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Extensions of inequalities for sector matrices

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Abstract

In this note, we first prove an inequality for sector matrices. This complements a result due to Kittaneh and Sakkijha (Linear Multilinear Algebra, 2018, https://doi.org/10.1080/03081087.2018.1441800) concerning accretive–dissipative matrices. And then we present two singular value inequalities for sector matrices which are similar to Yang and Lu's inequalities (J. Inequal. Appl. 2018:183, 2018).

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Keywords: Sector matrices; Accretive-dissipative matrices; Numerical ranges; Schatten p-norms

1 Introduction

We denote by $\mathbb{M}_n(\mathbb{C})$ the set of $n \times n$ complex matrices. For $A \in \mathbb{M}_n(\mathbb{C})$, the conjugate transpose of A is denoted by A^*, and the matrices $\Re A = \frac{1}{2}(A + A^*)$ and $\Im A = \frac{1}{2i}(A - A^*)$ are called the real part and imaginary part of A, respectively (e.g., [1, p. 6] and [6, p. 7]). Recall that a norm $\|\cdot\|$ on $\mathbb{M}_n(\mathbb{C})$ is unitarily invariant if $\|UAV\| = \|A\|$ for any $A \in \mathbb{M}_n(\mathbb{C})$ and unitarily matrices $U, V \in \mathbb{M}_n(\mathbb{C})$. For $p \geq 1$, the Schatten p-norm of $A \in \mathbb{M}_n(\mathbb{C})$ is defined as $\|A\|_p = (\sum_{j=1}^n \sigma_j^p(A))^{\frac{1}{p}}$. If the eigenvalues of a square matrix $A \in \mathbb{M}_n(\mathbb{C})$ are all real, then we denote $\lambda_j(A)$ the jth largest eigenvalue of A. The singular values of a complex matrix $A \in \mathbb{M}_n$ are the eigenvalues of $|A| := (A^*A)^{\frac{1}{2}}$, and we denote $\sigma_j(A) := \lambda_j(|A|)$ the jth largest singular value of A. A positive semidefinite matrix A will be expressed as $A \geq 0$. Likewise, we write $A > 0$ to refer that A is a positive definite matrix.

$A \in \mathbb{M}_n(\mathbb{C})$ is an accretive-dissipative matrix if $\Re A$ and $\Im A$ are both positive semidefinite. This class of matrices has recently been considered by Lin [9] and Lin and Zhou [11].

Let $H \in \mathbb{M}_n(\mathbb{C})$ be a Hermitian matrix and let f be a real-valued function defined on an interval containing all the eigenvalues of H. Then $f(H)$ is well defined through spectral decomposition. f is called matrix concave if $f(\alpha A + (1-\alpha)B) \geq \alpha f(A) + (1-\alpha)f(B)$ for any two Hermitian matrices $A, B \in \mathbb{M}_n(\mathbb{C})$ and all $\alpha \in [0, 1]$.

The numerical range of $A \in \mathbb{M}_n(\mathbb{C})$ is defined by

$$W(A) = \{x^*Ax | x \in \mathbb{C}^n, x^*x = 1\}.$$

For $\alpha \in [0, \pi/2)$, let

$$S_\alpha = \{z \in \mathbb{C} | \Re z \geq 0, |\Im z| \leq (\Re z)\tan(\alpha)\}$$

be a sector region on the complex plane. A matrix whose numerical range is contained in a sector region S_α is called a sector matrix [10]. Recent research interest in this class of matrices starts with a resolution of a problem from numerical analysis [3]. Some research results on sector matrices can be found in [3, 7, 8, 10, 13].

Kittaneh and Sakkijha [7] proved the following Schatten-p norm inequalities.

Theorem 1.1 (see [7, Theorem 2.7]) *Let $S, T \in \mathbb{M}_n(\mathbb{C})$ be accretive-dissipative. Then*

$$2^{\frac{-p}{2}}\left(\|S\|_p^p + \|T\|_p^p\right) \le \|S+T\|_p^p \le 2^{\frac{3p}{2}-1}\left(\|S\|_p^p + \|T\|_p^p\right) \quad \textit{for } p \ge 1.$$

Recently, Yang and Lu [12] gave a generalization of Theorem 1.1.

Theorem 1.2 (see [12, Theorem 2.3]) *Let $A_1, \ldots, A_n \in \mathbb{M}_n(\mathbb{C})$ be accretive-dissipative. Then*

$$2^{\frac{-p}{2}}\sum_{j=1}^n \|A_j\|_p^p \le \left\|\sum_{j=1}^n A_j\right\|_p^p \le \frac{(2n^2)^{\frac{p}{2}}}{n}\sum_{j=1}^n \|A_j\|_p^p \quad \textit{for } p \ge 1.$$

In [5], Garg and Aujla presented the following inequalities:

$$\begin{aligned} &\prod_{j=1}^k \sigma_j\left(|A+B|^r\right) \\ &\quad \le \prod_{j=1}^k \sigma_j\left(I_n + |A|^r\right)\prod_{j=1}^k \sigma_j\left(I_n + |B|^r\right) \quad \text{for } 1 \le k \le n, 1 \le r \le 2; \end{aligned} \tag{1}$$

$$\begin{aligned} &\prod_{j=1}^k \sigma_j\left(I_n + f\left(|A+B|\right)\right) \\ &\quad \le \prod_{j=1}^k \sigma_j\left(I_n + f\left(|A|\right)\right)\prod_{j=1}^k \sigma_j\left(I_n + f\left(|B|\right)\right) \quad \text{for } 1 \le k \le n, \end{aligned} \tag{2}$$

where $A, B \in \mathbb{M}_n(\mathbb{C})$ and $f : [0,\infty) \to [0,\infty)$ is a matrix concave function.

If $A, B \ge 0$, $r = 1$ and $f(X) = X$ for any $X \in \mathbb{M}_n(\mathbb{C})$ in (1) and (2), then

$$\prod_{j=1}^k \sigma_j(A+B) \le \prod_{j=1}^k \sigma_j(I_n + A)\prod_{j=1}^k \sigma_j(I_n + B) \quad \text{for } 1 \le k \le n$$

and

$$\prod_{j=1}^k \sigma_j(I_n + A + B) \le \prod_{j=1}^k \sigma_j(I_n + A)\prod_{j=1}^k \sigma_j(I_n + B) \quad \text{for } 1 \le k \le n. \tag{3}$$

Based on the above inequalities, Yang and Lu (see [12, Theorem 2.7]) gave the inequalities for sector matrices which removed the absolute values in (1) and (2) from the right sides as follows.

Theorem 1.3 (see [12, Theorem 2.7]) *Let $A, B \in \mathbb{M}_n(\mathbb{C})$ be such that $W(A), W(B) \subseteq S_\alpha$. Then, for $1 \le k \le n$,*

$$\prod_{j=1}^{k} \sigma_j(A+B) \le \sec^{2k}(\alpha) \prod_{j=1}^{k} \sigma_j(I_n + A) \prod_{j=1}^{k} \sigma_j(I_n + B) \tag{4}$$

and

$$\prod_{j=1}^{k} \sigma_j(I_n + A+B) \le \sec^{2k}(\alpha) \prod_{j=1}^{k} \sigma_j(I_n + A) \prod_{j=1}^{k} \sigma_j(I_n + B). \tag{5}$$

By Fan's dominance principle [1, p. 93], the theorem below follows from (4) and (5).

Theorem 1.4 (see [12, Corollary 2.8]) *Let $A, B \in \mathbb{M}_n(\mathbb{C})$ be such that $W(A), W(B) \subseteq S_\alpha$. Then*

$$\|A+B\| \le \sec^2(\alpha) \|I_n + A\| \|I_n + B\| \tag{6}$$

and

$$\|I_n + A + B\| \le \sec^2(\alpha) \|I_n + A\| \|I_n + B\|. \tag{7}$$

In this paper, we will extend Theorem 1.2 to sector matrices. Furthermore, we present some inequalities for sector matrices which are similar to the inequalities (4) and (5). However, in some cases, our results are stronger than (4) and (5), respectively.

2 Main results

We begin this section with some lemmas which are useful to establish our main results.

Lemma 2.1 (see [1, p. 73]) *Let $A \in \mathbb{M}_n(\mathbb{C})$. Then*

$$\lambda_j(\Re A) \le \sigma_j(A), \quad j = 1, 2, \ldots, n. \tag{8}$$

Consequently,

$$\|\Re A\| \le \|A\|. \tag{9}$$

Lemma 2.2 (see [13, Lemma 3.1]) *Let $A \in \mathbb{M}_n(\mathbb{C})$ be such that $W(A) \subseteq S_\alpha$ for some $\alpha \in [0, \pi/2)$. Then*

$$\|A\| \le \sec(\alpha) \|\Re A\|. \tag{10}$$

Lemma 2.3 (see [2, (4)]) *Let $A_1, \ldots, A_n \in \mathbb{M}_n(\mathbb{C})$ be positive semidefinite. Then, for $p \ge 1$,*

$$\sum_{j=1}^{n} \|A_j\|_p^p \le \left\| \sum_{j=1}^{n} A_j \right\|_p^p \le n^{p-1} \sum_{j=1}^{n} \|A_j\|_p^p. \tag{11}$$

Lemma 2.4 (see [4, Theorem 4.1]) *Let $A \in \mathbb{M}_n(\mathbb{C})$ be such that $W(A) \subseteq S_\alpha$. Then*

$$\sigma_j(A) \le \sec^2(\alpha)\lambda_j(\Re A), \quad j = 1, 2, \dots, n.$$

The above inequality implies that there exists a unitary matrix $U \in \mathbb{M}_n(\mathbb{C})$ such that

$$|A| \le \sec^2(\alpha) U \Re A U^*. \tag{12}$$

Lemma 2.5 (see [1, Theorem III.5.6]) *Let $A, B \in \mathbb{M}_n(\mathbb{C})$. Then there exist unitary matrices $U, V \in \mathbb{M}_n(\mathbb{C})$ such that*

$$|A + B| \le U|A|U^* + V|B|V^*. \tag{13}$$

For the above preparation, we present the first main result which is an extension of Theorem 1.1.

Theorem 2.6 *Let $A_1, \dots, A_n \in \mathbb{M}_n(\mathbb{C})$ be sector matrices. Then*

$$\cos^p(\alpha) \sum_{j=1}^n \|A_j\|_p^p \le \left\| \sum_{j=1}^n A_j \right\|_p^p \le \sec^p(\alpha) n^{p-1} \sum_{j=1}^n \|A_j\|_p^p \quad \textit{for } p \ge 1. \tag{14}$$

Proof Let $A_j = B_j + iC_j$ be the Cartesian decompositions of A_j, $j = 1, \dots, n$. Then we have

$$\begin{aligned}
\left\| \sum_{j=1}^n A_j \right\|_p^p &= \left\| \sum_{j=1}^n (B_j + iC_j) \right\|_p^p \\
&= \left\| \sum_{j=1}^n B_j + i \sum_{j=1}^n C_j \right\|_p^p \\
&\ge \left\| \sum_{j=1}^n B_j \right\|_p^p \quad \text{(by (9))} \\
&\ge \sum_{j=1}^n \|B_j\|_p^p \quad \text{(by (11))} \\
&\ge \cos^p(\alpha) \sum_{j=1}^n \|A_j\|_p^p \quad \text{(by (10))},
\end{aligned}$$

which proves the first inequality.

To prove the second inequality, compute

$$\begin{aligned}
\left\| \sum_{j=1}^n A_j \right\|_p^p &= \left\| \sum_{j=1}^n (B_j + iC_j) \right\|_p^p \\
&= \left\| \sum_{j=1}^n B_j + i \sum_{j=1}^n C_j \right\|_p^p \\
&\le \sec^p(\alpha) \left\| \sum_{j=1}^n B_j \right\|_p^p \quad \text{(by (10))}
\end{aligned}$$

$$\le \sec^p(\alpha) n^{p-1} \sum_{j=1}^{n} \|B_j\|_p^p \quad (\text{by } (11))$$

$$\le \sec^p(\alpha) n^{p-1} \sum_{j=1}^{n} \|A_j\|_p^p \quad (\text{by } (9)),$$

which completes the proof. □

Remark 2.7 Theorem 1.2 is immediate by setting $\alpha = \frac{\pi}{4}$ in (14). Moreover, when $n = 2$ and $\alpha = \frac{\pi}{4}$ in Theorem 2.6, our result is Theorem 1.1.

Next, we end this section with a generalization of singular value inequality for two positive semidefinite matrices to sector matrices.

Theorem 2.8 *Let $A, B \in \mathbb{M}_n(\mathbb{C})$ be such that $W(A), W(B) \subseteq S_\alpha$. Then, for $1 \le k \le n$, the following assertions hold*:

$$\prod_{j=1}^{k} \sigma_j(A+B) \le \prod_{j=1}^{k} \sigma_j\big(I_n + \sec^2(\alpha)A\big) \prod_{j=1}^{k} \sigma_j\big(I_n + \sec^2(\alpha)B\big) \tag{15}$$

and

$$\prod_{j=1}^{k} \sigma_j(I_n + A + B) \le \prod_{j=1}^{k} \sigma_j\big(I_n + \sec^2(\alpha)A\big) \prod_{j=1}^{k} \sigma_j\big(I_n + \sec^2(\alpha)B\big). \tag{16}$$

Proof Compute

$$\begin{aligned}
\prod_{j=1}^{k} \sigma_j(A+B) &\le \prod_{j=1}^{k} \sigma_j\big(I_n + |A|\big) \prod_{j=1}^{k} \sigma_j\big(I_n + |B|\big) \quad (\text{by } (1)) \\
&\le \prod_{j=1}^{k} \sigma_j\big(I_n + \sec^2(\alpha) U \Re A U^*\big) \prod_{j=1}^{k} \sigma_j\big(I_n + \sec^2(\alpha) V \Re B V^*\big) \\
&\quad (\text{by } (12)) \\
&\le \prod_{j=1}^{k} \sigma_j\big(I_n + \sec^2(\alpha)A\big) \prod_{j=1}^{k} \sigma_j\big(I_n + \sec^2(\alpha)B\big) \quad (\text{by } (8)),
\end{aligned}$$

where U, V are unitary matrices.

To prove (16), compute

$$\begin{aligned}
\prod_{j=1}^{k} \sigma_j\big(|I_n + A + B|\big) &\le \prod_{j=1}^{k} \sigma_j\big(U_1|I_n|U_1^* + V_1|A+B|V_1^*\big) \quad (\text{by } (13)) \\
&= \prod_{j=1}^{k} \sigma_j\big(I_n + |A+B|\big) \\
&\le \prod_{j=1}^{k} \sigma_j\big(I_n + U_2|A|U_2^* + V_2|B|V_2^*\big) \quad (\text{by } (13))
\end{aligned}$$

$$\leq \prod_{j=1}^{k} \sigma_j\bigl(I_n + \sec^2(\alpha) U_3 \Re A U_3^* + \sec^2(\alpha) V_3 \Re B V_3^*\bigr)$$

(by (12))

$$\leq \prod_{j=1}^{k} \sigma_j\bigl(I_n + \sec^2(\alpha)\Re A\bigr) \prod_{j=1}^{k} s_j\bigl(I_n + \sec^2(\alpha)\Re B\bigr)$$

(by (3))

$$\leq \prod_{j=1}^{k} \sigma_j\bigl(I_n + \sec^2(\alpha) A\bigr) \prod_{j=1}^{k} \sigma_j\bigl(I_n + \sec^2(\alpha) B\bigr),$$

(by (8)),

where U_j and V_j, $j = 1, 2, 3$, are unitary matrices.

Thus

$$\prod_{j=1}^{k} \sigma_j(I_n + A + B) \leq \prod_{j=1}^{k} \sigma_j\bigl(I_n + \sec^2(\alpha) A\bigr) \prod_{j=1}^{k} \sigma_j\bigl(I_n + \sec^2(\alpha) B\bigr).$$

This completes the proof. □

The following examples show that neither (4) nor (15) is uniformly better than the other.

Example 2.9 Let

$$A = e^{-\frac{\pi i}{4}} \begin{pmatrix} \frac{1}{8} & 0 \\ 0 & \frac{1}{8} \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{2}}{16} - \frac{\sqrt{2}}{16} i & 0 \\ 0 & \frac{\sqrt{2}}{16} - \frac{\sqrt{2}}{16} i \end{pmatrix}$$

and

$$B = e^{-\frac{\pi i}{4}} \begin{pmatrix} \frac{1}{16} & 0 \\ 0 & \frac{1}{16} \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{2}}{32} - \frac{\sqrt{2}}{32} i & 0 \\ 0 & \frac{\sqrt{2}}{32} - \frac{\sqrt{2}}{32} i \end{pmatrix}$$

be such that $W(A), W(B) \subseteq S_{\frac{\pi}{4}}$.

For the right side of the inequality (4),

$$\sec^4(\alpha) \prod_{j=1}^{2} \sigma_j(I_2 + A) \prod_{j=1}^{2} \sigma_j(I_2 + B) = 5.2098.$$

When $k = 2$, for the right side of the inequality (15),

$$\prod_{j=1}^{2} \sigma_j\bigl(I_2 + \sec^2(\alpha) A\bigr) \prod_{j=1}^{2} \sigma_j\bigl(I_2 + \sec^2(\alpha) B\bigr) = 1.6886.$$

This shows that (15) is stronger than (4).

Example 2.10 If

$$A = e^{-\frac{\pi i}{4}} \begin{pmatrix} 1+\frac{1}{2}i & 0 \\ 0 & 1+\frac{1}{2}i \end{pmatrix} = \begin{pmatrix} \frac{3\sqrt{2}}{4} - \frac{\sqrt{2}}{4}i & 0 \\ 0 & \frac{3\sqrt{2}}{4} - \frac{\sqrt{2}}{4}i \end{pmatrix}$$

and

$$B = e^{-\frac{\pi i}{4}} \begin{pmatrix} 1+\frac{1}{3}i & 0 \\ 0 & 1+\frac{1}{3}i \end{pmatrix} = \begin{pmatrix} \frac{2\sqrt{2}}{3} - \frac{\sqrt{2}}{3}i & 0 \\ 0 & \frac{2\sqrt{2}}{3} - \frac{\sqrt{2}}{3}i \end{pmatrix}$$

are such that $W(A), W(B) \subseteq S_{\frac{\pi}{4}}$, we also suppose $k = 2$.

For the right side of the inequality (4),

$$\sec^4(\alpha) \prod_{j=1}^{2} \sigma_j(I_2 + A) \prod_{j=1}^{2} \sigma_j(I_2 + B) = 69.8872.$$

For the right side of the inequality (15),

$$\prod_{j=1}^{2} \sigma_j\bigl(I_2 + \sec^2(\alpha)A\bigr) \prod_{j=1}^{2} \sigma_j\bigl(I_2 + \sec^2(\alpha)B\bigr) = 94.3901.$$

The example implies that the bound in (15) is weaker than that in (4).

Remark 2.11 Actually, the above examples also show that the inequalities (5) and (16) are not comparable.

Corollary 2.12 *Let $A, B \in \mathbb{M}_n(\mathbb{C})$ be such that $W(A), W(B) \subseteq S_\alpha$. Then, for all unitarily invariant norms $\|\cdot\|$ on $\mathbb{M}_n(\mathbb{C})$,*

$$\|A + B\| \leq \bigl\|I_n + \sec^2(\alpha)A\bigr\| \bigl\|I_n + \sec^2(\alpha)B\bigr\| \tag{17}$$

and

$$\|I_n + A + B\| \leq \bigl\|I_n + \sec^2(\alpha)A\bigr\| \bigl\|I_n + \sec^2(\alpha)B\bigr\|. \tag{18}$$

Proof From (15) and (16), we obtain for $1 \leq k \leq n$

$$\prod_{j=1}^{k} \sigma_j^{\frac{1}{2}}(A + B) \leq \prod_{j=1}^{k} \sigma_j^{\frac{1}{2}}\bigl(I_n + \sec^2(\alpha)A\bigr)\sigma_j^{\frac{1}{2}}\bigl(I_n + \sec^2(\alpha)B\bigr)$$

and

$$\prod_{j=1}^{k} \sigma_j^{\frac{1}{2}}(I_n + A + B) \leq \prod_{j=1}^{k} \sigma_j^{\frac{1}{2}}\bigl(I_n + \sec^2(\alpha)A\bigr)\sigma_j^{\frac{1}{2}}\bigl(I_n + \sec^2(\alpha)B\bigr).$$

By the property that weak log-majorization implies weak majorization, we have for $1 \leq k \leq n$

$$\sum_{j=1}^{k} \sigma_j^{\frac{1}{2}}(A+B) \leq \sum_{j=1}^{k} \sigma_j^{\frac{1}{2}}\big(I_n + \sec^2(\alpha)A\big)\sigma_j^{\frac{1}{2}}\big(I_n + \sec^2(\alpha)B\big)$$

and

$$\sum_{j=1}^{k} \sigma_j^{\frac{1}{2}}(I_n + A + B) \leq \sum_{j=1}^{k} \sigma_j^{\frac{1}{2}}\big(I_n + \sec^2(\alpha)A\big)\sigma_j^{\frac{1}{2}}\big(I_n + \sec^2(\alpha)B\big).$$

Then, by the Cauchy–Schwarz inequality, for $1 \leq k \leq n$

$$\sum_{j=1}^{k} \sigma_j^{\frac{1}{2}}(A+B) \leq \left(\sum_{j=1}^{k} \sigma_j\big(I_n + \sec^2(\alpha)A\big)\right)^{\frac{1}{2}} \left(\sum_{j=1}^{k} \sigma_j\big(I_n + \sec^2(\alpha)B\big)\right)^{\frac{1}{2}}$$

and

$$\sum_{j=1}^{k} \sigma_j^{\frac{1}{2}}(I_n + A + B) \leq \left(\sum_{j=1}^{k} \sigma_j\big(I_n + \sec^2(\alpha)A\big)\right)^{\frac{1}{2}} \left(\sum_{j=1}^{k} \sigma_j\big(I_n + \sec^2(\alpha)B\big)\right)^{\frac{1}{2}}.$$

By Fan's dominance principle [1, p. 93], we have

$$\big\| |A+B|^{\frac{1}{2}} \big\|^2 \leq \big\| I_n + \sec^2(\alpha)A \big\| \big\| I_n + \sec^2(\alpha)B \big\| \tag{19}$$

and

$$\big\| |I_n + A+B|^{\frac{1}{2}} \big\|^2 \leq \big\| I_n + \sec^2(\alpha)A \big\| \big\| I_n + \sec^2(\alpha)B \big\|. \tag{20}$$

Let $A + B = U|A + B|, I_n + A + B = V|I_n + A + B|$ be the polar decomposition of $A + B$ and $I_n + A + B$, respectively, where U and V are unitary matrices. Thus, by (19), we have

$$\begin{aligned}
\big\| |A+B| \big\| &= \big\| U|A+B| \big\| \\
&= \big\| \big(|A+B|^{\frac{1}{2}}\big)^2 \big\| \\
&\leq \big\| |A+B|^{\frac{1}{2}} \big\|^2 \\
&\leq \big\| I_n + \sec^2(\alpha)A \big\| \big\| I_n + \sec^2(\alpha)B \big\|.
\end{aligned}$$

Similarly, by (20) we have

$$\|I_n + A + B\| \leq \big\| I_n + \sec^2(\alpha)A \big\| \big\| I_n + \sec^2(\alpha)B \big\|,$$

which completes the proof. □

Remark 2.13 By computing Examples 2.9 and 2.10, it should be noticed here that neither (6) nor (17) is uniformly better than the other. When comparing the inequality (7) with (18), the same conclusion can be drawn.

Taking $k = n$ in Theorem 2.8, we get the following corollary.

Corollary 2.14 *Let $A, B \in \mathbb{M}_n(\mathbb{C})$ be such that $W(A), W(B) \subseteq S_\alpha$. Then*

$$\left|\det(A+B)\right| \le \left|\det\bigl(I_n + \sec^2(\alpha)A\bigr)\right|\left|\det\bigl(I_n + \sec^2(\alpha)B\bigr)\right|$$

and

$$\left|\det(I_n + A + B)\right| \le \left|\det\bigl(I_n + \sec^2(\alpha)A\bigr)\right|\left|\det\bigl(I_n + \sec^2(\alpha)B\bigr)\right|.$$

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