can be found in [23].

In fact, one remarkable feature of passivity is that the passive system utilizes the product of input and output as the energy provision, and it embodies the energy attenuation character. A passive system only burns energy, without energy production, *i.e.*, passivity represents the property of energy consumption of the system. In addition, the passivity analysis for NNs can help us understand the complex brain functionalities. It is worthy pointing out that, in most existing works on the passivity of delayed NNs, the diffusion effects were not discussed. The reaction-diffusion NNs (RDNNs) were firstly introduced by Chua in order to study passivity and complexity in [17]. What is more, in man-made NNs, strictly speaking, the diffusion phenomena cannot be ignored when electrons are moving in asymmetric electromagnetic fields, thus we must consider the space is varying with time [3, 8–10, 17, 24, 25]. It is also common to consider the diffusion effect in biological systems such as immigration [26, 27]. The stability of NNs with diffusion terms, which is expressed by partial differential equations, are discussed in [3, 8–10, 25]. Furthermore, the input and output variables are also dependent on the time and space in many practical situations. To the best of our knowledge, few authors considered the passivity problem for delayed RDNNs. It is interesting and important to discuss the passivity of RDNNs, in which the input and output variables are varied with the time and space variables. In NNs, besides the diffusion phenomena, there might also be some uncertainties due to the existence of modeling errors, external disturbance, and parameter fluctuation, which might lead to undesirable dynamic network behaviors such as oscillation and instability [24, 25]. Therefore, it is important to consider robust passivity of RDNNs against these uncertainties.

Based on the above discussion, there are many results as regards the passivity problem for delayed NNs. To the best of our knowledge, a passivity analysis has seldom been reported for the class of delayed BAM NNs with reaction-diffusion terms. In the theory of partial differential equations, Poincaré integral inequality is often used in the deduction of a diffusion operator [28]. This paper will investigate the passivity problem for the class of BAM RDNNs with mixed delays and Neumann boundary conditions, which is very important in theories and applications and also is a very challenging problem. Several sufficient general conditions are derived for the robust passivity of delayed BAM NNs with reaction-diffusion terms by using Lyapunov functional method and Poincaré integral inequality, which are very convenient to verify. Finally, a numerical example is illustrated to show the usefulness of the proposed criteria.

2 Model description and preliminaries

In this paper, we consider the BAM NNs model described by the following partial differential equations:

$$\begin{cases} \frac{\partial u_i}{\partial t} = \sum_{k=1}^{l} \frac{\partial}{\partial x_k}(D_{ik}\frac{\partial u_i}{\partial x_k}) - p_i u_i(t,x) + \sum_{j=1}^{n} b_{ji} f_j(v_j(t,x)) \\ \quad + \sum_{j=1}^{n} \tilde{b}_{ji} f_j(v_j(t-\theta_{ji}(t),x)) + \sum_{j=1}^{n} \bar{b}_{ji} \int_{-\infty}^{t} k_{ji}(t-s) f_j(v_j(s,x))\,ds + \sigma_i(t,x), \\ \frac{\partial v_j}{\partial t} = \sum_{k=1}^{l} \frac{\partial}{\partial x_k}(D_{jk}^{*}\frac{\partial v_j}{\partial x_k}) - q_j v_j(t,x) + \sum_{i=1}^{m} d_{ij} g_i(u_i(t,x)) \\ \quad + \sum_{i=1}^{m} \tilde{d}_{ij} g_i(u_i(t-\tau_{ij}(t),x)) + \sum_{i=1}^{m} \bar{d}_{ij} \int_{-\infty}^{t} \bar{k}_{ij}(t-s) g_i(u_i(s,x))\,ds + \vartheta_j(t,x), \\ y_i(t,x) = a_i u_i(t,x) + c_i \sigma_i(t,x), \\ z_j(t,x) = \bar{a}_j v_j(t,x) + \bar{c}_j \vartheta_j(t,x), \end{cases} \tag{1}$$

where $x = (x_1, x_2, \dots, x_l)^T \in \Omega \subset R^l$, Ω is a compact set with smooth boundary $\partial\Omega$ and $\operatorname{mes}\Omega > 0$ in space R^l; $u = (u_1, u_2, \dots, u_m)^T \in R^m$, $v = (v_1, v_2, \dots, v_n)^T \in R^n$, $u_i(t,x)$ and $v_j(t,x)$ represent the states of the ith neuron and the jth neuron at time t and in space x, respectively. b_{ji}, $\tilde{b}_{ji}$, $\bar{b}_{ji}$, d_{ij}, $\bar{d}_{ij}$, and $\tilde{d}_{ij}$ are known constants denoting the synaptic connection strengths between the neurons, respectively; f_j and g_i denote the activation functions of the neurons and the signal propagation functions, respectively; $\sigma_i(t,x)$ and $\vartheta_j(t,x)$ denote inputs of the ith neuron and the jth neuron at time t and in space x, respectively; p_i and q_j represent the rate with which the ith neuron and the jth neuron will reset its potential to the resting state when disconnected from the networks and external inputs in space, respectively; $\tau_{ij}(t)$ and $\theta_{ji}(t)$ represent continuous time-varying discrete delays, respectively; $D_{ik} \ge 0$ and $D_{jk}^{*} \ge 0$ stand for the transmission diffusion coefficient along the ith neuron and jth neuron, respectively; $y_i(t,x)$ and $z_j(t,x)$ denote outputs of the ith neuron and the jth neuron at time t and in space x, respectively. Finally, $i = 1, 2, \dots, m$, $k = 1, 2, \dots, l$ and $j = 1, 2, \dots, n$.

System (1) is supplemented with the following boundary conditions and initial values:

$$\begin{aligned} &\frac{\partial u_i}{\partial \bar{n}} := \left(\frac{\partial u_i}{\partial x_1}, \frac{\partial u_i}{\partial x_2}, \dots, \frac{\partial u_i}{\partial x_l}\right)^T = 0, \\ &\frac{\partial v_j}{\partial \bar{n}} := \left(\frac{\partial v_j}{\partial x_1}, \frac{\partial v_j}{\partial x_2}, \dots, \frac{\partial v_j}{\partial x_l}\right)^T = 0, \quad t \ge 0, x \in \partial\Omega, \end{aligned} \tag{2}$$

$$u_i(s,x) = \varphi_{ui}(s,x), \qquad v_j(s,x) = \varphi_{vj}(s,x), \quad (s,x) \in (-\infty, 0] \times \Omega \tag{3}$$

for any $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$, where $\bar{n}$ is the outer normal vector of $\partial\Omega$, $\varphi = \binom{\varphi_u}{\varphi_v} = (\varphi_{u1}, \dots, \varphi_{um}, \varphi_{v1}, \dots, \varphi_{vn})^T \in C$ are bounded and continuous, where

$$C = \left\{ \varphi \,\middle|\, \varphi = \begin{pmatrix} \varphi_u \\ \varphi_v \end{pmatrix}, \varphi : \begin{pmatrix} (-\infty, 0] \times R^m \\ (-\infty, 0] \times R^n \end{pmatrix} \to R^{m+n} \right\}.$$

It is the Banach space of continuous functions which map $\binom{(-\infty,0]}{(-\infty,0]}$ into R^{m+n} with the topology of uniform convergence for the norm

$$\|\varphi\| = \left\| \begin{pmatrix} \varphi_u \\ \varphi_v \end{pmatrix} \right\| = \sup_{-\infty \le s \le 0} \left[\int_{\Omega} \sum_{i=1}^{m} |\varphi_{ui}|^2\,dx \right] + \sup_{-\infty \le s \le 0} \left[\int_{\Omega} \sum_{j=1}^{n} |\varphi_{vj}|^2\,dx \right].$$

For system (1), the following assumptions are made for each subsystem in this paper:

(A1) The functions $\tau_{ij}(t)$, $\theta_{ji}(t)$ are piecewise-continuous of class C^1 on the closure of each continuity subinterval and satisfy

$$0 \le \tau_{ij}(t) \le \tau_{ij}, \qquad 0 \le \theta_{ji}(t) \le \theta_{ji}, \qquad \dot{\tau}_{ij}(t) \le \mu_\tau < 1, \qquad \dot{\theta}_{ji}(t) \le \mu_\theta < 1,$$
$$\tau = \max_{1\le i\le m, 1\le j\le n}\{\tau_{ij}\}, \qquad \theta = \max_{1\le i\le m, 1\le j\le n}\{\theta_{ji}\},$$

with some constants $\tau_{ij} \ge 0$, $\theta_{ji} \ge 0$, $\tau > 0$, $\theta > 0$, for all $t \ge 0$.

(A2) The activation functions are bounded and Lipschitz continuous, *i.e.*, there exist positive constants L_j^f and L_i^g such that for all $\eta_1, \eta_2 \in R$

$$\bigl|f_j(\eta_1) - f_j(\eta_2)\bigr| \le L_j^f|\eta_1 - \eta_2|, \qquad \bigl|g_i(\eta_1) - g_i(\eta_2)\bigr| \le L_i^g|\eta_1 - \eta_2|. \tag{4}$$

(A3) The delay kernels $K_{ji}(s), \bar{K}_{ij}(s) : [0,\infty) \to [0,\infty)$ ($i = 1,2,\dots,m$, $j = 1,2,\dots,n$) are real-valued non-negative continuous functions that satisfy the following conditions:

(i) $\int_0^{+\infty} K_{ji}(s)\,ds = 1$, $\int_0^{+\infty} \bar{K}_{ji}(s)\,ds = 1$,

(ii) $\int_0^{+\infty} sK_{ji}(s)\,ds < \infty$, $\int_0^{+\infty} s\bar{K}_{ij}(s)\,ds < \infty$.

Definition 1 A system with inputs $\sigma(t,x)$, $\vartheta(t,x)$ and outputs $y(t,x)$, $z(t,x)$ where $\sigma(t,x) \in R^m$, $\vartheta(t,x) \in R^n$, $y(t,x) \in R^m$, $z(t,x) \in R^m$ is said to be passive if there are constants $\gamma \ge 0$ and $\beta \in R$ such that

$$\begin{aligned} &2\int_0^{t_p}\int_\Omega \bigl(y(t,x)^T\sigma(t,x) + z(t,x)^T\vartheta(t,x)\bigr)\,dx \\ &\quad \ge -\beta^2 - \gamma\int_0^{t_p}\int_\Omega \bigl(\sigma(t,x)^T\sigma(t,x) + \vartheta(t,x)^T\vartheta(t,x)\bigr)\,dx \end{aligned} \tag{5}$$

for all $t_p \ge 0$, where Ω is a bounded compact set.

Remark 1 According to passivity property, the authors in [29, 30] addressed the passive stability, control, and synchronization of complex networks, in which the input and output variables are only dependent on the time. However, the input and output variables in many systems are varied with the space and time variables. To the best of our knowledge, for this case, there are few results of passivity that have been proposed [24]. As a natural extension of the definition of passivity in [20, 24, 31], we propose Definition 1, which extends the definition of passivity in [20, 24, 31].

Lemma 1 [28] (Poincaré integral inequality) *Let Ω be a bounded domain of R^m with a smooth boundary $\partial\Omega$ of class C^2 by Ω. $u(x)$ is a real-valued function belonging to $H_0^1(\Omega)$ and*

$$\frac{\partial u(x)}{\partial \bar{n}}\bigg|_{\partial\Omega} = 0.$$

Then

$$\int_\Omega \bigl|u(x)\bigr|^2\,dx \le \frac{1}{\lambda_1}\int_\Omega \bigl|\nabla u(x)\bigr|^2\,dx,$$

in which λ_1 *is the lowest positive eigenvalue of the Neumann boundary problem*

$$\begin{cases} -\Delta\varphi(x) = \lambda_1\varphi(x), & x \in \Omega, \\ \frac{\partial u(x)}{\partial \vec{n}}|_{\partial\Omega} = 0, & x \in \partial\Omega. \end{cases} \tag{6}$$

3 Passive analysis

Theorem 1 *Under the assumptions* (A1)-(A3), *system* (1) *is passive if there exist constants* $w_i, w_{m+j} > 0$ *and* $\gamma > 0$ *such that*

$$\begin{pmatrix} \Xi_i & w_i - a_i \\ w_i - a_i & -\gamma - 2c_i \end{pmatrix} \le 0, \tag{7}$$

and

$$\begin{pmatrix} \Theta_j & w_{m+j} - \bar{a}_j \\ w_{m+j} - \bar{a}_j & -\gamma - 2\bar{c}_j \end{pmatrix} \le 0 \tag{8}$$

in which

$$\begin{aligned} \Xi_i &= w_i\left(-2D_i\lambda_1 - 2p_i + \sum_{j=1}^n |b_{ji}|L_j^f + \sum_{j=1}^n |\tilde{b}_{ji}| + \sum_{j=1}^n |\bar{b}_{ji}|\right) + L_i^g \sum_{j=1}^n w_{m+j}|d_{ij}| \\ &\quad + \frac{1}{1-\mu_\tau}\left(L_i^g\right)^2 \sum_{j=1}^n w_{m+j}|\tilde{d}_{ij}| + \left(L_i^g\right)^2 \sum_{j=1}^n w_{m+j}|\bar{d}_{ij}|, \\ \Theta_j &= \left(L_j^f\right)^2 \sum_{i=1}^m w_i|\bar{b}_{ji}| + L_j^f \sum_{i=1}^m w_i|b_{ji}| \\ &\quad + w_{m+j}\left(-2D_j^*\lambda_1 - 2q_j + \sum_{i=1}^m |d_{ij}|L_i^g + \sum_{i=1}^m |\tilde{d}_{ij}| + 2\sum_{i=1}^m |\bar{d}_{ij}|\right) \\ &\quad + \frac{1}{1-\mu_\theta}\left(L_j^f\right)^2 \sum_{i=1}^m w_i|\tilde{b}_{ji}|, \end{aligned}$$

$i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$, L_j^f *and* L_i^g *are Lipschitz constants*, $D_i = \min_{1\le k\le l} D_{ik}$, $D_j^* = \min_{1\le k\le l} D_{jk}^*$, λ_1 *is the lowest positive eigenvalue of problem* (6).

Proof Multiplying both sides of the first equation of (1) by $u_i(t,x)$ and integrating over Ω yields

$$\begin{aligned} &\frac{1}{2}\frac{d}{dt}\int_\Omega u_i(t,x)^2\,dx \\ &\quad = \int_\Omega \sum_{k=1}^l u_i(t,x)\frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial u_i(t,x)}{\partial x_k}\right)dx - p_i\int_\Omega u_i(t,x)^2\,dx \\ &\qquad + \int_\Omega \sum_{j=1}^n b_{ji}u_i(t,x)f_j\big(v_j(t,x)\big)\,dx + \sum_{j=1}^n \int_\Omega \tilde{b}_{ji}u_i(t,x)f_j\big(v_j\big(t-\theta_{ji}(t),x\big)\big)\,dx \\ &\qquad + \sum_{j=1}^n \int_\Omega \bar{b}_{ji}u_i(t,x)\int_{-\infty}^t k_{ji}(t-s)f_j\big(v_j(s,x)\big)\,ds\,dx + \int_\Omega u_i(t,x)\sigma_i(t,x)\,dx. \end{aligned} \tag{9}$$

It is easy to calculate by the Neumann boundary conditions (2) that

$$\begin{aligned}
&\int_\Omega \sum_{k=1}^{l} u_i(t,x)\frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial u_i(t,x)}{\partial x_k}\right)dx \\
&\quad = \int_\Omega \sum_{k=1}^{l} u_i(t,x)\nabla\left(D_{ik}\frac{\partial u_i(t,x)}{\partial x_k}\right)dx \\
&\quad = \int_{\partial\Omega} \sum_{k=1}^{l} u_i(t,x)D_{ik}\frac{\partial u_i(t,x)}{\partial x_k}\,dx - \int_\Omega \sum_{k=1}^{l} D_{ik}\left(\frac{\partial u_i(t,x)}{\partial x_k}\right)^2 dx \\
&\quad = -\sum_{k=1}^{l}\int_\Omega D_{ik}\left(\frac{\partial u_i(t,x)}{\partial x_k}\right)^2 dx. \qquad (10)
\end{aligned}$$

Moreover, from Lemma 1, we can derive

$$-\sum_{k=1}^{l}\int_\Omega D_{ik}\left(\frac{\partial u_i(t,x)}{\partial x_k}\right)^2 dx \le -\sum_{k=1}^{l}\int_\Omega D_{i}\left(\frac{\partial u_i(t,x)}{\partial x_k}\right)^2 dx \le -D_i\lambda_1\left\|u_i(t,x)\right\|_2^2. \qquad (11)$$

From (9)-(11), we obtain

$$\begin{aligned}
&\frac{d}{dt}\int_\Omega\left|u_i(t,x)\right|^2 dx \\
&\quad \le -2D_i\lambda_1\int_\Omega\left|u_i(t,x)\right|^2 dx - 2p_i\int_\Omega\left|u_i(t,x)\right|^2 dx \\
&\qquad + 2\int_\Omega\sum_{j=1}^{n}|b_{ji}|\left|u_i(t,x)\right|L_j^f\left|v_j(t,x)\right|dx + 2\sum_{j=1}^{n}\int_\Omega|\tilde{b}_{ji}|\left|u_i(t,x)\right|\left|f_j\left(v_j\left(t-\theta_{ji}(t),x\right)\right)\right|dx \\
&\qquad + 2\sum_{j=1}^{n}\int_\Omega|\bar{b}_{ji}|\int_{-\infty}^{t}k_{ji}(t-s)\left|u_i(t,x)\right|\left|f_j\left(v_j(s,x)\right)\right|ds\,dx \\
&\qquad + 2\int_\Omega u_i(t,x)\sigma_i(t,x)\,dx. \qquad (12)
\end{aligned}$$

Multiplying both sides of the second equation of system (1) by $v_j(t,x)$, similarly, we also have

$$\begin{aligned}
\frac{d}{dt}\int_\Omega\left|v_j(t,x)\right|^2 dx \le{}& -2D_j^*\lambda_1\int_\Omega\left|v_j(t,x)\right|^2 dx - 2q_j\int_\Omega\left|v_j(t,x)\right|^2 dx \\
&+ 2\int_\Omega\sum_{i=1}^{m}|d_{ij}|L_i^g\left|u_i(t,x)\right|\left|v_j(t,x)\right|dx \\
&+ 2\sum_{i=1}^{m}\int_\Omega|\tilde{d}_{ij}|\left|g_i\left(u_i\left(t-\tau_{ij}(t),x\right)\right)\right|\left|v_j(t,x)\right|dx \\
&+ 2\sum_{i=1}^{m}\int_\Omega|\bar{d}_{ij}|\int_{-\infty}^{t}\bar{k}_{ij}(t-s)\left|g_i\left(u_i(s,x)\right)\right|\left|v_j(t,x)\right|ds\,dx \\
&+ 2\int_\Omega v_j(t,x)\vartheta_j(t,x)\,dx. \qquad (13)
\end{aligned}$$

Let us construct the following Lyapunov functional:

$$\begin{aligned}
V(t) = {} & \int_\Omega \sum_{i=1}^m w_i \Biggl[u_i(t,x)^2 + \sum_{j=1}^n |\tilde{b}_{ji}| \frac{1}{1-\mu_\theta} \int_{t-\theta_{ji}(t)}^t \bigl|f_j\bigl(v_j(\xi,x)\bigr)\bigr|^2 \, d\xi \\
& + \sum_{j=1}^n |\bar{b}_{ji}| \int_0^{+\infty} k_{ji}(s) \int_{t-s}^t \bigl|f_j\bigl(v_j(\xi,x)\bigr)\bigr|^2 \, d\xi \, ds \Biggr] dx \\
& + \int_\Omega \sum_{j=1}^n w_{m+j} \Biggl[v_j(t,x)^2 + \sum_{i=1}^m |\tilde{d}_{ij}| \frac{1}{1-\mu_\tau} \int_{t-\tau_{ij}(t)}^t \bigl|g_i\bigl(u_i(\xi,x)\bigr)\bigr|^2 \, d\xi \\
& + \sum_{i=1}^m |\bar{d}_{ij}| \int_0^{+\infty} \bar{k}_{ij}(s) \int_{t-s}^t \bigl|g_i\bigl(u_i(\xi,x)\bigr)\bigr|^2 \, d\xi \, ds \Biggr] dx.
\end{aligned}$$

Its upper Dini-derivative along the solution to system (1) can be calculated as

$$\begin{aligned}
D^+V(t) \le {} & \int_\Omega \sum_{i=1}^m w_i \Biggl[-2D_i\lambda_1 \bigl|u_i(t,x)\bigr|^2 - 2p_i \bigl|u_i(t,x)\bigr|^2 \\
& + 2\sum_{j=1}^n |b_{ji}| \bigl|u_i(t,x)\bigr| L_j^f \bigl|v_j(t,x)\bigr| + 2\sum_{j=1}^n |\tilde{b}_{ji}| \bigl|u_i(t,x)\bigr| \bigl|f_j\bigl(v_j\bigl(t-\theta_{ji}(t),x\bigr)\bigr)\bigr| \\
& + 2\sum_{j=1}^n |\bar{b}_{ji}| \int_{-\infty}^t k_{ji}(t-s) \bigl|u_i(t,x)\bigr| \bigl|f_j\bigl(v_j(s,x)\bigr)\bigr| \, ds + 2u_i(t,x)\sigma_i(t,x) \\
& + \sum_{j=1}^n |\tilde{b}_{ji}| \frac{1}{1-\mu_\theta} \bigl|f_j\bigl(v_j(t,x)\bigr)\bigr|^2 + \sum_{j=1}^n |\bar{b}_{ji}| \int_0^{+\infty} k_{ji}(s) \bigl|f_j\bigl(v_j(t,x)\bigr)\bigr|^2 \, ds \\
& - \sum_{j=1}^n |\tilde{b}_{ji}| \frac{1}{1-\mu_\theta} \bigl(1-\dot{\theta}_{ji}(t)\bigr) \bigl|f_j\bigl(v_j\bigl(t-\theta_{ji}(t),x\bigr)\bigr)\bigr|^2 \\
& - \sum_{j=1}^n |\bar{b}_{ji}| \int_0^{+\infty} k_{ji}(s) \bigl|f_j\bigl(v_j(t-s,x)\bigr)\bigr|^2 \, ds \Biggr] dx \\
& + \int_\Omega \sum_{j=1}^n w_{m+j} \Biggl[-2D_j^*\lambda_1 \bigl|v_j(t,x)\bigr|^2 \\
& - 2q_j \bigl|v_j(t,x)\bigr|^2 + 2\sum_{i=1}^m |d_{ij}| L_i^g \bigl|u_i(t,x)\bigr| \bigl|v_j(t,x)\bigr| \\
& + 2\sum_{i=1}^m |\tilde{d}_{ij}| \bigl|g_i\bigl(u_i\bigl(t-\tau_{ij}(t),x\bigr)\bigr)\bigr| \bigl|v_j(t,x)\bigr| \\
& + 2\sum_{i=1}^m |\bar{d}_{ij}| \int_{-\infty}^t \bar{k}_{ij}(t-s) \bigl|g_i\bigl(u_i(s,x)\bigr)\bigr| \bigl|v_j(t,x)\bigr| \, ds + 2v_j(t,x)\vartheta_j(t,x) \\
& + \sum_{i=1}^m |\tilde{d}_{ij}| \frac{1}{1-\mu_\tau} \bigl|g_i\bigl(u_i(t,x)\bigr)\bigr|^2 \\
& - \sum_{i=1}^m |\tilde{d}_{ij}| \frac{1}{1-\mu_\tau} \bigl(1-\dot{\tau}_{ij}(t)\bigr) \bigl|g_i\bigl(u_i\bigl(t-\tau_{ij}(t),x\bigr)\bigr)\bigr|^2
\end{aligned}$$

$$
\begin{aligned}
&+\sum_{i=1}^{m}|\bar{d}_{ij}|\int_{0}^{+\infty}\bar{k}_{ij}(s)\bigl|g_i\bigl(u_i(t,x)\bigr)\bigr|^2\,ds \\
&-\sum_{i=1}^{m}|\bar{d}_{ij}|\int_{0}^{+\infty}\bar{k}_{ij}(s)\bigl|g_i\bigl(u_i(t-s,x)\bigr)\bigr|^2\,ds\Biggr]\,dx. \qquad (14)
\end{aligned}
$$

According to (12)-(14) and (A1)-(A3), we can derive

$$
\begin{aligned}
D^+V(t) \le{}& \int_\Omega \sum_{i=1}^{m} w_i\Biggl[-2D_i\lambda_1\bigl|u_i(t,x)\bigr|^2 - 2p_i\bigl|u_i(t,x)\bigr|^2 \\
&+\sum_{j=1}^{n}|b_{ji}|L_j^f\bigl(\bigl|u_i(t,x)\bigr|^2+\bigl|v_j(t,x)\bigr|^2\bigr) \\
&+\sum_{j=1}^{n}|\tilde{b}_{ji}|\bigl(\bigl|u_i(t,x)\bigr|^2+\bigl|f_j\bigl(v_j\bigl(t-\theta_{ji}(t),x\bigr)\bigr)\bigr|^2\bigr) \\
&+\sum_{j=1}^{n}|\bar{b}_{ji}|\int_{-\infty}^{t}k_{ji}(t-s)\bigl(\bigl|u_i(t,x)\bigr|^2+\bigl|f_j\bigl(v_j(s,x)\bigr)\bigr|^2\bigr)\,ds + 2u_i(t,x)\sigma_i(t,x) \\
&+\sum_{j=1}^{n}|\tilde{b}_{ji}|\frac{1}{1-\mu_\theta}\bigl|f_j\bigl(v_j(t,x)\bigr)\bigr|^2+\sum_{j=1}^{n}|\bar{b}_{ji}|\int_0^{+\infty}k_{ji}(s)\bigl|f_j\bigl(v_j(t,x)\bigr)\bigr|^2\,ds \\
&-\sum_{j=1}^{n}|\tilde{b}_{ji}|\bigl|f_j\bigl(v_j\bigl(t-\theta_{ji}(t),x\bigr)\bigr)\bigr|^2-\sum_{j=1}^{n}|\bar{b}_{ji}|\int_0^{+\infty}k_{ji}(s)\bigl|f_j\bigl(v_j(t-s,x)\bigr)\bigr|^2\,ds\Biggr]\,dx \\
&+\int_\Omega\sum_{j=1}^{n}w_{m+j}\Biggl[-2D_j^*\lambda_1\bigl|v_j(t,x)\bigr|^2-2q_j\bigl|v_j(t,x)\bigr|^2 \\
&+\sum_{i=1}^{m}|d_{ij}|L_i^g\bigl(\bigl|u_i(t,x)\bigr|^2+\bigl|v_j(t,x)\bigr|^2\bigr) \\
&+\sum_{i=1}^{m}|\tilde{d}_{ij}|\bigl(\bigl|g_i\bigl(u_i\bigl(t-\tau_{ij}(t),x\bigr)\bigr)\bigr|^2+\bigl|v_j(t,x)\bigr|^2\bigr) \\
&+2\sum_{i=1}^{m}|\bar{d}_{ij}|\int_{-\infty}^{t}\bar{k}_{ij}(t-s)\bigl(\bigl|g_i\bigl(u_i(s,x)\bigr)\bigr|^2+\bigl|v_j(t,x)\bigr|^2\bigr)\,ds+2v_j(t,x)\vartheta_j(t,x) \\
&+\sum_{i=1}^{m}|\tilde{d}_{ij}|\frac{1}{1-\mu_\tau}\bigl|g_i\bigl(u_i(t,x)\bigr)\bigr|^2-\sum_{i=1}^{m}|\tilde{d}_{ij}|\bigl|g_i\bigl(u_i\bigl(t-\tau_{ij}(t),x\bigr)\bigr)\bigr|^2 \\
&+\sum_{i=1}^{m}|\bar{d}_{ij}|\int_0^{+\infty}\bar{k}_{ij}(s)\bigl|g_i\bigl(u_i(t,x)\bigr)\bigr|^2\,ds \\
&-\sum_{i=1}^{m}|\bar{d}_{ij}|\int_0^{+\infty}\bar{k}_{ij}(s)\bigl|g_i\bigl(u_i(t-s,x)\bigr)\bigr|^2\,ds\Biggr]\,dx \\
={}&\int_\Omega\sum_{i=1}^{m}\Biggl\{w_i\Biggl[-2D_i\lambda_1-2p_i+\sum_{j=1}^{n}|b_{ji}|L_j^f+\sum_{j=1}^{n}|\tilde{b}_{ji}|+\sum_{j=1}^{n}|\bar{b}_{ji}|\Biggr] \\
&+L_i^g\sum_{j=1}^{n}w_{m+j}|d_{ij}|+\frac{1}{1-\mu_\tau}\bigl(L_i^g\bigr)^2\sum_{j=1}^{n}w_{m+j}|\tilde{d}_{ij}|+\bigl(L_i^g\bigr)^2\sum_{j=1}^{n}w_{m+j}|\bar{d}_{ij}|\Biggr\}
\end{aligned}
$$

$$\begin{aligned}
&\times \bigl|u_i(t,x)\bigr|^2\,dx \\
&+ \int_\Omega \sum_{j=1}^{n} \Biggl\{ w_{m+j}\Biggl[-2D_j^*\lambda_1 - 2q_j + \sum_{i=1}^{m} |d_{ij}|L_i^g + \sum_{i=1}^{m} |\tilde{d}_{ij}| + 2\sum_{i=1}^{m} |\bar{d}_{ij}| \Biggr] \\
&+ \frac{1}{1-\mu_\theta}\bigl(L_j^f\bigr)^2 \sum_{i=1}^{m} w_i|\tilde{b}_{ji}| + \bigl(L_j^f\bigr)^2 \sum_{i=1}^{m} w_i|\bar{b}_{ji}| \int_0^{+\infty} k_{ji}(s)\,ds + L_j^f \sum_{i=1}^{m} w_i|b_{ji}| \Biggr\} \\
&\times \bigl|v_j(t,x)\bigr|^2\,dx \\
&+ 2\int_\Omega \Biggl[\sum_{i=1}^{m} w_i u_i(t,x)\sigma_i(t,x) + \sum_{j=1}^{n} w_{m+j} v_j(t,x)\vartheta_j(t,x) \Biggr]\,dx. \qquad (15)
\end{aligned}$$

Thus, we have

$$\begin{aligned}
&D^+V(t) - 2\int_\Omega \Biggl[\sum_{i=1}^{m} y_i(t,x)\sigma_i(t,x) + \sum_{j=1}^{n} z_j(t,x)\vartheta_j(t,x) \Biggr]\,dx \\
&\quad - \gamma \int_\Omega \Biggl[\sum_{i=1}^{m} \sigma_i(t,x)^2 + \sum_{j=1}^{n} \vartheta_j(t,x)^2 \Biggr]\,dx \\
&\le \int_\Omega \sum_{i=1}^{m} \Biggl\{ w_i\Biggl[-2D_i\lambda_1 - 2p_i + \sum_{j=1}^{n} |b_{ji}|L_j^f + \sum_{j=1}^{n} |\tilde{b}_{ji}| + \sum_{j=1}^{n} |\bar{b}_{ji}| \Biggr] + L_i^g \sum_{j=1}^{n} w_{m+j}|d_{ij}| \\
&\quad + \frac{1}{1-\mu_\tau}\bigl(L_i^g\bigr)^2 \sum_{j=1}^{n} w_{m+j}|\tilde{d}_{ij}| + \bigl(L_i^g\bigr)^2 \sum_{j=1}^{n} w_{m+j}|\bar{d}_{ij}| \Biggr\} \bigl|u_i(t,x)\bigr|^2\,dx \\
&\quad + \int_\Omega \sum_{j=1}^{n} \Biggl\{ w_{m+j}\Biggl[-2D_j^*\lambda_1 - 2q_j + \sum_{i=1}^{m} |d_{ij}|L_i^g + \sum_{i=1}^{m} |\tilde{d}_{ij}| + 2\sum_{i=1}^{m} |\bar{d}_{ij}| \Biggr] \\
&\quad + \frac{1}{1-\mu_\theta}\bigl(L_j^f\bigr)^2 \sum_{i=1}^{m} w_i|\tilde{b}_{ji}| + \bigl(L_j^f\bigr)^2 \sum_{i=1}^{m} w_i|\bar{b}_{ji}| + L_j^f \sum_{i=1}^{m} w_i|b_{ji}| \Biggr\} \bigl|v_j(t,x)\bigr|^2\,dx \\
&\quad + 2\int_\Omega \Biggl[\sum_{i=1}^{m} w_i u_i(t,x)\sigma_i(t,x) + \sum_{j=1}^{n} w_{m+j} v_j(t,x)\vartheta_j(t,x) \Biggr]\,dx \\
&\quad - 2\int_\Omega \Biggl[\sum_{i=1}^{m} y_i(t,x)\sigma_i(t,x) + \sum_{j=1}^{n} z_j(t,x)\vartheta_j(t,x) \Biggr]\,dx \\
&\quad - \gamma \int_\Omega \Biggl[\sum_{i=1}^{m} \sigma_i(t,x)^2 + \sum_{j=1}^{n} \vartheta_j(t,x)^2 \Biggr]\,dx \\
&= \int_\Omega \sum_{i=1}^{m} \Biggl\{ \Biggl[w_i\Biggl(-2D_i\lambda_1 - 2p_i + \sum_{j=1}^{n} |b_{ji}|L_j^f + \sum_{j=1}^{n} |\tilde{b}_{ji}| + \sum_{j=1}^{n} |\bar{b}_{ji}| \Biggr) \\
&\quad + L_i^g \sum_{j=1}^{n} w_{m+j}|d_{ij}| + \frac{1}{1-\mu_\tau}\bigl(L_i^g\bigr)^2 \sum_{j=1}^{n} w_{m+j}|\tilde{d}_{ij}| + \bigl(L_i^g\bigr)^2 \sum_{j=1}^{n} w_{m+j}|\bar{d}_{ij}| \Biggr] \bigl|u_i(t,x)\bigr|^2 \\
&\quad + \bigl[2(w_i - a_i)u_i(t,x)\sigma_i(t,x) + (-\gamma - 2c_i)\sigma_i(t,x)^2\bigr] \Biggr\}\,dx
\end{aligned}$$

$$
\begin{aligned}
&+\int_\Omega \sum_{j=1}^{n}\Bigg\{\Bigg[w_{m+j}\Bigg(-2D_j^*\lambda_1-2q_j+\sum_{i=1}^{m}|d_{ij}|L_i^g+\sum_{i=1}^{m}|\tilde{d}_{ij}|+2\sum_{i=1}^{m}|\bar{d}_{ij}|\Bigg)\\
&+\frac{1}{1-\mu_\theta}\big(L_j^f\big)^2\sum_{i=1}^{m}w_i|\tilde{b}_{ji}|+\big(L_j^f\big)^2\sum_{i=1}^{m}w_i|\bar{b}_{ji}|+L_j^f\sum_{i=1}^{m}w_i|b_{ji}|\Bigg]\big|v_j(t,x)\big|^2\\
&+\big[2(w_{m+j}-\bar{a}_i)v_j(t,x)\vartheta_j(t,x)+(-\gamma-2\bar{c}_i)\vartheta_j(t,x)^2\big]\Bigg\}\,dx\\
&=\int_\Omega\sum_{i=1}^{m}\begin{pmatrix}u_i(t,x)\\ \sigma_i(t,x)\end{pmatrix}^T\begin{pmatrix}\Xi_i & w_i-a_i\\ w_i-a_i & -\gamma-2c_i\end{pmatrix}\begin{pmatrix}u_i(t,x)\\ \sigma_i(t,x)\end{pmatrix}dx\\
&+\int_\Omega\sum_{j=1}^{n}\begin{pmatrix}v_j(t,x)\\ \vartheta_j(t,x)\end{pmatrix}^T\begin{pmatrix}\Theta_j & w_{m+j}-\bar{a}_j\\ w_{m+j}-\bar{a}_j & -\gamma-2\bar{c}_j\end{pmatrix}\begin{pmatrix}v_j(t,x)\\ \vartheta_j(t,x)\end{pmatrix}dx.
\end{aligned}
\tag{16}
$$

According to (16), we can derive

$$
\begin{aligned}
&D^+V(t)-2\int_\Omega\Bigg[\sum_{i=1}^{m}y_i(t,x)\sigma_i(t,x)+\sum_{j=1}^{n}z_j(t,x)\vartheta_j(t,x)\Bigg]dx\\
&-\gamma\int_\Omega\Bigg[\sum_{i=1}^{m}\sigma_i(t,x)^2+\sum_{j=1}^{n}\vartheta_j(t,x)^2\Bigg]dx\le 0.
\end{aligned}
\tag{17}
$$

By integrating the above inequality with respect to t over the time period 0 to t_p, we have

$$
\begin{aligned}
&2\int_0^{t_p}\int_\Omega\Bigg[\sum_{i=1}^{m}y_i(t,x)\sigma_i(t,x)+\sum_{j=1}^{n}z_j(t,x)\vartheta_j(t,x)\Bigg]dx\,dt\\
&\quad\ge V(t_p)-V(0)-\gamma\int_0^{t_p}\int_\Omega\Bigg[\sum_{i=1}^{m}\sigma_i(t,x)^2+\sum_{j=1}^{n}\vartheta_j(t,x)^2\Bigg]dx\,dt.
\end{aligned}
\tag{18}
$$

Since $V(t_p)\ge 0$ and $V(0)\ge 0$, we have

$$
\begin{aligned}
&2\int_0^{t_p}\int_\Omega\Bigg[\sum_{i=1}^{m}y_i(t,x)\sigma_i(t,x)+\sum_{j=1}^{n}z_j(t,x)\vartheta_j(t,x)\Bigg]dx\,dt\\
&\quad\ge -\beta^2-\gamma\int_0^{t_p}\int_\Omega\Bigg[\sum_{i=1}^{m}\sigma_i(t,x)^2+\sum_{j=1}^{n}\vartheta_j(t,x)^2\Bigg]dx\,dt
\end{aligned}
\tag{19}
$$

for all $t_p\ge 0$, where $\beta=\sqrt{V(0)}$. This completes the proof of Theorem 1. □

Remark 2 In Theorem 1, the Poincaré integral inequality is utilized firstly. This is a very key step. Thus, the obtained passivity condition includes diffusion terms, which is novel for delayed RDNNs.

Remark 3 References [20, 32] investigated the passivity of NNs with time-varying delay, in which the input and output variables are only dependent on the time. Theorem 1 gives a passivity condition of BAM RDNNs with Neumann boundary conditions and mixed time-varying delays. This condition not only connects with the delays, but it also relates to the diffusion effect. There are few results on the passivity analysis of BAM RDNNs with Neumann boundary conditions and mixed time-varying delays. Therefore, the passivity criterion provided in Theorem 1 is essentially new compared with those given in [17, 24].

4 Robust passive analysis

Because of the presence of unavoidable factors, such as modeling errors, external perturbations, and parameter fluctuations during the physical implementation, the NNs model certainly involve uncertainties such as perturbations and component variations, which will affect the passivity of the whole system. In this regard, to analyze the uncertainty of BAM RDNNs, one reasonable method is to assume parameters in certain intervals. In order to establish the passive condition for system (1), the quantities may be considered as intervals as follows:

$$\begin{aligned}
&D_I := \bigl\{D = (D_{ik})_{m\times l} : \widehat{D} \le D \le \breve{D}, i.e., \widehat{D}_{ik} \le D_{ik} \le \breve{D}_{ik}, k = 1,\dots,l, i = 1,\dots,m\bigr\},\\
&D_I^* := \bigl\{D^* = \bigl(D_{jk}^*\bigr)_{n\times l} : \widehat{D}^* \le D^* \le \breve{D}^*, i.e., \widehat{D}_{jk}^* \le D_{jk}^* \le \breve{D}_{jk}^*, k = 1,\dots,l, j = 1,\dots,n\bigr\},\\
&P_I := \bigl\{P = \operatorname{diag}(p_i) : \widehat{P} \le P \le \breve{P}, i.e., \widehat{p}_i \le p_i \le \breve{p}_i, i = 1,\dots,m\bigr\},\\
&Q_I := \bigl\{Q = \operatorname{diag}(q_j) : \widehat{Q} \le Q \le \breve{Q}, i.e., \widehat{q}_j \le q_j \le \breve{q}_j, j = 1,\dots,n\bigr\},\\
&B_I := \bigl\{B = (b_{ji})_{n\times m} : \widehat{B} \le B \le \breve{B}, i.e., \widehat{b}_{ji} \le b_{ji} \le \breve{b}_{ji}, i = 1,\dots,m, j = 1,\dots,n\bigr\},\\
&\tilde{B}_I := \bigl\{\tilde{B} = (\tilde{b}_{ji})_{n\times m} : \widehat{\tilde{B}} \le \tilde{B} \le \breve{\tilde{B}}, i.e., \widehat{\tilde{b}}_{ji} \le \tilde{b}_{ji} \le \breve{\tilde{b}}_{ji}, i = 1,\dots,m, j = 1,\dots,n\bigr\},\\
&\bar{B}_I := \bigl\{\bar{B} = (\bar{b}_{ji})_{n\times m} : \widehat{\bar{B}} \le \bar{B} \le \breve{\bar{B}}, i.e., \widehat{\bar{b}}_{ji} \le \bar{b}_{ji} \le \breve{\bar{b}}_{ji}, i = 1,\dots,m, j = 1,\dots,n\bigr\},\\
&D_I := \bigl\{D = (d_{ij})_{m\times n} : \widehat{D} \le D \le \breve{D}, i.e., \widehat{d}_{ij} \le d_{ij} \le \breve{d}_{ij}, i = 1,\dots,m, j = 1,\dots,n\bigr\},\\
&\tilde{D}_I := \bigl\{\tilde{D} = (\tilde{d}_{ij})_{m\times n} : \widehat{\tilde{D}} \le \tilde{D} \le \breve{\tilde{D}}, i.e., \widehat{\tilde{d}}_{ij} \le \tilde{d}_{ij} \le \breve{\tilde{d}}_{ij}, i = 1,\dots,m, j = 1,\dots,n\bigr\},\\
&\bar{D}_I := \bigl\{\bar{D} = (\bar{d}_{ij})_{m\times n} : \widehat{\bar{D}} \le \bar{D} \le \breve{\bar{D}}, i.e., \widehat{\bar{d}}_{ij} \le \bar{d}_{ij} \le \breve{\bar{d}}_{ij}, i = 1,\dots,m, j = 1,\dots,n\bigr\}.
\end{aligned} \tag{20}$$

Definition 2 System (1) with the parameter ranges defined by (20) is called robustly passive, if, for all $D \in D_I$, $D^* \in D_I^*$, $P \in P_I$, $Q \in Q_I$, $B \in B_I$, $\tilde{B} \in \tilde{B}_I$, $\bar{B} \in \bar{B}_I$, $D \in D_I$, $\tilde{D} \in \tilde{D}_I$, and $\bar{D} \in \bar{D}_I$, there are constants $\gamma \ge 0$ and $\beta \in R$ such that

$$\begin{aligned}
&2\int_0^{t_p}\int_\Omega \bigl(y(t,x)^T\sigma(t,x) + z(t,x)^T\vartheta(t,x)\bigr)\,dx\\
&\quad \ge -\beta^2 - \gamma\int_0^{t_p}\int_\Omega \bigl(\sigma(t,x)^T\sigma(t,x) + \vartheta(t,x)^T\vartheta(t,x)\bigr)\,dx
\end{aligned} \tag{21}$$

for all $t_p \ge 0$.

Theorem 2 *Under the assumptions* (A1)-(A3), *system* (1) *with the parameter ranges defined by* (20) *is robustly passive if there exist constants* $w_i, w_{m+j} > 0$ *and* $\gamma > 0$ *such that*

$$-\gamma - 2\widehat{c}_i < 0, \qquad -\gamma - 2\widehat{\bar{c}}_j < 0, \tag{22}$$

$$\Xi_i^* - a_i^{*2}(-\gamma - 2\widehat{c}_i)^{-1} \le 0, \tag{23}$$

and

$$\Theta_j^* - \bar{a}_j^{*2}(-\gamma - 2\widehat{\bar{c}}_j)^{-1} \le 0, \tag{24}$$

in which

$$b_{ij}^* = \max\bigl(|\widehat{b}_{ij}|, |\breve{b}_{ij}|\bigr), \qquad \tilde{b}_{ij}^* = \max\bigl(|\widehat{\tilde{b}}_{ij}|, |\breve{\tilde{b}}_{ij}|\bigr), \qquad \bar{b}_{ij}^* = \max(\widehat{\bar{b}}_{ij}, \breve{\bar{b}}_{ij}),$$

$$d_{ij}^* = \max\bigl(|\widehat{d}_{ij}|, |\breve{d}_{ij}|\bigr), \qquad \tilde{d}_{ij}^* = \max\bigl(|\widehat{\tilde{d}}_{ij}|, |\breve{\tilde{d}}_{ij}|\bigr), \qquad \bar{d}_{ij}^* = \max\bigl(|\widehat{\bar{d}}_{ij}|, |\breve{\bar{d}}_{ij}|\bigr),$$

$$a_i^* = \max\bigl(|w_i - \widehat{a}_i|, |w_i - \breve{a}_i|\bigr), \qquad \bar{a}_j^* = \bigl(|w_{m+j} - \widehat{\bar{a}}_j|, |w_{m+j} - \breve{\bar{a}}_j|\bigr),$$

$$\begin{aligned} \Xi_i^* &= w_i\left(-2\widehat{D}_i\lambda_1 - 2\widehat{p}_i + \sum_{j=1}^n b_{ji}^* L_j^f + \sum_{j=1}^n \tilde{b}_{ji}^* + \sum_{j=1}^n \bar{b}_{ji}^*\right) + L_i^g \sum_{j=1}^n w_{m+j} d_{ij}^* \\ &\quad + \frac{1}{1-\mu_\tau}\bigl(L_i^g\bigr)^2 \sum_{j=1}^n w_{m+j}\tilde{d}_{ij}^* + \bigl(L_i^g\bigr)^2 \sum_{j=1}^n w_{m+j}\bar{d}_{ij}^*, \\ \Theta_j^* &= \bigl(L_j^f\bigr)^2 \sum_{i=1}^m w_i \bar{b}_{ji}^* + L_j^f \sum_{i=1}^m w_i b_{ji}^* \\ &\quad + w_{m+j}\left(-2\widehat{D}_j^*\lambda_1 - 2\widehat{q}_j + \sum_{i=1}^m d_{ij}^* L_i^g + \sum_{i=1}^m \tilde{d}_{ij}^* + 2\sum_{i=1}^m \bar{d}_{ij}^*\right) + \frac{1}{1-\mu_\theta}\bigl(L_j^f\bigr)^2 \sum_{i=1}^m w_i \tilde{b}_{ji}^*, \end{aligned}$$

$i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$, $\widehat{D}_i = \min_{1\le k\le l} \widehat{D}_{ik}$, $\widehat{D}_j^* = \min_{1\le k\le l} \widehat{D}_{jk}^*$, L_j^f *and* L_i^g *are Lipschitz constants,* λ_1 *is the lowest positive eigenvalue of problem* (6).

Proof Let us construct the following Lyapunov functional:

$$\begin{aligned} V(t) = {} & \int_\Omega \sum_{i=1}^m w_i\left[u_i(t,x)^2 + \sum_{j=1}^n \tilde{b}_{ji}^* \frac{1}{1-\mu_\theta} \int_{t-\theta_{ji}(t)}^t \bigl|f_j\bigl(v_j(\xi,x)\bigr)\bigr|^2 \, d\xi \right. \\ & \left. + \sum_{j=1}^n \bar{b}_{ji}^* \int_0^{+\infty} k_{ji}(s) \int_{t-s}^t \bigl|f_j\bigl(v_j(\xi,x)\bigr)\bigr|^r \, d\xi \, ds\right] dx \\ & + \int_\Omega \sum_{j=1}^n w_{m+j}\left[v_j(t,x)^2 + \sum_{i=1}^m \tilde{d}_{ij}^* \frac{1}{1-\mu_\tau} \int_{t-\tau_{ij}(t)}^t \bigl|g_i\bigl(u_i(\xi,x)\bigr)\bigr|^2 \, d\xi \right. \\ & \left. + \sum_{i=1}^m \bar{d}_{ij}^* \int_0^{+\infty} \bar{k}_{ij}(s) \int_{t-s}^t \bigl|g_i\bigl(u_i(\xi,x)\bigr)\bigr|^2 \, d\xi \, ds\right] dx. \end{aligned}$$

From (9)-(13), its upper Dini-derivative along the solution to system (1) can be calculated as

$$\begin{aligned} D^+V(t) \le {} & \int_\Omega \sum_{i=1}^m w_i\left[-2\widehat{D}_i\lambda_1 \bigl|u_i(t,x)\bigr|^2 - 2\widehat{p}_i \bigl|u_i(t,x)\bigr|^2 \right. \\ & + 2\sum_{j=1}^n b_{ji}^* \bigl|u_i(t,x)\bigr| L_j^f \bigl|v_j(t,x)\bigr| + 2\sum_{j=1}^n \tilde{b}_{ji}^* \bigl|u_i(t,x)\bigr| \bigl|f_j\bigl(v_j\bigl(t-\theta_{ji}(t),x\bigr)\bigr)\bigr| \\ & + 2\sum_{j=1}^n \bar{b}_{ji}^* \int_{-\infty}^t k_{ji}(t-s) \bigl|u_i(t,x)\bigr| \bigl|f_j\bigl(v_j(s,x)\bigr)\bigr| \, ds + 2u_i(t,x)\sigma_i(t,x) \end{aligned}$$

$$
\begin{aligned}
&+\sum_{j=1}^{n}\tilde{b}_{ji}^{*}\frac{1}{1-\mu_\theta}\left|f_j\big(v_j(t,x)\big)\right|^2+\sum_{j=1}^{n}\bar{b}_{ji}^{*}\int_0^{+\infty}k_{ji}(s)\left|f_j\big(v_j(t,x)\big)\right|^2\,ds\\
&-\sum_{j=1}^{n}\tilde{b}_{ji}^{*}\frac{1}{1-\mu_\theta}\big(1-\dot{\theta}_{ji}(t)\big)\left|f_j\big(v_j\big(t-\theta_{ji}(t),x\big)\big)\right|^2\\
&-\sum_{j=1}^{n}\bar{b}_{ji}^{*}\int_0^{+\infty}k_{ji}(s)\left|f_j\big(v_j(t-s,x)\big)\right|^2\,ds\Bigg]\,dx\\
&+\int_\Omega\sum_{j=1}^{n}w_{m+j}\Bigg[-2\widehat{D}_j^{*}\lambda_1\left|v_j(t,x)\right|^2-2\widehat{q}_j\left|v_j(t,x)\right|^2\\
&+2\sum_{i=1}^{m}d_{ij}^{*}L_i^{g}\left|u_i(t,x)\right|\left|v_j(t,x)\right|+2\sum_{i=1}^{m}\tilde{d}_{ij}^{*}\left|g_i\big(u_i\big(t-\tau_{ij}(t),x\big)\big)\right|\left|v_j(t,x)\right|\\
&+2\sum_{i=1}^{m}\bar{d}_{ij}^{*}\int_{-\infty}^{t}\bar{k}_{ij}(t-s)\left|g_i\big(u_i(s,x)\big)\right|\left|v_j(t,x)\right|\,ds+2v_j(t,x)\vartheta_j(t,x)\\
&+\sum_{i=1}^{m}\tilde{d}_{ij}^{*}\frac{1}{1-\mu_\tau}\left|g_i\big(u_i(t,x)\big)\right|^2-\sum_{i=1}^{m}\tilde{d}_{ij}^{*}\frac{1}{1-\mu_\tau}\big(1-\dot{\tau}_{ij}(t)\big)\left|g_i\big(u_i\big(t-\tau_{ij}(t),x\big)\big)\right|^2\\
&+\sum_{i=1}^{m}\bar{d}_{ij}^{*}\int_0^{+\infty}\bar{k}_{ij}(s)\left|g_i\big(u_i(t,x)\big)\right|^2\,ds\\
&-\sum_{i=1}^{m}\bar{d}_{ij}^{*}\int_0^{+\infty}\bar{k}_{ij}(s)\left|g_i\big(u_i(t-s,x)\big)\right|^2\,ds\Bigg]\,dx. \qquad (25)
\end{aligned}
$$

According to (12)-(13), (25) and (A1)-(A3), we can get

$$
\begin{aligned}
D^{+}V(t)\le{}&\int_\Omega\sum_{i=1}^{m}w_i\Bigg[-2\widehat{D}_i\lambda_1\left|u_i(t,x)\right|^2-2\widehat{p}_i\left|u_i(t,x)\right|^2\\
&+\sum_{j=1}^{n}b_{ji}^{*}L_j^{f}\big(\left|u_i(t,x)\right|^2+\left|v_j(t,x)\right|^2\big)+\sum_{j=1}^{n}\tilde{b}_{ji}^{*}\big(\left|u_i(t,x)\right|^2+\left|f_j\big(v_j\big(t-\theta_{ji}(t),x\big)\big)\right|^2\big)\\
&+\sum_{j=1}^{n}\bar{b}_{ji}^{*}\int_{-\infty}^{t}k_{ji}(t-s)\big(\left|u_i(t,x)\right|^2+\left|f_j\big(v_j(s,x)\big)\right|^2\big)\,ds+2u_i(t,x)\sigma_i(t,x)\\
&+\sum_{j=1}^{n}\tilde{b}_{ji}^{*}\frac{1}{1-\mu_\theta}\left|f_j\big(v_j(t,x)\big)\right|^2+\sum_{j=1}^{n}\bar{b}_{ji}^{*}\int_0^{+\infty}k_{ji}(s)\left|f_j\big(v_j(t,x)\big)\right|^2\,ds\\
&-\sum_{j=1}^{n}\tilde{b}_{ji}^{*}\left|f_j\big(v_j\big(t-\theta_{ji}(t),x\big)\big)\right|^2-\sum_{j=1}^{n}\bar{b}_{ji}^{*}\int_0^{+\infty}k_{ji}(s)\left|f_j\big(v_j(t-s,x)\big)\right|^2\,ds\Bigg]\,dx\\
&+\int_\Omega\sum_{j=1}^{n}w_{m+j}\Bigg[-2\widehat{D}_j^{*}\lambda_1\left|v_j(t,x)\right|^2-2\widehat{q}_j\left|v_j(t,x)\right|^2\\
&+\sum_{i=1}^{m}d_{ij}^{*}L_i^{g}\big(\left|u_i(t,x)\right|^2+\left|v_j(t,x)\right|^2\big)\\
&+\sum_{i=1}^{m}\tilde{d}_{ij}^{*}\big(\left|g_i\big(u_i\big(t-\tau_{ij}(t),x\big)\big)\right|^2+\left|v_j(t,x)\right|^2\big)
\end{aligned}
$$

$$
\begin{aligned}
&+ 2\sum_{i=1}^{m}\bar{d}_{ij}^{*}\int_{-\infty}^{t}\bar{k}_{ij}(t-s)\bigl(\bigl|g_i\bigl(u_i(s,x)\bigr)\bigr|^2+\bigl|v_j(t,x)\bigr|^2\bigr)\,ds+2v_j(t,x)\vartheta_j(t,x)\\
&+\sum_{i=1}^{m}\tilde{d}_{ij}^{*}\frac{1}{1-\mu_\tau}\bigl|g_i\bigl(u_i(t,x)\bigr)\bigr|^2-\sum_{i=1}^{m}\tilde{d}_{ij}^{*}\bigl|g_i\bigl(u_i\bigl(t-\tau_{ij}(t),x\bigr)\bigr)\bigr|^2\\
&+\sum_{i=1}^{m}\bar{d}_{ij}^{*}\int_{0}^{+\infty}\bar{k}_{ij}(s)\bigl|g_i\bigl(u_i(t,x)\bigr)\bigr|^2\,ds\\
&-\sum_{i=1}^{m}\bar{d}_{ij}^{*}\int_{0}^{+\infty}\bar{k}_{ij}(s)\bigl|g_i\bigl(u_i(t-s,x)\bigr)\bigr|^2\,ds\Bigg]\,dx\\
=&\int_\Omega\sum_{i=1}^{m}\Bigg\{w_i\Bigg[-2\widehat{D}_i\lambda_1-2\widehat{p}_i+\sum_{j=1}^{n}b_{ji}^{*}L_j^f+\sum_{j=1}^{n}\tilde{b}_{ji}^{*}+\sum_{j=1}^{n}\bar{b}_{ji}^{*}\int_{-\infty}^{t}k_{ji}(t-s)\,ds\Bigg]\\
&+L_i^g\sum_{j=1}^{n}w_{m+j}d_{ji}^{*}+\frac{1}{1-\mu_\tau}\bigl(L_i^g\bigr)^2\sum_{j=1}^{n}w_{m+j}\tilde{d}_{ji}^{*}\\
&+\bigl(L_i^g\bigr)^2\sum_{j=1}^{n}w_{m+j}\bar{d}_{ji}^{*}\int_{0}^{+\infty}\bar{k}_{ij}(s)\,ds\Bigg\}\bigl|u_i(t,x)\bigr|^2\,dx\\
&+\int_\Omega\sum_{j=1}^{n}\Bigg\{w_{m+j}\Bigg[-2\widehat{D}_j^{*}\lambda_1-2\widehat{q}_j+\sum_{i=1}^{m}d_{ij}^{*}L_i^g+\sum_{i=1}^{m}\tilde{d}_{ij}^{*}\\
&+2\sum_{i=1}^{m}\bar{d}_{ij}^{*}\int_{-\infty}^{t}\bar{k}_{ij}(t-s)\,ds\Bigg]\\
&+\frac{1}{1-\mu_\theta}\bigl(L_j^f\bigr)^2\sum_{i=1}^{m}w_i\tilde{b}_{ij}^{*}+\bigl(L_j^f\bigr)^2\sum_{i=1}^{m}w_i\bar{b}_{ij}^{*}\int_{0}^{+\infty}k_{ji}(s)\,ds+L_j^f\sum_{i=1}^{m}w_ib_{ij}^{*}\Bigg\}\\
&\times\bigl|v_j(t,x)\bigr|^2\,dx\\
&+2\int_\Omega\Bigg[\sum_{i=1}^{m}w_iu_i(t,x)\sigma_i(t,x)+\sum_{j=1}^{n}w_{m+j}v_j(t,x)\vartheta_j(t,x)\Bigg]\,dx.
\end{aligned}
\tag{26}
$$

Thus, we get

$$
\begin{aligned}
&D^{+}V(t)-2\int_\Omega\Bigg[\sum_{i=1}^{m}y_i(t,x)\sigma_i(t,x)+\sum_{j=1}^{n}z_j(t,x)\vartheta_j(t,x)\Bigg]\,dx\\
&\quad-\gamma\int_\Omega\Bigg[\sum_{i=1}^{m}\sigma_i(t,x)^2+\sum_{j=1}^{n}\vartheta_j(t,x)^2\Bigg]\,dx\\
&\le\int_\Omega\sum_{i=1}^{m}\Bigg\{w_i\Bigg[-2\widehat{D}_i\lambda_1-2\widehat{p}_i+\sum_{j=1}^{n}b_{ji}^{*}L_j^f+\sum_{j=1}^{n}\tilde{b}_{ji}^{*}+\sum_{j=1}^{n}\bar{b}_{ji}^{*}\int_{-\infty}^{t}k_{ji}(t-s)\,ds\Bigg]\\
&\quad+L_i^g\sum_{j=1}^{n}w_{m+j}d_{ij}^{*}+\frac{1}{1-\mu_\tau}\bigl(L_i^g\bigr)^2\sum_{j=1}^{n}w_{m+j}\tilde{d}_{ij}^{*}\\
&\quad+\bigl(L_i^g\bigr)^2\sum_{j=1}^{n}w_{m+j}\bar{d}_{ij}^{*}\int_{0}^{+\infty}\bar{k}_{ij}(s)\,ds\Bigg\}\bigl|u_i(t,x)\bigr|^2\,dx
\end{aligned}
$$

$$
\begin{aligned}
&+\int_\Omega \sum_{j=1}^{n}\Bigg\{ w_{m+j}\Bigg[-2\widehat{D}_j^*\lambda_1 - 2\widehat{q}_j + \sum_{i=1}^{m} d_{ij}^* L_i^g + \sum_{i=1}^{m}\tilde{d}_{ij}^* + 2\sum_{i=1}^{m}\bar{d}_{ij}^*\int_{-\infty}^{t}\bar{k}_{ij}(t-s)\,ds\Bigg] \\
&+\frac{1}{1-\mu_\theta}\big(L_j^f\big)^2\sum_{i=1}^{m} w_i\tilde{b}_{ji}^* + \big(L_j^f\big)^2\sum_{i=1}^{m} w_i\bar{b}_{ji}^*\int_0^{+\infty} k_{ji}(s)\,ds + L_j^f\sum_{i=1}^{m} w_i b_{ji}^*\Bigg\}\big|v_j(t,x)\big|^2\,dx \\
&+2\int_\Omega\Bigg[\sum_{i=1}^{m} w_i u_i(t,x)\sigma_i(t,x) + \sum_{j=1}^{n} w_{m+j}v_j(t,x)\vartheta_j(t,x)\Bigg]dx \\
&-2\int_\Omega\Bigg[\sum_{i=1}^{m} y_i(t,x)\sigma_i(t,x) + \sum_{j=1}^{n} z_j(t,x)\vartheta_j(t,x)\Bigg]dx \\
&-\gamma\int_\Omega\Bigg[\sum_{i=1}^{m}\sigma_i(t,x)^2 + \sum_{j=1}^{n}\vartheta_j(t,x)^2\Bigg]dx \\
=&\int_\Omega\sum_{i=1}^{m}\Bigg\{\Bigg[w_i\Bigg(-2\widehat{D}_i\lambda_1 - 2\widehat{p}_i + \sum_{j=1}^{n} b_{ji}^* L_j^f + \sum_{j=1}^{n}\tilde{b}_{ji}^* + \sum_{j=1}^{n}\bar{b}_{ji}^*\Bigg) \\
&+L_i^g\sum_{j=1}^{n} w_{m+j}d_{ij}^* + \frac{1}{1-\mu_\tau}\big(L_i^g\big)^2\sum_{j=1}^{n} w_{m+j}\tilde{d}_{ij}^* + \big(L_i^g\big)^2\sum_{j=1}^{n} w_{m+j}\bar{d}_{ij}^*\Bigg]\big|u_i(t,x)\big|^2 \\
&+\big[2(w_i-a_i)u_i(t,x)\sigma_i(t,x) + (-\gamma-2c_i)\sigma_i(t,x)^2\big]\Bigg\}\,dx \\
&+\int_\Omega\sum_{j=1}^{n}\Bigg\{\Bigg[w_{m+j}\Bigg(-2\widehat{D}_j^*\lambda_1 - 2\widehat{q}_j + \sum_{i=1}^{m} d_{ij}^* L_i^g + \sum_{i=1}^{m}\tilde{d}_{ij}^* + 2\sum_{i=1}^{m}\bar{d}_{ij}^*\Bigg) \\
&+\frac{1}{1-\mu_\theta}\big(L_j^f\big)^2\sum_{i=1}^{m} w_i\tilde{b}_{ji}^* + \big(L_j^f\big)^2\sum_{i=1}^{m} w_i\bar{b}_{ji}^* + L_j^f\sum_{i=1}^{m} w_i b_{ji}^*\Bigg]\big|v_j(t,x)\big|^2 \\
&+\big[2(w_{m+j}-\bar{a}_i)v_j(t,x)\vartheta_j(t,x) + (-\gamma-2\bar{c}_i)\vartheta_j(t,x)^2\big]\Bigg\}\,dx \\
=&\int_\Omega\sum_{i=1}^{m}\begin{pmatrix}u_i(t,x)\\ \sigma_i(t,x)\end{pmatrix}^T\begin{pmatrix}\Xi_i^* & w_i-a_i\\ w_i-a_i & -\gamma-2c_i\end{pmatrix}\begin{pmatrix}u_i(t,x)\\ \sigma_i(t,x)\end{pmatrix}dx \\
&+\int_\Omega\sum_{j=1}^{n}\begin{pmatrix}v_j(t,x)\\ \vartheta_j(t,x)\end{pmatrix}^T\begin{pmatrix}\Theta_j^* & w_{m+j}-\bar{a}_j\\ w_{m+j}-\bar{a}_j & -\gamma-2\bar{c}_j\end{pmatrix}\begin{pmatrix}v_j(t,x)\\ \vartheta_j(t,x)\end{pmatrix}dx. \qquad (27)
\end{aligned}
$$

Since $-\gamma - 2c_i < -\gamma - 2\widehat{c_i} < 0$ and $-\gamma - 2\bar{c}_j < -\gamma - 2\widehat{\bar{c}_j} < 0$, it is easy to see that

$$
\begin{pmatrix}\Xi_i^* & w_i-a_i\\ w_i-a_i & -\gamma-2c_i\end{pmatrix} \le 0 \text{ is equivalent to } \Xi_i^* - (w_i-a_i)^2(-\gamma-2c_i)^{-1} \le 0. \qquad (28)
$$

Similarly, we see that

$$
\begin{pmatrix}\Theta_j^* & w_{m+j}-\bar{a}_j\\ w_{m+j}-\bar{a}_j & -\gamma-2\bar{c}_j\end{pmatrix} \le 0 \text{ is equivalent to } \Theta_j^* - (w_{m+j}-\bar{a}_j)^2(-\gamma-2\bar{c}_j)^{-1} \le 0. \qquad (29)
$$

In addition,

$$\Xi_i^* - (w_i - a_i)^2(-\gamma - 2c_i)^{-1} \leq \Xi_i^* - a_i^{*2}(-\gamma - 2\widehat{c}_i)^{-1}, \tag{30}$$

and

$$\Theta_j^* - (w_{m+j} - \bar{a}_j)^2(-\gamma - 2\bar{c}_j)^{-1} \leq \Theta_j^* - \bar{a}_j^{*2}(-\gamma - 2\widehat{\bar{c}}_j)^{-1}. \tag{31}$$

According to (27)-(31), we can derive

$$\begin{aligned} &D^+V(t) - 2\int_\Omega \Biggl[\sum_{i=1}^m y_i(t,x)\sigma_i(t,x) + \sum_{j=1}^n z_j(t,x)\vartheta_j(t,x)\Biggr] dx \\ &\quad - \gamma\int_\Omega \Biggl[\sum_{i=1}^m \sigma_i(t,x)^2 + \sum_{j=1}^n \vartheta_j(t,x)^2\Biggr] dx \leq 0. \end{aligned} \tag{32}$$

By integrating (32) with respect to t over the time period 0 to t_p, we have

$$\begin{aligned} &2\int_0^{t_p}\int_\Omega \Biggl[\sum_{i=1}^m y_i(t,x)\sigma_i(t,x) + \sum_{j=1}^n z_j(t,x)\vartheta_j(t,x)\Biggr] dx\,dt \\ &\quad \geq V(t_p) - V(0) - \gamma\int_0^{t_p}\int_\Omega \Biggl[\sum_{i=1}^m \sigma_i(t,x)^2 + \sum_{j=1}^n \vartheta_j(t,x)^2\Biggr] dx\,dt. \end{aligned} \tag{33}$$

Since $V(t_p) \geq 0$ and $V(0) \geq 0$, thus

$$\begin{aligned} &2\int_0^{t_p}\int_\Omega \Biggl[\sum_{i=1}^m y_i(t,x)\sigma_i(t,x) + \sum_{j=1}^n z_j(t,x)\vartheta_j(t,x)\Biggr] dx\,dt \\ &\quad \geq -\beta^2 - \gamma\int_0^{t_p}\int_\Omega \Biggl[\sum_{i=1}^m \sigma_i(t,x)^2 + \sum_{j=1}^n \vartheta_j(t,x)^2\Biggr] dx\,dt \end{aligned} \tag{34}$$

for all $t_p \geq 0$, where $\beta = \sqrt{V(0)}$. This completes the proof of Theorem 1. □

5 Illustration example

To illustrate the effectiveness of our criterion, we give the following example.

Example 1 Consider the following BAM RDNNs with mixed delays and Neumann boundary conditions:

$$\begin{cases} \frac{\partial u_i}{\partial t} = \frac{\partial}{\partial x_1}(D_{i1}\frac{\partial u_i}{\partial x_1}) - p_i u_i(t,x) + \sum_{j=1}^n b_{ji}f_j(v_j(t,x)) \\ \quad + \sum_{j=1}^n \tilde{b}_{ji}f_j(v_j(t-\theta_{ji}(t),x)) + \sum_{j=1}^n \bar{b}_{ji}\int_{-\infty}^t k_{ji}(t-s)f_j(v_j(s,x))\,ds + \sigma_i(t,x), \\ \frac{\partial v_j}{\partial t} = \frac{\partial}{\partial x_1}(D_{j1}^*\frac{\partial v_j}{\partial x_1}) - q_j v_j(t,x) + \sum_{i=1}^m d_{ij}g_i(u_i(t,x)) \\ \quad + \sum_{i=1}^m \tilde{d}_{ij}g_i(u_i(t-\tau_{ij}(t),x)) + \sum_{i=1}^m \bar{d}_{ij}\int_{-\infty}^t \bar{k}_{ij}(t-s)g_i(u_i(s,x))\,ds + \vartheta_j(t,x), \\ y_i(t,x) = a_i u_i(t,x) + c_i\sigma_i(t,x), \\ z_j(t,x) = \bar{a}_j v_j(t,x) + \bar{c}_j\vartheta_j(t,x), \end{cases} \tag{35}$$

where $\Omega = \{x_1 | 0 < x_1 < \sqrt{0.2}\pi\} \subset R$, $D_{i1} = D^*_{j1} = 1$, $n = m = r = 2$, $k_{ji}(t) = \bar{k}_{ij}(t) = te^{-t}$, $f_j(\eta) = g_i(\eta) = \frac{1}{2}(|\eta + 1| + |\eta - 1|)$, $L^f_j = L^{\bar{f}}_j = L^g_i = L^{\bar{g}}_i = 1$, $\lambda_1 = 3$, $\tau = \theta = \ln 2$, $a_i = \bar{a}_j = 1$, $c_i = \bar{c}_j = 0.5$, $p_i = 4$, $q_j = 3$, $i, j = 1, 2$, $\mu_\tau = \mu_\theta = 0.2$, $d_{11} = 0.5$, $d_{12} = 1$, $d_{21} = 0.5$, $d_{22} = 0.2$, $\bar{d}_{11} = 0.2$, $\bar{d}_{12} = 0.6$, $\bar{d}_{21} = 0.5$, $\tilde{d}_{11} = 0.3$, $\tilde{d}_{12} = 0.8$, $\tilde{d}_{21} = 0.1$, $\tilde{d}_{22} = 0.2$, $\bar{d}_{22} = 0.8$, $b_{11} = 0.5$, $b_{12} = 0.6$, $b_{21} = 1$, $b_{22} = -0.8$, $\bar{b}_{11} = -1$, $\bar{b}_{12} = 0.2$, $\bar{b}_{21} = 0.5$, $\bar{b}_{22} = 0.4$, $\tilde{b}_{11} = -0.5$, $\tilde{b}_{12} = 0.1$, $\tilde{b}_{21} = 0.3$, $\tilde{b}_{22} = 0.5$. By a simple calculation with $w_1 = w_2 = w_3 = w_4 = 1$ and $\gamma = 1$, we get

$$\begin{pmatrix} \Xi_1 & w_1 - a_1 \\ w_1 - a_1 & -\gamma - 2c_1 \end{pmatrix} = \begin{pmatrix} -6.525 & 0 \\ 0 & -2 \end{pmatrix} \leq 0, \tag{36}$$

$$\begin{pmatrix} \Xi_2 & w_2 - a_2 \\ w_2 - a_2 & -\gamma - 2c_2 \end{pmatrix} = \begin{pmatrix} -9.225 & 0 \\ 0 & -2 \end{pmatrix} \leq 0, \tag{37}$$

$$\begin{pmatrix} \Theta_1 & w_3 - \bar{a}_1 \\ w_3 - \bar{a}_1 & -\gamma - 2\bar{c}_1 \end{pmatrix} = \begin{pmatrix} -5.15 & 0 \\ 0 & -2 \end{pmatrix} \leq 0, \tag{38}$$

and

$$\begin{pmatrix} \Theta_2 & w_4 - \bar{a}_2 \\ w_4 - \bar{a}_2 & -\gamma - 2\bar{c}_2 \end{pmatrix} = \begin{pmatrix} -3.3 & 0 \\ 0 & -2 \end{pmatrix} \leq 0. \tag{39}$$

That is, (7) and (8) hold. Therefore, it follows from Theorem 1 that system (35) is passive.

6 Conclusions

In this paper, we have investigated the passivity analysis problem for a class of spatially and temporally BAM NNs with mixed time delays. The model suggested in this paper is comprehensive, since it simultaneously incorporates reaction-diffusion terms and mixed time delays. We have not only developed several novel sufficient conditions to ensure the passivity of BAM NNs with reaction-diffusion terms and mixed time delays but also investigated the robust passivity of the corresponding system with unknown parameters. In particular, many techniques and approaches, such as the Lyapunov functional and Poincaré integral inequality, have been successfully applied in this paper. Moreover, our obtained passivity conditions depend on time delays and reaction-diffusion terms, and thus are less conservative than delay-independent and reaction-diffusion-independent criteria. Therefore, the results obtained in this paper are less conservative, and they generalize and improve many earlier results. Finally, a numerical example has been presented to show the effectiveness of the derived results.

Competing interests
The author declares to have no competing interests.

Acknowledgements
This work is partially supported by the Natural Science Foundation of Shannxi Province under Grant No. 2015JM1015, Doctor Introduced Project of Xianyang Normal University under Grant No. 12XSYK008 and University Innovation and Entrepreneurship Training Program Project of Xianyang Normal University under Grant No. 048.

Received: 5 February 2015 Accepted: 8 September 2015 Published online: 25 September 2015

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