Journal of Inequalities and Applications
a SpringerOpen Journal

RESEARCH Open Access

Inequalities and pth moment exponential stability of impulsive delayed Hopfield neural networks

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Abstract

In this paper, the pth moment exponential stability for a class of impulsive delayed Hopfield neural networks is investigated. Some concise algebraic criteria are provided by a new method concerned with impulsive integral inequalities. Our discussion neither requires a complicated Lyapunov function nor the differentiability of the delay function. In addition, we also summarize a new result on the exponential stability of a class of impulsive integral inequalities. Finally, one example is given to illustrate the effectiveness of the obtained results.

Keywords: pth moment exponential stability; Integral inequality; Impulse; Hopfield neural networks; Delay

1 Introduction

In the past few years, the artificial neural networks introduced by Hopfield [1, 2] have become a significant research topic due to their wide applications in various areas such as signal and image processing, associative memory, combinatorial optimization, pattern classification, etc. [3–5]. All the applications of Hopfield neural networks (HNNs) depend on qualitative behavior such as stability, existence and uniqueness, convergence, oscillation, and so on [6–10]. Particularly, the stability property is a major concern in the design and applications of neural networks. Therefore, many researchers have been paying much attention to the stability study of HNNs.

In addition, since time delays are frequently encountered for the finite switching speed of neurons and amplify in implementation of neural networks, it is meaningful to discuss the effect of time delays on the stability of HNNs. Consequently, the scientists put forward the model of delayed Hopfield neural networks (DHNNs) and made great efforts for the stability research (see e.g. [11, 12]).

Furthermore, it is worth noting that impulsive effects are also a common phenomenon in many engineering systems, that is, instantaneous jump or reset of system states of automobile industry, network control, video coding, etc. Hence, the model of impulsive delayed Hopfield neural networks (IDHNNs) is more representative, and it is necessary to probe

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the stability of IDHNNs theoretically and practically. So far, there have been a number of research achievements (see e.g. [13–17])

Among the existing stability results of impulsive delayed systems, one powerful technique is Lyapunov method (see e.g. [18–24]). Wei et al. [18] studied the global exponential stability in the mean-square sense of a class of stochastic impulsive reaction-diffusion systems with S-type distributed delays based on a Lyapunov–Krasovskii functional and an impulsive inequality. Ren et al. [19] considered the mean-square exponential input-to-state stability for a class of delayed stochastic neural networks with impulsive effects driven by G-Brownian motion by constructing an appropriate G-Lyapunov–Krasovskii functional, mathematical induction approach, and some inequality techniques.

It should be pointed out that the key to the Lyapunov method is to construct a suitable Lyapunov function or functional. However, finding a suitable Lyapunov function or functional often involves some mathematical difficulties.

On the other hand, an alternative technique for stability analysis of impulsive delayed systems has been developed based on the fixed point theorem (see e.g.[25–29]). Zhang et al. [25] studied the application of the fixed point theory to the stability analysis of a class of impulsive delayed neural networks. By employing the contraction mapping principle, some novel and concise sufficient conditions have been presented to ensure the existence and uniqueness of solution and the global exponential stability of the considered system.

However, the fixed point method has its disadvantage due to using Holder inequalities at an inappropriate time.

Motivated by the above discussion, we attempt to study the stability of IDHNNs by a new method different from the Lyapunov method and the fixed point method. As we all know, there are many works focused on discussion to mean-square stability of complex dynamical systems. However, mean-square stability is actually a special case of pth moment stability by choosing $p = 2$, so the study of pth moment stability will be more representative. In our paper, we investigate the pth moment exponential stability of IDHNNs with the help of impulsive integral inequalities. Compared with the Lyapunov method and the fixed point theory, our method has two advantages. One is no demand of Lyapunov functions and the differentiability of the delay function. The other is no demand of seeking the appropriate time to use Holder inequalities. Furthermore, a new criterion for the exponential stability of impulsive integral inequalities is provided based on our discussion.

The contents of this paper are organized as follows. In Sect. 2, some notations, the model description, and a useful lemma are introduced. In Sect. 3, we consider the pth moment exponential stability of IDHNNs and obtain some new sufficient conditions. Inspired by Sect. 3, we discuss the exponential stability of a class of impulsive integral inequalities in Sect. 4 and give an algebraic criterion. In Sect. 5, one example is given to illustrate the effectiveness of our results.

2 Preliminaries

Notations: Let R^n denote the n-dimensional Euclidean space. $|\cdot|$ represents the Euclidean norm for vectors or absolute value for real numbers. $\mathcal{N} \stackrel{\Delta}{=} \{1, 2, \ldots, n\}$. $\mathrm{R}_+ = [0, \infty)$. $C[X, Y]$ stands for the space of continuous mappings from the topological space X to the topological space Y. For some $\tau > 0$, let $C[[-\tau, 0], \mathrm{R}]$ be the family of all continuous real-valued functions ϕ defined in $[-\tau, 0]$ equipped with the norm $\|\phi\| = \sup_{s\in[-\tau,0]} |\phi(s)|$.

Consider a class of impulsive delayed Hopfield neural network described by

$$\begin{aligned}
&\frac{dx_i(t)}{dt} = -a_i x_i(t) + \sum_{j=1}^{n} b_{ij} f_j\big(x_j(t)\big) + \sum_{j=1}^{n} c_{ij} g_j\big(x_j\big(t-\tau_j(t)\big)\big), \quad t \geq 0, t \neq t_k, \\
&\Delta x_i(t_k) = x_i(t_k+0) - x_i(t_k) = I_{ik}\big(x_i(t_k)\big), \quad k = 1,2,\ldots, \\
&x_i(s) = \varphi_i(s), \quad -\tau \leq s \leq 0, i \in \mathcal{N},
\end{aligned} \tag{2.1}$$

where $i \in \mathcal{N}$ and n is the number of neurons in the neural network. $x_i(t)$ stands for the state of the ith neuron at time t. $f_j(\bullet), g_j(\bullet) \in C[\mathrm{R},\mathrm{R}]$, $f_j(x_j(t))$ is the activation function of the jth neuron at time t and $g_j(x_j(t-\tau_j(t)))$ is the activation function of the jth neuron at time $t-\tau_j(t)$, where $\tau_j(t) \in C[\mathrm{R}^+,\mathrm{R}^+]$ denotes the transmission delay along the axon of the jth neuron and satisfies $0 \leq \tau_j(t) \leq \tau_j$ (τ_j is a constant). The constant $a_i > 0$ stands for the rate with which the ith neuron will reset its potential to the resting state when disconnected from the network and external inputs. The constant b_{ij} represents the connection weight of the jth neuron on the ith neuron at time t. The constant c_{ij} denotes the connection strength of the jth neuron on the ith neuron at time $t-\tau_j(t)$. The fixed impulsive moments t_k ($k = 1,2,\ldots$) satisfy $0 = t_0 < t_1 < t_2 < \cdots$ and $\lim_{k\to\infty} t_k = \infty$. $x_i(t_k+0)$ and $x_i(t_k-0)$ stand for the right-hand and left-hand limit of $x_i(t)$ at time t_k, respectively. $I_{ik}(x_i(t_k))$ shows the abrupt change of $x_i(t)$ at the impulsive moment t_k and $I_{ik}(\bullet) \in C[\mathrm{R},\mathrm{R}]$. $\varphi_i(s) \in C[[-\tau,0],\mathrm{R}]$ and $\tau = \max_{j\in\mathcal{N}}\{\tau_j\}$.

Denote by $\mathbf{x}(t;\varphi) = (x_1(t;\varphi_1),\ldots,x_n(t;\varphi_n))^{\mathrm{T}} \in \mathrm{R}^n$ the solution of system (2.1), where $\varphi(s) = (\varphi_1(s),\ldots,\varphi_n(s))^{\mathrm{T}} \in \mathrm{R}^n$. The solution $\mathbf{x}(t;\varphi)$ of system (2.1) is, for time variable t, a piecewise continuous vector-valued function with the first kind discontinuity at the points t_k ($k = 1,2,\ldots$), where it is left-continuous i.e. the following relations are valid:

$$x_i(t_k-0) = x_i(t_k), \qquad x_i(t_k+0) = x_i(t_k) + I_{ik}\big(x_i(t_k)\big), \quad i \in \mathcal{N}, k = 1,2,\ldots.$$

Throughout this paper, we always assume that $f_j(0) = g_j(0) = I_{jk}(0) = 0$ for $j \in \mathcal{N}$ and $k = 1,2,\ldots$. Then system (2.1) admits a trivial solution with initial value $\varphi = 0$.

Definition 2.1 The trivial solution of system (2.1) is said to be pth ($p \geq 1$) moment exponentially stable if there exists a pair of positive constants λ and C such that

$$\big|x_i(t;\varphi)\big|^p \leq C \max_{i\in\mathcal{N}}\big\{\|\varphi_i\|^p\big\} e^{-\lambda t}, \quad t \geq 0,$$

holds for any $\varphi_i(s) \in C[[-\tau,0],\mathrm{R}]$ and $i \in \mathcal{N}$.

Lemma 2.1 *Suppose* $0 < \theta < 1$ *and* $\lambda\theta(t-s) < 1-\theta$. *Then* $\int_s^t e^{\lambda x}\,dx > \theta(t-s)e^{\lambda t}$ *holds for* $t > s$ *and* $\lambda > 0$.

Proof Construct function $F(t) = \int_s^t e^{\lambda x}\,dx - \theta(t-s)e^{\lambda t}$. For fixed s, it is easy to find that $F(s) = 0$ and

$$F'(t) = e^{\lambda t} - \theta e^{\lambda t} - \lambda\theta(t-s)e^{\lambda t} = e^{\lambda t}\big[1-\theta-\lambda\theta(t-s)\big] > 0.$$

So, $F(t) > F(s) = 0$ as $t > s$, which means $\int_s^t e^{\lambda x}\,dx > \theta(t-s)e^{\lambda t} (t > s)$. □

3 *p*th moment exponential stability of IDHNNs

In this section, we develop a new method to discuss the pth moment exponential stability of system (2.1). Before proceeding, we introduce some hypotheses listed as follows:

(H1) There exist nonnegative constants α_j such that, for any $x_j^{(1)}, x_j^{(2)} \in \mathrm{R}$,

$$\left|f_j\left(x_j^{(1)}\right) - f_j\left(x_j^{(2)}\right)\right| \le \alpha_j \left|x_j^{(1)} - x_j^{(2)}\right|, \quad j \in \mathcal{N}.$$

(H2) There exist nonnegative constants β_j such that, for any $x_j^{(1)}, x_j^{(2)} \in \mathrm{R}$,

$$\left|g_j\left(x_j^{(1)}\right) - g_j\left(x_j^{(2)}\right)\right| \le \beta_j \left|x_j^{(1)} - x_j^{(2)}\right|, \quad j \in \mathcal{N}.$$

(H3) There exist nonnegative constants P_{jk} such that, for any $x_j^{(1)}, x_j^{(2)} \in \mathrm{R}$,

$$\left|I_{jk}\left(x_j^{(1)}\right) - I_{jk}\left(x_j^{(2)}\right)\right| \le P_{jk} \left|x_j^{(1)} - x_j^{(2)}\right|, \quad j \in \mathcal{N}, k = 1, 2, \ldots.$$

Theorem 3.1 *Suppose that*

(i) there exist constants $\mu > 0$ and $\theta \in (0,1)$ such that $\inf_{k=1,2,\ldots}\{\theta(t_k - t_{k-1})\} \ge \mu$ and $\max_{k=1,2,\ldots}\{t_k - t_{k-1}\} < \frac{1-\theta}{\theta a_i}$,

(ii) there exist constants P_i such that $P_{ik} \le P_i \mu$ for $i \in \mathcal{N}$ and $k = 1, 2, \ldots$,

(iii)

$$3^{p-1}\left\{a_i^{1-p}\left(\sum_{j=1}^n |b_{ij}\alpha_j|\right)^p + a_i^{1-p}\left(\sum_{j=1}^n |c_{ij}\beta_j|\right)^p + a_i^{1-p}P_i^p\right\} < a_i.$$

Then system (2.1) is globally exponentially stable in the pth ($p \ge 1$) moment.

Proof Multiplying both sides of system (2.1) with $\mathrm{e}^{a_i t}$ and integrating from $t_{k-1} + \varepsilon$ ($\varepsilon > 0$) to $t \in (t_{k-1}, t_k)$ yields

$$\begin{aligned} x_i(t)\mathrm{e}^{a_i t} &= x_i(t_{k-1} + \varepsilon)\mathrm{e}^{a_i(t_{k-1}+\varepsilon)} \\ &\quad + \int_{t_{k-1}+\varepsilon}^{t} \mathrm{e}^{a_i s}\left\{\sum_{j=1}^n b_{ij}f_j\left(x_j(s)\right) + \sum_{j=1}^n c_{ij}g_j\left(x_j\left(s - \tau_j(s)\right)\right)\right\} ds. \end{aligned} \tag{3.1}$$

Letting $\varepsilon \to 0^+$ in (3.1), we have, for $t \in (t_{k-1}, t_k)$ ($k = 1, 2, \ldots$),

$$\begin{aligned} x_i(t)\mathrm{e}^{a_i t} &= x_i(t_{k-1} + 0)\mathrm{e}^{a_i t_{k-1}} \\ &\quad + \int_{t_{k-1}}^{t} \mathrm{e}^{a_i s}\left\{\sum_{j=1}^n b_{ij}f_j\left(x_j(s)\right) + \sum_{j=1}^n c_{ij}g_j\left(x_j\left(s - \tau_j(s)\right)\right)\right\} ds. \end{aligned} \tag{3.2}$$

Setting $t = t_k - \varepsilon'$ ($\varepsilon' > 0$) in (3.2), we get

$$\begin{aligned} x_i\left(t_k - \varepsilon'\right)\mathrm{e}^{a_i(t_k-\varepsilon')} &= x_i(t_{k-1} + 0)\mathrm{e}^{a_i t_{k-1}} \\ &\quad + \int_{t_{k-1}}^{t_k-\varepsilon'} \mathrm{e}^{a_i s}\left\{\sum_{j=1}^n b_{ij}f_j\left(x_j(s)\right) + \sum_{j=1}^n c_{ij}g_j\left(x_j\left(s - \tau_j(s)\right)\right)\right\} ds, \end{aligned}$$

which generates, by letting $\varepsilon' \to 0^+$,

$$x_i(t_k - 0)\mathrm{e}^{a_i t_k} = x_i(t_{k-1} + 0)\mathrm{e}^{a_i t_{k-1}} + \int_{t_{k-1}}^{t_k} \mathrm{e}^{a_i s} \left\{ \sum_{j=1}^{n} b_{ij} f_j\bigl(x_j(s)\bigr) + \sum_{j=1}^{n} c_{ij} g_j\bigl(x_j\bigl(s - \tau_j(s)\bigr)\bigr) \right\} ds. \tag{3.3}$$

As $x_i(t_k - 0) = x_i(t_k)$, (3.3) can be rearranged as

$$x_i(t_k)\mathrm{e}^{a_i t_k} = x_i(t_{k-1} + 0)\mathrm{e}^{a_i t_{k-1}} + \int_{t_{k-1}}^{t_k} \mathrm{e}^{a_i s} \left\{ \sum_{j=1}^{n} b_{ij} f_j\bigl(x_j(s)\bigr) + \sum_{j=1}^{n} c_{ij} g_j\bigl(x_j\bigl(s - \tau_j(s)\bigr)\bigr) \right\} ds. \tag{3.4}$$

Combining (3.2) and (3.4), we derive, for $t \in (t_{k-1}, t_k]$ $(k = 1, 2, \ldots)$,

$$x_i(t)\mathrm{e}^{a_i t} = x_i(t_{k-1} + 0)\mathrm{e}^{a_i t_{k-1}} + \int_{t_{k-1}}^{t} \mathrm{e}^{a_i s} \left\{ \sum_{j=1}^{n} b_{ij} f_j\bigl(x_j(s)\bigr) + \sum_{j=1}^{n} c_{ij} g_j\bigl(x_j\bigl(s - \tau_j(s)\bigr)\bigr) \right\} ds.$$

This leads to, for $t \in (t_{k-1}, t_k]$ $(k = 1, 2, \ldots)$,

$$x_i(t)\mathrm{e}^{a_i t} = x_i(t_{k-1})\mathrm{e}^{a_i t_{k-1}} + \int_{t_{k-1}}^{t} \mathrm{e}^{a_i s} \left\{ \sum_{j=1}^{n} b_{ij} f_j\bigl(x_j(s)\bigr) + \sum_{j=1}^{n} c_{ij} g_j\bigl(x_j\bigl(s - \tau_j(s)\bigr)\bigr) \right\} ds + I_{i(k-1)}\bigl(x_i(t_{k-1})\bigr)\mathrm{e}^{a_i t_{k-1}}.$$

Hence,

$$\begin{aligned} x_i(t_{k-1})\mathrm{e}^{a_i t_{k-1}} &= x_i(t_{k-2})\mathrm{e}^{a_i t_{k-2}} + \int_{t_{k-2}}^{t_{k-1}} \mathrm{e}^{a_i s} \left\{ \sum_{j=1}^{n} b_{ij} f_j\bigl(x_j(s)\bigr) + \sum_{j=1}^{n} c_{ij} g_j\bigl(x_j\bigl(s - \tau_j(s)\bigr)\bigr) \right\} ds \\ &\quad + I_{i(k-2)}\bigl(x_i(t_{k-2})\bigr)\mathrm{e}^{a_i t_{k-2}}, \\ &\vdots \\ x_i(t_2)\mathrm{e}^{a_i t_2} &= x_i(t_1)\mathrm{e}^{a_i t_1} + \int_{t_1}^{t_2} \mathrm{e}^{a_i s} \left\{ \sum_{j=1}^{n} b_{ij} f_j\bigl(x_j(s)\bigr) + \sum_{j=1}^{n} c_{ij} g_j\bigl(x_j\bigl(s - \tau_j(s)\bigr)\bigr) \right\} ds \\ &\quad + I_{i1}\bigl(x_i(t_1)\bigr)\mathrm{e}^{a_i t_1}, \\ x_i(t_1)\mathrm{e}^{a_i t_1} &= \varphi_i(0) + \int_{0}^{t_1} \mathrm{e}^{a_i s} \left\{ \sum_{j=1}^{n} b_{ij} f_j\bigl(x_j(s)\bigr) + \sum_{j=1}^{n} c_{ij} g_j\bigl(x_j\bigl(s - \tau_j(s)\bigr)\bigr) \right\} ds. \end{aligned}$$

By induction, we obtain that, for $t > 0$,

$$x_i(t) = \varphi_i(0)\mathrm{e}^{-a_i t} + \mathrm{e}^{-a_i t} \int_{0}^{t} \mathrm{e}^{a_i s} \left\{ \sum_{j=1}^{n} b_{ij} f_j\bigl(x_j(s)\bigr) + \sum_{j=1}^{n} c_{ij} g_j\bigl(x_j\bigl(s - \tau_j(s)\bigr)\bigr) \right\} ds + \mathrm{e}^{-a_i t} \sum_{0 < t_k < t} \bigl\{ I_{ik}\bigl(x_i(t_k)\bigr)\mathrm{e}^{a_i t_k} \bigr\}.$$

From (H1)–(H3), we know, for $t > 0$,

$$\begin{aligned}\left|x_i(t)\right| &\le \left|\varphi_i(0)\right|\mathrm{e}^{-a_i t} + \mathrm{e}^{-a_i t}\sum_{j=1}^{n}|b_{ij}\alpha_j|\int_0^t \mathrm{e}^{a_i s}\left|x_j(s)\right|ds \\ &\quad + \mathrm{e}^{-a_i t}\sum_{j=1}^{n}|c_{ij}\beta_j|\int_0^t \mathrm{e}^{a_i s}\sup_{s-\tau_j(s)\le\upsilon\le s}\left|x_j(\upsilon)\right|ds + \mathrm{e}^{-a_i t}\sum_{0<t_k<t}\left\{P_{ik}\left|x_i(t_k)\right|\mathrm{e}^{a_i t_k}\right\}.\end{aligned}$$

Denote

$$\begin{aligned}&I_{i1} = \left|\varphi_i(0)\right|\mathrm{e}^{-a_i t}, \qquad I_{i2} = \mathrm{e}^{-a_i t}\sum_{j=1}^{n}|b_{ij}\alpha_j|\int_0^t \mathrm{e}^{a_i s}\left|x_j(s)\right|ds, \\ &I_{i3} = \mathrm{e}^{-a_i t}\sum_{j=1}^{n}|c_{ij}\beta_j|\int_0^t \mathrm{e}^{a_i s}\sup_{s-\tau_j(s)\le\upsilon\le s}\left|x_j(\upsilon)\right|ds, \qquad I_{i4} = \mathrm{e}^{-a_i t}\sum_{0<t_k<t}\left\{P_{ik}\left|x_i(t_k)\right|\mathrm{e}^{a_i t_k}\right\}.\end{aligned}$$

Condition (iii) implies that there exists $\chi \in (0,1)$ such that

$$\frac{3^{p-1}}{(1-\chi)^{p-1}}\left\{a_i^{1-p}\left(\sum_{j=1}^{n}|b_{ij}\alpha_j|\right)^p + a_i^{1-p}\left(\sum_{j=1}^{n}|c_{ij}\beta_j|\right)^p + a_i^{1-p}P_i^p\right\} < a_i. \tag{3.5}$$

By employing Holder's inequality, we get

$$\begin{aligned}\left|x_i(t)\right|^p &\le \chi^{1-p}I_{i1}^p + (1-\chi)^{1-p}(I_{i2}+I_{i3}+I_{i4})^p \\ &\le \chi^{1-p}I_{i1}^p + 3^{p-1}(1-\chi)^{1-p}I_{i2}^p + 3^{p-1}(1-\chi)^{1-p}I_{i3}^p + 3^{p-1}(1-\chi)^{1-p}I_{i4}^p.\end{aligned}$$

Moreover, it follows from Holder's inequality that

$$\begin{aligned}I_{i2}^p &= \left\{\sum_{j=1}^{n}|b_{ij}\alpha_j|^{\frac{p-1}{p}}|b_{ij}\alpha_j|^{\frac{1}{p}}\int_0^t \mathrm{e}^{-a_i(t-s)}\left|x_j(s)\right|ds\right\}^p \\ &\le \left(\sum_{j=1}^{n}|b_{ij}\alpha_j|\right)^{p-1}\sum_{j=1}^{n}|b_{ij}\alpha_j|\left\{\int_0^t \mathrm{e}^{-a_i(t-s)}\left|x_j(s)\right|ds\right\}^p \\ &= \left(\sum_{j=1}^{n}|b_{ij}\alpha_j|\right)^{p-1}\sum_{j=1}^{n}|b_{ij}\alpha_j|\left\{\int_0^t \mathrm{e}^{\frac{-(p-1)a_i(t-s)}{p}}e^{\frac{-a_i(t-s)}{p}}\left|x_j(s)\right|ds\right\}^p \\ &\le \left(\sum_{j=1}^{n}|b_{ij}\alpha_j|\right)^{p-1}\left\{\int_0^t \mathrm{e}^{-a_i(t-s)}ds\right\}^{p-1}\left(\sum_{j=1}^{n}|b_{ij}\alpha_j|\int_0^t e^{-a_i(t-s)}\left|x_j(s)\right|^p ds\right) \\ &\le a_i^{1-p}\left(\sum_{j=1}^{n}|b_{ij}\alpha_j|\right)^{p-1}\left(\sum_{j=1}^{n}|b_{ij}\alpha_j|\int_0^t e^{-a_i(t-s)}\left|x_j(s)\right|^p ds\right).\end{aligned}$$

Similarly, we get

$$I_{i3}^p \le a_i^{1-p}\left(\sum_{j=1}^{n}|c_{ij}\beta_j|\right)^{p-1}\left(\sum_{j=1}^{n}|c_{ij}\beta_j|\int_0^t e^{-a_i(t-s)}\sup_{s-\tau_j(s)\le\upsilon\le s}\left|x_j(\upsilon)\right|^p ds\right).$$

In addition, Lemma 2.1, conditions (i)–(ii), and Holder's inequality yield

$$\begin{aligned}
I_{i4}^p &\le \Bigl\{P_i \sum_{0<t_k<t} \bigl\{\theta(t_k - t_{k-1})\bigl|x_i(t_k)\bigr|\mathrm{e}^{-a_i(t-t_k)}\bigr\}\Bigr\}^p \le \Bigl(P_i \int_0^t e^{-a_i(t-s)}\bigl|x_i(s)\bigr|\,ds\Bigr)^P \\
&= P_i^p \Bigl\{\int_0^t \mathrm{e}^{\frac{-(p-1)a_i(t-s)}{p}} e^{\frac{-a_i(t-s)}{p}} \bigl|x_i(s)\bigr|\,ds\Bigr\}^p \\
&\le P_i^p \Bigl(\int_0^t \mathrm{e}^{-a_i(t-s)}\,ds\Bigr)^{p-1} \Bigl(\int_0^t e^{-a_i(t-s)}\bigl|x_i(s)\bigr|^p\,ds\Bigr) \\
&\le a_i^{1-p} P_i^p \Bigl(\int_0^t e^{-a_i(t-s)}\bigl|x_i(s)\bigr|^p\,ds\Bigr).
\end{aligned}$$

Therefore,

$$\begin{aligned}
\bigl|x_i(t)\bigr|^p \le{}& \chi^{1-p}\bigl|\varphi_i(0)\bigr|^p \mathrm{e}^{-a_i t} + 3^{p-1}(1-\chi)^{1-p} a_i^{1-p} \Bigl(\sum_{j=1}^n |b_{ij}\alpha_j|\Bigr)^{p-1} \\
&\times \Bigl(\sum_{j=1}^n |b_{ij}\alpha_j| \int_0^t e^{-a_i(t-s)}\bigl|x_j(s)\bigr|^p\,ds\Bigr) \\
&+ 3^{p-1}(1-\chi)^{1-p} a_i^{1-p} \Bigl(\sum_{j=1}^n |c_{ij}\beta_j|\Bigr)^{p-1} \\
&\times \Bigl(\sum_{j=1}^n |c_{ij}\beta_j| \int_0^t e^{-a_i(t-s)} \sup_{s-\tau_j(s)\le\upsilon\le s}\bigl|x_j(\upsilon)\bigr|^p\,ds\Bigr) \\
&+ 3^{p-1}(1-\chi)^{1-p} a_i^{1-p} P_i^p \Bigl(\int_0^t e^{-a_i(t-s)}\bigl|x_i(s)\bigr|^p\,ds\Bigr).
\end{aligned} \tag{3.6}$$

For each $i \in \mathcal{N}$, define the following function:

$$\begin{aligned}
G_i(\lambda) ={}& (\lambda - a_i) + 3^{p-1}\bigl(a_i(1-\chi)\bigr)^{1-p}\Bigl(\sum_{j=1}^n |b_{ij}\alpha_j|\Bigr)^p \\
&+ 3^{p-1}\bigl(a_i(1-\chi)\bigr)^{1-p}\Bigl(\sum_{j=1}^n |c_{ij}\beta_j|\Bigr)^{p-1}\Bigl(\sum_{j=1}^n |c_{ij}\beta_j| e^{\lambda\tau_j}\Bigr) + 3^{p-1}\bigl(a_i(1-\chi)\bigr)^{1-p} P_i^p.
\end{aligned}$$

From (3.5), we know $G_i(0) < 0$. Further, $G_i(\lambda)$ is continuous on R_+, $G_i(+\infty) = +\infty$, and $G_i'(\lambda) > 0$ for $\lambda \in \mathrm{R}_+$, so for each $i \in \mathcal{N}$, the equation $G_i(\lambda) = 0$ has a unique solution $\lambda_i \in \mathrm{R}_+$. Choosing $\vartheta = \min_{i\in\mathcal{N}}\{\lambda_i\}$, we get, for $i \in \mathcal{N}$,

$$\begin{aligned}
&\frac{3^{p-1}}{(1-\chi)^{p-1}}\Bigl\{a_i^{1-p}\Bigl(\sum_{j=1}^n |b_{ij}\alpha_j|\Bigr)^p + a_i^{1-p}\Bigl(\sum_{j=1}^n |c_{ij}\beta_j|\Bigr)^{p-1}\Bigl(\sum_{j=1}^n |c_{ij}\beta_j| e^{\vartheta\tau_j}\Bigr) + a_i^{1-p} P_i^p\Bigr\} \\
&\quad\le a_i - \vartheta.
\end{aligned} \tag{3.7}$$

Let $u(t) = \chi^{1-p}\max_{i\in\mathcal{N}}\{\|\varphi_i\|^p\}e^{-\vartheta t}$, $t \in [-\tau, +\infty)$. Obviously, $u(s) = u(t)e^{\vartheta(t-s)}$ holds for $-\tau \le s \le t$, and $\sup_{t-\tau_j(t)\le s\le t} u(s) \le u(t)e^{\vartheta\tau_j}$ is true for each $j \in \mathcal{N}$ and $t \in \mathrm{R}_+$. Denote

$$\begin{aligned}
L_1 &= u(0)\mathrm{e}^{-a_i t},\\
L_2 &= 3^{p-1}(1-\chi)^{1-p}a_i^{1-p}\left(\sum_{j=1}^n |b_{ij}\alpha_j|\right)^{p-1}\left(\sum_{j=1}^n |b_{ij}\alpha_j| \int_0^t e^{-a_i(t-s)}u(s)\,ds\right),\\
L_3 &= 3^{p-1}(1-\chi)^{1-p}a_i^{1-p}\left(\sum_{j=1}^n |c_{ij}\beta_j|\right)^{p-1}\left(\sum_{j=1}^n |c_{ij}\beta_j| \int_0^t e^{-a_i(t-s)}\sup_{s-\tau_j(s)\le\upsilon\le s} u(\upsilon)\,ds\right),\\
L_4 &= 3^{p-1}(1-\chi)^{1-p}a_i^{1-p}P_i^p\left(\int_0^t e^{-a_i(t-s)}u(s)\,ds\right).
\end{aligned}$$

As

$$\begin{aligned}
L_1 &= u(0)\mathrm{e}^{-a_i t} = u(t)e^{(\vartheta-a_i)t},\\
L_2 &= u(t)3^{p-1}(1-\chi)^{1-p}a_i^{1-p}\left(\sum_{j=1}^n |b_{ij}\alpha_j|\right)^{p} e^{(\vartheta-a_i)t}\int_0^t e^{(a_i-\vartheta)s}\,ds,\\
L_3 &\le u(t)3^{p-1}(1-\chi)^{1-p}a_i^{1-p}\left(\sum_{j=1}^n |c_{ij}\beta_j|\right)^{p-1}\left(\sum_{j=1}^n |c_{ij}\beta_j|e^{\vartheta\tau_j}\right)e^{(\vartheta-a_i)t}\int_0^t e^{(a_i-\vartheta)s}\,ds,\\
L_4 &= u(t)3^{p-1}(1-\chi)^{1-p}a_i^{1-p}P_i^p e^{(\vartheta-a_i)t}\int_0^t e^{(a_i-\vartheta)s}\,ds,
\end{aligned}$$

we obtain from (3.7) that

$$\begin{aligned}
&L_1 + L_2 + L_3 + L_4\\
&\quad\le u(t)e^{(\vartheta-a_i)t}\\
&\qquad + u(t)\frac{(1-e^{\vartheta-a_i t})}{a_i-\vartheta}\left\{\begin{array}{c}
3^{p-1}(1-\chi)^{1-p}a_i^{1-p}(\sum_{j=1}^n |b_{ij}\alpha_j|)^p\\
+3^{p-1}(1-\chi)^{1-p}a_i^{1-p}(\sum_{j=1}^n |c_{ij}\beta_j|)^{p-1}(\sum_{j=1}^n |c_{ij}\beta_j|e^{\vartheta\tau_j})\\
+3^{p-1}(1-\chi)^{1-p}a_i^{1-p}P_i^p
\end{array}\right\}\\
&\quad\le u(t)e^{(\vartheta-a_i)t} + u(t)\bigl(1-e^{\vartheta-a_i t}\bigr) = u(t).
\end{aligned} \tag{3.8}$$

Finally, we prove that $|x_i(t)|^p \le u(t)$ for all $t \ge -\tau$ and $i \in \mathcal{N}$ by a contradiction. Obviously, $|x_i(t)|^p \le u(t)$ holds for $t \in [-\tau, 0]$ and $i \in \mathcal{N}$. For each i, assume that there exist $t_i > 0$ and $\varepsilon > 0$ such that $|x_i(t)|^p < u(t) + \varepsilon$ as $t \in [0, t_i)$ and $|x_i(t_i)|^p = u(t_i) + \varepsilon$. Choose $t^* \triangleq t_{i^*} = \min_{i\in\mathcal{N}}\{t_i\}$. Obviously, $\chi^{1-p}|\varphi_{i^*}(0)|^p < u(0) + \varepsilon$, from (3.6) and (3.8), we get

$$\begin{aligned}
&\bigl|x_{i^*}\bigl(t^*\bigr)\bigr|^p - u\bigl(t^*\bigr)\\
&\quad\le 3^{p-1}(1-\chi)^{1-p}a_{i^*}^{1-p}\left(\sum_{j=1}^n |b_{i^*j}\alpha_j|\right)^{p-1}\\
&\qquad\times\left(\sum_{j=1}^n |b_{i^*j}\alpha_j| \int_0^{t^*} e^{-a_{i^*}(t^*-s)}\bigl[\bigl|x_j(s)\bigr|^p - u(s)\bigr]\,ds\right)
\end{aligned}$$

$$
\begin{aligned}
&+ 3^{p-1}(1-\chi)^{1-p} a_{i^*}^{1-p} \left(\sum_{j=1}^{n} |c_{i^*j}\beta_j| \right)^{p-1} \\
&\times \left(\sum_{j=1}^{n} |c_{i^*j}\beta_j| \int_0^{t^*} e^{-a_{i^*}(t^*-s)} \sup_{s-\tau_j(s)\le \upsilon \le s} \left[\left|x_j(\upsilon)\right|^p - u(\upsilon)\right] ds \right) \\
&+ 3^{p-1}(1-\chi)^{1-p} a_{i^*}^{1-p} P_{i^*}^{p} \left(\int_0^{t^*} e^{-a_{i^*}(t^*-s)} \left[\left|x_{i^*}(s)\right|^p - u(s)\right] ds \right) \\
&+ \left[\chi^{1-p} \left|\varphi_{i^*}(0)\right|^p - u(0)\right] \mathrm{e}^{-a_{i^*}t^*} \\
\le{}& \varepsilon \mathrm{e}^{-a_i t} + \varepsilon 3^{p-1}(1-\chi)^{1-p} a_{i^*}^{1-p} \left(\sum_{j=1}^{n} |b_{i^*j}\alpha_j| \right)^{p} \int_0^{t^*} e^{-a_{i^*}(t^*-s)} \, ds \\
&+ \varepsilon 3^{p-1}(1-\chi)^{1-p} a_{i^*}^{1-p} \left(\sum_{j=1}^{n} |c_{i^*j}\beta_j| \right)^{p} \int_0^{t^*} e^{-a_{i^*}(t^*-s)} \, ds \\
&+ \varepsilon 3^{p-1}(1-\chi)^{1-p} a_{i^*}^{1-p} P_{i^*}^{p} \left(\int_0^{t^*} e^{-a_{i^*}(t^*-s)} \, ds \right) \\
={}& \varepsilon \mathrm{e}^{-a_{i^*}t^*} + \varepsilon 3^{p-1}(1-\chi)^{1-p} a_{i^*}^{1-p} \left(\sum_{j=1}^{n} |b_{i^*j}\alpha_j| \right)^{p} \frac{(1-\mathrm{e}^{-a_{i^*}t^*})}{a_{i^*}} \\
&+ \varepsilon 3^{p-1}(1-\chi)^{1-p} a_{i^*}^{1-p} \left(\sum_{j=1}^{n} |c_{i^*j}\beta_j| \right)^{p} \frac{(1-\mathrm{e}^{-a_{i^*}t^*})}{a_{i^*}} \\
&+ \varepsilon 3^{p-1}(1-\chi)^{1-p} a_{i^*}^{1-p} P_{i^*}^{p} \frac{(1-\mathrm{e}^{-a_{i^*}t^*})}{a_{i^*}} \\
={}& \varepsilon \mathrm{e}^{-a_{i^*}t^*} + \frac{3^{p-1}(1-\chi)^{1-p}\{a_{i^*}^{1-p}(\sum_{j=1}^{n}|b_{i^*j}\alpha_j|)^p + a_{i^*}^{1-p}(\sum_{j=1}^{n}|c_{i^*j}\beta_j|)^p + a_{i^*}^{1-p}P_{i^*}^{p}\}}{a_{i^*}} \\
&\times \left(1-\mathrm{e}^{-a_{i^*}t^*}\right)\varepsilon \\
<{}& \varepsilon,
\end{aligned}
$$

which is a contradiction. This shows that $|x_i(t)|^p \le u(t)$ for all $t \ge -\tau$ and $i \in \mathcal{N}$, which means that $|x_i(t;\varphi)|^p \le \chi^{1-p} \max_{i\in\mathcal{N}}\{\|\varphi_i\|^p\} e^{-\vartheta t}$ for $t \in [-\tau, +\infty)$. □

As a special case, we give the following theorem.

Theorem 3.2 *Suppose that*

(i) there exists constant $\mu > 0$ *such that* $\inf_{k=1,2,\dots}\{\frac{t_k - t_{k-1}}{2}\} \ge \mu$ *and* $\max_{k=1,2,\dots}\{t_k - t_{k-1}\} < \frac{1}{a_i}$,
(ii) there exist constants P_i *such that* $P_{ik} \le P_i\mu$ *for* $i \in \mathcal{N}$ *and* $k = 1, 2, \dots,$
(iii) $-a_i + \sum_{j=1}^{n} |b_{ij}\alpha_j| + \sum_{j=1}^{n} |c_{ij}\beta_j| + P_i < 0$.
Then system (2.1) *is globally exponentially stable.*

Proof Let $p = 1$ and $\theta = \frac{1}{2}$ in Theorem 3.1. □

Remark 3.1 In [25], the fixed point theory was employed to study system (2.1), and the research shows that system (2.1) is globally exponentially stable on the condition that $\sum_{i=1}^{n}\{\frac{1}{a_i}\max_{j\in\mathcal{N}}|b_{ij}l_j| + \frac{1}{a_i}\max_{j\in\mathcal{N}}|c_{ij}k_j|\} + \max_{i\in\mathcal{N}}\{p_i(\mu + \frac{1}{a_i})\} < 1$. Obviously, condition (iii) in Theorem 3.2 is weaker.

4 Exponential stability of impulsive integral inequalities

Consider the following impulsive integral inequalities:

$$
\begin{aligned}
y_i(t) &\le C\phi_i(0)\mathrm{e}^{-a_i t} + \sum_{j=1}^{n}\alpha_{ij}\int_0^t \mathrm{e}^{-a_i(t-s)}y_j(s)\,ds + \sum_{j=1}^{n}\beta_{ij}\int_0^t \mathrm{e}^{-a_i(t-s)}\sup_{s-\tau_j(s)\le\upsilon\le s} y_j(\upsilon)\,ds \\
&\quad + \sum_{0<t_k<t}\bigl\{P_{ik}y_i(t_k)\mathrm{e}^{-a_i(t-t_k)}\bigr\}, \quad t\ge 0, \\
y_i(t) &= \phi_i(0) \in C\bigl([-\tau,0],R^+\bigr), \quad t\in[-\tau,0],
\end{aligned}
\tag{4.1}
$$

where $C\ge 1$, and for each $i,j\in\mathcal{N}$, $y_i(t)\ge 0$ for $t\ge -\tau$, $0\le\tau_j(s)\le\tau_j\le\tau$ for $s\ge 0$, and $a_i>0$, $\alpha_{ij}\ge 0$, $\beta_{ij}\ge 0$, $P_{ik}\ge 0$, $k=1,2,\ldots$.

Theorem 4.1 *Suppose that*

(*i*) *there exist constants $\mu>0$ and $\theta\in(0,1)$ such that $\inf_{k=1,2,\ldots}\{\theta(t_k-t_{k-1})\}\ge\mu$ and $\max_{k=1,2,\ldots}\{t_k-t_{k-1}\}<\frac{1-\theta}{\theta a_i}$,*

(*ii*) *there exist nonnegative constants P_i such that $P_{ik}\le P_i\mu$ for $i\in\mathcal{N}$ and $k=1,2,\ldots$,*

(*iii*) $-a_i+\sum_{j=1}^n\alpha_{ij}+\sum_{j=1}^n\beta_{ij}+P_i<0$.

Then there exist positive constant C and λ^* such that

$$
\max_{i\in\mathcal{N}} y_i(t)\le C\max_{i\in\mathcal{N}}\bigl\{\|\phi_i\|\bigr\}e^{-\lambda^* t}, \quad t\in[-\tau,+\infty),
$$

where λ^* is the minimum solution of the following equations:

$$
\lambda - a_i + \sum_{j=1}^{n}\alpha_{ij} + \sum_{j=1}^{n}\beta_{ij}e^{\lambda\tau_j} + P_i = 0.
$$

Proof For each $i\in\mathcal{N}$, define the following function:

$$
F_i(\lambda) = \lambda - a_i + \sum_{j=1}^{n}\alpha_{ij} + \sum_{j=1}^{n}\beta_{ij}e^{\lambda\tau_j} + P_i.
$$

Note that $F_i(\lambda)$ is continuous on R_+, $F_i(0)=-a_i+\sum_{j=1}^n\alpha_{ij}+\sum_{j=1}^n\beta_{ij}+P_i<0$, $F_i(+\infty)=+\infty$, and $F_i'(\lambda)>0$ for $\lambda\in\mathrm{R}_+$, so for each $i\in\mathcal{N}$, the equation $F_i(\lambda)=0$ has a unique solution $\lambda_i\in\mathrm{R}_+$. Choosing $\lambda^*=\min_{i\in\mathcal{N}}\{\lambda_i\}$, we get

$$
\lambda^* - a_i + \sum_{j=1}^{n}|b_{ij}\alpha_j| + \sum_{j=1}^{n}|c_{ij}\beta_j|e^{\lambda^*\tau_j} + P_i \le 0, \quad i\in\mathcal{N}.
$$

Let $u(t)=C\max_{i\in\mathcal{N}}\{\|\phi_i\|\}e^{-\lambda^* t}$, $t\in[-\tau,+\infty)$. Similar to Theorem 3.1, we get

$$
\begin{aligned}
&\mathrm{e}^{-a_i t}u(0) + \mathrm{e}^{-a_i t}\sum_{j=1}^{n}\alpha_{ij}\int_0^t \mathrm{e}^{a_i s}u(s)\,ds + \mathrm{e}^{-a_i t}\sum_{j=1}^{n}\beta_{ij}\int_0^t \mathrm{e}^{a_i s}\sup_{s-\tau_j(s)\le\upsilon\le s}u(\upsilon)\,ds \\
&\quad + \mathrm{e}^{-a_i t}\sum_{0<t_k<t}\bigl\{P_{ik}u(t_k)\mathrm{e}^{a_i t_k}\bigr\} \le u(t), \quad t\ge 0.
\end{aligned}
$$

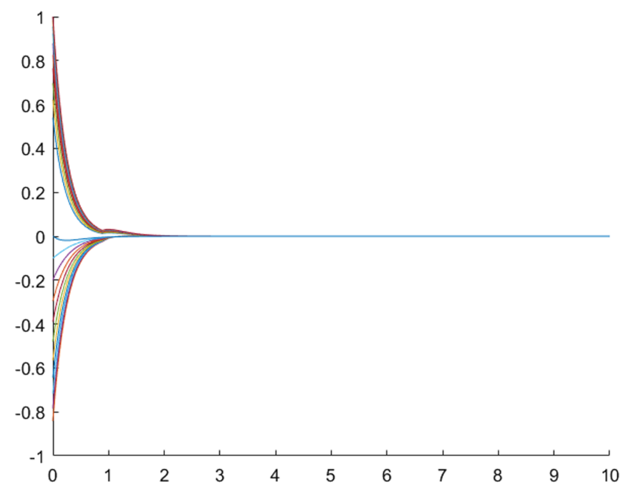

Figure 1 The simulation in the example

Finally, we prove that $y_i(t) \leq u(t)$ for all $t \geq -\tau$ and $i \in \mathcal{N}$ by a contradiction. This is similar to the proof of Theorem 3.1, so we omit it here now. This shows that $y_i(t) \leq u(t)$ for all $t \geq -\tau$ and $i \in \mathcal{N}$, which means that $\max_{i\in\mathcal{N}} y_i(t) \leq C \max_{i\in\mathcal{N}}\{\|\phi_i\|\}e^{-\lambda^* t}$ for $t \in [-\tau, +\infty)$. □

Remark 4.1 Inequalities (4.1) can be considered as multidimensional Halanay inequalities with impulses. In [30–32], the authors used the one-dimensional Halanay inequality to consider the stability of delayed neural networks. However, they did not consider the impulse effect, and they needed to construct a complicated Lyapunov function [30, 31] or define a complicated matrix norm [32]; in addition, their results are not easy to verify in practice. The advantages of our multidimensional Halanay inequalities with impulses are that we take into account the impulse effects, and we neither require to construct a complicated Lyapunov function nor to define the adaptive matrix form; furthermore, our results are easy to verify.

5 Example

Consider the following two-dimensional impulsive delayed Hopfield neural network:

$$\frac{dx_i(t)}{dt} = -a_i x_i(t) + \sum_{j=1}^{2} b_{ij} f_j\bigl(x_j(t)\bigr) + \sum_{j=1}^{2} c_{ij} g_j\bigl(x_j\bigl(t-\tau_j(t)\bigr)\bigr), \quad t \geq 0, t \neq t_k,$$

$$\Delta x_i(t_k) = x_i(t_k+0) - x_i(t_k) = I_{ik}\bigl(x_i(t_k)\bigr), \quad t_k = 0.25k, k = 1, 2, \ldots,$$

with the initial conditions $x_1(s) = \cos(s)$, $x_2(s) = \sin(s)$ on $-\tau \leq s \leq 0$, where $a_1 = a_2 = 4$, $b_{11} = 0$, $b_{12} = 0.1$, $b_{21} = -0.2$, $b_{22} = 0$, $c_{11} = 0.2$, $c_{12} = 0$, $c_{21} = 0$, $c_{22} = -0.1$, $f_j(s) = g_j(s) = (|s+1| - |s-1|)/2$ $(j = 1, 2)$, $I_{ik}(x_i(t_k)) = \arctan(0.45x_i(t_k))$ for $i = 1, 2$ and $k = 1, 2, \ldots$ $(k = 1, 2, \ldots)$. It is easy to find that $\mu = 0.125$, $\alpha_j = \beta_j = 1$, and $P_{ik} = 0.45$. Select $P_i = 3.6$ and then compute $-a_i + \sum_{j=1}^{n} |b_{ij}\alpha_j| + \sum_{j=1}^{n} |c_{ij}\beta_j| + P_i = -0.1 < 0$. From Theorem 3.2, we know this system is globally exponentially stable (Fig. 1).

Acknowledgements
The authors would like to thank the editors and reviewers for their valuable contributions, which greatly improved the readability of this paper.

Funding
This work is supported by the National Natural Science Foundation of China with Grant Nos. 61573193, 61473213, 61671338, by Hubei Province Key Laboratory of Systems Science in Metallurgical Process (Wuhan University of Science and Technology) with Grant No. Z201901, and the Joint Key Grant of National Natural Science Foundation of China and Zhejiang Province (U1509217).

Availability of data and materials
Not applicable.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
The authors have equally made contributions. All authors read and approved the final manuscript.

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Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Received: 10 September 2020 Accepted: 2 June 2021 Published online: 27 June 2021

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