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A technique of tripled coincidence points for solving a system of nonlinear integral equations in POCML spaces

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Abstract

This manuscript aims to initiate some recent theoretical consequences related to tripled coincidence points for non-self mappings via the notion of C-type functions in partially ordered complete metric-like space (for short, POCML space). Our contributions unify and expand some previous studies in this line. Moreover, some corollaries and suitable examples are presented to demonstrate the novelty of the results established. Ultimately, two applications are given here to boost our theoretical consequences, the first one about the contributions of the integral type to obtain a triple coincidence points and the other application is about solving a system of nonlinear integral equations.

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Keywords: Tripled coincidence point; Partially ordered set; Metric-like spaces; Nonlinear integral equations

1 Introduction

In the past, it was believed that scientific disciplines are completely separate; but now, after the tremendous development and modern theories in basic science techniques, they have become completely connected. For example, mathematics in which the level of development in different disciplines has varied dramatically in contemporary time. As an interesting example, fixed-point technologies offer a focal concept with many diverse usages. It has been and still is an important theoretical tool in many fields and various disciplines such as topology, game theory, optimal control, artificial intelligence, logic programming, dynamical systems (and chaos), functional analysis, differential equations, and economics. More clearly, for example, the technique of fixed point is applied for finding the solution of the equilibrium troubles in economics and game theory. In nonlinear integral equations, it is used to find analytical and numerical solutions to Fredholm integral equations [1–5], etc.

The ideas of mixed-monotone functions and coupled fixed point were initiated in the paper [6]. Under these ideas, some main results in partially ordered metric spaces have been driven by the authors [6]. For enjoyable specifics on coupled fixed point consequences and related topics in abstract spaces, the reader can refer to [7–27].

Pivotal results related to a triple fixed point (established in 2011 by Berinde and Borcut [28]) were presented in partially ordered metric spaces. For more topics of this notion, we cite papers [29–35].

Definition 1.1 ([28]) It is said that a trio $(\wp, \hbar, \eth) \in \chi^3$ is a tripled fixed point of a self-mapping $\Re : \chi^3 \to \chi$ if $\wp = \Re(\wp, \hbar, \eth)$, $\hbar = \Re(\hbar, \wp, \hbar)$, and $\eth = \Re(\eth, \hbar, \wp)$.

Definition 1.2 ([29]) A trio $(\wp, \hbar, \eth) \in \chi^3$ on a nonempty set χ is called a tripled coincidence point of the two self-mappings $\Re : \chi^3 \to \chi$ and $\Theta : \chi \to \chi$ if $\Im\wp = \Re(\wp, \hbar, \eth)$, $\Im\hbar = \Re(\hbar, \wp, \hbar)$, and $\Im\eth = \Re(\eth, \hbar, \wp)$.

Definition 1.3 ([29]) Consider that $\chi \neq \emptyset$ is a set, a trio $(\wp, \hbar, \eth) \in \chi^3$ is said to be a tripled common fixed point of $\Re : \chi \to \chi$ and $\Theta : \chi \to \chi$ if $\wp = \Im\wp = \Re(\wp, \hbar, \eth)$, $\hbar = \Im\hbar = \Re(\hbar, \wp, \hbar)$, and $\eth = \Im\eth = \Re(\eth, \hbar, \wp)$.

Definition 1.4 ([31]) Suppose that $\chi \neq \emptyset$ is a set, the mappings $\Re : \chi^3 \to \chi$ and $\Theta : \chi \to \chi$ are commutative $\Im(\Re(\wp, \hbar, \eth)) = \Re(\Im\wp, \Im\wp, \Im\wp)$ for all $\wp, \hbar, \eth \in \chi$.

Definition 1.5 ([28]) A mapping $\Re : \chi^3 \to \chi$ on a partially ordered set $(\chi, \precsim)$ has a mixed-monotone property if, for any $\wp, \hbar, \eth \in \chi$,

$$\begin{aligned}
&\wp_1, \wp_2 \in \chi, \wp_1 \precsim \wp_2 \text{ implies } \quad \Re(\wp_1, \hbar, \eth) \precsim \Re(\wp_2, \hbar, \eth),\\
&\hbar_1, \hbar_2 \in \chi, \hbar_1 \precsim \hbar_2 \text{ implies } \quad \Re(\wp, \hbar_1, \eth) \succsim \Re(\wp, \hbar_2, \eth),\\
&\eth_1, \eth_2 \in \chi, \eth_1 \precsim \eth_2 \text{ implies } \quad \Re(\wp, \hbar, \eth_1) \precsim \Re(\wp, \hbar, \eth_2).
\end{aligned}$$

Recently, Aydi et al. extended the property of mixed-monotone to $\Im$-mixed-monotone as follows.

Definition 1.6 ([36]) A mapping $\Re : \chi^3 \to \chi$ on a partially ordered set $(\chi, \precsim)$ has a mixed $\Im$-monotone property where $\Im : \chi \to \chi$ if, for any $\wp, \hbar, \eth \in \chi$,

$$\begin{aligned}
&\wp_1, \wp_2 \in \chi, \Im\wp_1 \precsim \Im\wp_2 \text{ implies } \quad \Re(\wp_1, \hbar, \eth) \precsim \Re(\wp_2, \hbar, \eth),\\
&\hbar_1, \hbar_2 \in \chi, \Im\hbar_1 \precsim \Im\hbar_2 \text{ implies } \quad \Re(\wp, \hbar_1, \eth) \succsim \Re(\wp, \hbar_2, \eth),\\
&\eth_1, \eth_2 \in \chi, \Im\eth_1 \precsim \Im\eth_2 \text{ implies } \quad \Re(\wp, \hbar, \eth_1) \precsim \Re(\wp, \hbar, \eth_2).
\end{aligned}$$

The first theorem concerned with a tripled fixed point of the mapping which has a mixed-monotone property in a partially ordered set was introduced as follows.

Theorem 1.7 ([28]) *Let $(\chi, \precsim, \xi)$ be a POCM space. Consider the mapping $\Re : \chi^3 \to \chi$ such that*:

(i) *$\Re$ has a mixed-monotone property*;

(ii) *Either $\Re$ is continuous or χ has the following properties*:

  (a) *$l_n \precsim l$ if a nondecreasing sequence $l_n \to l$ for all n*,

  (b) *$j_n \succsim j$ if a nonincreasing sequence $j_n \to j$ for all n*;

(iii) *There are* $\alpha, \beta, \gamma \geq 0$ *with* $\alpha + \beta + \gamma < 1$ *such that*

$$\xi\big(\Re(\wp, \hbar, \eth), \Re(x, y, z)\big) \leq \alpha\xi(\wp, x) + \beta\xi(\hbar, y) + \gamma\xi(c, z)$$

for any $\wp, \hbar, c, x, y, z \in \chi$, *for which* $\wp \precsim x$, $y \precsim \hbar$, *and* $\eth \precsim z$. *If there exist* $\wp_\circ, \hbar_\circ, \eth_\circ \in \chi$ *such that* $\wp_\circ \precsim \Re(\wp_\circ, \hbar_\circ, \eth_\circ)$, $\hbar_\circ \succsim \Re(\hbar_\circ, \wp_\circ, \hbar_\circ)$, *and* $\eth_\circ \precsim \Re(\eth_\circ, \hbar_\circ, \wp_\circ)$. *Then* $\Re$ *has a tripled fixed point.*

2 Metric-like spaces and *C*-class functions

In 1994, the notion of spaces with the same nonzero distance from the points was shown by Matthews [37]. The authors [38] improved it in dislocated and dislocated quasi-metric spaces. In [39] the concept of a metric-like space was discussed, which is an important extension of the spaces defined in [38, 39].

Very recently, many fixed point results on metric-like spaces have been provided. For more specifics, see [40–49].

Now, we state some basic significance of metric-like spaces.

Definition 2.1 ([39]) A mapping $\xi : \chi \times \chi \to [0, +\infty)$ on a nonempty set χ is called metric-like if, for all $\wp, \hbar, \eth \in \chi$, the following assumptions hold:

(ξ_1) $\xi(\wp, \hbar) = 0$ implies $\wp = \hbar$;

(ξ_2) $\xi(\wp, \hbar) = \xi(\hbar, \wp)$;

(ξ_3) $\xi(\wp, \eth) \leq \xi(\wp, \hbar) + \xi(\hbar, \eth)$.

Thus, the parenthesis (χ, ξ) is called a metric-like space.

For $\wp \in \chi$, $\xi(\wp, \wp)$ may be positive except that a metric-like mapping satisfies all the assumptions of ordinary metric.

Definition 2.2 ([39]) Let a sequence of points $\{\wp_n\}$ be on χ in a metric-like space (χ, ξ). A point $\wp \in \chi$ is called the limit of the sequence $\{\wp_n\}$ if $\lim_{n\to\infty} \xi(\wp, \wp_n) = \xi(\wp, \wp)$, and we say that the point $\wp$ is a limit point of the sequence $\{\wp_n\}$.

Definition 2.3 ([39]) Assume that (χ, ξ) is a metric-like space.

(i) A sequence $\{\wp_n\}$ is called a ξ-Cauchy sequence if $\lim_{m,n\to+\infty} \xi(\wp_m, \wp_n)$ exists and is finite;

(ii) If every ξ-Cauchy sequence $\{\wp_n\}$ in χ converges to $\wp \in \chi$, with $\lim_{m,n\to+\infty} \xi(\wp_m, \wp_n) = \omega(\wp, \wp) = \lim_{n\to+\infty} \xi(\wp_n, \wp)$, then the space (χ, ξ) is called complete.

Lemma 2.4 ([40]) *Consider* (χ, ξ) *to be a metric-like space and* $\{\wp_n\}$ *be a sequence of* χ *such that* $a_n \to a$ *as* $n \to +\infty$ *and* $\xi(\wp, \wp) = 0$. *Then* $\lim_{n\to+\infty} \xi(\wp_n, \hbar) = \xi(\wp, \hbar)$ *for all* $\hbar \in \chi$.

Lemma 2.5 ([40]) *Suppose that* (χ, ξ) *is a metric-like space. Then*

(i) *if* $\xi(\wp, \hbar) = 0$, *then* $\xi(\wp, \wp) = \xi(\hbar, \hbar) = 0$;

(ii) *if* $\{\wp_n\}$ *is a sequence such that* $\lim_{n\to+\infty} \xi(\wp_n, \wp_{n+1}) = 0$, *then*

$$\lim_{n\to+\infty} \xi(\wp_n, \wp_n) = \lim_{n\to+\infty} \xi(\wp_{n+1}, \wp_{n+1}) = 0;$$

(iii) *if* $\wp \neq \hbar$, *then* $\xi(\wp,\hbar) > 0$;
(iv) $\xi(\wp,\wp) \leq \frac{2}{n}\sum_{i=1}^{n}\xi(\wp,\wp_i)$ *holds for all* $\wp,\wp_i \in \chi$, *where* $1 \leq i \leq n$.

Here, we assume that $\Pi = \{\pi : [0,+\infty) \to [0,+\infty)$ is a nondecreasing function and lower semi-continuous such that $\pi(\nu) = 0 \Leftrightarrow \nu = 0\}$.

In 2014, the idea of C-type functions which cover a large class of contractive conditions was presented by Ansari [50] as follows.

Definition 2.6 ([50]) A mapping $\Lambda : [0,+\infty)^2 \to \mathbb{R}$ is called C-type function if it is continuous and fulfills the following hypotheses:
(1) $\Lambda(\lambda,\mu) \leq \lambda$;
(2) $\Lambda(\lambda,\mu) = \lambda$ implies that either $\lambda = 0$ or $\mu = 0$ for all $\lambda,\mu \in [0,\infty)$.
We symbolize the C-type functions as $\mathtt{C}$.

Example 2.7 For all $\lambda,\mu \in [0,\infty)$, the following functions $\Re : [0,\infty)^2 \to \mathbb{R}$ are elements of $\mathtt{C}$:

- $\Lambda(\lambda,\mu) = \lambda - \mu$;
- $\Lambda(\lambda,\mu) = \varsigma\lambda$, $0 < \varsigma < 1$;
- $\Lambda(\lambda,\mu) = \frac{\lambda}{(1+\mu)^{\varepsilon}}$; $\varepsilon \in (0,+\infty)$;
- $\Lambda(\lambda,\mu) = \frac{\log(\mu+c^{\lambda})}{(1+\mu)}$, $c > 1$;
- $\Lambda(\lambda,1) = \frac{\ln(1+d^{\lambda})}{2}$, $d > e$;
- $\Lambda(\lambda,\mu) = \theta(\mu)$, where $\theta : [0,+\infty) \to [0,+\infty)$ is an upper semi-continuous function such that $\theta(0) = 0$, and $\theta(\mu) < \mu$ for $\mu > 0$;
- $\Lambda(\lambda,\mu) = \lambda\Omega(\lambda)$, $\Omega : [0,1) \to [0,1)$;
- $\Lambda(\lambda,\mu) = \lambda - \theta(\lambda)$;
- $\Lambda(\lambda,\mu) = \lambda - \frac{\mu}{1-\mu}$;
- $\Lambda(\lambda,\mu) = \lambda q(\lambda,\mu)$, where $q : [0,1) \times [0,1) \to [0,1)$ is a continuous function such that for all $\lambda,\mu > 0$, $q(\lambda,\mu) < 1$.

In this article, some new tripled coincidence point consequences for mixed-monotone mappings via the notion of C-type functions in POCbML spaces are introduced. Some examples to back our work are showed. Also, some theoretical results under various contractive conditions are discussed as corollaries. Eventually, some important results in integral types and the existence of solutions of a system of nonlinear integral equations are presented here as applications.

3 Main theorems

Theorem 3.1 *Assume that* $\Re : \chi^3 \to \chi$ *and* $\Theta : \chi \to \chi$ *are two mappings on a POCML space* $(\chi,\precsim,\xi)$ *such that*:
(i) $\Re(\chi^3) \subseteq \Im(\chi)$;
(ii) $\Re$ *is continuous*;
(iii) $\Im$ *is continuous and commutes with* $\Re$;
(iv) $\Re$ *has a mixed* $\Im$-*monotone property*;

(v) *there are* $\pi \in \Pi$, $\zeta \geq 0$, *and* $\Lambda \in \complement$ *such that*

$$\begin{aligned}&\xi\big(\Re(\wp,\hbar,\eth),\Re(x,y,z)\big)\\&\quad\leq \Lambda\begin{pmatrix}\pi\left(\max\{\xi(\Im(\wp),\Im(x)),\xi(\Im(\hbar),\Im(y)),\xi(\Im(\eth),\Im(z))\}\right),\\ \zeta\max\{\xi(\Im(\wp),\Im(x)),\xi(\Im(\hbar),\Im(y)),\xi(\Im(\eth),\Im(z))\}\end{pmatrix}\end{aligned} \tag{1}$$

for any $\wp,\hbar,\eth,x,y,z \in \chi$, *for which* $\Im(\wp) \precsim \Im(x)$, $\Im(y) \precsim \Im(\hbar)$, *and* $\Im(\eth) \precsim \Im(z)$. *If there exist* $\wp_\circ,\hbar_\circ,\eth_\circ \in \chi$ *such that* $\Im(\wp_\circ) \precsim \Re(\wp_\circ,\hbar_\circ,\eth_\circ)$, $\Im(\hbar_\circ) \succsim \Re(\hbar_\circ,\wp_\circ,\eth_\circ)$, *and* $\Im(\eth_\circ) \precsim \Re(\eth_\circ,\hbar_\circ,\wp_\circ)$, *then* $\Re$ *and* $\Im$ *have a tripled coincidence point.*

Proof Let $\wp_\circ,\hbar_\circ,\eth_\circ \in \chi$ with $\Im(\wp_\circ) \precsim \Re(\wp_\circ,\hbar_\circ,\eth_\circ)$, $\Im(\hbar_\circ) \succsim \Re(\hbar_\circ,\wp_\circ,\eth_\circ)$, and $\Im(\eth_\circ) \precsim \Re(\eth_\circ,\hbar_\circ,\wp_\circ)$. Since $\Re(\chi^3) \subseteq \Im(\chi)$, there exist $\wp_1,\hbar_1,\eth_1 \in \chi$ such that

$$\Im(\wp_1) = \Re(\wp_\circ,\hbar_\circ,\eth_\circ), \qquad \Im(\hbar_1) = \Re(\hbar_\circ,\wp_\circ,\eth_\circ), \quad \text{and} \quad \Im(\eth_1) = \Re(\eth_\circ,\hbar_\circ,\wp_\circ). \tag{2}$$

Continuing with the same scenario, there are $\{\wp_n\}$, $\{\hbar_n\}$, and $\{\eth_n\}$ in χ such that

$$\begin{aligned}&\Im(\wp_{n+1}) = \Re(\wp_n,\hbar_n,\eth_n), \qquad \Im(\hbar_{n+1}) = \Re(\hbar_n,\wp_n,\eth_n), \quad \text{and}\\&\Im(\eth_{n+1}) = \Re(\eth_n,\hbar_n,\wp_n).\end{aligned} \tag{3}$$

By induction, we shall show that

$$\begin{aligned}&\Im(\wp_n) \precsim \Im(\wp_{n+1}), \qquad \Im(\hbar_{n+1}) \precsim \Im(\hbar_n), \quad \text{and}\\&\Im(\eth_n) \precsim \Im(\eth_{n+1}) \quad \text{for all } n \in \mathbb{N}\cup\{0\}.\end{aligned} \tag{4}$$

Since $\Im(\wp_\circ) \precsim \Re(\wp_\circ,\hbar_\circ,\eth_\circ)$, $\Im(\hbar_\circ) \succsim \Re(\hbar_\circ,\wp_\circ,\eth_\circ)$, and $\Im(\eth_\circ) \precsim \Re(\eth_\circ,\hbar_\circ,\wp_\circ)$, and by (2), we have

$$\Im(\wp_\circ) \precsim \Im(\wp_1), \qquad \Im(\hbar_\circ) \succsim \Im(\hbar_1), \quad \text{and} \quad \Im(\eth_\circ) \precsim \Im(\eth_1).$$

This leads to (4) fulfilled for $n = 0$. Consider (4) to be realized for some fixed $n \in \mathbb{N}$. Because $\Re$ has a mixed $\Im$-monotone property, we have

$$\begin{aligned}\Im(\wp_{n+1}) &= \Re(\wp_n,\hbar_n,\eth_n) \precsim \Re(\wp_{n+1},\hbar_n,\eth_n) \precsim \Re(\wp_{n+1},\hbar_n,\eth_{n+1})\\&\precsim \Re(\wp_{n+1},\hbar_{n+1},\eth_{n+1}) = \Im(\wp_{n+2}),\\ \Im(\hbar_{n+2}) &= \Re(\hbar_{n+1},\wp_{n+1},\hbar_{n+1}) \precsim \Re(\hbar_{n+1},\wp_n,\hbar_{n+1}) \precsim \Re(\hbar_n,\wp_n,\hbar_{n+1})\\&\precsim \Re(\hbar_n,\wp_n,\hbar_n) = \Im(\hbar_{n+1}),\end{aligned}$$

and

$$\begin{aligned}\Im(\eth_{n+1}) &= \Re(\eth_n,\hbar_n,\wp_n) \precsim \Re(\eth_{n+1},\hbar_n,\wp_n) \precsim \Re(\eth_{n+1},\hbar_{n+1},\wp_n)\\&\precsim \Re(\eth_{n+1},\hbar_{n+1},\wp_{n+1}) = \Im(\eth_{n+2}).\end{aligned}$$

Thus, (4) is fulfilled. For each $n \in \mathbb{N}$, suppose that

$$\Im(\wp_n) = \Im(\wp_{n+1}), \qquad \Im(\hbar_n) = \Im(\hbar_{n+1}), \quad \text{and} \quad \Im(\eth_n) = \Im(\eth_{n+1}).$$

By (3), we deduce that a trio $(\wp_n, \hbar_n, \eth_n)$ is a coincidence point of $\Re$ and $\Im$. Now, consider at least for any $n \in \mathbb{N}$

$$\Im(\wp_n) \neq \Im(\wp_{n+1}), \qquad \Im(\hbar_n) \neq \Im(\hbar_{n+1}), \quad \text{and} \quad \Im(\eth_n) \neq \Im(\eth_{n+1}).$$

Applying (1), and since π is nondecreasing, we can get

$$\begin{aligned}
&\xi\left(\Im(\wp_n), \Im(\wp_{n+1})\right) \\
&\quad = \xi\left(\Re(\wp_{n-1}, \hbar_{n-1}, \eth_{n-1}), \Re(\wp_n, \hbar_n, \eth_n)\right) \\
&\quad \leq \Lambda \begin{pmatrix} \pi(\max\{\xi(\Im(\wp_{n-1}), \Im(\wp_n)), \xi(\Im(\hbar_{n-1}), \Im(\hbar_n)), \xi(\Im(\eth_{n-1}), \Im(\eth_n))\}), \\ \zeta \max\{\xi(\Im(\wp_{n-1}), \Im(\wp_n)), \xi(\Im(\hbar_{n-1}), \Im(\hbar_n)), \xi(\Im(\eth_{n-1}), \Im(\eth_n))\} \end{pmatrix} \\
&\quad \leq \pi\left(\max\left\{\xi\left(\Im(\wp_{n-1}), \Im(\wp_n)\right), \xi\left(\Im(\hbar_{n-1}), \Im(\hbar_n)\right), \xi\left(\Im(\eth_{n-1}), \Im(\eth_n)\right)\right\}\right), \qquad (5) \\
&\xi\left(\Im(\hbar_{n+1}), \Im(\hbar_n)\right) \\
&\quad = \xi\left(\Re(\hbar_n, \wp_n, \hbar_n), \Re(\hbar_{n-1}, \wp_{n-1}, \hbar_{n-1})\right) \\
&\quad \leq \Lambda \begin{pmatrix} \pi(\max\{\xi(\Im(\hbar_{n-1}), \Im(\hbar_n)), \xi(\Im(\wp_{n-1}), \Im(\wp_n)), \xi(\Im(\hbar_{n-1}), \Im(\hbar_n))\}), \\ \zeta \max\{\xi(\Im(\hbar_{n-1}), \Im(\hbar_n)), \xi(\Im(\wp_{n-1}), \Im(\wp_n)), \xi(\Im(\hbar_{n-1}), \Im(\hbar_n))\} \end{pmatrix} \\
&\quad = \Lambda \begin{pmatrix} \pi(\max\{\xi(\Im(\hbar_{n-1}), \Im(\hbar_n)), \xi(\Im(\wp_{n-1}), \Im(\wp_n))\}), \\ \zeta \max\{\xi(\Im(\hbar_{n-1}), \Im(\hbar_n)), \xi(\Im(\wp_{n-1}), \Im(\wp_n))\} \end{pmatrix} \\
&\quad \leq \pi\left(\max\left\{\xi\left(\Im(\hbar_{n-1}), \Im(\hbar_n)\right), \xi\left(\Im(\wp_{n-1}), \Im(\wp_n)\right)\right\}\right), \qquad (6)
\end{aligned}$$

and

$$\begin{aligned}
&\xi\left(\Im(\eth_n), \Im(\eth_{n+1})\right) \\
&\quad = \xi\left(\Xi(\eth_{n-1}, \hbar_{n-1}, a_{n-1}), \Xi(\eth_n, \hbar_n, a_n)\right) \\
&\quad \leq \Lambda \begin{pmatrix} \pi(\max\{\xi(\Im(\eth_{n-1}), \Im(\eth_n)), \xi(\Im(\hbar_{n-1}), \Im(\hbar_n)), \xi(\Im(\wp_{n-1}), \Im(\wp_n))\}), \\ \zeta \max\{\xi(\Im(\eth_{n-1}), \Im(\eth_n)), \xi(\Im(\hbar_{n-1}), \Im(\hbar_n)), \xi(\Im(\wp_{n-1}), \Im(\wp_n))\} \end{pmatrix} \\
&\quad \leq \pi\left(\max\left\{\xi\left(\Im(\eth_{n-1}), \Im(\eth_n)\right), \xi\left(\Im(\hbar_{n-1}), \Im(\hbar_n)\right), \xi\left(\Im(\wp_{n-1}), \Im(\wp_n)\right)\right\}\right). \qquad (7)
\end{aligned}$$

Take into account that $\pi(\nu) < \nu$ for all $\nu > 0$, then by (5), (6), and (7), one can get

$$\begin{aligned}
0 &< \max\left\{\xi\left(\Im(\wp_n), \Im(\wp_{n+1})\right), \xi\left(\Im(\hbar_{n+1}), \Im(\hbar_n)\right), \xi\left(\Im(\eth_n), \Im(\eth_{n+1})\right)\right\} \\
&\leq \pi\left(\max\left\{\xi\left(\Im(\eth_{n-1}), \Im(\eth_n)\right), \xi\left(\Im(\hbar_{n-1}), \Im(\hbar_n)\right), \xi\left(\Im(\wp_{n-1}), \Im(\wp_n)\right)\right\}\right) \\
&< \max\left\{\xi\left(\Im(\eth_{n-1}), \Im(\eth_n)\right), \xi\left(\Im(\hbar_{n-1}), \Im(\hbar_n)\right), \xi\left(\Im(\wp_{n-1}), \Im(\wp_n)\right)\right\}. \qquad (8)
\end{aligned}$$

It follows by (8) that

$$\begin{aligned}\Delta_n &= \max\{\xi(\Im(\wp_n),\Im(\wp_{n+1})),\xi(\Im(\hbar_{n+1}),\Im(\hbar_n)),\xi(\Im(\eth_n),\Im(\eth_{n+1}))\}\\ &< \max\{\xi(\Im(\eth_{n-1}),\Im(\eth_n)),\xi(\Im(\hbar_{n-1}),\Im(\hbar_n)),\xi(\Im(\wp_{n-1}),\Im(\wp_n))\}.\end{aligned}$$

Thus, Δ_n is a positive decreasing sequence. So there is $\kappa \geq 0$ such that

$$\lim_{n\to+\infty}\Delta_n = \kappa.$$

Consider $\kappa > 0$ and $n \to +\infty$ in (8), we can write

$$\begin{aligned}\kappa &\leq \lim_{n\to+\infty}\pi\big(\max\{\xi(\Im(\eth_{n-1}),\Im(\eth_n)),\xi(\Im(\hbar_{n-1}),\Im(\hbar_n)),\xi(\Im(\wp_{n-1}),\Im(\wp_n))\}\big)\\ &< \lim_{n\to+\infty}\Delta_{n-1} = \kappa.\end{aligned}$$

This is an inconsistency, hence

$$\lim_{n\to+\infty}\Delta_n = 0. \tag{9}$$

Now, we shall demonstrate that $\{\Im(\wp_n)\}$, $\{\Im(\hbar_n)\}$, and $\{\Im(\eth_n)\}$ are Cauchy sequences by an inconsistency method. So, let one of them not be Cauchy, that is,

$$\lim_{n,m\to+\infty}\xi(\Im(\wp_n),\Im(\wp_m)) \neq 0, \quad \text{or} \quad \lim_{n,m\to+\infty}\xi(\Im(\hbar_n),\Im(\hbar_m)) \neq 0, \quad \text{or}$$
$$\lim_{n,m\to+\infty}\xi(\Im(\eth_n),\Im(\eth_m)) \neq 0.$$

In other words, there are $\epsilon > 0$ and integers subsequences $\{m_k\}$ and $\{n_k\}$ with $n_k > m_k > k$ such that

$$\max\{\xi(\Im(\wp_{m_k}),\Im(\wp_{n_k})),\xi(\Im(\hbar_{m_k}),\Im(\hbar_{n_k})),\xi(\Im(\eth_{m_k}),\Im(\eth_{n_k}))\} \geq \epsilon. \tag{10}$$

If we adopt m_k the little integer with $n_k > m_k$ satisfying (10), then the following connection holds:

$$\max\{\xi(\Im(\wp_{m_k}),\Im(\wp_{n_k})),\xi(\Im(\hbar_{m_k}),\Im(\hbar_{n_k})),\xi(\Im(\eth_{m_k}),\Im(\eth_{n_k}))\} < \epsilon. \tag{11}$$

Thus, by (9), stipulation (ξ_3), and (11), we have

$$\begin{aligned}\lim_{k\to+\infty}\xi(\Im(\wp_{m_k}),\Im(\wp_{n_k})) &\leq \lim_{k\to+\infty}\big(\xi(\Im(\wp_{m_k}),\Im(\wp_{n_k-1})) + \xi(\Im(\wp_{n_k-1}),\Im(\wp_{n_k}))\big)\\ &\leq \lim_{k\to+\infty}\xi(\Im(\wp_{m_k}),\Im(\wp_{n_k-1})) \leq \epsilon.\end{aligned}$$

By the same logic, we can get

$$\lim_{k\to+\infty}\xi(\Im(\hbar_{m_k}),\Im(\hbar_{n_k})) \leq \lim_{k\to+\infty}\xi(\Im(\hbar_{m_k}),\Im(\hbar_{n_k-1})) \leq \epsilon,$$
$$\lim_{k\to+\infty}\xi(\Im(\eth_{m_k}),\Im(\eth_{n_k})) \leq \lim_{k\to+\infty}\xi(\Im(\eth_{m_k}),\Im(\eth_{n_k-1})) \leq \epsilon.$$

Once more, by (11), we can note

$$\begin{aligned}\xi\left(\Im(\wp_{m_k}),\Im(\wp_{n_k})\right) &\le \xi\left(\Im(\wp_{m_k}),\Im(\wp_{m_k-1})\right)\\ &\quad+\xi\left(\Im(\wp_{m_k-1}),\Im(\wp_{n_k-1})\right)+\xi\left(\Im(\wp_{n_k-1}),\Im(\wp_{n_k})\right)\\ &\le \xi\left(\Im(\wp_{m_k}),\Im(\wp_{m_k-1})\right)+\xi\left(\Im(\wp_{m_k-1}),\Im(\wp_{m_k})\right)\\ &\quad+\xi\left(\Im(\wp_{m_k}),\Im(\wp_{n_k-1})\right)+\xi\left(\Im(\wp_{n_k-1}),\Im(\wp_{n_k})\right)\\ &< \xi\left(\Im(\wp_{m_k}),\Im(\wp_{m_k-1})\right)+\xi\left(\Im(\wp_{m_k-1}),\Im(\wp_{m_k})\right)\\ &\quad+\epsilon+\xi\left(\Im(\wp_{n_k-1}),\Im(\wp_{n_k})\right).\end{aligned}$$

If $k\to+\infty$, and by (9), we can record

$$\lim_{k\to+\infty}\xi\left(\Im(\wp_{m_k}),\Im(\wp_{n_k})\right)\le\lim_{k\to+\infty}\xi\left(\Im(\wp_{m_k-1}),\Im(\wp_{n_k-1})\right)\le\epsilon. \tag{12}$$

Likewise, we have

$$\lim_{k\to+\infty}\xi\left(\Im(\hbar_{m_k}),\Im(\hbar_{n_k})\right)\le\lim_{k\to+\infty}\xi\left(\Im(\hbar_{m_k-1}),\Im(\hbar_{n_k-1})\right)\le\epsilon \tag{13}$$

and

$$\lim_{k\to+\infty}\xi\left(\Im(\eth_{m_k}),\Im(\eth_{n_k})\right)\le\lim_{k\to+\infty}\xi\left(\Im(\eth_{m_k-1}),\Im(\eth_{n_k-1})\right)\le\epsilon. \tag{14}$$

Applying (10) and (12)–(14), we observe that

$$\lim_{k\to+\infty}\max\left\{\begin{array}{c}\xi(\Im(\wp_{m_k-1}),\Im(\wp_{n_k-1})),\\ \xi(\Im(\hbar_{m_k-1}),\Im(\hbar_{n_k-1})),\\ \xi(\Im(\eth_{m_k-1}),\Im(\eth_{n_k-1}))\end{array}\right\}=\epsilon. \tag{15}$$

Now, by stipulation (1), we can get

$$\begin{aligned}&\xi\left(\Im(\wp_{m_k}),\Im(\wp_{n_k})\right)\\ &\quad=\xi\left(\Re(\wp_{m_k-1},\hbar_{m_k-1},\eth_{m_k-1}),\Re(\wp_{n_k-1},\hbar_{n_k-1},\eth_{n_k-1})\right)\\ &\quad\le\Lambda\begin{pmatrix}\pi(\max\{\xi(\Im(\wp_{m_k-1}),\Im(\wp_{n_k-1})),\xi(\Im(\hbar_{m_k-1}),\Im(\hbar_{n_k-1})),\xi(\Im(\eth_{m_k-1}),\Im(\eth_{n_k-1}))\}),\\ \zeta\max\{\xi(\Im(\wp_{m_k-1}),\Im(\wp_{n_k-1})),\xi(\Im(\hbar_{m_k-1}),\Im(\hbar_{n_k-1})),\xi(\Im(\eth_{m_k-1}),\Im(\eth_{n_k-1}))\}\end{pmatrix}\\ &\quad\le\pi\left(\max\left\{\xi\left(\Im(\wp_{m_k-1}),\Im(\wp_{n_k-1})\right),\xi\left(\Im(\hbar_{m_k-1}),\right.\right.\right.\\ &\qquad\left.\left.\left.\Im(\hbar_{n_k-1})\right),\xi\left(\Im(\eth_{m_k-1}),\Im(\eth_{n_k-1})\right)\right\}\right),\end{aligned} \tag{16}$$

$$\begin{aligned}&\xi\left(\Im(\hbar_{m_k}),\Im(\hbar_{n_k})\right)\\ &\quad=\xi\left(\Re(\hbar_{m_k-1},\wp_{m_k-1},\hbar_{m_k-1}),\Re(\hbar_{n_k-1},\wp_{n_k-1},\hbar_{n_k-1})\right)\\ &\quad\le\Lambda\begin{pmatrix}\pi(\max\{\xi(\Im(\hbar_{m_k-1}),\Im(\hbar_{n_k-1})),\xi(\Im(\wp_{m_k-1}),\Im(\wp_{n_k-1}))\}),\\ \zeta\max\{\xi(\Im(\hbar_{m_k-1}),\Im(\hbar_{n_k-1})),\xi(\Im(\wp_{m_k-1}),\Im(\wp_{n_k-1}))\}\end{pmatrix}\\ &\quad\le\pi\left(\max\left\{\xi\left(\Im(\hbar_{m_k-1}),\Im(\hbar_{n_k-1})\right),\xi\left(\Im(\wp_{m_k-1}),\Im(\wp_{n_k-1})\right)\right\}\right),\end{aligned} \tag{17}$$

and

$$\begin{aligned}
&\xi\left(\Im(\eth_{m_k}),\Im(\eth_{n_k})\right)\\
&\quad=\xi\left(\Re(\eth_{m_k-1},\hbar_{m_k-1},\wp_{m_k-1}),\Re(\eth_{n_k-1},\hbar_{n_k-1},\wp_{n_k-1})\right)\\
&\quad\le\Lambda\begin{pmatrix}\pi(\max\{\xi(\Im(\eth_{m_k-1}),\Im(\eth_{n_k-1})),\xi(\Im(\hbar_{m_k-1}),\Im(\hbar_{n_k-1})),\xi(\Im(\wp_{m_k-1}),\Im(\wp_{n_k-1}))\}),\\ \zeta\max\{\xi(\Im(\eth_{m_k-1}),\Im(\eth_{n_k-1})),\xi(\Im(\hbar_{m_k-1}),\Im(\hbar_{n_k-1})),\xi(\Im(\wp_{m_k-1}),\Im(\wp_{n_k-1}))\}\end{pmatrix}\\
&\quad\le\pi\left(\max\left\{\xi\left(\Im(\eth_{m_k-1}),\Im(\eth_{n_k-1})\right),\xi\left(\Im(\hbar_{m_k-1}),\Im(\hbar_{n_k-1})\right),\right.\right.\\
&\qquad\left.\left.\xi\left(\Im(\wp_{m_k-1}),\Im(\wp_{n_k-1})\right)\right\}\right).
\end{aligned}\tag{18}$$

The three inequalities (16)–(18) say that

$$\begin{aligned}
&\max\left\{\xi\left(\Im(\wp_{m_k}),\Im(\wp_{n_k})\right),\xi\left(\Im(\hbar_{m_k}),\Im(\hbar_{n_k})\right),\xi\left(\Im(\eth_{m_k}),\Im(\eth_{n_k})\right)\right\}\\
&\quad\le\pi\left(\max\begin{Bmatrix}\xi(\Im(\eth_{m_k-1}),\Im(\eth_{n_k-1})),\xi(\Im(\hbar_{m_k-1}),\Im(\hbar_{n_k-1})),\\ \xi(\Im(\wp_{m_k-1}),\Im(\wp_{n_k-1}))\end{Bmatrix}\right).
\end{aligned}\tag{19}$$

Letting $k\to+\infty$ in (19) and having in mind (15), we get

$$0<\epsilon\le\lim_{k\to+\infty}\max\begin{Bmatrix}\xi(\Im(\wp_{m_k-1}),\Im(\wp_{n_k-1})),\\ \xi(\Im(\hbar_{m_k-1}),\Im(\hbar_{n_k-1})),\\ \xi(\Im(\eth_{m_k-1}),\Im(\eth_{n_k-1}))\end{Bmatrix}\le\lim_{\nu\to\epsilon^+}\pi(\nu)<\epsilon.$$

Incompatibility. Hence $\{\Im(\wp_n)\}$, $\{\Im(\hbar_n)\}$, and $\{\Im(\eth_n)\}$ are Cauchy sequences in a POCML space. By completeness, there are $\wp,\hbar,\eth\in\chi$ such that

$$\lim_{n\to+\infty}\Im(\wp_n)=\wp,\qquad\lim_{n\to+\infty}\Im(\hbar_n)=\hbar,\quad\text{and}\quad\lim_{n\to+\infty}\Im(\eth_n)=\eth.\tag{20}$$

Applying the thought of continuity of $\Im$ on (20), we can get

$$\lim_{n\to+\infty}\Im(\Im\wp_n)=\Im\wp,\qquad\lim_{n\to+\infty}\Im(\Im\hbar_n)=\Im\hbar,\quad\text{and}\quad\lim_{n\to+\infty}\Im(\Im\eth_n)=\Im\eth.\tag{21}$$

Since $\Im$ commutes with $\Re$, then by (3) we can write

$$\begin{cases}\Im(\Im\wp_{n+1})=\Im(\Re(\wp_n,\hbar_n,\eth_n))=\Re(\Im\wp_n,\Im\hbar_n,\Im\eth_n),\\ \Im(\Im\hbar_{n+1})=\Im(\Re(\hbar_n,\wp_n,\hbar_n))=\Re(\Im\hbar_n,\Im\wp_n,\Im\hbar_n),\\ \Im(\Im\eth_{n+1})=\Im(\Re(\eth_n,\hbar_n,\wp_n))=\Re(\Im\eth_n,\Im\hbar_n,\Im\wp_n).\end{cases}\tag{22}$$

Letting $n\to+\infty$ in (22) and taking into account relations (20)–(21) and the continuity of $\Re$, we have

$$\begin{aligned}
\Im\wp&=\lim_{n\to+\infty}\Im(\Im\wp_{n+1})=\lim_{n\to+\infty}\Im\left(\Re(\wp_n,\hbar_n,\eth_n)\right)\\
&=\Re\left(\lim_{n\to+\infty}\Im\wp_n,\lim_{n\to+\infty}\Im\hbar_n,\lim_{n\to+\infty}\Im\eth_n\right)=\Re(\wp,\hbar,\eth),\\
\Im\hbar&=\lim_{n\to+\infty}\Im(\Im\hbar_{n+1})=\lim_{n\to+\infty}\Im\left(\Re(\hbar_n,\wp_n,\hbar_n)\right)
\end{aligned}$$

$$= \Re\Big(\lim_{n\to+\infty}\Im\hbar_n, \lim_{n\to+\infty}\Im\wp_n, \lim_{n\to+\infty}\Im\hbar_n\Big) = \Re(\hbar,\wp,\hbar),$$

$$\Im\eth = \lim_{n\to+\infty}\Im(\Im\eth_{n+1}) = \lim_{n\to+\infty}\Im\big(\Re(\eth_n,\hbar_n,\wp_n)\big)$$

$$= \Re\Big(\lim_{n\to+\infty}\Im\eth_n, \lim_{n\to+\infty}\Im\hbar_n, \lim_{n\to+\infty}\Im\wp_n\Big) = \Re(\eth,\hbar,\wp).$$

Thus, there is a tripled coincidence point of $\Re$ and $\Im$. This ends the demonstration. □

The question arises here. What happens when you omit the continuity stipulation of the mapping $\Re$? To answer this query, we give the following theorem.

Theorem 3.2 *Let $(\chi,\precsim,\xi)$ be a POCML space. Assume that $\Re:\chi^3\to\chi$ and $\Im:\chi\to\chi$ fulfill the following hypotheses*:

(i) $\Re(\chi^3)\subseteq\Im(\chi)$;

(ii) *$\Re$ has a mixed $\Im$-monotone property*;

(iii) *$(\Im(\chi),\xi)$ is a complete metric-like space and χ is obligated by the following assumptions*:

  (I) *$l_n\precsim l$ if a nondecreasing sequence $l_n\to l$, $n\to+\infty$*,

  (II) *$j_n\succsim j$ if a nonincreasing sequence $j_n\to j$, $n\to+\infty$*;

(iv) *$\Im$ is continuous and commutes with $\Re$*;

(v) *There are $\pi\in\Pi$, $\zeta\geq 0$, and $\Lambda\in\complement$ such that*

$$\xi\big(\Re(\wp,\hbar,\eth),\Re(x,y,z)\big) \leq \Lambda\begin{pmatrix}\pi(\max\{\xi(\Im(\wp),\Im(x)),\xi(\Im(\hbar),\Im(y)),\xi(\Im(\eth),\Im(z))\}),\\ \zeta\max\{\xi(\Im(\wp),\Im(x)),\xi(\Im(\hbar),\Im(y)),\xi(\Im(\eth),\Im(z))\}\end{pmatrix}.$$

If there exist $\wp_\circ,\hbar_\circ,\eth_\circ\in\chi$ such that $\Im(\wp_\circ)\precsim\Re(\wp_\circ,\hbar_\circ,\eth_\circ)$, $\Im(\hbar_\circ)\succsim\Re(\hbar_\circ,\wp_\circ,\eth_\circ)$, and $\Im(\eth_\circ)\precsim\Re(\eth_\circ,\hbar_\circ,\wp_\circ)$, then $\Re$ and $\Im$ have a tripled coincidence point.

Proof The same scenario of Theorem 3.1 implies that $\{\Im(\wp_n)\}$, $\{\Im(\hbar_n)\}$, and $\{\Im(\eth_n)\}$ are Cauchy sequences in a complete metric-like space $(\Im(\chi),\xi)$. Then there are $\wp,\hbar,\eth\in\chi$ such that (20) is achieved. By nondecreasing of $\{\Im(\wp_n)\}$ and $\{\Im(\eth_n)\}$, nonincreasing of $\{\Im(\hbar_n)\}$, and properties (I) and (II) of χ, we can write

$$\Im(\wp_n)\precsim\Im(\wp),\qquad \Im(\hbar_n)\succsim\Im(\hbar),\quad\text{and}\quad \Im(\eth_n)\precsim\Im(\eth)\quad\text{for all } n\geq 0.$$

If $\Im(\wp_n)=\Im(\wp)$, $\Im(\hbar_n)=\Im(\hbar)$, and $\Im(\eth_n)=\Im(\eth)$, then $\Im(\wp)=\Im(\wp_n)\precsim\Im(\wp_{n+1})\precsim\Im(\wp)=\Im(\wp_n)$, $\Im(\hbar)\precsim\Im(\hbar_{n+1})\precsim\Im(\hbar_n)=\Im(\hbar)$, and $\Im(\eth)=\Im(\eth_n)\precsim\Im(\eth_{n+1})\precsim\Im(\eth)=\Im(\eth_n)$. This leads to

$$\Im(\wp_n)=\Im(\wp_{n+1})=\Re(\wp_n,\hbar_n,\eth_n),\qquad \Im(\hbar_n)=\Im(\hbar_{n+1})=\Re(\hbar_n,\eth_n,\wp_n),$$

and

$$\Im(\eth_n)=\Im(\eth_{n+1})=\Re(\eth_n,\wp_n,\hbar_n).$$

Accordingly, the trio $(\wp_n, \hbar_n, \eth_n)$ is a tripled coincidence point of $\Im$ and $\Re$. So, we postulate, for all $n \geq 0$, $(\Im\wp_n, \Im\hbar_n, \Im\eth_n) \neq (\Im\wp, \Im\hbar, \Im\eth)$; consequently, by (1) we can get

$$\begin{aligned}
&\xi\left(\Im\wp, \Re(\wp, \hbar, \eth)\right) \\
&\quad \leq \xi(\Im\wp, \Im\wp_{n+1}) + \xi\left(\Im\wp_{n+1}, \Re(\wp, \hbar, \eth)\right) \\
&\quad = \xi(\Im\wp, \Im\wp_{n+1}) + \xi\left(\Re(\wp_n, \hbar_n, \eth_n), \Re(\wp, \hbar, \eth)\right) \\
&\quad \leq \xi(\Im\wp, \Im\wp_{n+1}) + \Lambda\begin{pmatrix} \pi(\max\{\xi(\Im(\wp_n), \Im(\wp)), \xi(\Im(\hbar_n), \Im(\hbar)), \xi(\Im(\eth_n), \Im(\eth))\}), \\ \zeta\max\{\xi(\Im(\wp_n), \Im(\wp)), \xi(\Im(\hbar_n), \Im(\hbar)), \xi(\Im(\eth_n), \Im(\eth))\} \end{pmatrix} \\
&\quad \leq \xi(\Im\wp, \Im\wp_{n+1}) + \pi\left(\max\left\{\xi\left(\Im(\wp_n), \Im(\wp)\right), \xi\left(\Im(\hbar_n), \Im(\hbar)\right), \xi\left(\Im(\eth_n), \Im(\eth)\right)\right\}\right).
\end{aligned} \tag{23}$$

Letting $n \to +\infty$ in (23) and using (20), we deduce that $\xi(\Im\wp, \Re(\wp, \hbar, \eth)) = 0$, thus $\Im\wp = \Re(\wp, \hbar, \eth)$. By the same manner, we can write $\Im\hbar = \Re(\hbar, \wp, \hbar)$ and $\Im\eth = \Re(\eth, \hbar, \wp)$. Then we have reached the end of the proof. □

Mathematicians in this direction can stir up the following: What about the structure and exclusivity of a tripled combined fixed point? To reduce this excitement, we recognize a partial ordering $(\chi, \leq)$ as follows: For all $(\wp, \hbar, \eth)$ and (x, y, z) belonging to the product χ^3,

$$(\wp, \hbar, \eth) \leq (x, y, z) \quad \text{if and only if} \quad \wp \leq x, \qquad \hbar \geq y, \qquad \eth \leq z.$$

Let us say that the trios $(\wp, \hbar, \eth)$ and (x, y, z) are comparable if

$$(\wp, \hbar, \eth) \leq (x, y, z) \quad \text{or} \quad (x, y, z) \leq (\wp, \hbar, \eth);$$

also, $(\wp, \hbar, \eth)$ is equal to (x, y, z) iff $\wp = x$, $\hbar = y$, and $\eth = z$.

Now, the excitement is killed by the following important theorem.

Theorem 3.3 *Besides the presumptions of Theorem 3.1, postulate for all* $(\wp, \hbar, \eth), (x, y, z) \in \chi^3$ *there exists* $(\alpha, \beta, \gamma) \in \chi^3$ *such that* $(\Re(\alpha, \beta, \gamma), \Re(\beta, \alpha, \beta), \Re(\gamma, \beta, \alpha))$ *is comparable to* $(\Re(\wp, \hbar, \eth), \Re(\hbar, \wp, \hbar), \Re(\eth, \hbar, \wp))$ *and* $(\Re(x, y, z), \Re(y, x, y), \Re(z, y, x))$. *Therefore, there is a unique tripled combined fixed point* $(\wp, \hbar, \eth)$ *for the mappings* $\Re$ *and* $\Im$, *i.e.,*

$$\wp = \Im\wp = \Re(\wp, \hbar, \eth), \qquad \hbar = \Im\hbar = \Re(\hbar, \wp, \hbar), \quad \textit{and} \quad \eth = \Im\eth = \Re(\eth, \hbar, \wp)).$$

Proof According to Theorem 3.1, the set of tripled coincidence points of $\Re$ and $\Im$ is nonempty. Thence, we assume that $(\wp, \hbar, \eth)$ and (x, y, z) are two tripled coincidence points of $\Re$ and $\Im$, i.e.,

$$\begin{aligned}
&\Re(\wp, \hbar, \eth) = \Im\wp, \qquad \Re(x, y, z) = \Im x, \\
&\Re(\hbar, \wp, \hbar) = \Im\hbar, \qquad \Re(y, x, y) = \Im y, \\
&\Re(\eth, \hbar, \wp) = \Im\eth, \qquad \Re(z, y, x) = \Im z.
\end{aligned} \tag{24}$$

First, we shall show that $(\Im\wp, \Im\hbar, \Im\eth)$ equals $(\Im x, \Im y, \Im z)$. Consider the hypothesis of comparable fulfilled and define sequences $\{\Im\alpha_n\}$, $\{\Im\beta_n\}$, and $\{\Im\gamma_n\}$ such that

$$\alpha_n = \alpha_\circ, \qquad \beta_n = \beta_\circ, \qquad \gamma_n = \gamma_\circ,$$

and for all $n \geq 1$,

$$\Im\alpha_n = \Re(\alpha_{n-1}, \beta_{n-1}, \gamma_{n-1}),$$
$$\Im\beta_n = \Re(\beta_{n-1}, \alpha_{n-1}, \beta_{n-1}),$$
$$\Im\gamma_n = \Re(\gamma_{n-1}, \beta_{n-1}, \alpha_{n-1}).$$

On the other hand, appoint $\wp_\circ = \wp$, $\hbar_\circ = \hbar$, $\eth = \eth_\circ$, $x_\circ = x$, $y_\circ = y$, and $z = z_\circ$, and by the same manner, define the sequences $\{\Im\wp_n\}$, $\{\Im\hbar_n\}$, $\{\Im\eth_n\}$, $\{\Im x_n\}$, $\{\Im y_n\}$, and $\{\Im z_n\}$. Then it is easy to conclude that

$$\Im\wp_n = \Re(\wp, \hbar, \eth), \qquad \Im x_n = \Re(x, y, z),$$
$$\Im\hbar_n = \Re(\hbar, \wp, \hbar), \qquad \Im y_n = \Re(y, x, y),$$
$$\Im\eth_n = \Re(\eth, b, \wp), \qquad \Im z_n = \Re(z, y, x),$$

for all $n \geq 1$. Since $(\Re(\wp,\hbar,\eth), \Re(\hbar,\wp,\hbar), \Re(\eth,\hbar,\wp)) = (\Im\wp_1, \Im\hbar_1, \Im\eth_1) = (\Im\wp, \Im\hbar, \Im\eth)$ is comparable to $(\Re(\alpha,\beta,\gamma), \Re(\beta,\alpha,\beta), \Re(\gamma,\beta,\alpha)) = (\Im\alpha_1, \Im\beta_1, \Im\gamma_1)$, therefore $(\Im\wp, \Im\hbar, \Im\eth) \geq (\Im\alpha_1, \Im\beta_1, \Im\eth_1)$, by repeating for all n, we get

$$(\Im\wp, \Im\hbar, \Im\eth) \succsim (\Im\alpha_n, \Im\beta_n, \Im\gamma_n). \tag{25}$$

Applying (1), (25), and (24), we can write

$$\begin{aligned}
\xi(\Im\alpha_{n+1}, \Im\wp) &= \xi\big(\Re(\alpha_n, \beta_n, \gamma_n), \Re(\wp, \hbar, \eth)\big) \\
&\leq \Lambda\begin{pmatrix} \pi(\max\{\xi(\Im((\alpha_n), \Im(\wp)), \xi(\Im(\beta_n), \Im(\hbar)), \xi(\Im(\gamma_n), \Im(\eth))\}), \\ \zeta \max\{\xi(\Im((\alpha_n), \Im(\wp)), \xi(\Im(\beta_n), \Im(\hbar)), \xi(\Im(\gamma_n), \Im(\eth))\} \end{pmatrix} \\
&\leq \pi(\max\{\xi(\Im((\alpha_n), \Im(\wp)), \xi(\Im(\beta_n), \Im(\hbar)), \xi(\Im(\gamma_n), \Im(\eth))\}),
\end{aligned} \tag{26}$$

$$\begin{aligned}
\xi(\Im\hbar, \Im\beta_{n+1}) &= \xi\big(\Re(\hbar, \wp, \hbar), \Re(\beta_n, \alpha_n, \beta_n)\big) \\
&\leq \Lambda\begin{pmatrix} \pi(\max\{\xi(\Im(\hbar), \Im(\beta_n)), \xi(\Im(\wp), \Im(\alpha_n))\}), \\ \zeta \max\{\xi(\Im(\hbar), \Im(\beta_n)), \xi(\Im(\wp), \Im(\alpha_n))\} \end{pmatrix} \\
&\leq \pi\big(\max\{\xi(\Im(\hbar), \Im(\beta_n)), \xi(\Im(\wp), \Im(\alpha_n))\}\big) \\
&\leq \pi\left(\max\begin{Bmatrix} \xi(\Im(\hbar), \Im(\beta_n)), \xi(\Im(\wp), \Im(\alpha_n)), \\ \xi(\Theta(\eth), \Theta(\gamma_n)) \end{Bmatrix}\right),
\end{aligned} \tag{27}$$

and

$$\begin{aligned}
\xi(\Im\eth, \Im\gamma_{n+1}) &= \xi\big(\Re(\eth, \hbar, \wp), \Re(\gamma_n, \beta_n, \alpha_n)\big) \\
&\leq \Lambda\begin{pmatrix} \pi(\max\{\xi(\Im(\eth), \Im(\gamma_n)), \xi(\Im(\hbar), \Im(\beta_n)), \xi(\Im(\wp), \Im(\alpha_n))\}), \\ \zeta \max\{\xi(\Im(\eth), \Im(\gamma_n)), \xi(\Im(\hbar), \Im(\beta_n)), \xi(\Im(\wp), \Im(\alpha_n))\} \end{pmatrix} \\
&\leq \pi\big(\max\{\xi(\Im(\eth), \Im(\gamma_n)), \xi(\Im(\hbar), \Im(\beta_n)), \xi(\Im(\wp), \Im(\alpha_n))\}\big).
\end{aligned} \tag{28}$$

Inequalities (26)–(28) indicate that

$$\begin{aligned}&\max\{\xi(\Im\alpha_{n+1},\Im\wp),\xi(\Im\hbar,\Im\beta_{n+1}),\xi(\Im\eth,\Im\gamma_{n+1})\}\\&\quad\leq\pi\big(\max\{\xi(\Im(\eth),\Im(\gamma_n)),\xi(\Im(\hbar),\Im(\beta_n)),\xi(\Im(\wp),\Im(\alpha_n))\}\big).\end{aligned}$$

Subsequently, for all $n\geq 1$, we can get

$$\begin{aligned}&\max\{\xi(\Im(\eth),\Im(\gamma_n)),\xi(\Im(\hbar),\Im(\beta_n)),\xi(\Im(\wp),\Im(\alpha_n))\}\\&\quad\leq\pi^n\big(\max\{\xi(\Im(\eth),\Im(\gamma_\circ)),\xi(\Im(\hbar),\Im(\beta_\circ)),\xi(\Im(\wp),\Im(\alpha_\circ))\}\big).\end{aligned}\tag{29}$$

Since $\pi(\iota)<\iota$ and $\lim_{\rho\to\iota^+}\pi(\rho)<\iota$, then for all $\iota>0$, $\lim_{n\to+\infty}\pi^n(\iota)=0$. Thus, assign this on (29), after letting $n\to+\infty$, we have

$$\lim_{n\to+\infty}\max\{\xi(\Im(\eth),\Im(\gamma_n)),\xi(\Im(\hbar),\Im(\beta_n)),\xi(\Im(\wp),\Im(\alpha_n))\}=0$$

yields

$$\lim_{n\to+\infty}\xi(\Im(\eth),\Im(\gamma_n))=\lim_{n\to+\infty}\xi(\Im(\hbar),\Im(\beta_n))=\lim_{n\to+\infty}\xi(\Im(\wp),\Im(\alpha_n))=0.\tag{30}$$

In a similar way, we can write

$$\lim_{n\to+\infty}\xi(\Im z,\Im\gamma_n)=\lim_{n\to+\infty}\xi(\Im y,\Im\beta_n)=\lim_{n\to+\infty}\xi(\Im\alpha_n,\Im x)=0.\tag{31}$$

Combining (30) and (31), we deduce that $(\Im\wp,\Im\hbar,\Im\eth)$ and $(\Im x,\Im y,\Im z)$ are equal. As $\Im\wp=\Re(\wp,\hbar,\eth)$, $\Im\hbar=\Re(\hbar,\wp,\hbar)$, $\Im c=\Re(\eth,\hbar,\wp)$ and $\Im$, $\Re$ are commutes, then we have

$$\begin{aligned}\Im\wp^*&=\Im(\Im\wp)=\Im\big(\Re(\wp,\hbar,\eth)\big)=\Re(\Im\wp,\Im\hbar,\Im\eth),\\\Im\hbar^*&=\Im(\Im\hbar)=\Im\big(\Re(\hbar,\wp,\hbar)\big)=\Re(\Im\hbar,\Im\wp,\Im\hbar),\\\Im\eth^*&=\Im(\Im\eth)=\Im\big(\Re(\eth,\hbar,\wp)\big)=\Re(\Im\eth,\Im\hbar,\Im\wp),\end{aligned}$$

where $a^*=\Im\wp$, $\hbar^*=\Im\hbar$, and $\eth^*=\Im\eth$. Therefore the trio $(\wp^*,\hbar^*,\eth^*)$ is a tripled coincidence point of $\Im$ and $\Re$. Hence, $(\Im\wp^*,\Im\hbar^*,\Im\eth^*)$ and $(\Im\wp,\Im\hbar,\Im\eth)$ are equal, so, one can write

$$\Im\wp^*=\Im\wp=\wp^*,\qquad\Im\hbar^*=\Im\hbar=\hbar^*,\quad\text{and}\quad\Im\eth^*=\Im\eth=\eth^*.$$

Thus, $(\wp^*,\hbar^*,\eth^*)$ is a tripled common fixed of $\Im$ and $\Re$. The uniqueness follows immediately by (1). □

It is known that the numerical examples clarify and strengthen the theoretical results, so we shall present some examples in what follows.

Example 3.4 Let $\Lambda(\lambda,\mu)=\varrho\lambda$, $0<\varrho<1$, and $\chi=\mathbb{R}$ be equipped with

$$\xi(\wp,\hbar)=\wp+\hbar$$

for all $\wp, \hbar \in \chi$. Define the order relation $\preceq$ by

$$(\wp \preceq_\chi \hbar \Longleftrightarrow \wp = \hbar) \quad \text{or} \quad \bigl(\wp, \hbar \in [0,1] \text{ and } \wp \leq \hbar\bigr).$$

It is clear that $(\Im(\chi), \xi)$ is a complete metric-like space. Define $\Im : \chi \to \chi$ and $\Re : \chi^3 \to \chi$ by

$$\Im(\wp) = \begin{cases} \frac{3}{2}\wp & \text{if } \wp < 0, \\ \wp & \text{if } \wp \in [0,1], \\ \frac{4}{3}\wp & \text{if } \wp > 1, \end{cases}$$

$$\Re(\wp, \hbar, \eth) = \frac{\wp + \hbar + \eth}{8}.$$

It is obvious that $\Re(\chi^3) \subset \Im(\chi)$, $\Re$ has a mixed $\Im$-monotone property. Now we will go to investigate the contractive condition of Theorem 3.2 for all $\wp, \hbar, \eth, x, y, z \in \chi$ such that $\Im\wp \preceq_\chi \Im x$, $\Im y \preceq_\chi \Im\hbar$, and $\Im\eth \preceq_\Upsilon \Im z$. Take $\pi(\nu) = \frac{1}{2}\nu$ for all $\nu \in [0, +\infty)$, $\zeta \geq 0$, and $\varrho = \frac{3}{4}$. Now, we check the following statuses:

► *Status i.* If $\wp, \hbar, \eth, x, y, z \in [0,1]$, we have

$$\begin{aligned} \xi\bigl(\Re(\wp, \hbar, \eth), \Re(x, y, z)\bigr) &= \frac{\wp + \hbar + \eth}{8} + \frac{x + y + z}{8} \\ &= \frac{\wp + x}{8} + \frac{\hbar + y}{8} + \frac{c + z}{8} \\ &\leq \frac{3}{8}\max\{\wp + x, \hbar + x, \eth + z\} \\ &= \frac{3}{4} \times \frac{1}{2}\max\{\wp + x, \hbar + y, \eth + z\} \\ &= \frac{3}{4}\pi\bigl(\max\bigl\{\xi\bigl(\Im(\wp), \Im(x)\bigr), \xi\bigl(\Im(\hbar), \Im(y)\bigr), \xi\bigl(\Im(\eth), \Im(z)\bigr)\bigr\}\bigr) \\ &= \Lambda\begin{pmatrix} \pi(\max\{\xi(\Im(\wp), \Im(x)), \xi(\Im(\hbar), \Im(y)), \xi(\Im(\eth), \Im(z))\}), \\ \zeta\max\{\xi(\Im(\wp), \Im(x)), \xi(\Im(\hbar), \Im(y)), \xi(\Im(\eth), \Im(z))\} \end{pmatrix}. \end{aligned}$$

► *Status ii.* If $\wp, x, \hbar, y \in [0,1]$, $\eth, z \notin [0,1]$, here $\Im\eth, \Im z \notin [0,1]$ and since they must be comparable, $\Im(z) = \Im\eth$ and $z = \eth$,

$$\begin{aligned} \xi\bigl(\Re(\wp, \hbar, \eth), \Re(x, y, z)\bigr) &= \frac{\wp + \hbar + \eth}{8} + \frac{x + y + z}{8} \\ &= \frac{\wp + x}{8} + \frac{\hbar + y}{8} + \frac{2z}{8} \\ &\leq \frac{3}{8}\max\{\wp + x, \hbar + x, 2z\} \\ &\leq \frac{3}{8}\begin{cases} \max\{\wp + x, \hbar + x, 3z\} & \text{if } \wp < 0, \\ \max\{\wp + x, \hbar + x, \frac{8}{3}z\} & \text{if } \wp > 1 \end{cases} \\ &= \frac{3}{4} \times \frac{1}{2}\max\bigl\{\xi\bigl(\Im(\wp), \Im(x)\bigr), \xi\bigl(\Im(\hbar), \Im(y)\bigr), \xi\bigl(\Im(\eth), \Im(z)\bigr)\bigr\} \end{aligned}$$

$$
\begin{aligned}
&= \frac{3}{4}\pi\left(\max\left\{\xi\left(\Im(\wp),\Im(x)\right),\xi\left(\Im(\hbar),\Im(y)\right),\xi\left(\Im(\eth),\Im(z)\right)\right\}\right)\\
&= \Lambda\begin{pmatrix}\pi(\max\{\xi(\Im(\wp),\Im(x)),\xi(\Im(\hbar),\Im(y)),\xi(\Im(\eth),\Im(z))\}),\\ \zeta\max\{\xi(\Im(\wp),\Im(x)),\xi(\Im(\hbar),\Im(y)),\xi(\Im(\eth),\Im(z))\}\end{pmatrix}.
\end{aligned}
$$

► *Status iii*. If ($\wp, x, \eth, z \in [0,1]$ and $\hbar, y \notin [0,1]$), or ($\hbar, y, \eth, z \in [0,1]$ and $\wp, x \notin [0,1]$), we have the same results of Status ii.

► *Status iv*. If $\wp, x \in [0,1]$ and $\hbar, y, \eth, z \notin [0,1]$, here $\Im\hbar, \Im y, \Im\eth, \Im z \notin [0,1]$ and since they must be comparable, $\Im\hbar = \Im y$ and $\Im\eth = \Im z$, so $\hbar = y$ and $\eth = z$,

$$
\begin{aligned}
\xi\left(\Re(\wp,\hbar,\eth),\Re(x,y,z)\right) &= \frac{\wp+\hbar+\eth}{8}+\frac{x+y+z}{8}\\
&= \frac{\wp+x}{8}+\frac{2y}{8}+\frac{2z}{8}\\
&\le \frac{3}{8}\max\{a+x,2y,2z\}\\
&\le \frac{3}{8}\begin{cases}\max\{\wp+x,3y,3z\} & \text{if } a<0,\\ \max\{\wp+x,\frac{8}{3}y,\frac{8}{3}z\} & \text{if } a>1\end{cases}\\
&= \frac{3}{4}\times\frac{1}{2}\max\left\{\xi\left(\Im(\wp),\Im(x)\right),\xi\left(\Im(b),\Im(y)\right),\xi\left(\Im(\eth),\Im(z)\right)\right\}\\
&= \frac{3}{4}\pi\left(\max\left\{\xi\left(\Im(\wp),\Im(x)\right),\xi\left(\Im(b),\Im(y)\right),\xi\left(\Im(\eth),\Im(z)\right)\right\}\right)\\
&= \Lambda\begin{pmatrix}\pi(\max\{\xi(\Im(\wp),\Im(x)),\xi(\Im(b),\Im(y)),\xi(\Im(\eth),\Im(z))\}),\\ \zeta\max\{\xi(\Im(\wp),\Im(x)),\xi(\Im(b),\Im(y)),\xi(\Im(\eth),\Im(z))\}\end{pmatrix}.
\end{aligned}
$$

► *Status v*. If ($\hbar, y \in [0,1]$ and $\wp, x, \eth, z \notin [0,1]$) or ($\eth, z \in [0,1]$ and $\wp, x, \hbar, y \notin [0,1]$), we treat it analogously to Status iv.

► *Status vi*. If $\wp, x, \hbar, y, \eth, z \notin [0,1]$, then the only possibility for $\Im x$, $\Im\wp$ as well as $\Im y$, $\Im\hbar$ and $\Im z$, $\Im\eth$ to be comparable is that $x = \wp$, $y = \hbar$, and $z = \eth$,

$$
\begin{aligned}
\xi\left(\Re(\wp,\hbar,\eth),\Re(x,y,z)\right) &= \frac{\wp+\hbar+\eth}{8}+\frac{x+y+z}{8}\\
&= \frac{2x}{8}+\frac{2y}{8}+\frac{2z}{8}\\
&\le \frac{3}{8}\max\{2x,2y,2z\}\\
&\le \frac{3}{8}\begin{cases}\max\{3x,3y,3z\} & \text{if } a<0,\\ \max\{\frac{8}{3}x,\frac{8}{3}y,\frac{8}{3}z\} & \text{if } a>1\end{cases}\\
&= \frac{3}{4}\times\frac{1}{2}\max\left\{\xi\left(\Im(\wp),\Im(x)\right),\xi\left(\Im(\hbar),\Im(y)\right),\xi\left(\Im(\eth),\Im(z)\right)\right\}\\
&= \frac{3}{4}\pi\left(\max\left\{\xi\left(\Im(\wp),\Im(x)\right),\xi\left(\Im(\hbar),\Im(y)\right),\xi\left(\Im(\eth),\Im(z)\right)\right\}\right)\\
&= \Lambda\begin{pmatrix}\pi(\max\{\xi(\Im(\wp),\Im(x)),\xi(\Im(\hbar),\Im(y)),\xi(\Im(\eth),\Im(z))\}),\\ \zeta\max\{\xi(\Im(\wp),\Im(x)),\xi(\Im(\hbar),\Im(y)),\xi(\Im(\eth),\Im(z))\}\end{pmatrix}.
\end{aligned}
$$

The six statuses complete postulates of Theorem 3.2 and $(0,0,0)$ is a tripled coincidence common fixed point of $\Im$ and $\Re$.

Example 3.5 Let $\Lambda(\lambda,\mu)=\lambda-\mu$ and $\chi=[0,+\infty)$ be equipped with

$$\xi(\wp,\hbar)=\max\{\wp,\hbar\}$$

for all $\wp,\hbar\in\chi$. Define the order relation by

$$\wp,\hbar\in\Upsilon,\quad (\wp\leq_\chi\hbar\Longleftrightarrow\wp=\hbar=0)\quad\text{or}\quad\bigl(\wp,\hbar\in(0,+\infty)\text{ and }\wp\leq\hbar\bigr).$$

It is obvious that $(\Im(\chi),\xi)$ is a complete metric-like space. Let us define $\Im:\chi\to\chi$ and $\Re:\chi^3\to\chi$ by

$$\chi(\wp)=\frac{1}{128}\wp,\quad\text{and}\quad\Re(\wp,\hbar,\eth)=\begin{cases}\frac{1}{130} & \text{if }\wp\hbar\eth\neq0,\\ 0 & \text{if }\wp\hbar\eth=0\end{cases}$$

for all $\wp,\hbar,\eth\in\chi$. It is evident that $\Re(\chi^3)\subset\Im(\chi)$, Ξ has a mixed Θ-monotone property. Now, for all $\wp,\hbar,\eth,x,y,z\in\chi$, if $\wp\hbar\eth=0$ or $xyz=0$, then the contractive condition of Theorem 3.2 is verified directly, so we discuss it when $\wp\hbar\eth\neq0$ or $xyz\neq0$. Take into account $\pi(\nu)=\frac{1}{8}\nu$ for all $\nu\in[0,+\infty)$ and $\zeta=\frac{15}{128}$. Thus, we have

$$\begin{aligned}\xi\bigl(\Re(\wp,\hbar,\eth),\Re(x,y,z)\bigr)&=\frac{1}{130}\\&\leq\frac{1}{128}\max\bigl\{\max\{\wp,x\},\max\{\hbar,y\},\max\{\eth,z\}\bigr\}\\&=\left(\frac{1}{8}-\frac{15}{128}\right)\max\bigl\{\xi\bigl(\Im(\wp),\Im(x)\bigr),\xi\bigl(\Im(\hbar),\Im(y)\bigr),\xi\bigl(\Im(\eth),\Im(z)\bigr)\bigr\}\\&=\pi\bigl(\max\bigl\{\xi\bigl(\Im(\wp),\Im(x)\bigr),\xi\bigl(\Im(\hbar),\Im(y)\bigr),\xi\bigl(\Im(\eth),\Im(z)\bigr)\bigr\}\bigr)\\&\quad-\zeta\max\bigl\{\xi\bigl(\Im(\wp),\Im(x)\bigr),\xi\bigl(\Im(\hbar),\Im(y)\bigr),\xi\bigl(\Im(\eth),\Im(z)\bigr)\bigr\}\\&=\Lambda\begin{pmatrix}\pi(\max\{\xi(\Im(\wp),\Im(x)),\xi(\Im(\hbar),\Im(y)),\xi(\Im(\eth),\Im(z))\}),\\ \zeta\max\{\xi(\Im(\wp),\Im(x)),\xi(\Im(\hbar),\Im(y)),\xi(\Im(\eth),\Im(z))\}\end{pmatrix}.\end{aligned}$$

Thus, all the suppositions of Theorem 3.2 are contented and $(0,0,0)$ is a tripled coincidence common fixed point of $\Im$ and $\Re$.

Example 3.6 Assume that all data of Example 3.1 are validated except the mappings as follows: $\Im:\chi\to\chi$ and $\Xi:\chi^3\to\chi$ defined by

$$\Im(\wp)=\frac{1}{10}\wp,\qquad\Re(\wp,\hbar,\eth)=\frac{\wp-\hbar+\eth}{40}.$$

Then

$$\begin{aligned}\bigl(\Re(\wp,\hbar,\eth),\Re(x,y,z)\bigr)&=\frac{\wp-\hbar+\eth}{80}+\frac{x-y+z}{80}\\&=\frac{\wp+x}{80}-\frac{\hbar+y}{80}+\frac{\eth+z}{80}\end{aligned}$$

$$
\begin{aligned}
&\leq \frac{\wp + x}{80} + \frac{\hbar + y}{80} + \frac{\eth + z}{80} \\
&= \frac{1}{8}\left\{\left(\frac{\wp + x}{10}\right) + \left(\frac{\hbar + y}{10}\right) + \left(\frac{\eth + z}{10}\right)\right\} \\
&\leq \frac{3}{8}\max\left\{\frac{\wp + x}{10}, \frac{\hbar + y}{10}, \frac{\eth + z}{10}\right\} \\
&\leq \frac{3}{4} \times \frac{1}{2}\max\left\{\xi\left(\Im(\wp), \Im(x)\right), \xi\left(\Im(\hbar), \Im(y)\right), \xi\left(\Im(\eth), \Im(z)\right)\right\} \\
&= \frac{3}{4}\pi\left(\max\left\{\xi\left(\Im(\wp), \Im(x)\right), \xi\left(\Im(\hbar), \Im(y)\right), \xi\left(\Im(\eth), \Im(z)\right)\right\}\right) \\
&= \Lambda\begin{pmatrix} \pi(\max\{\xi(\Im(\wp), \Im(x)), \xi(\Im(\hbar), \Im(y)), \xi(\Im(\eth), \Im(z))\}), \\ \zeta\max\{\xi(\Im(\wp), \Im(x)), \xi(\Im(\hbar), \Im(y)), \xi(\Im(\eth), \Im(z))\} \end{pmatrix}.
\end{aligned}
$$

Thus, all the suppositions of Theorem 3.2 are contented and $(0,0,0)$ is a tripled coincidence common fixed point of $\Im$ and $\Re$.

4 Consequences of the main results

This section is devoted to discussing some immediate consequences of the above theorems as follows:

If we devote $\Lambda(\lambda, \mu) = \lambda - \mu$ for all $\lambda, \mu \in \chi$ in Theorems 3.1 and 3.2, we get the following.

Corollary 4.1 *Assume that* $\Xi : \chi^3 \to \chi$ *and* $\Theta : \chi \to \chi$ *are two mappings on a POCML space* $(\chi, \precsim, \xi)$ *such that*:

(i) $\Re(\chi^3) \subseteq \Im(\chi)$;
(ii) $\Re$ *is continuous*;
(iii) $\Im$ *is continuous and commutes with* $\Re$;
(iv) $\Re$ *has a mixed* $\Im$*-monotone property*;
(v) *there are* $\pi \in \Pi$, $\zeta \geq 0$, *and* $\Lambda \in \complement$ *such that*

$$
\begin{aligned}
\xi\left(\Re(\wp, \hbar, \eth), \Re(x, y, z)\right) \leq{}& \pi\left(\max\left\{\xi\left(\Im(\wp), \Im(x)\right), \xi\left(\Im(\hbar), \Im(y)\right), \xi\left(\Im(\eth), \Im(z)\right)\right\}\right) \\
&- \zeta\max\left\{\xi\left(\Im(\wp), \Im(x)\right), \xi\left(\Im(\hbar), \Im(y)\right), \xi\left(\Im(\eth), \Im(z)\right)\right\}
\end{aligned}
$$

for any $\wp, \hbar, \eth, x, y, z \in \chi$, *for which* $\Im(\wp) \precsim \Im(x)$, $\Im(y) \precsim \Im(\hbar)$, *and* $\Im(\eth) \precsim \Im(z)$. *If there exist* $\wp_\circ, \hbar_\circ, \eth_\circ \in \chi$ *such that* $\Im(\wp_\circ) \precsim \Re(\wp_\circ, \hbar_\circ, \eth_\circ)$, $\Im(\hbar_\circ) \succsim \Re(\hbar_\circ, \wp_\circ, \eth_\circ)$, *and* $\Im(\eth_\circ) \precsim \Re(\eth_\circ, \hbar_\circ, \wp_\circ)$. *Then* $\Re$ *and* $\Im$ *have a tripled coincidence point.*

Corollary 4.2 *Let* $(\chi, \precsim, \xi)$ *be a POCML space. Assume that* $\Xi : \Upsilon \times \Upsilon \times \Upsilon \to \Upsilon$ *and* $\Theta : \Upsilon \to \Upsilon$ *fulfill the following hypotheses*:

(i) $\Re(\chi^3) \subseteq \Im(\chi)$;
(ii) $\Re$ *has a mixed* $\Im$*-monotone property*;
(iii) $(\Im(\chi), \xi)$ *is a complete metric-like space and* χ *is obligated by the following assumptions*:
  (I) $l_n \precsim l$ *if a nondecreasing sequence* $l_n \to l$, $n \to +\infty$,
  (II) $j_n \succsim j$ *if a nonincreasing sequence* $j_n \to j$, $n \to +\infty$;
(iv) $\Im$ *is continuous and commutes with* $\Re$;

(v) *there are* $\pi \in \Pi$, $\zeta \geq 0$, *and* $\Lambda \in \complement$ *such that*

$$\xi\big(\Re(\wp,\hbar,\eth),\Re(x,y,z)\big) \leq \pi\Big(\max\big\{\xi\big(\Im(\wp),\Im(x)\big),\xi\big(\Im(\hbar),\Im(y)\big),\xi\big(\Im(\eth),\Im(z)\big)\big\}\Big) - \zeta\max\big\{\xi\big(\Im(\wp),\Im(x)\big),\xi\big(\Im(\hbar),\Im(y)\big),\xi\big(\Im(\eth),\Im(z)\big)\big\}.$$

If there exist $\wp_\circ,\hbar_\circ,\eth_\circ \in \chi$ *such that* $\Im(\wp_\circ) \precsim \Re(\wp_\circ,\hbar_\circ,\eth_\circ)$, $\Im(\hbar_\circ) \succsim \Re(\hbar_\circ,\wp_\circ,\eth_\circ)$, *and* $\Im(\eth_\circ) \precsim \Re(\eth_\circ,\hbar_\circ,\wp_\circ)$, *then* $\Re$ *and* $\Im$ *have a tripled coincidence point.*

Corollary 4.3 *Assume that* $\Xi : \chi^3 \to \chi$ *and* $\Theta : \chi \to \chi$ *are two mappings on a POCML space* $(\chi,\precsim,\xi)$ *such that*:

(i) $\Re(\chi^3) \subseteq \Im(\chi)$;

(ii) $\Re$ *is continuous*;

(iii) $\Im$ *is continuous and commutes with* $\Re$;

(iv) $\Re$ *has a mixed* $\Im$*-monotone property*;

(v) *there are* $\pi \in \Pi$, $\zeta \geq 0$, *and* $\Lambda \in \complement$ *such that*

$$\xi\big(\Re(\wp,\hbar,\eth),\Re(x,y,z)\big) \leq \pi\left(\frac{\xi(\Im(\wp),\Im(x)) + \xi(\Im(\hbar),\Im(y)) + \xi(\Im(\eth),\Im(z))}{3}\right) - \zeta\left(\frac{\xi(\Im(\wp),\Im(x)) + \xi(\Im(\hbar),\Im(y)) + \xi(\Im(\eth),\Im(z))}{3}\right)$$

for any $\wp,\hbar,\eth,x,y,z \in \chi$, *for which* $\Im(\wp) \precsim \Im(x)$, $\Im(y) \precsim \Im(\hbar)$, *and* $\Im(\eth) \precsim \Im(z)$. *If there exist* $\wp_\circ,\hbar_\circ,\eth_\circ \in \chi$ *such that* $\Im(\wp_\circ) \precsim \Re(\wp_\circ,\hbar_\circ,\eth_\circ)$, $\Im(\hbar_\circ) \succsim \Re(\hbar_\circ,\wp_\circ,\eth_\circ)$, *and* $\Im(\eth_\circ) \precsim \Re(\eth_\circ,\hbar_\circ,\wp_\circ)$, *then* $\Re$ *and* $\Im$ *have a tripled coincidence point.*

Proof It is sufficient to note that

$$\frac{\xi(\Im(\wp),\Im(x)) + \xi(\Im(\hbar),\Im(y)) + \xi(\Im(\eth),\Im(z))}{3} \leq \max\left\{\begin{matrix}\xi(\Im(\wp),\Im(x)),\\ \xi(\Im(\hbar),\Im(y)),\\ \xi(\Im(\eth),\Im(z))\end{matrix}\right\}.$$

Since π is nondecreasing, so we can apply Corollary 4.1. □

Corollary 4.4 *Let* $(\chi,\precsim,\xi)$ *be a POCML space. Assume that* $\Xi : \Upsilon \times \Upsilon \times \Upsilon \to \Upsilon$ *and* $\Theta : \Upsilon \to \Upsilon$ *fulfill the following hypotheses*:

(i) $\Re(\chi^3) \subseteq \Im(\chi)$;

(ii) $\Re$ *has a mixed* $\Im$*-monotone property*;

(iii) $(\Im(\chi),\xi)$ *is a complete metric-like space and* χ *is obligated by the following assumptions*:

  (I) $l_n \precsim l$ *if a nondecreasing sequence* $l_n \to l$, $n \to +\infty$,

  (II) $j_n \succsim j$ *if a nonincreasing sequence* $j_n \to j$, $n \to +\infty$;

(iv) $\Im$ *is continuous and commutes with* $\Re$;

(v) *there are* $\pi \in \Pi$, $\zeta \geq 0$, *and* $\Lambda \in \complement$ *such that*

$$\xi\big(\Re(\wp,\hbar,\eth),\Re(x,y,z)\big) \leq \pi\left(\frac{\xi(\Im(\wp),\Im(x)) + \xi(\Im(\hbar),\Im(y)) + \xi(\Im(\eth),\Im(z))}{3}\right) - \zeta\left(\frac{\xi(\Im(\wp),\Im(x)) + \xi(\Im(\hbar),\Im(y)) + \xi(\Im(\eth),\Im(z))}{3}\right).$$

If there exist $\wp_\circ, \hbar_\circ, \eth_\circ \in \chi$ *such that* $\Im(\wp_\circ) \precsim \Re(\wp_\circ, \hbar_\circ, \eth_\circ)$, $\Im(b_\circ) \succsim \Re(\hbar_\circ, \wp_\circ, \eth_\circ)$, *and* $\Im(\eth_\circ) \precsim \Re(\eth_\circ, \hbar_\circ, \wp_\circ)$, *then* $\Re$ *and* $\Im$ *have a tripled coincidence point.*

Proof It is enough to remark

$$\frac{\xi(\Im(\wp), \Im(x)) + \xi(\Im(\hbar), \Im(y)) + \xi(\Im(\eth), \Im(z))}{3} \leq \max \begin{Bmatrix} \xi(\Im(\wp), \Im(x)), \\ \xi(\Im(\hbar), \Im(y)), \\ \xi(\Im(\eth), \Im(z)) \end{Bmatrix}.$$

So we can use Corollary 4.2 to complete the required. □

Customizing $\Lambda(\lambda, \mu) = \varrho\lambda$, $0 < \varrho < 1$ in Theorems 3.1 and 3.2, we have the following.

Corollary 4.5 *Assume that* $\Xi : \chi^3 \to \chi$ *and* $\Theta : \chi \to \chi$ *are two mappings on a POCML space* $(\chi, \precsim, \xi)$ *such that*:

(i) $\Re(\chi^3) \subseteq \Im(\chi)$;
(ii) $\Re$ *is continuous*;
(iii) $\Im$ *is continuous and commutes with* $\Re$;
(iv) $\Re$ *has a mixed* $\Im$*-monotone property*;
(v) *there are* $\pi \in \Pi$, $\zeta \geq 0$, *and* $\Lambda \in \complement$ *such that*

$$\xi\left(\Re(\wp, \hbar, \eth), \Re(x, y, z)\right) \leq \varrho\pi\left(\max\left\{\xi\left(\Im(a), \Im(x)\right), \xi\left(\Im(b), \Im(y)\right), \xi\left(\Im(c), \Im(z)\right)\right\}\right)$$

for any $\wp, \hbar, \eth, x, y, z \in \chi$, *for which* $\Im(\wp) \precsim \Im(x)$, $\Im(y) \precsim \Im(\hbar)$, *and* $\Im(\eth) \precsim \Im(z)$. *If there exist* $\wp_\circ, \hbar_\circ, \eth_\circ \in \chi$ *such that* $\Im(\wp_\circ) \precsim \Re(\wp_\circ, \hbar_\circ, \eth_\circ)$, $\Im(\hbar_\circ) \succsim \Re(\hbar_\circ, \wp_\circ, \eth_\circ)$, *and* $\Im(\eth_\circ) \precsim \Re(\eth_\circ, \hbar_\circ, \wp_\circ)$, *then* $\Re$ *and* $\Im$ *have a tripled coincidence point.*

Corollary 4.6 *Let* $(\chi, \precsim, \xi)$ *be a POCML space. Assume that* $\Xi : \Upsilon \times \Upsilon \times \Upsilon \to \Upsilon$ *and* $\Theta : \Upsilon \to \Upsilon$ *fulfill the following hypotheses*:

(i) $\Re(\chi^3) \subseteq \Im(\chi)$;
(ii) $\Re$ *has a mixed* $\Im$*-monotone property*;
(iii) $(\Im(\chi), \xi)$ *is a complete metric-like space and* χ *is obligated by the following assumptions*:
   (I) $l_n \precsim l$ *if a nondecreasing sequence* $l_n \to l$, $n \to +\infty$,
   (II) $j_n \succsim j$ *if a nonincreasing sequence* $j_n \to j$, $n \to +\infty$;
(iv) $\Im$ *is continuous and commutes with* $\Re$;
(v) *there are* $\pi \in \Pi$, $\zeta \geq 0$, *and* $\Lambda \in \complement$ *such that*

$$\xi\left(\Re(\wp, \hbar, \eth), \Re(x, y, z)\right) \leq \varrho\pi\left(\max\left\{\xi\left(\Im(\wp), \Im(x)\right), \xi\left(\Im(\hbar), \Im(y)\right), \xi\left(\Im(\eth), \Im(z)\right)\right\}\right).$$

If there exist $\wp_\circ, \hbar_\circ, \eth_\circ \in \chi$ *such that* $\Im(\wp_\circ) \precsim \Re(\wp_\circ, \hbar_\circ, \eth_\circ)$, $\Im(\hbar_\circ) \succsim \Re(\hbar_\circ, \wp_\circ, \eth_\circ)$, *and* $\Im(\eth_\circ) \precsim \Re(\eth_\circ, \hbar_\circ, \wp_\circ)$, *then* $\Re$ *and* $\Im$ *have a tripled coincidence point.*

Now, if we choose $\Im = I_\chi$ (where I_χ is the identity mapping) and replace a mixed-monotone property with a monotone-increasing one in Theorem 3.1, we get the following important result.

Corollary 4.7 *Let* $(\Re, \precsim, \xi)$ *be a POCML space. Suppose that* $\Re : \chi^3 \to \chi$ *is a mapping such that*:

(i) $\Re$ *is continuous*;

(ii) $\Re$ *is nondecreasing with respect to* $\precsim$;

(iii) *there exist three elements* $\wp_\circ, \hbar_\circ, c_\circ \in \chi$ *such that* $\wp_\circ \precsim \Re(\wp_\circ, \hbar_\circ, \eth_\circ)$, $\hbar_\circ \succsim \Re(\hbar_\circ, \wp_\circ, \hbar_\circ)$, *and* $c_\circ \precsim \Re(c_\circ, \hbar_\circ, \wp_\circ)$;

(iv) *there are* $\pi \in \Pi$, $\zeta \geq 0$, *and* $\Lambda \in \complement$ *such that*

$$\begin{aligned} &\xi\big(\Re(\wp, \hbar, \eth), \Re(x, y, z)\big) \\ &\quad \leq \Lambda\big(\pi\big(\max\big\{\xi(\wp, x), \xi(\hbar, y), \xi(\eth, z)\big\}\big), \zeta \max\big\{\xi(\wp, x), \xi(\hbar, y), \xi(\eth, z)\big\}\big) \end{aligned} \tag{32}$$

for any $\wp, \hbar, \eth, x, y, z \in \chi$, *and for which* $\wp \precsim x$, $y \precsim \hbar$, *and* $\eth \precsim z$. *Then there is a tripled coincidence point of* $\Re$.

5 Applications

This part is considered as the mainstay of this paper because it indicates the applications that contribute to solving some nonlinear integral systems that attract many readers and researchers and show the importance of the fixed point theory in many areas.

5.1 Some contributions of integral type

Let Ω be a class of functions $\varpi : [0, +\infty) \to [0, +\infty)$ fulfilling the following postulates:

(i) For each compact subset of $[0, +\infty)$, ϖ is a positive Lebesgue integrable mapping;

(ii) $\int_0^\epsilon \varpi(\ell)\, d\ell > 0$ for all $\epsilon > 0$.

Corollary 5.1 *Let* $\Lambda(\lambda, \mu) = \lambda - \mu$. *Exchange stipulation* (1) *of Theorem* 3.1 *by the formula*

$$\int_0^{\xi(\Re(\wp,\hbar,\eth),\Re(x,y,z))} \varpi(\ell)\, d\ell \leq \int_0^{\pi(\varphi(\wp,x,\hbar,y,\eth,z)) - \zeta\varphi(\wp,x,\hbar,y,\eth,z)} \varpi(\ell)\, d\ell \tag{33}$$

for all $\varpi \in \Omega$, *where* $\varphi(\wp, x, \hbar, y, \eth, z) = \max\{\xi(\Im(\wp), \Im(x)), \xi(\Im(\hbar), \Im(y)), \xi(\Im(\eth), \Im(z))\}$. *If other hypotheses of Theorem* 3.1 *are fulfilled, then there is a tripled coincidence point of the mentioned mappings*.

Proof Suppose the function $\Upsilon(\wp) = \int_0^\wp \varpi(\ell)\, d\ell$, then (33) becomes

$$\Upsilon\big(\xi\big(\Re(\wp, \hbar, \eth), \Re(x, y, z)\big)\big) \leq \Upsilon\pi\big(\varphi(\wp, x, \hbar, y, \eth, z)\big) - \Upsilon\big(\zeta\varphi(\wp, x, \hbar, y, \eth, z)\big).$$

Letting $\pi_1 = \Upsilon \circ \pi$, we have $\pi_1 \in \Pi$, since $\Upsilon\zeta \geq 0$, then the proof is quickly completed from Theorem 3.1. □

Corollary 5.2 *Let* $\Lambda(\lambda, \mu) = \lambda - \mu$. *Exchange stipulation* (1) *of Theorem* 3.1 *by the formula*

$$\begin{aligned} &\int_0^{\xi(\Re(\wp,\hbar,\eth),\Re(x,y,z))} \varpi(\ell)\, d\ell \\ &\quad \leq \pi\left(\int_0^{\varphi(\wp,x,\hbar,y,\eth,z)} \varpi(\ell)\, d\ell\right) - \zeta \int_0^{\varphi(\wp,x,\hbar,y,\eth,z)} \varpi(\ell)\, d\ell \end{aligned} \tag{34}$$

for each $\varpi \in \Omega$, If other hypotheses of Theorem 3.1 are fulfilled, then there is a tripled coincidence point of the mentioned mappings.

Proof As in Corollary 5.1, define the function $\Upsilon(\wp) = \int_0^{\wp} \varpi(\ell)\,d\ell$, then (34) is

$$\begin{aligned}&\Upsilon\big(\xi\big(\Im(\wp,\hbar,\eth),\Re(x,y,z)\big)\big)\\&\quad\le \pi\big(\Upsilon\big(\varphi(\wp,x,\hbar,y,\eth,z)\big)\big) - \zeta\,\Upsilon\big(\varphi(\wp,x,\hbar,y,\eth,z)\big).\end{aligned}$$

Putting $\pi_2 = \pi \circ \Upsilon$, we get $\pi_2 \in \Pi$, since $\zeta\Upsilon \ge 0$, then the proof is quickly completed from Theorem 3.1. □

In the same line of [51], let a fixed number $\nabla \in \mathbb{N}$. Suppose that $\{\varpi_j\}_{1\le j\le\nabla}$ is a collection of ∇ functions which belong to Ω. For each $\ell \ge 0$, we define

$$\begin{aligned}J_1(\ell) &= \int_0^{\ell} \varpi_1(\rho)\,d\rho,\\ J_2(\ell) &= \int_0^{J_1(\ell)} \varpi_2(\rho)\,d\rho = \int_0^{\int_0^{\ell}\varpi_1(\rho)\,d\rho,} \varpi_2(\rho)\,d\rho,\\ J_3(\ell) &= \int_0^{J_2(\ell)} \varpi_3(\rho)\,d\rho = \int_0^{\int_0^{\int_0^{\ell}\varpi_1(\rho)\,d\rho,}\varpi_2(\rho)\,d\rho} \varpi_3(\rho)\,d\rho,\\ &\dots\\ J_{\nabla}(\ell) &= \int_0^{J_{(\nabla-1)}(\ell)} \varpi_{\nabla}(\rho)\,d\rho.\end{aligned}$$

We have the following consequence.

Corollary 5.3 *Let $\Lambda(\lambda,\mu) = \lambda - \mu$. Replace inequality (1) of Theorem 3.1 by the the following assumption: There is $\varpi \in \Omega$ such that*

$$J_{\nabla}\big(\xi\big(\Re(\wp,\hbar,\eth),\Re(x,y,z)\big)\big) \le \pi\big(J_{\nabla}\big(\varphi(\wp,x,\hbar,y,\eth,z)\big) - \zeta J_{\nabla}(\varphi(\wp,x,\hbar,y,\eth,z). \tag{35}$$

If the remaining conditions of Theorem 3.1 are true, then there is a tripled coincidence point of $\Re$ and $\Im$.

Proof Specify $\pi_3 = \pi \circ J_{\nabla}$ and $\pi_4 = \zeta \circ J_{\nabla}$, then inequality (35) takes the form

$$J_{\nabla}\big(\xi\big(\Re(\wp,\hbar,\eth),\Re(x,y,z)\big)\big) \le \pi_3(\varphi(\wp,x,\hbar,y,\eth,z) - \psi_4(\varphi(\wp,x,\hbar,y,\eth,z).$$

Applying Theorem 3.1, we obtain the desired result because $\pi_2 \in \Pi$ and $\pi_4 = \zeta J_{\nabla} \ge 0$. □

5.2 Solve a system of nonlinear integral equations

Let Ω be a class functions $\omega : [0,+\infty) \to [0,+\infty)$ such that ω is increasing and there exist $\pi \in \Pi$, $\zeta \ge 0$, and $\Lambda \in \complement$ such that $\omega(\mu) = \frac{1}{3}\Lambda(\pi(\mu),\zeta\mu)$ for all $\mu \in [0,+\infty)$.

Consider the following problem:

$$\begin{aligned}\wp(\nu) = \varphi(\nu) + \int_p^q \big(r_1(\nu,\rho) + r_2(\nu,\rho) + r_3(\nu,\rho)\big) \\ \times \big[p_1\big(\rho,\wp(\rho)\big) + p_2\big(\rho,\hbar(\rho)\big) + p_3\big(\rho,\eth(\rho)\big)\big]\,d\rho \end{aligned} \tag{36}$$

for all $\nu \in [p,q]$. We postulate that the following assumptions hold:

(i) $\varphi : [p,q] \to \mathbb{R}$ is continuous;

(ii) $p_i, r_i (i = 1,2,3) : [p,q] \times \mathbb{R} \to \mathbb{R}$ are continuous;

(iii) For all $\wp, \eth \in \mathbb{R}$, there are $\varkappa$, τ, σ such that

$$0 \le p_1(\rho,\wp) - p_1(\rho,\eth) \le \varkappa\omega(\wp - \eth),$$
$$0 \le p_2(\rho,\wp) - p_2(\rho,\eth) \le \tau\omega(\wp - \eth),$$

and

$$0 \le p_3(\rho,\wp) - p_3(\rho,c) \le \sigma\omega(\wp - \eth);$$

(iv) We assume that

$$\max\{\varkappa,\tau,\sigma\}\left(\sup_{t\in[p,q]}\int_p^q \big[r_1(\nu,\rho) + r_2(\nu,\rho) + r_3(\nu,\rho)\big]\,d\rho\right) \le 1;$$

(v) There are continuous functions $\alpha, \beta, \gamma : [p,q] \to \mathbb{R}$ such that

$$\begin{aligned}\alpha(\nu) \le{}& \int_p^q r_1(\nu,\rho)\big[p_1\big(\rho,\alpha(\rho)\big) + p_2\big(\rho,\beta(\rho)\big) + p_3\big(\rho,\gamma(\rho)\big)\big]\,d\rho \\ &+ \int_p^q r_2(\nu,\rho)\big[p_1\big(\rho,\delta(\rho)\big) + p_2\big(\rho,\eta(\rho)\big) + p_3\big(\rho,\delta(\rho)\big)\big]\,d\rho \\ &+ \int_p^q r_3(\nu,\rho)\big[p_1\big(\rho,\gamma(\rho)\big) + p_2\big(\rho,\beta(\rho)\big) + p_3\big(\rho,\alpha(\rho)\big)\big]\,d\rho, \\ \beta(\nu) \le{}& \int_p^q r_1(\nu,\rho)\big[p_1\big(\rho,\beta(\rho)\big) + p_2\big(\rho,\eta(\rho)\big) + p_3\big(\rho,\beta(\rho)\big)\big]\,d\rho \\ &+ \int_p^q r_2(\nu,\rho)\big[p_1\big(\rho,\gamma(\rho)\big) + p_2\big(\rho,\beta(\rho)\big) + p_3\big(\rho,\alpha(\rho)\big)\big]\,d\rho \\ &+ \int_p^q r_3(\nu,\rho)\big[p_1\big(\rho,\alpha(\rho)\big) + p_2\big(\rho,\beta(\rho)\big) + p_3\big(\rho,\gamma(\rho)\big)\big],\end{aligned}$$

and

$$\begin{aligned}\gamma(\nu) \le{}& \int_p^q r_1(\nu,\rho)\big[p_1\big(\rho,\gamma(\rho)\big) + p_2\big(\rho,\beta(\rho)\big) + p_3\big(\rho,\alpha(\rho)\big)\big]\,d\rho \\ &+ \int_p^q r_2(\nu,\rho)\big[p_1\big(\rho,\alpha(\rho)\big) + p_2\big(\rho,\beta(\rho)\big) + p_3\big(\rho,\gamma(\rho)\big)\big]\,d\rho \\ &+ \int_p^q r_3(\nu,\rho)\big[p_1\big(\rho,\beta(\rho)\big) + p_2\big(\rho,\alpha(\rho)\big) + p_3\big(\rho,\beta(\rho)\big)\big]\,d\rho.\end{aligned}$$

Let $\chi = C([p,q],\mathbb{R})$ be the set of real continuous functions on $[p,q]$ endowed with

$$\xi(\wp,\hbar) = \|\wp - \hbar\|_\infty = \sup_{\nu\in[p,q]}\{|\wp(\nu) - \hbar(\nu)|\}$$

for all $\wp,\hbar \in \chi$. Then the pair (χ,ξ) is a complete metric-like space. We endow χ with the partial order $\precsim$ as follows:

$$\wp \precsim \hbar \quad \Leftrightarrow \quad \wp(\nu) \le \hbar(\nu), \quad \forall\nu \in [p,q].$$

Subsequently, $(\chi,\precsim,\xi)$ is a POCML space if $\wp \precsim x, y \precsim \hbar$, and $\eth \precsim z$ whenever $\wp(\nu) \le x(\nu)$, $y(\nu) \le \hbar(\nu)$, and $\eth(\nu) \le z(\nu)$ for all $\wp,\hbar,\eth,x,y,z \in \chi$ and $\nu \in [p,q]$.

Now, we can state and prove our main theorem of this section.

Theorem 5.4 *Under hypotheses* (i)–(v), *problem* (36) *has a solution in* χ^3, *where* $\chi = C([p,q],\mathbb{R})$.

Proof Define an operator $\Re : \chi^3 \to \chi$ by

$$\begin{aligned}
&\Re(\wp,\hbar,\eth)(\nu)\\
&\quad = \varphi(\nu) + \int_p^q r_1(\nu,\rho)\big[p_1\big(\rho,\wp(\rho)\big) + p_2\big(\rho,\hbar(\rho)\big) + p_3\big(\rho,\eth(\rho)\big)\big]\,d\rho\\
&\qquad + \int_p^q r_2(\nu,\rho)\big[p_1\big(\rho,\hbar(\rho)\big) + p_2\big(\rho,\wp(\rho)\big) + p_3\big(\rho,\hbar(\rho)\big)\big]\,d\rho\\
&\qquad + \int_p^q r_3(\nu,\rho)\big[p_1\big(\rho,\eth(\rho)\big) + p_2\big(\rho,\hbar(\rho)\big) + p_3\big(\rho,\wp(\rho)\big)\big]\,d\rho
\end{aligned}$$

for all $\nu \in [p,q]$ and $\wp,\hbar,\eth \in \chi$. It is clear that if the mapping Ξ has a tripled coincidence point in $\Upsilon = C([p,q],\mathbb{R})$, then it is a solution of problem (36).

Now, we shall prove the increasing property of the mapping Ξ with $\wp_1 \precsim \wp_2$, so $\wp_1(\nu) \le \wp_2(\nu)$ for all $\nu \in [p,q]$, we get

$$\begin{aligned}
&\Re(\wp_1,\hbar,\eth)(\nu) - \Xi(\wp_2,\hbar,\eth)(\nu)\\
&\quad = \int_p^q r_1(\nu,\rho)\big[p_1\big(\rho,\wp_1(\rho)\big) + p_2\big(\rho,\hbar(\rho)\big) + p_3\big(\rho,\eth(\rho)\big)\big]\,d\rho\\
&\qquad + \int_p^q r_2(\nu,\rho)\big[p_1\big(\rho,\hbar(\rho)\big) + p_2\big(\rho,\wp_1(\rho)\big) + p_3\big(\rho,\hbar(\rho)\big)\big]\,d\rho\\
&\qquad + \int_p^q r_3(\nu,\rho)\big[p_1\big(\rho,\eth(\rho)\big) + p_2\big(\rho,\hbar(\rho)\big) + p_3\big(\rho,\wp_1(\rho)\big)\big]\,d\rho\\
&\qquad - \int_p^q r_1(\nu,\rho)\big[p_1\big(\rho,\wp_2(\rho)\big) + p_2\big(\rho,\hbar(\rho)\big) + p_3\big(\rho,\eth(\rho)\big)\big]\,d\rho\\
&\qquad - \int_p^q r_2(\nu,\rho)\big[p_1\big(\rho,\hbar(\rho)\big) + p_2\big(\rho,\wp_2(\rho)\big) + p_3\big(\rho,\hbar(\rho)\big)\big]\,d\rho\\
&\qquad - \int_p^q r_3(\nu,\rho)\big[p_1\big(\rho,\eth(\rho)\big) + p_2\big(\rho,\hbar(\rho)\big) + p_3\big(\rho,\wp_2(\rho)\big)\big]\,d\rho
\end{aligned}$$

$$
\begin{aligned}
&= \int_p^q r_1(\nu,\rho)\big[p_1\big(\rho,\wp_1(\rho)\big) - p_1\big(\rho,\wp_2(\rho)\big)\big]\,d\rho \\
&\quad + \int_p^q r_2(\nu,\rho)\big[p_2\big(\rho,\wp_1(\rho)\big) - p_2\big(\rho,\wp_2(\rho)\big)\big]\,d\rho \\
&\quad + \int_p^q r_3(\nu,\rho)\big[p_3\big(\rho,\wp_1(\rho)\big) - p_3\big(\rho,\wp_2(\rho)\big)\big]\,d\rho \\
&\leq 0.
\end{aligned}
$$

Hence, $\Re(\wp_1,\hbar,\eth)(\nu) \leq \Re(\wp_2,\hbar,\eth)(\nu)$ for all $\nu \in [p,q]$. Subsequently, $\Re(\wp_1,\hbar,\eth) \precsim \Re(\wp_2,\hbar,\eth)$.

Again, if $\hbar_1 \precsim \hbar_2$, so $\hbar_1(\nu) \leq \hbar_2(\nu)$ for all $\nu \in [p,q]$, we can get

$$
\begin{aligned}
&\Re(\wp,\hbar_1,\eth)(\nu) - \Re(\wp,\hbar_2,\eth)(\nu) \\
&= \int_p^q r_1(\nu,\rho)\big[p_1\big(\rho,\wp(\rho)\big) + p_2\big(\rho,\hbar_1(\rho)\big) + p_3\big(\rho,\eth(\rho)\big)\big]\,d\rho \\
&\quad + \int_p^q r_2(\nu,\rho)\big[p_1\big(\rho,\hbar_1(\rho)\big) + p_2\big(\rho,\wp(\rho)\big) + p_3\big(\rho,\hbar_1(\rho)\big)\big]\,d\rho \\
&\quad + \int_p^q r_3(\nu,\rho)\big[p_1\big(\rho,\eth(\rho)\big) + p_2\big(\rho,\hbar_1(\rho)\big) + p_3\big(\rho,\wp(\rho)\big)\big]\,d\rho \\
&\quad - \int_p^q r_1(\nu,\rho)\big[p_1\big(\rho,\wp(\rho)\big) + p_2\big(\rho,\hbar_2(\rho)\big) + p_3\big(\rho,\eth(\rho)\big)\big]\,d\rho \\
&\quad - \int_p^q r_2(\nu,\rho)\big[p_1\big(\rho,\hbar_2(\rho)\big) + p_2\big(\rho,\wp(\rho)\big) + p_3\big(\rho,\hbar_2(\rho)\big)\big]\,d\rho \\
&\quad - \int_p^q r_3(\nu,\rho)\big[p_1\big(\rho,\eth(\rho)\big) + p_2\big(\rho,\hbar_2(\rho)\big) + p_3\big(\rho,\wp(\rho)\big)\big]\,d\rho \\
&= \int_p^q r_1(\nu,\rho)\big[p_2\big(\rho,\hbar_1(\rho)\big) - p_2\big(\rho,\hbar_2(\rho)\big)\big]\,d\rho \\
&\quad + \int_p^q r_2(\nu,\rho)\begin{bmatrix} \{p_1(\rho,\hbar_1(\rho)) - p_1(\rho,\hbar_2(\rho))\} \\ +\{p_3(\rho,\hbar_1(\rho)) - p_3(\rho,\hbar_2(\rho))\} \end{bmatrix} d\rho \\
&\quad + \int_p^q r_3(\nu,\rho)\big[p_2\big(\rho,\hbar_1(\rho)\big) - p_2\big(\rho,\hbar_2(\rho)\big)\big]\,d\rho \\
&\leq 0.
\end{aligned}
$$

So, $\Re(\wp,\hbar_1,\eth) \leq \Re(\wp,\hbar_2,\eth)(\nu)$ for all $\nu \in [p,q]$. Hence $\Re(\wp,\hbar_1,\eth) \precsim \Re(\wp,\hbar_2,\eth)$. In the same manner, we can write $\Re(\wp,\hbar,\eth_1) \precsim \Re(\wp_2,\hbar,\eth_2)$ if $\eth_1 \precsim \eth_2$. From the above inequalities, we observe that the mapping $\Re$ is increasing with respect to the variables $\wp$, $\hbar$, and $\eth$.

Finally, we shall verify contractive condition (32) of Corollary 4.7 for all $\wp,\hbar,\eth,x,y,z \in \chi$ such that $\wp \precsim x$, $y \precsim \hbar$, and $\eth \precsim z$,

$$
\begin{aligned}
&\xi\big(\Re(\wp,\hbar,\eth),\Re(x,y,z)\big) \\
&\quad = \sup_{\nu\in[p,q]} \big\{\big|\Re(\wp,\hbar,\eth)(\nu) - \Re(x,y,z)(\nu)\big|\big\}
\end{aligned}
$$

$$= \sup_{\nu\in[p,q]} \left\{ \begin{array}{l} |\int_p^q r_1(\nu,\rho)[p_1(\rho,\wp(\rho)) + p_2(\rho,\hbar(\rho)) + p_3(\rho,\eth(\rho))]\,d\rho \\ + \int_p^q r_2(\nu,\rho)[p_1(\rho,\hbar(\rho)) + p_2(\rho,\wp(\rho)) + p_3(\rho,\hbar(\rho))]\,d\rho \\ + \int_p^q r_3(\nu,\rho)[p_1(\rho,\eth(\rho)) + p_2(\rho,\hbar(\rho)) + p_3(\rho,\wp(\rho))]\,d\rho \\ - \int_p^q r_1(\nu,\rho)[p_1(\rho,x(\rho)) + p_2(\rho,y(\rho)) + p_3(\rho,z(\rho))]\,d\rho \\ - \int_p^q r_2(\nu,\rho)[p_1(\rho,y(\rho)) + p_2(\rho,x(\rho)) + p_3(\rho,y(\rho))]\,d\rho \\ - \int_p^q r_3(\nu,\rho)[p_1(\rho,z(\rho)) + p_2(\rho,y(\rho)) + p_3(\rho,x(\rho))]\,d\rho| \end{array} \right\}$$

$$= \sup_{\nu\in[p,q]} \left\{ \begin{array}{l} \left| \int_p^q r_1(\nu,\rho) \begin{pmatrix} [p_1(\rho,\wp(\rho)) - p_1(\rho,x(\rho))] \\ +[p_2(\rho,\hbar(\rho)) - p_2(\rho,y(\rho))] \\ +[p_3(\rho,\eth(\rho)) - p_3(\rho,z(\rho))] \end{pmatrix} d\rho \right. \\ + \int_p^q r_2(\nu,\rho) \begin{pmatrix} [p_1(\rho,\hbar(\rho)) - p_1(\rho,y(\rho))] \\ +[p_2(\rho,\wp(\rho)) - p_2(\rho,x(\rho))] \\ +[p_3(\rho,\hbar(\rho)) - p_3(\rho,y(\rho))] \end{pmatrix} d\rho \\ \left. + \int_p^q r_3(\nu,\rho) \begin{pmatrix} [p_1(\rho,\eth(\rho)) - p_1(\rho,z(\rho))] \\ +[p_2(\rho,\hbar(\rho)) - p_2(\rho,y(\rho))] \\ +[p_3(\rho,\wp(\rho)) - p_3(\rho,x(\rho))] \end{pmatrix} d\rho \right| \end{array} \right\}.$$

Applying hypothesis (iii), we get

$$\begin{aligned} &\xi\big(\Re(\wp,\hbar,\eth),\Re(x,y,z)\big) \\ &\quad\le \sup_{\nu\in[p,q]} \left\{ \begin{array}{l} |\int_p^q r_1(\nu,\rho)[\varkappa\omega(\wp(\rho)-x(\rho)) + \tau\omega(\hbar(\rho)-y(\rho)) + \sigma\omega(\eth(\rho)-z(\rho))]\,d\rho \\ + \int_p^q r_2(\nu,\rho)[\varkappa\omega(\hbar(\rho)-y(\rho)) + \tau\omega(\wp(\rho)-x(\rho)) + \sigma\omega(\hbar(\rho)-y(\rho))]\,d\rho \\ + \int_p^q r_3(\nu,\rho)[\varkappa\omega(\eth(\rho)-z(\rho)) + \tau\omega(\hbar(\rho)-y(\rho)) + \sigma\omega(\wp(\rho)-x(\rho))]\,d\rho| \end{array} \right\} \\ &\quad\le \max\{\varkappa,\tau,\sigma\} \\ &\qquad\times \sup_{\nu\in[p,q]} \left\{ \begin{array}{c} \int_p^q (r_1(\nu,\rho) + r_2(\nu,\rho) + r_3(\nu,\rho))\times \\ [\omega(|\wp(\rho)-x(\rho)|) + \omega(|\hbar(\rho)-y(\rho)|) + \omega(|\eth(\rho)-z(\rho)|)]\,d\rho \end{array} \right\}. \end{aligned} \tag{37}$$

By the characterizations of the function ω and the distance ξ, one can write, for all $\rho \in [p,q]$,

$$\begin{aligned} &\omega\big(\big|\wp(\rho)-x(\rho)\big|\big) \le \omega\xi(\wp,x), \\ &\omega\big(\big|\hbar(\rho)-y(\rho)\big|\big) \le \omega\xi(\hbar,y), \\ &\omega\big(\big|\eth(\rho)-z(\rho)\big|\big) \le \omega\xi(\eth,z). \end{aligned} \tag{38}$$

It follows from (37), (38) and assumption (iv) that

$$\begin{aligned} &\xi\big(\Re(\wp,\hbar,\eth),\Re(x,y,z)\big) \\ &\quad\le \max\{\varkappa,\tau,\sigma\} \times \big(\omega\xi(\wp,x) + \omega\xi(\hbar,y) + \omega\xi(\eth,z)\big) \\ &\qquad\times \left(\sup_{\nu\in[p,q]} \int_p^q \big(r_1(\nu,\rho) + r_2(\nu,\rho) + r_3(\nu,\rho)\big)\,d\rho \right) \\ &\quad\le \omega\xi(\wp,x) + \omega\xi(\hbar,y) + \omega\xi(\eth,z) \\ &\quad\le 3\omega\big(\max\big\{\xi(\wp,x),\xi(\hbar,y),\xi(\hbar,y)\big\}\big) \end{aligned}$$

$$\leq 3 \times \frac{1}{3} \Lambda\Big(\pi\Big(\max\big\{\xi(\wp,x),\xi(\hbar,y),\xi(\hbar,y)\big\}\Big),\zeta \max\big\{\xi(\wp,x),\xi(\hbar,y),\xi(\hbar,y)\big\}\Big)$$
$$= \Lambda\Big(\pi\Big(\max\big\{\xi(\wp,x),\xi(\hbar,y),\xi(\hbar,y)\big\}\Big),\zeta \max\big\{\xi(\wp,x),\xi(\hbar,y),\xi(\hbar,y)\big\}\Big).$$

Also, condition (v) tells us $\alpha(\nu) \leq \Re(\alpha,\beta,\gamma)(\nu)$, $\beta(\nu) \geq \Re(\beta,\alpha,\beta)(\nu)$, and $\gamma(\nu) \leq \Re(\gamma,\beta,\alpha)(\nu)$ for all $\nu \in [p,q]$. This yields $\alpha \precsim \Re(\alpha,\beta,\gamma)$, $\beta \succsim \Re(\beta,\alpha,\beta,\gamma)$, and $\gamma \precsim \Re(\gamma,\beta,\alpha)$. Applying Corollary 4.7, we deduce the existence solution of problem (36). □

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