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Generic stability of the solution mapping for set-valued optimization problems

Xian-Jun Long*, Ying-Quan Huang and Li-Ping Tang

*Correspondence:
xianjunlong@hotmail.com
College of Mathematics and Statistics, Chongqing Technology and Business University, Chongqing, 400067, P.R. China

Abstract

In this paper, we consider the generic stability of the weakly efficient solution mapping for set-valued optimization problems. Firstly, we obtain the upper semicontinuity of the weakly efficient solution mapping for set-valued optimization problems. Secondly, we show that, in the sense of Baire category, most set-valued optimization problems are stable. Finally, we give sufficient conditions ensuring the existence of essential. Our results extend and improve the corresponding results of Song *et al.* (J. Optim. Theory Appl. 156:591-599, 2013).

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Keywords: generic stability; set-valued optimization problem; weakly efficient solution; essential; semicontinuity

1 Introduction

Set-valued optimization is a vibrant and expanding branch of applied mathematics that deals with optimization problems where the objective function is a set-valued map acting between abstract spaces. Set-valued optimization provides an important generalization and unification of scalar as well as vector optimization problems. Therefore, this relatively new discipline has justifiably attracted a great deal of attention in recent years (see [1–21]).

Stability is very interesting and important in optimization theory and applications. It may be understood as the solution set having some topological properties such as semicontinuity, well-posedness, essential stability and so on. Essential stability was firstly introduced by Fort [22] for the study of fixed points of a continuous mapping. Since then, essentiality was applied in many nonlinear problems such as KKM points, vector equilibrium problems and Nash equilibrium problems (see [22–27]). Recently, Xiang and Zhou [28] obtained the essential stability of efficient solution sets for continuous vector optimization problems. Very recently, Song *et al.* [29] generalized the results obtained by Xiang and Zhou [28] to a set-valued case. They obtained the essential stability of efficient solution sets for set-valued optimization problems with the only perturbation of the objective function in compact metric spaces.

In this paper, we consider the stability of a weakly efficient solution mapping for set-valued optimization problems with the perturbation of both the objective function and the constraint set in noncompact Banach spaces. In Section 2 we recall some basic definitions and some known results. In Section 3 we obtain the upper semicontinuity of the weakly efficient solution mapping for set-valued optimization problems. Moreover, we show that, in the sense of Baire category, most set-valued optimization problems are stable. Finally, we give sufficient conditions ensuring the existence of essential. Our results extend and improve the corresponding results of Song *et al.* [29].

2 Preliminaries

Let X and Y be two topological vector spaces. Let $C \subset Y$ be a closed convex pointed cone with $\operatorname{int} C \neq \emptyset$, where $\operatorname{int} C$ denotes the interior of C. Let $A \subset Y$ be a nonempty subset. We denote by

$$\operatorname{WMin} A := \{y \in A : (A - y) \cap -\operatorname{int} C = \emptyset\}$$

the set of weakly efficient elements of A and by

$$\operatorname{Min} A := \{y \in A : (A - y) \cap -C = \{0\}\}$$

the set of efficient elements of A.

Let $F : X \to 2^Y$ be a set-valued map, $K \subseteq X$ be a nonempty subset. We consider the following set-valued optimization problem (in short, SOP):

$$\min_C F(x) \text{ subject to } x \in K.$$

We denote

$$F(K) = \bigcup_{x \in K} F(x).$$

Definition 2.1 A point $x_0 \in K$ is said to be a weakly efficient (resp. an efficient) solution of problem (SOP) iff there exists $y_0 \in F(x_0)$ such that $y_0 \in \operatorname{WMin} F(K)$ (resp. $y_0 \in \operatorname{Min} F(K)$).

Definition 2.2 [2] Let $G : X \to 2^Y$ be a set-valued map. T is said to be
(1) upper semicontinuous at $x_0 \in X$ if, for any open set V containing $G(x_0)$, there exists a neighborhood $U(x_0)$ of x_0 such that $G(x) \subset V$ for all $x \in U(x_0)$; G is said to be upper semicontinuous on X if it is upper semicontinuous at each $x \in X$;
(2) lower semicontinuous at $x_0 \in X$ if, for any open set V with $G(x_0) \cap V \neq \emptyset$, there exists a neighborhood $U(x_0)$ of x_0 such that $G(x) \cap V \neq \emptyset$ for all $x \in U(x_0)$; G is said to be lower semicontinuous on X if it is lower semicontinuous at each $x \in X$;
(3) continuous on X if it is both upper semicontinuous and lower semicontinuous on X;
(4) closed if $\operatorname{Graph}(G) := \{(x, y) : x \in X, y \in G(x)\}$ is a closed set in $X \times Y$.

Lemma 2.1 [2] *Let $G : X \to 2^Y$ be a set-valued map. If G is upper semicontinuous and for any $x \in X$, $G(x)$ is a closed set, then G is closed.*

Definition 2.3 Let (X, d) be a metric space and let A, B be nonempty subsets of X. The Hausdorff distance $H(\cdot, \cdot)$ between A and B is defined by

$$H(A, B) = \max\{e(A, B), e(B, A)\},$$

where $e(A,B) = \sup_{a\in A} d(a,B)$ with $d(a,B) = \inf_{b\in B} \|a - b\|$. Let $\{A_n\}$ be a sequence of nonempty subsets of X. We say that A_n converges to A in the sense of Hausdorff distance (denoted by $A_n \to A$) if $H(A_n, A) \to 0$. It is easy to see that $e(A_n, A) \to 0$ if and only if $d(a_n, A) \to 0$ for all selection $a_n \in A_n$. For more details on this topic, we refer the readers to [30, 31].

Lemma 2.2 [32] *Let A and A_n $(n = 1, 2, \ldots)$ all be nonempty compact subsets of the Hausdorff topological space X with $A_n \to A$. Then the following statements hold*:

(i) $\bigcup_{n=1}^{+\infty} A_n \cup A$ *is also a nonempty compact subset of X.*

(ii) *If $x_n \in A_n$ converging to x, then $x \in A$.*

A topological space X is said to be a Baire space if the following condition holds: given any countable collection $\{A_n\}_{n=1}^{+\infty}$ of the closed subsets of X each of which has empty interior in X, their union $\cup A_n$ also has empty interior in X. A subset G of X is called residual if it contains a countable intersection of open dense subsets of X.

Lemma 2.3 (Baire category theorem) *If X is a compact Hausdorff space or a complete metric space, then X is a Baire space.*

Lemma 2.4 ([22], Theorem 2) *Let X be a Baire space, Y be a metric space and $G : X \to 2^Y$ be upper semicontinuous with compact values. Then there exists a dense residual subset Q of X such that G is lower semicontinuous at each $x \in Q$.*

For convenience in the later presentation, denote by $K(X)$ and $K(Y)$ all nonempty compact subsets of X and Y, respectively.

Lemma 2.5 [31] *Let (X, d) be a metric space and H be Hausdorff distance on X. Then $(K(X), H)$ is complete if and only if (X, d) is complete.*

The next lemma is a special case of Lemma 2.4 in [24].

Lemma 2.6 *Let K be a nonempty compact subset of X and $G : K \to 2^Y$ be a set-valued map with nonempty compact values. Then G is continuous if and only if for any $x_0 \in K$, $x \to x_0$ implies $G(x) \to G(x_0)$.*

Lemma 2.7 [29] *Let $F_n \to F$, $n = 1, 2, \ldots$, where $F_n, F : X \to 2^Y$ are continuous on X and have nonempty compact values. If $y_n \in F_n(x_n)$, $x_n \to x^*$ and $y_n \to y^*$, then $y^* \in F(x^*)$.*

Lemma 2.8 *Let $F_n \to F$, $n = 1, 2, \ldots$, where $F_n, F : X \to 2^Y$ are continuous on X and have nonempty compact values. Then, for any $x \in X$, $y \in F(x)$, $x_n \to x$, there exists $y_n \in F_n(x_n)$ such that $y_n \to y$.*

Proof Since $F_n \to F$, $H(F_n(x), F(x)) \to 0$ for any $x \in X$. Note that

$$H\bigl(F_n(x_n), F(x)\bigr) \le H\bigl(F_n(x_n), F(x_n)\bigr) + H\bigl(F(x_n), F(x)\bigr).$$

By the continuity of F and Lemma 2.6, $H(F_n(x_n), F(x)) \to 0$. Therefore, for any $y \in F(x)$, there exists $y_n \in F_n(x_n)$ such that $y_n \to y$. The proof is complete. □

3 Main results

Throughout this section, let X and Y be two real Banach spaces, K be a nonempty subset of X, $C \subset Y$ be a closed convex pointed cone with $\operatorname{int} C \neq \emptyset$.

The space M of the problem (SOP) is defined by

$$M := \left\{ u = (F,K) : \begin{array}{l} F : K \to 2^Y \text{ is continuous and has nonempty compact values,} \\ K \text{ is a nonempty compact subset of } X. \end{array} \right\}.$$

For any $u_1 = (F_1, K_1), u_2 = (F_2, K_2) \in M$, we define the metric ρ as follows:

$$\rho(u_1, u_2) := \sup_{x \in K} H_F\bigl(F_1(x), F_2(x)\bigr) + H_K(K_1, K_2),$$

where H_F, H_K are two Hausdorff distances on Y and X, respectively.

Lemma 3.1 *(M, ρ) is a complete metric space.*

Proof Clearly, (M, ρ) is a metric space. We only need to show that (M, ρ) is complete. Let $\{u_n\}$ be a Cauchy sequence of M, where $u_n = (F_n, K_n)$. Then, for any $\varepsilon > 0$, there exists a positive integer N_1 such that

$$\rho(u_n, u_m) < \frac{\varepsilon}{3} \quad \text{for all } m, n \geq N_1.$$

It follows that for any $x \in K$,

$$H_F\bigl(F_n(x), F_m(x)\bigr) < \frac{\varepsilon}{3} \quad \text{and} \quad H_K(K_n, K_m) < \frac{\varepsilon}{3}. \tag{1}$$

This implies that $\{F_n(x)\}$ is a Cauchy sequence in $K(Y)$ and $\{K_n\}$ is a Cauchy sequence in $K(X)$. By the assumption and Lemma 2.5, $(K(Y), H_F)$ and $(K(X), H_K)$ are complete. It follows that there exist $F(x) \in K(Y)$ and $K \in K(X)$ such that

$$F_n(x) \to F(x) \quad \text{and} \quad K_n \to K. \tag{2}$$

For fixed $n \geq N_1$ and any $x \in K$, let $m \to +\infty$ in (1), we have

$$H_F\bigl(F_n(x), F(x)\bigr) < \frac{\varepsilon}{3} \quad \text{and} \quad H_K(K_n, K) < \frac{\varepsilon}{3}. \tag{3}$$

We now show that F is continuous. In fact, by the continuity of F_n and Lemma 2.6, there exist a neighborhood $U(x_0)$ of x_0 and a positive integer N_2 such that

$$H_F\bigl(F_n(x), F_n(x_0)\bigr) < \frac{\varepsilon}{3} \quad \text{for all } x \in U(x_0) \cap K, \text{ for any } n \geq N_2. \tag{4}$$

Let $N = \max\{N_1, N_2\}$. Combining with (2), (3) and (4) yields

$$H_F\bigl(F(x), F(x_0)\bigr) \leq H_F\bigl(F(x), F_n(x)\bigr) + H_F\bigl(F_n(x), F_n(x_0)\bigr) + H_F\bigl(F_n(x_0), F(x_0)\bigr) < \varepsilon$$

for all $x \in U(x_0) \cap K$ and for any $n \geq N$. By Lemma 2.6, F is continuous on K. Set $u = (F, K)$ and so $u \in M$. It follows that

$$\rho(u_n, u) = \sup_{x \in K} H_F\big(F_n(x), F(x)\big) + H_K(K_n, K) < \varepsilon,$$

which implies $u_n \xrightarrow{\rho} u$. Therefore, (M, ρ) is a complete metric space. The proof is complete. □

For any $u = (F, K) \in M$, we denote by $S(u)$ and $S_w(u)$ the efficient solution set and the weakly efficient solution set of problem (SOP), respectively. Then S and S_w define two set-valued maps from M to X. By the compactness of K and the continuity of F, the set $\mathrm{Min}(F(X))$ is nonempty, and so $S(u)$ is nonempty for any $u \in M$. Moreover, $S_w(u)$ is nonempty since $S(u) \subset S_w(u)$.

Theorem 3.1 *The set-valued map $S_w : M \to 2^X$ is upper semicontinuous with compact values.*

Proof For any $u = (F, K) \in M$, we prove that the set

$$S_w(u) = \big\{x \in K : \big(F(K) - y\big) \cap -\operatorname{int} C = \emptyset, \exists y \in F(x)\big\}$$

is compact. In fact, let $\{x_n\} \subseteq S_w(u)$ with $x_n \to x_0$. Then $x_n \in K$ and there exists $y_n \in F(x_n)$ such that

$$\big(F(K) - y_n\big) \cap -\operatorname{int} C = \emptyset. \tag{5}$$

Note that K is a compact set. It follows that $x_0 \in K$. Since $F(K) \supset F(x_n)$ is compact, there exists a subsequence of $\{y_n\}$ which converges to y_0. Without loss of generality, we may assume that $y_n \to y_0$. By the continuity of F, $y_0 \in F(x_0)$. This fact together with (5) yields $x_0 \in S_w(u)$. It follows that $S_w(u)$ is closed. Therefore, $S_w(u)$ is compact since K is compact.

Next, we prove that S_w is upper semicontinuous on M. Suppose by contradiction that there exists $u = (F, K) \in M$ such that S_w is not upper semicontinuous at u. Then there exists an open neighborhood U in X with $U \supset S_w(u)$ such that, for each $n = 1, 2, \ldots$ and each open neighborhood $V_n := \{u' = (F', K') \in M : \rho(u', u) < \frac{1}{n}\}$ of u, there exist $u_n = (F_n, K_n) \in V_n$ and $x_n \in S_w(u_n)$ but $x_n \notin U$.

From $u_n = (F_n, K_n) \in V_n$ for each $n = 1, 2, \ldots$, we have $\rho(u_n, u) < \frac{1}{n} \to 0$. This implies

$$F_n \to F \quad \text{and} \quad K_n \to K.$$

As $x_n \in S_w(u_n)$, we have $x_n \in K_n$ and there exists $y_n \in F_n(x_n)$ such that

$$\big(F_n(K_n) - y_n\big) \cap -\operatorname{int} C = \emptyset.$$

By the compactness of K and K_n and Lemma 2.2(i), $\bigcup_{n=1}^{+\infty} K_n \cup K$ is compact. Note that $\{x_n\} \subseteq \bigcup_{n=1}^{+\infty} K_n \cup K$. Then $\{x_n\}$ has a convergent subsequence. Without loss of generality, we may assume that $\{x_n\}$ is convergent. By Lemma 2.2(ii) and the uniqueness of the limit of $\{x_n\}$, $x_n \to x^* \in K$. Since $x_n \notin U$ and U is open, $x^* \notin U$. From $S_w(u) \subset U$, we have $x^* \notin S_w(u)$. It follows that

$$\bigl(F(K) - y\bigr) \cap -\operatorname{int} C \neq \emptyset, \quad \forall y \in F\bigl(x^*\bigr). \tag{6}$$

On the other hand, since $y_n \in F_n(x_n)$ and $F_n(x_n)$ is compact for any n, there exists y_0 such that $y_n \to y_0$. By Lemma 2.7, $y_0 \in F(x^*)$. Note that $K_n \to K$. Then, for any $z \in K$, there exists a sequence $\{z_n\}$ such that $z_n \in K_n$ and $z_n \to z$. By Lemma 2.8, for any $w \in F(z)$, there exists $w_n \in F_n(z_n)$ such that $w_n \to w$. Since $(F_n(K_n) - y_n) \cap -\operatorname{int} C = \emptyset$, one has

$$w_n - y_n \notin -\operatorname{int} C.$$

It follows that

$$w - y_0 \notin -\operatorname{int} C.$$

This contradicts (6). Therefore, S_w is upper semicontinuous on M. The proof is complete. □

From the proof of Theorem 3.1, we obtain that for any $u \in M$, the weakly efficient solution set $S_w(u)$ is closed. By Lemma 2.1, we have the following corollary.

Corollary 3.1 *The set-valued map $S_w : M \to 2^X$ is closed.*

Remark 3.1 Corollary 3.1 generalizes and improves the corresponding result of Song *et al.* [29], Theorem 3.1, in the following four aspects:

(1) the assumption that the metric space is compact is removed;
(2) the setting of Euclidean spaces is generalized to Banach spaces;
(3) the order cone $\mathbb{R}^n_+$ is generalized to any closed convex pointed cone;
(4) we not only consider the perturbation of the set-valued map, but also consider the perturbation of the feasible set; while Song *et al.* [29] only considered the former.

Definition 3.1 Let $u \in M$. The weakly efficient solution set $S_w(u)$ is called stable if the set-valued map S_w is continuous at u.

Remark 3.2 The following example shows that there exists $u \in M$ such that $S_w(u)$ is not stable.

Example 3.1 Let $X = \mathbb{R}$, $Y = \mathbb{R}^2$, $C = \mathbb{R}^2_+$, $K = [0,1]$ and $K_n = [\frac{1}{n}, 1]$. Define set-valued mappings $F, F_n : X \to 2^{\mathbb{R}^2}$ such that for any $x \in X$,

$$F(x) = [0,1] \times [x,1] \quad \text{and} \quad F_n(x) = \left[\frac{x}{n}, 1\right] \times [x,1].$$

Then $F_n \to F$ and $K_n \to K$ when $n \to +\infty$. By a simple computation,

$$S_w(u) = [0,1], \qquad u = (F,K),$$

$$S_w(u_n) = \frac{1}{n}, \qquad u_n = (F_n, K_n).$$

It is easy to see that S_w is upper semicontinuous at u. However, S_w is not lower semicontinuous at u. In fact, let $x_0 = 1 \in S_w(u)$, one can easily find that for small enough neighborhood $U(x_0)$ of x_0 and large enough n, $S_w(u_n) \cap U(x_0) = \emptyset$. Therefore, S_w is not stable at u.

Definition 3.2 For $u \in M$, a point $x \in S_w(u)$ is said to be essential if, for any open neighborhood U of x in X, there exists an open neighborhood V of u in M such that $S_w(u') \cap U \neq \emptyset$ for all $u' \in V$. u is said to be essential if every $x \in S_w(u)$ is essential.

From Definition 3.2, it is easy to see that the following lemma holds, so we omit its proof.

Lemma 3.2 *The set-valued map S_w is lower semicontinuous at $u \in M$ if and only if u is essential.*

We now give a generic stability result for set-valued optimization problems.

Theorem 3.2 *There exists a dense residual subset Q of M such that, for every $u \in Q$, u is essential.*

Proof By Lemmas 3.1 and 2.3, M is a Baire space. By Theorem 3.1, the set-valued map $S_w : M \to 2^X$ is upper semicontinuous with compact values. By Lemma 2.4, there exists a dense residual subset Q of M such that S_w is lower semicontinuous at each $u \in Q$. Therefore, the conclusion holds by Lemma 3.2. □

Remark 3.3 Example 3.1 shows that there exists $u \in M$ such that u is not essential.

The following theorem gives a sufficient condition that $u \in M$ is essential.

Theorem 3.3 *If $u \in M$ and $S_w(u)$ is a singleton set, then u is essential.*

Proof Suppose that $S_w(u) = \{x_0\}$. Let U be any open set in X such that $S_w(u) \cap U \neq \emptyset$. Then $x_0 \in U$ and $S_w(u) \subset U$. By Theorem 3.1, S_w is upper semicontinuous at $u \in M$. It follows that there exists an open neighborhood V of u in M such that $S_w(u') \subset U$ for each $u' \in V$. This implies that $S_w(u') \cap U \neq \emptyset$ for each $u' \in V$. Thus, S_w is lower semicontinuous at u. By Lemma 3.2, u is essential. □

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors carried out the proof. All authors conceived of the study and participated in its design and coordination. All authors read and approved the final manuscript.

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