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Effect of rotation, magnetic field and a periodic loading on radial vibrations thermo-viscoelastic non-homogeneous media

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Abstract

In this paper, an analytical solution for the effect of the rotation in a magneto-thermo-viscoelastic non-homogeneous medium with a spherical cavity subjected to periodic loading is presented. The distribution of displacements, temperature, and stresses in the non-homogeneous medium in the context of generalized thermo-elasticity using GL (Green-Lindsay) theory is discussed and obtained in analytical form. The results are displayed graphically to illustrate the effect of rotation, relaxation, magnetic field, viscoelasticity, and non-homogeneity. Comparisons are made with the previous work in the absence of rotation and initial stress.

Keywords: rotation; viscoelasticity; periodic loading; non-homogeneity; magneto-thermo-elasticity

1 Introduction

In recent years, the theory of magneto-thermo-viscoelasticity, which deals the interactions among strain, temperature, and electromagnetic fields has drawn the attention of many researchers because of its extensive use in diverse fields, such as geophysics for understanding the effect of the Earth's magnetic field on seismic waves, damping of acoustic waves in a magnetic field, emission of electromagnetic radiations from nuclear devices, development of a highly sensitive superconducting magnetometer, electrical power engineering, optics, *etc.*; see [1–4]. Mahmoud *et al.* [5, 6] investigated the effect of the rotation on plane vibrations in a transversely isotropic infinite hollow cylinder, the effect of the rotation on wave motion through a cylindrical bore in a micropolar porous cubic crystal and he investigated the effect of a magnetic field and non-homogeneity on the radial vibrations in a hollow rotating elastic cylinder. Abd-Alla *et al.* [7–10] investigated the effect of the rotation on a non-homogeneous infinite cylinder of orthotropic material, influence of rotation, radial vibrations in a non-homogeneous orthotropic elastic hollow sphere subjected to rotation, and they investigated the magneto-thermo-elastic problem in a rotating non-homogeneous orthotropic hollow cylinder in the hyperbolic heat conduction model. Mahmoud [11, 12] investigated the analytical solution for an electrostatic potential on wave propagation modeling in human long wet bones, and they studied the influence of rotation and generalized magneto-thermo-elastics on Rayleigh waves in a granular medium under

the effect of initial stress and a gravity field. Abd-Alla and Mahmoud [13] investigated the analytical solution of wave propagation in non-homogeneous orthotropic rotating elastic media. Abd-Alla *et al.* [14–16] investigated some problems like the propagation of an S-wave in a non-homogeneous anisotropic incompressible and initially stressed medium under the influence of a gravity field, the generalized magneto-thermo-elastic Rayleigh waves in a granular medium under the influence of a gravity field and initial stress, and they also investigated the problem of transient coupled thermo-elasticity of an annular fin. Some problems of thermo-elasticity and wave propagation modeling in a cylinder are investigated by Abd-Alla *et al.* [17, 18], respectively. Mukhopadhyay [19] investigated the effects of thermal relaxations on thermo-viscoelastic interactions in an unbounded body with a spherical cavity subjected to a periodic loading on the boundary. The effects of thermal relaxations on thermo-elastic interactions in an unbounded body with a spherical cavity or cylindrical hole subjected to a periodic loading on the boundary, respectively, were investigated by Roychoudhuri and Mukhopadhyay [20]. The thermally induced vibrations in a generalized thermo-elastic solid with a cavity have been investigated by Erbay *et al.* [21] and Li and Qi [22]. Mahmoud [23] investigated the analytical solution for free vibrations of an elasto-dynamic orthotropic hollow sphere under the influence of rotation.

In this paper, rotation and the magneto-thermo-elastic equation of a spherical cavity are decomposed into a non-homogeneous equation with boundary conditions. The effect of thermal relaxation times on the wave propagation in the magneto-thermo-viscoelastic case using the GL theory will be discussed. We take the material of the spherical cavity to be of Kelvin-Voigt type. Thus, the exact expressions for the transient response of displacement, stresses, and temperature in a spherical cavity are obtained. The numerical calculations will be investigated for the displacement, temperature, and the components of stresses, and we explain the special case from this study when the magnetic field and non-homogeneity are neglected. Finally, numerical results are calculated and discussed.

2 Formulation of the problem

We shall consider the spherical coordinates of any representing point to be (r, θ, ϕ) and assume that the spherical cavity is subjected to a rapid change in temperature $T(r, t)$ and magnetic field $\overline{H}(0, 0, H_0)$, for the axisymmetric plane strain problem, and the components of the displacement $\overline{u} = \overline{u}(u_r, u_\theta, u_\phi)$ are expressed as $u_\theta = u_\phi = 0$, and $u_r = u_r(r, t)$. Let us consider an infinite isotropic non-homogeneous viscoelastic solid, and the viscoelastic nature of the material is described by the Voigt type of linear viscoelasticity. The medium is assumed to have a spherical cavity of radius a. For a spherical symmetric system, the non-vanishing stress components are expressed as

$$\tau_{rr} = \tau_m(\lambda + 2\mu)\frac{\partial u_r}{\partial r} + 2\lambda\tau_m\frac{u_r}{r} - \gamma(T + \tau_2\dot{T}),$$

$$\tau_{\theta\theta} = 2\tau_m(\lambda + \mu)\frac{u_r}{r} + \lambda\tau_m\frac{\partial u_r}{\partial r} - \gamma(T + \tau_2\dot{T}),$$

$$\tau_{\varphi\varphi} = 2\tau_m(\lambda + \mu)\frac{u_r}{r} + \lambda\tau_m\frac{\partial u_r}{\partial r} - \gamma(T + \tau_2\dot{T}),$$

$$\tau_{r\varphi} = \tau_{r\theta} = \tau_{\theta\varphi} = 0,$$

(1)

where τ_{rr}, $\tau_{\theta\theta}$, and $\tau_{\varphi\varphi}$ are the normal mechanical stresses, $\tau_{r\varphi}$, $\tau_{r\theta}$, $\tau_{\theta\varphi}$ are shear mechanical stresses. $\tau_m = 1 + \tau_0\frac{\partial}{\partial t}$ and τ_0 are the mechanical relaxation times due to the viscosity.

The magneto-elasto-dynamic equation of the non-homogeneity in the spherical case if $u_r = u_r(r, t)$, becomes

$$\frac{\partial \tau_{rr}}{\partial r} + \frac{2}{r}\tau_{rr} - \frac{1}{r}\tau_{\theta\theta} - \frac{1}{r}\tau_{\varphi\varphi} + \mu_e(\overline{J} \times \overline{H}) - \rho(\overleftarrow{\Omega} \times \overleftarrow{\Omega} \times \overleftarrow{u})_r - \rho(2\overleftarrow{\Omega} \times \overleftarrow{u})_r$$

$$= \rho \frac{\partial^2 u_r}{\partial t^2}, \tag{2}$$

where $\overleftarrow{\Omega} = (\Omega, 0, 0)$, $\overleftarrow{\Omega} \times \overleftarrow{\Omega} \times \overleftarrow{u}$ is the centripetal acceleration due to the time varying motion only and $2\overleftarrow{\Omega} \times \overleftarrow{u}$ is the Coriolis acceleration, we have the Lorentz force [18], which may be written as

$$f_r = \mu_e(\overline{J} \times \overline{H}) = \mu_e H_0^2 \frac{\partial}{\partial r}\left(\frac{\partial u_r}{\partial r} + \frac{2u_r}{r}\right),$$

$$\overline{h} = \mathrm{curl}(\overline{u} \times \overline{H}) = \left(0, 0, -H_0\left(\frac{\partial u_r}{\partial r} + \frac{2u_r}{r}\right)\right), \qquad \overline{J} = \mathrm{curl}\,\overline{h} = \left(0, \frac{\partial h_\phi}{\partial r}, 0\right), \tag{3}$$

where $\overline{h}$ is the perturbed magnetic field over the primary magnetic field, $\overline{E}$ is the electric intensity, $\overline{J}$ is the electric current density, μ_e is the magnetic permeability, $\overline{H}$ is the constant primary magnetic field, and $\overline{u}$ is the displacement vector.

The magneto-thermo-elasto-dynamic equations of the non-homogeneity sphere may be written as

$$\frac{\partial \tau_{rr}}{\partial r} + \frac{2}{r}\tau_{rr} - \frac{1}{r}\tau_{\theta\theta} - \frac{1}{r}\tau_{\varphi\varphi} + \mu_e H_0^2 \frac{\partial}{\partial r}\left(\frac{\partial u_r}{\partial r} + \frac{2u_r}{r}\right) + \rho\Omega^2 u_r = \rho\frac{\partial^2 u_r}{\partial t^2}, \tag{4a}$$

$$L\left(\frac{\partial^2 \theta}{\partial r^2} + \frac{2}{r}\frac{\partial \theta}{\partial r}\right) = \rho c_v \left(\frac{\partial \theta}{\partial t} + \tau_1 \frac{\partial^2 \theta}{\partial t^2}\right) + \gamma T_0 \left[\frac{\partial}{\partial r} + \frac{2}{r}\right]\dot{u}_r, \tag{4b}$$

where L is the thermal conductivity, $\gamma = \alpha_t(3\lambda + 2\mu)$, Ω is the rotation, ρ is the density of the material, c_v is the specific heat of the material per unit mass, τ_1, τ_2 are thermal relaxation parameters, α_t is the coefficient of linear thermal expansion, λ, μ are Lame elastic constants, T_1 is the absolute temperature, T_0 is a reference temperature of solid, θ is temperature difference $(T_1 - T_0)$, and τ_0 is the mechanical relaxation time due to the viscosity. In the above formula, the non-homogeneity of the material is characterized by a special law as follows:

$$\lambda = \lambda_0 r^{2n}, \qquad \mu = \mu_0 r^{2n}, \qquad \rho = \rho_0 r^{2n}, \qquad \mu_e = \mu_e^0 r^{2n}, \qquad \gamma = \gamma_0 r^{2n}, \tag{5}$$

where λ_0, μ_0, ρ_0 and μ_e^0 are the Lame's constant, shear modulus, mass density, pressure, and magnetic permeability coefficient of the homogeneous material, respectively, and n expresses a non-homogeneous exponent of the material, which is an arbitrary real number. Substituting (5) into (1) yields

$$\tau_{rr} = r^{2n}\left[\tau_m(\lambda_0 + 2\mu_0)\frac{\partial u_r}{\partial r} + 2(\lambda_0)\tau_m \frac{u_r}{r} - \gamma_0(\theta + \tau_2\dot{\theta})\right],$$

$$\tau_{\theta\theta} = r^{2n}\left[2\tau_m(\lambda_0 + \mu_0)\frac{u_r}{r} + (\lambda_0)\tau_m \frac{\partial u_r}{\partial r} - \gamma_0(\theta + \tau_2\dot{\theta})\right],$$

$$\tau_{\varphi\varphi} = r^{2n}\left[2\tau_m(\lambda_0 + \mu_0)\frac{u_r}{r} + \lambda_0\tau_m\frac{\partial u_r}{\partial r} - \gamma_0(\theta + \tau_2\dot{\theta})\right], \tag{6}$$

$$\tau_{r\varphi} = \tau_{r\theta} = \tau_{\theta\varphi} = 0,$$

$$\sigma_{rr}^* = \mu_e^0 H_0^2 r^{2n}\left(\frac{\partial u_r}{\partial r} + \frac{2u_r}{r}\right),$$

where σ_{rr}^* is the Maxwell stress tensor. From plugging (5) and (6) into (2) we have

$$\left[\tau_m + \frac{\mu_e^0 H_\phi^2}{(\lambda_0 + 2\mu_0)}\right]\frac{\partial^2 u_r}{\partial r^2} + \left[2(n+1)\tau_m + \frac{\mu_e^0 H_\phi^2}{(\lambda_0 + 2\mu_0)}\right]\frac{1}{r}\frac{\partial u_r}{\partial r}$$
$$+ \left[\frac{4n\lambda_0\tau_m}{(\lambda_0 + 2\mu_0)} - 2\tau_m - \frac{\mu_e^0 H_\phi^2}{(\lambda_0 + 2\mu_0)}\right]\frac{u_r}{r^2} - \left[\frac{2n}{r} + \frac{\partial}{\partial r}\right]\frac{\gamma_0}{(\lambda_0 + 2\mu_0)}(\theta + \tau_2\dot{\theta})$$
$$+ \rho_0 r^{2n}\Omega^2 u = \frac{\rho_0}{(\lambda_0 + 2\mu_0)}\frac{\partial^2 u}{\partial t^2}. \tag{7}$$

Let $c_0 = \frac{\lambda_0}{(\lambda_0 + 2\mu_0)}$, $c_2 = \frac{\gamma_0}{(\lambda_0 + 2\mu_0)}$, $c_3 = \frac{\mu_e^0 H_\phi^2}{(\lambda_0 + 2\mu_0)}$, $c_v = \sqrt{\frac{(\lambda_0 + 2\mu_0)}{\rho_0}}$.
Then the elasto-dynamic equation (7) becomes

$$(\tau_m + c_3)\frac{\partial^2 u_r}{\partial r^2} + \left[2(n+1)\tau_m + c_3\right]\frac{1}{r}\frac{\partial u_r}{\partial r} + [4nc_0\tau_m - 2\tau_m - c_3]\frac{u_r}{r^2}$$
$$- c_2\left[\frac{2n}{r} + \frac{\partial}{\partial r}\right](\theta + \tau_2\dot{\theta}) + \rho_0\Omega^2 u_r = \frac{1}{c_v^2}\frac{\partial^2 u_r}{\partial t^2}. \tag{8}$$

In addition, the heat conduction equation is

$$L\left(\frac{\partial^2\theta}{\partial r^2} + \frac{2}{r}\frac{\partial\theta}{\partial r}\right) = \rho_0 r^{2n} c_v\left(\frac{\partial\theta}{\partial t} + \tau_1\frac{\partial^2\theta}{\partial t^2}\right) + \gamma_0 r^{2n} T_0\left[\frac{\partial}{\partial r} + \frac{2}{r}\right]\dot{u}_r. \tag{9}$$

We now use the following dimensionless quantities:

$$U = \frac{u_r}{a}, \qquad l = \frac{L}{\rho_0 c_v}, \qquad t' = \frac{kc_v}{a}t, \qquad T = \frac{\theta}{T_0}, \qquad \tau_0' = \frac{c_v}{a}\tau_0, \qquad \tau_1' = \frac{c_v}{a}\tau_1,$$

$$\tau_2' = \frac{kc_v}{a}\tau_2, \qquad r' = \frac{r}{a}, \qquad \Omega^* = \frac{\Omega}{a}, \tag{10}$$

$$\sigma_{rr} = \frac{\tau_{rr}}{(\lambda_0 + 2\mu_0)}, \qquad \sigma_{\theta\theta} = \frac{\tau_{\theta\theta}}{(\lambda_0 + 2\mu_0)}.$$

In the following discussion the primes are neglected for r'.
The normal stresses relations can be set right in non-dimensional form as:

$$\sigma_{rr} = (ar)^{2n}\left[\left(1 + \tau_0'\frac{\partial}{\partial t'}\right)\frac{\partial U}{\partial r} + 2c_1\left(1 + \tau_0'\frac{\partial}{\partial t'}\right)\frac{U}{r} - T_0 c_2\left(1 + \tau_2'\frac{\partial}{\partial t'}\right)T\right],$$

$$\sigma_{\theta\theta} = (ar)^{2n}\left[2\left(1 + \tau_0'\frac{\partial}{\partial t'}\right)\frac{U}{r} + c_4\left(1 + \tau_0'\frac{\partial}{\partial t'}\right)\frac{\partial U}{\partial r} - T_0 c_5\left(1 + \tau_2'\frac{\partial}{\partial t'}\right)T\right], \tag{11}$$

$$\sigma_{rr}^* = \mu_e^0 H_\phi^2(ar)^{2n}\left(\frac{\partial U}{\partial r} + \frac{2U}{r}\right), \qquad f_r = \frac{\mu_e^0 H_\phi^2(ar)^{2n}}{a}\left(\frac{\partial^2 U}{\partial r^2} + \frac{2}{r}\frac{\partial U}{\partial r} - \frac{2U}{r^2}\right),$$

where $c_4 = \frac{\lambda_0 + P_0^*}{(\lambda_0 + 2\mu_0)}$, $c_5 = \frac{\gamma_0}{(\lambda_0 + 2\mu_0)}$.

Substituting of (11) into (8) and (9) gives the displacement equation in the non-dimensional form of the non-homogeneous spherical as follows:

$$\left[\left(1 + \tau_0' \frac{\partial}{\partial t'}\right) + c_3\right]\frac{\partial^2 U}{\partial r^2} + \left[2(n+1)\left(1 + \tau_0' \frac{\partial}{\partial t'}\right) + c_3\right]\frac{1}{r}\frac{\partial U}{\partial r}$$

$$+ \left[(4nc_1 - 2)\left(1 + \tau_0' \frac{\partial}{\partial t'}\right) - c_3\right]\frac{U}{r^2}$$

$$- c_2 T_0 \left[1 + \tau_2' \frac{\partial}{\partial t'}\right]\left(\frac{2l}{r} + \frac{1}{c_L^2}\frac{\partial}{\partial r}\right)T + \rho_0 a c_1^2 \Omega^{*2} U = l^2 \frac{\partial^2 U}{\partial t'^2}. \tag{12}$$

The heat conduction equation in non-dimensional form is

$$\left(\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r}\right) = l_1\left(1 + \tau_1' \frac{\partial}{\partial t'}\right)\frac{\partial T}{\partial t'} + l_2\left[\frac{\partial}{\partial r} + \frac{2}{r}\right]\frac{\partial U}{\partial t'}, \tag{13}$$

where $l_1 = \frac{a c_v}{l}$, $l_2 = \frac{a \gamma_0}{\rho_0}$.

3 The problem solution

We seek the general solution to the basic equation (12) of magneto-thermo-elastic motion as a harmonic vibration in the form

$$U(r, t') = U^*(r)e^{i\omega t'}, \tag{14}$$

$$T(r, t') = T^*(r)e^{i\omega t'}. \tag{15}$$

From (14) the equation of motion becomes

$$\left[(1 + i\omega\tau_0') + c_3\right]\frac{d^2 U^*}{dr^2} + \left[2(n+1)(1 + i\omega\tau_0') + c_3\right]\frac{1}{r}\frac{dU^*}{dr}$$

$$+ \left[(4nc_1 - 2)(1 + i\omega\tau_0') - c_3\right]\frac{U^*}{r^2} + \rho_0 a c_1^2 \Omega^{*2} U^*$$

$$= -k^2\omega^2 U^* + c_2 T_0 \gamma'\left(\frac{2n}{r} + \frac{d}{dr}\right)T^*, \tag{16}$$

where $\gamma' = (1 + i\tau_2'\omega)$, or in the form

$$\frac{d^2 U^*}{dr^2} + \eta_1 \frac{1}{r}\frac{dU^*}{dr} + \eta_2 \frac{U^*}{r^2} + \rho_0 a c_1^2 \Omega^{*2} U^* = -m_1^2 U^* + \varepsilon\left(\frac{2n}{r} + \frac{d}{dr}\right)T^*,$$

where

$$\eta_1 = \frac{(2n+1)(1 + i\omega\tau_0')}{(1 + i\omega\tau_0') + c_3} + 1, \qquad \eta_2 = \frac{(4nc_1 - 1)(1 + i\omega\tau_0')}{(1 + i\omega\tau_0') + c_3} - 1,$$

$$\varepsilon = \frac{c_2 T_0 \gamma'}{(1 + i\omega\tau_0') + c_3}, \qquad m_1^2 = \frac{k^2\omega^2}{(1 + i\omega\tau_0') + c_3}, \qquad c_1 = \sqrt{\frac{\lambda + 2\mu}{\rho_0}}.$$

Also the heat conduction equation becomes

$$\left(\nabla^2 + \beta_1\right)T^* = \beta_2\left[\frac{d}{dr} + \frac{2}{r}\right]U^*, \tag{17}$$

where $\nabla^2 = \frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr}$, $\beta_1 = l_1(\omega^2\tau_1' - i\omega)$, $\beta_2 = i\omega l_2$.

To solve (16) and (17), we let

$$U^*(r) = \frac{d\xi(r)}{dr}, \tag{18}$$

$$\frac{d}{dr}\left[\frac{d^2\xi(r)}{dr^2} + \frac{\eta_1}{r}\frac{d\xi(r)}{dr} + \frac{\eta_2}{r^2}\xi(r)\right] + f_1\Omega^{*2}\frac{d\xi(r)}{dr} = -m_1^2\frac{d\xi(r)}{dr} + \varepsilon\left(\frac{2n}{r} + \frac{d}{dr}\right)T^*. \tag{19}$$

By comparing the coefficients of $\frac{d}{dr}$ in (19), we get

$$\frac{d^2\xi(r)}{dr^2} + \frac{\eta_1}{r}\frac{d\xi(r)}{dr} + \left[\frac{\eta_2}{r^2} + m_1^2\right]\xi(r) = \varepsilon T^*. \tag{20}$$

The heat conduction equation becomes

$$\left(\nabla^2 + \beta_1\right)T^* = \beta_2\nabla^2\xi(r). \tag{21}$$

From (20) and (21), we have

$$\frac{d^4\xi(r)}{dr^4} + [\eta_1 + 1]\frac{1}{r}\frac{d^3\xi(r)}{dr^3} + \left[\Gamma_1 + \frac{\eta_2 - \eta_1}{r^2}\right]\frac{d^2\xi(r)}{dr^2}$$
$$+ \left[\frac{\eta_2 + \eta_1}{r^3} + \frac{\Gamma_2}{r}\right]\frac{d\xi(r)}{dr} + \beta_1 N^2\xi + f_1\Omega^{*2}\frac{d\xi(r)}{dr} = 0, \tag{22}$$

where $\Gamma_1 = m_1^2 + \beta_1 - \varepsilon\beta_2$, $\Gamma_2 = m_1^2 + \beta_1\eta_1 - \varepsilon\beta_2$.

Decoupling (20) and (21), we obtain

$$\left(\nabla^2 + \chi_1^2\right)\left(\nabla^2 + \chi_2^2\right)\left(\xi, T^*\right) = 0, \tag{23}$$

where $\beta_4 = \frac{\beta_1}{l}$, χ_1^2 and χ_2^2 are the roots with positive real parts of the biquadratic equation:

$$\chi^4 + \left(m_1^2 + \beta_4^2 - \eta_1\eta_2\right)\chi^2 + m_1^2\beta_4^2 = 0. \tag{24}$$

Assuming the regularity conditions for ξ and T^* the solutions of (23) are obtained in terms of the spherical Hankel function, (23) representing an ordinary differential equation with variable coefficients of order four; from this equation we can determine the components of the displacement U and the temperature T, and finally determine the components of the stress. We have

$$\xi = K_1 h_0^{(2)}(\chi_1 r) + K_2 h_0^{(2)}(\chi_2 r), \tag{25a}$$

$$T^* = K_1 h_0^{(2)}(\chi_1 r) + K_2 h_0^{(2)}(\chi_2 r), \tag{25b}$$

where K_1 and K_2 are arbitrary constants and $h_0^{(2)}$ is the Hankel function of order zero and of the second kind. From (14), (18), (16), and (25a)-(25b), the solution for the displacement, temperature and the radial and hoop stresses are found to have the forms

$$U = \left\{A h_1^{(2)}(\chi_1 r) + B h_1^{(2)}(\chi_2 r)\right\}e^{i\omega t'}, \tag{26}$$

$$T = \left\{K_1 h_0^{(2)}(\chi_1 r) + K_2 h_0^{(2)}(\chi_2 r)\right\}e^{i\omega t'}, \tag{27}$$

$$\sigma_{rr} = \left\{ Q_1 h_0^{(2)}(\chi_1 r) + \frac{Q_2}{r} h_1^{(2)}(\chi_1 r) \right\} A e^{i\omega t'}$$

$$+ \left\{ Q_3 h_0^{(2)}(\chi_2 r) + \frac{Q_2}{r} h_1^{(2)}(\chi_2 r) \right\} B e^{i\omega t'}, \tag{28}$$

$$\sigma_{\theta\theta} = \left\{ Q_4 h_0^{(2)}(\chi_1 r) + \frac{Q_5}{r} h_1^{(2)}(\chi_1 r) \right\} A e^{i\omega t'}$$

$$+ \left\{ Q_6 h_0^{(2)}(\chi_2 r) + \frac{Q_5}{r} h_1^{(2)}(\chi_2 r) \right\} B e^{i\omega t'}. \tag{29}$$

4 Boundary conditions

Let us consider the corresponding boundary conditions,

$$u(1, t) = 0, \quad r = 1,$$
$$\sigma_{rr} + \sigma_{rr}^* = -\sigma_0 e^{i\omega t}, \quad r = 1, \tag{30}$$

where σ_0 is a constant; we get

$$z_i = \frac{\chi_i^2 - m_1^2}{t' \chi_i}, \qquad K_i = z_i A_i, \quad i = 1, 2,$$

$$A_1 = -\frac{\sigma_0' h_0^{(2)}(\chi_2)}{d_1}, \qquad A_2 = \frac{\sigma_0' z_1 h_0^{(2)}(\chi_1)}{d_1}, \qquad \sigma_0' = \frac{\sigma_0}{\gamma T_0},$$

$$d_1 = z_2 h_0^{(2)}(\chi_2) \left\{ Q_1 h_0^{(2)}(\chi_1) + Q_2 h_1^{(2)}(\chi_1) \right\}$$
$$\quad - z_1 h_0^{(2)}(\chi_1) \left\{ Q_3 h_0^{(2)}(\chi_2) + Q_2 h_1^{(2)}(\chi_2) \right\},$$

$$Q_1 = \left(1 + i\omega\tau_0' + \beta_4^2\right)\chi_1 - \left(1 + i\omega\tau_2'\right)z_1, \qquad Q_2 = \left(1 + i\omega\tau_0' + \beta_4^2\right)(2\lambda_e - 2),$$

$$Q_3 = \left(1 + i\omega\tau_0' + \beta_4^2\right)\chi_2 - \left(1 + i\omega\tau_2'\right)z_2, \qquad Q_4 = \left(1 + i\omega\tau_0' + \beta_4^2\right)\chi_1 - \left(1 + i\omega\tau_2'\right)z_1,$$

$$Q_5 = \left(1 + i\omega\tau_0' + \beta_4^2\right)(1 - \lambda_e), \qquad Q_6 = \lambda_e\left(1 + i\omega\tau_0' + \beta_4^2\right)\chi_2 - \left(1 + i\omega\tau_2'\right)z_2,$$

$$\beta_4^2 = \frac{\mu_e^0 H_\varphi^2}{\rho c^2}, \qquad m_1^2 = \frac{k^2 \omega^2}{a^2 c^2 (1 + i\omega\tau_0' + \beta_4^2)}, \qquad \lambda_e = \frac{\lambda}{\lambda + 2\mu + P}.$$

This is the solution of the current problem for the case of a non-homogeneous isotropic viscoelastic unbounded body with spherical cavity without the effect of a magnetic field; it coincides with one previously published.

5 Discussion and numerical results

The results presented in this paper should prove useful for researchers in material science, designers of new materials, low-temperature physicists as well as for those working on the development of magneto-thermo-viscoelastic theory. The copper material was used chosen for purposes of numerical evaluations. The constants of the problem are given [24]. The numerical technique outlined above was used to obtain the temperature, radial displacement, radial stress and hoop stress inside the sphere. These distributions are shown in Figures 1-4, respectively. Important phenomena are observed in all computations: It was found that for large values of time the coupled and the generalized give close results. The case is quite different when we consider a small value of time. The coupled theory

Figure 1 Variation of temperature *versus* the radius *r* at various values of rotation when $\tau_1 = 0.75$, $\tau_0 = 0.64$, $\omega = 2 \times 10^3$, $m = 0.5$, $H_0 = 1 \times 10^2$.

Figure 2 Variation of radial displacement *versus* the radius *r* at various values of rotation when $\tau_1 = 0.75$, $\tau_0 = 0.64$, $\omega = 2 \times 10^3$, $m = 0.5$, $H_0 = 1 \times 10^2$.
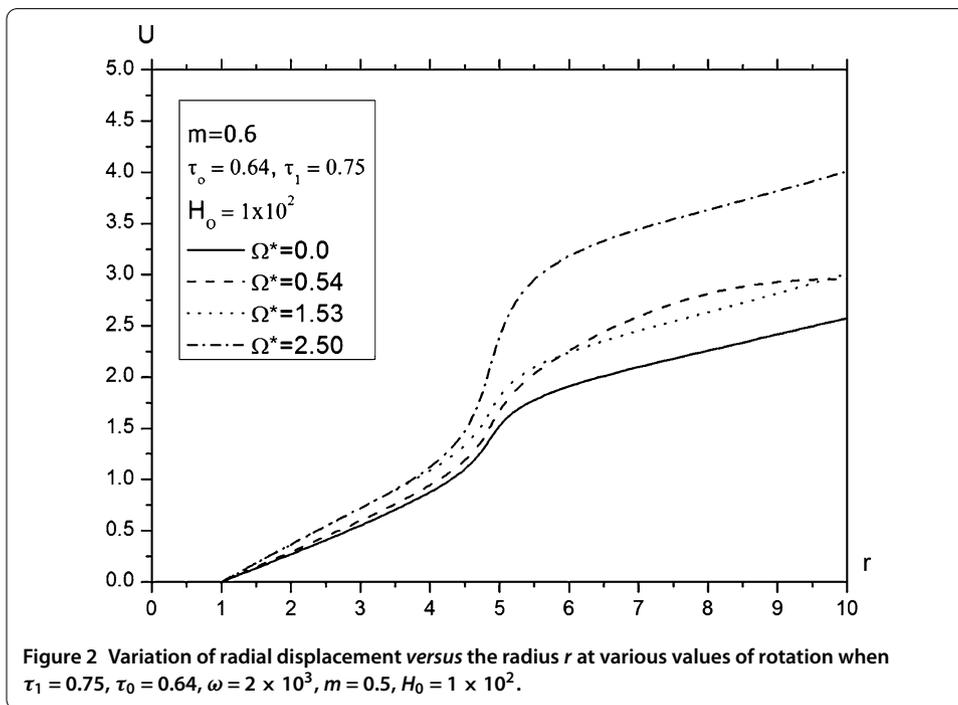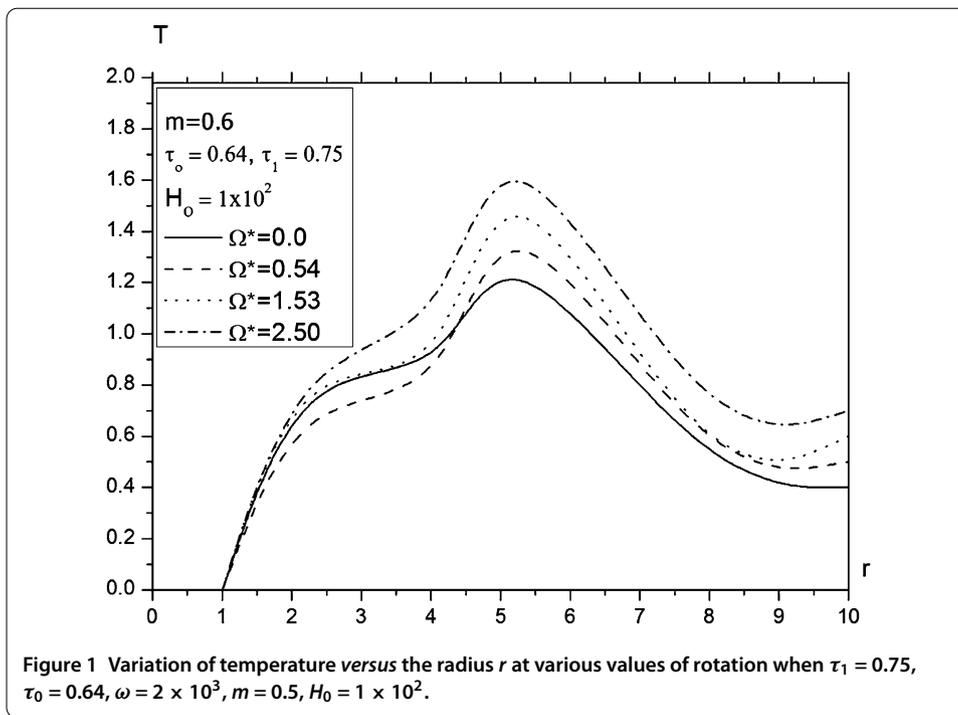
predicts infinite speeds of wave propagation. This is evident from the fact that the obtained solutions are not identically zero for any values of time but fade gradually to very small values at points for removed from the surface. The solutions obtained in the context of GL theory, however, exhibit the behavior of finite speeds of wave propagation. The computations were carried out for a thermal relaxation time of $\tau_1 = 0.75$, a magnetic field of $H_0 = 1 \times 10^2$, a mechanical relaxation time of $\tau_0 = 0.64$, and a frequency of $\omega = 2 \times 10^2$.

Figure 3 Variation of radial stress *versus* the radius *r* at various values of rotation when $\tau_1 = 0.75$, $\tau_0 = 0.64$, $\omega = 2 \times 10^3$, $m = 0.5$, $H_0 = 1 \times 10^2$.

Figure 4 Variation of hoop stress *versus* the radius *r* at various values of rotation when $\tau_1 = 0.75$, $\tau_0 = 0.64$, $\omega = 2 \times 10^3$, $m = 0.5$, $H_0 = 1 \times 10^2$.
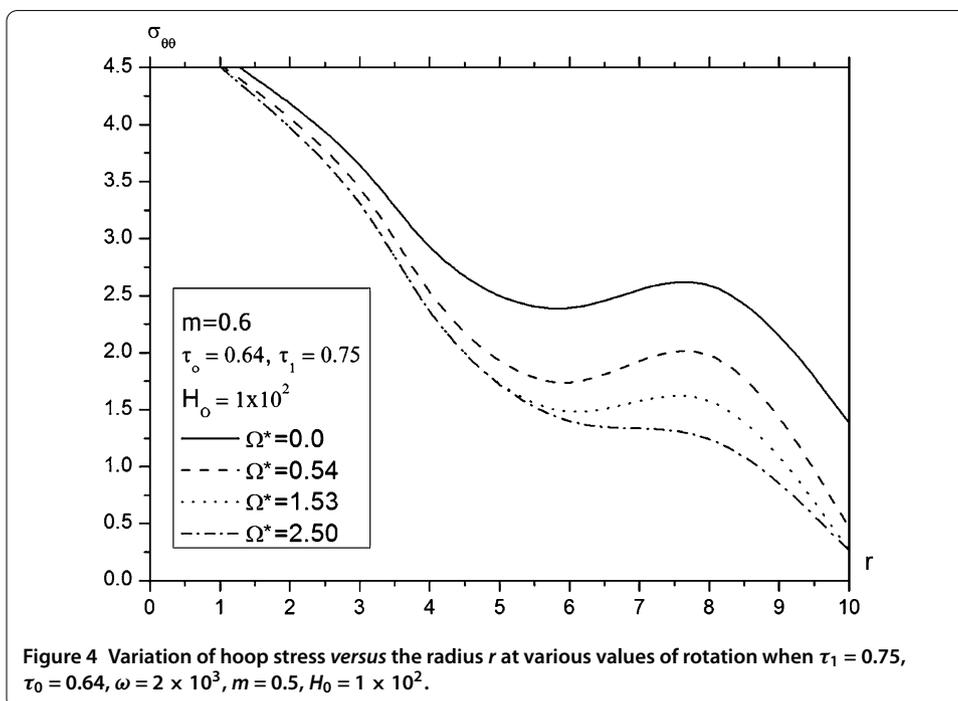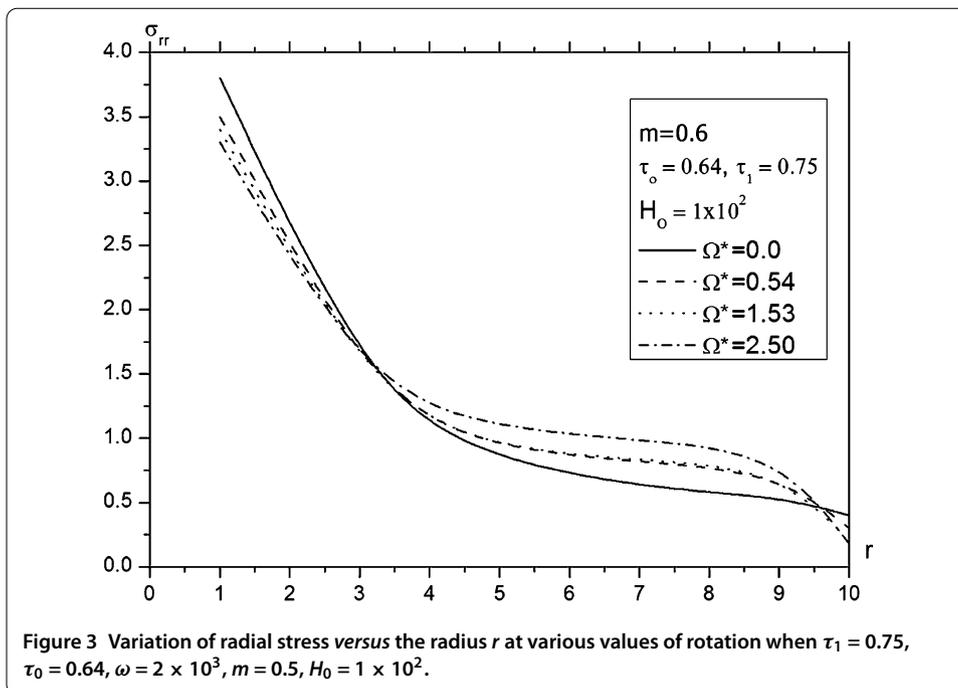
For the sake of brevity some computational results are not presented here. Figures 1-4 show the solution corresponding to the use of the non-homogeneous material ($m = 0.65$). Figure 1 shows the temperature distribution and the solution corresponding to the effect of rotation. Figure 2 shows the radial displacement in a generalized thermo-elastic non-homogeneous medium subjected to rotation, thermal relaxation, and a magnetic field. The figures indicate that the medium along the radius *r* undergoes expansion deformation because of these effects. The radial displacement increases with increasing rotation

and it increases with increasing radius r. Figure 3 shows the radial stress in a generalized thermo-elastic non-homogeneous medium subjected to rotation, thermal relaxation, and a magnetic field; in Figure 3 the radial stress decreases with increasing rotation when we have a small value of the radius r of less than 2. Also, Figure 3 represents the solution corresponding to the use of the effect of the magnetic field H_0. Figure 4 shows the hoop stress in a generalized thermo-elastic non-homogeneous medium subjected to rotation, thermal relaxation, and a magnetic field. From this figure, the hoop stress decreases with increasing radius r; Figure 4 represents the solution corresponding to the use of the effect of rotation; also Figure 4 represents the hoop stress corresponding to the use of the effect of the magnetic field H_0. It was found that near the surface cavity where the boundary conditions dominate the coupled and the generalized theories give very close results. Inside the sphere, the solution is markedly different. This is due to the fact that thermal waves in the coupled theory the traveled distance is not identically zero (though it may be very small) for any small of time. By comparing with results in [24] it was found that U has the same behavior in both media. But the values of u in a generalized thermo-elastic medium are larger in comparison with those in a thermo-elastic medium. The same remark can be made for σ_{rr} by comparing the figures. This is due to the influence of relaxation time, magnetic field, and frequency. Extensive literature on the topic is now available and we can only mention a few recent interesting investigations in [25–34].

6 Conclusions

The elasto-dynamic equations for the generalized thermo-viscoelastic theory under the effect of the non-homogeneous material, rotation, relaxation, and magnetic field have a complicated nature. The method used in this study provides a quite successful approach in dealing with such problems. The displacement, temperature, and stress components have been obtained in analytical form. This approach gives an exact solution in the Hankel transform domain that appears in the governing equations of the problem considered. Numerical results are calculated and discussed and illustrated graphically.

Authors' contributions
All authors, KSA, SRM and EOA contributed to each part of this work equally and read and approved the final version of the manuscript.

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