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A novel model for representing a plane target and finding the worst-case coverage in wireless sensor network based on Clifford algebra

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Abstract
Wireless ad hoc sensor networks have recently emerged as a premier research topic. They have great long-term economic potential and ability to transform our lives and pose many new system building challenges. Sensor networks also pose a number of new conceptual and optimization problems. Most of researches in wireless sensor networks are focused in obtaining better target coverage in order to reduce energy and cost of the network. The problem of planar target analysis is one of the crucial problems that should be considered while studying coverage problem of sensor networks. By combining computational geometry and graph theoretic techniques, specifically the Voronoi diagram and graph search algorithms, this paper introduces a novel sensor network coverage model that deals with plane target problem based on Clifford algebra which is a powerful tool that is coordinate free. Also, the calculations of the node coverage rate for the plane target in the sensor network using Clifford algebra are presented. Then, the maximum clearance path (worst-case coverage) of the sensor network for a plane target is proposed. The optimality and reliability of the proposed algorithm have been proved using simulation. Also, a comparison between the breach weight of the point target and the plane target is provided.

Keywords: Clifford algebra, Wireless sensor network, Coverage, Plane target, Point target

1 Introduction

There exist a several number of applications enabled by sensor networks: Surveillance systems, forest fire systems, Environmental monitoring, target tracking, robotics and military operations. This huge development in wireless sensor networks was due to its reliability and low cost. Each sensor node of the networks can measure targets states and communicate with each other to share collected information and doing computations [1, 2]. According to the deployment method of nodes, wireless sensor towers can be divided into random deployment deterministic deployment, according to the node mobile capabilities, wireless sensor networks can be divided into static networks and dynamic networks [3]. Coverage problem is the basic problem of any type of wireless sensor network. The coverage problems of static wireless sensor networks can be divided into three categories: point coverage, area coverage and barrier coverage [4]. It is required that when a moving target traverses the network deployment area along an arbitrary path, the probability of the target not to be discovered is the smallest. So, for any sensor network, the detection probability of the target in the network when it passes through the network needs to be examined. Measuring the quality of sensor network coverage provide the concept of worst-case coverage to characterize the network ability to perceive a target [5]. To measure the possibility of a certain target otherwise if it's detected by a sensor network or not, many documents have cited exposure. In the literature [6], the exposure degree of the target is calculated considering it as the target energy collected by the sensor network, and Dijkstra's algorithm is used to find the minimum exposure path of the target. In the literature [7], exposure is regarded as the distance between the standard path and the sensor node, two paths are calculated: the maximum clearance path and the maximum support path. In [8] a two-dimensional rectangular area is considered to be guarded by a set of sensors, that may be surveillance cameras or sensors. And the minimal exposure path is calculated by first computing an approximate "feasible region" of interest using the sensors' sensing ranges, and then searching for the minimum exposure path in a systematic manner using a grid within this feasible area. It should be noticed that [9] barrier sweep coverage was investigated using mobile sensors, with the barrier being modeled as a finite-length continuous curve on a plane. The sink node utilized sensor node exposure metrics to compute the lowest exposure path, and the function fitness was created using a combination of the computed minimal exposed path and the ratio of covered to uncovered grids in [10] algorithm. When using traditional methods to calculate the maximum clearance path, the planar target traveling on the path may be incorrectly considered as not covered by nodes. This paper proposes a coverage analysis for plane target based on Clifford algebra. So, its calculation is applied to the study of the traversal problem of two-dimensional planar targets. Therefore, using Clifford geometric algebra which is a tool that does not depend on a specific coordinate system can be proposed [11]. The coverage analysis model and the method that is consistent with different targets in dimensional space can effectively solve the problem of sensor network coverage performance analysis through the complete relative information between sensor nodes and targets. On this basis, this paper uses Clifford geometric algebra. Representation of the planar target and the rate of coverage for each node to the planar target are given, and a sensor network maximum clearance path algorithm based on the planar target is proposed. The Voronoi diagram of the network is implemented to represent the planar target traversing through the sensor network [12]. Experiments show that the use of Clifford geometric algebra effectively solves the problem of searching for the optimal path of the plane target in the sensor network, which reflects the network coverage performance.

Clifford algebra was founded by WK. Clifford at the end of the nineteenth century. It is also called exterior algebra. It was extended of Grassman algebra. Clifford algebra provides calculations for space geometry without depending on coordinates to obtain a geometric symbol representation. On the other hand, it can be easily extended to higher-dimensional space for geometric calculations and analysis [13]. It has become an important research tool in theoretical mathematics and physics [11, 14]. The most popular algebraic structure today for Euclidean n-space is the inner product space R_n. This powerful extension of this structure is represented in [15, 16].

Specifically, study contributions are as follows:

- This paper provides a novel method for representing a target crossing a sensor network, where the target is represented as a plane not as a point.
- Clifford algebra which is a powerful mathematical tool is used to represent the plane target, which is simpler and easier in calculations.
- This paper provides an algorithm which is used to get the maximal breach path for a wireless sensor network based on plane target.
- Breach weight for a wireless sensor network is obtained based on plane target representation, and compared with other related work that deal with the target crossing a sensor network as a point.

The remaining part of the paper is structured out as follows: In Sect. 3, the plane target in a wireless sensor network is presented and the coverage rate of the nodes in the wireless sensor network is calculated. In Sect. 4, the maximum clearance path is obtained based on the representation of the plane target using Clifford algebra. Experimental methods and presented algorithm are introduced in Sect. 5. Finally, Results and discussion are provided in Sect. 6.

2 Related work

For the coverage problem in wireless sensor networks (WSNs), the quality of service is one of the most important evaluations indexes, which can be measured by three common and important metrics, *k*-Coverage, *k*-Barrier-Coverage and maximal breach and support. Maximal support/breach is defined as the upper/lower bound on the distances of the sensors from any path of a potential moving target, and a substantial part of related researches paid a lot of attentions on the path-based coverage problems in WSNs. Meguerdichian et al. [17] proposed centralized algorithms for the coverage problem in two cases, the best-case coverage (maximal support) and the worst-case coverage (maximal breach). This work drew two important conclusions which make the coverage become a special path planning: an optimal solution for the maximal support problem is a path which must lie along the edges of the Delaunay triangulation, and an optimal one for the maximal breach problem is a path which lies along the edges of the Voronoi diagram [18]. This work also implied that a variation localized algorithm that can be utilized to solve the worst-case coverage problem.

An efficient algorithm for exposure calculation in sensor networks, which gave a guide for solving the worst-case exposure-based coverage. Megerian et al. [5] investigated the best and worst-case coverage in the isotropic sensor networks and directional sensor networks, respectively. The authors proposed two optimal polynomial time algorithms for computing the worst-case breach coverage. Duttagupta et al. [19] paid attentions on the geometric and combinatorial properties, which improved the results in [5]. Lee et al. [20] focused on the path-based coverage problem and ultimately aimed to find an optimal placement of k additional sensor nodes to improve the coverage for a given positive integer k. They proposed an algorithm based on Minimum Spanning Tree to find a maximal support path between any pair of points in the monitored region. The coverage problems for CSNs have received an increasing number of research efforts recently [21] by Mavrinac and Chen. A new concept, the full-view coverage, in which a target is full-view covered by a camera sensor only if the target is guaranteed to be captured no matter which direction it faces was proposed in [22] by Wang et al. Based on full-view coverage model, they studied the barrier coverage problem of CSNs and developed a theoretical framework for the coverage problem in CSNs in [23]. The result of [22] was extended by Ma et al. in [24], which dealt with a full-view barrier coverage problem in CSNs. Hong et al. introduced two new coverage problems in mission-driven camera sensor networks, the target-temporal effective-sensing coverage and desperate coverage and proposed a 2-approximation algorithm and heuristics in [25]. The problem of adjusting multiple cameras to maximize the total sensing quality was investigated by Johnson and Bar-Noy in [26]. Han et al. [27] focused on an optimization problem of finding a minimum subset of camera sensors to cover a given set of targets such that the camera sensors are connected to transmit the sensing data to the sink through a multi-hop path. Piciarelli et al. [28] considered Pan–Tilt–Zoom (PTZ) cameras and proposed an algorithm to automatically change the pan, tilt and zoom parameters with the objective of maximizing the area coverage with relevant portions. Dieber et al. [29] presented a formulation of the camera selection and task assignment problem for CSNs and proposed an approximation algorithm for the selection and scheduling of cameras to sufficiently monitor the area of interest. Different from the prior researches, based on the visual sensing model, Huang et al. considered the deployment problem of optimal cameras inside a complex indoor setting and gave a 2-approximation algorithm satisfying both visibility coverage and wireless connectivity in [30]. Enhancing localization precision [31–33], tackles acoustic sensor placement optimization in irregular and constrained 3D surfaces, for inverted ultra-short baseline approaches. In the above-related works, the maximal breach has rarely considered as the QoS index for coverage problem in CSNs and coverage issue for a particular monitoring region has not been practically solved yet.

3 Representation for plane target of sensor network

Assuming that the sensor nodes in the wireless sensor network are omnidirectional sensors, and the coverage of the nodes is a binary perception model. And, in two-dimensional plane, the coverage area of sensor nodes is a circle radius R. This area is called the "Sensing Disk" of the sensor node and R is the sensing range of the sensor node which is determined by the physical characteristics of the sensor node unit.

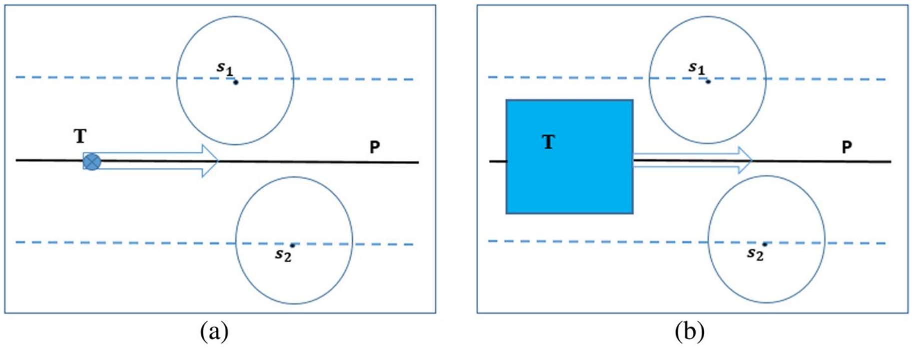

Fig. 1 **a** point target and **b** plane target through traversing path

In traditional computing sensor networks in the case of the maximal clearance path (worst-case coverage), the target is often regarded as a point target in calculations. As shown in Fig. 1a, but it's not true to deal with the target as a point. As shown in Fig. 1b, when the traditional method is used to calculate the maximum clearance path, the plane target T on the path P traveling will be wrongly calculated as not covered by a node, but in fact, when T is traveling along path P, it will be covered by nodes S_1 and S_2 [34, 35].

3.1 Plane target representation based on Clifford geometric algebra

In this section, Clifford algebra is used to provide an expression of plane targets in the sensor network. For a plane target it can be represented by the double direction of the tangent plane B [36].

$$x \wedge B = p \wedge B \tag{1}$$

where p is any point on the plane target, and formula (1) is the equation of all vectors of the tangent plane B and $p \wedge B$ into the point on B through p which is vertical. For the equation of all vectors, the support vector is $(p \wedge B)/B$, let $B = b_1 \wedge b_2$. The vector b_1, which is perpendicular to b_2. So, B can be written as $B = b_1 b_2$.

Theorem 1 *Clifford geometric algebra representation of the plane target in the sensor network is expressed as*:

$$x = \frac{p \wedge B}{B} + \tau_1 b_1 + \tau_2 b_2 \tag{2}$$

where τ_1 *and* τ_2 *are, respectively,* $\tau_1 = \left(x.b_1^{-1}\right), \tau_2 = \left(x.b_2^{-1}\right)$.

Proof Using the inner product formula $x.(b_1 b_2) = (x.b_1)b_2 - (x.b_2)b_1$, derive the parametric equation of the plane target.

$$
\begin{aligned}
x &= \frac{(x \wedge B)}{B} + \frac{(x.B)}{B} = \frac{(P \wedge B)}{B} + \frac{(x.B)}{B} \\
&= \frac{(P \wedge B)}{B} + (x.(b_1 b_2))B^{-1} = \frac{(P \wedge B)}{B} + ((x.b_1)b_2 - (x.b_2)b_1)B^{-1} \\
&= \frac{(P \wedge B)}{B} + (x.b_1)b_2B^{-1} - (x.b_2)b_1B^{-1} \\
&= \frac{(P \wedge B)}{B} + (x.b_1)b_2(b_1b_2)^{-1} - (x.b_2)b_1(b_1b_2)^{-1} \\
&= \frac{(P \wedge B)}{B} + (x.b_1)b_1^{-1} + (x.b_2)b_2^{-1} \\
&= \frac{(P \wedge B)}{B} + \left(x.b_1^{-1}\right)b_1 + \left(x.b_2^{-1}\right)b_2 \\
&= \frac{(P \wedge B)}{B} + \tau_1 b_1 + \tau_2 b_2
\end{aligned} \tag{3}
$$

Equation (3) gives the plane target parameter vector b_1 and b_2. So b_1 and b_2 can be used. The direction establishes an affine coordinate system about the plane target.

3.2 The coverage rate of nodes to plane targets

The necessary and sufficient conditions that show whether the target in the sensor network is covered by the sensor node S or not is provided in [37]. But in reality, if the plane target passes through the coverage area of the node S, it is completely covered by the node S, as shown in Fig. 2. So, it is important to calculate the coverage rate of a single S node to the plane target.

In Fig. 2, the parameter vectors b_1 and b_2 represent the plane target intersecting the coverage area of the sensor node, let the unit vectors of the coordinate system constructed are e_1 and e_2, and the point nearest to the origin of the coordinate system is p. In the same coordinate system, let the node center coordinate is S, that can be expressed as $s = s_1e_1 + s_2e_2$, where s_1 and s_2 are the components of the node in the e_1 and e_2 directions, respectively. Therefore, from Eq. (2) and the distance relationship between points in space, Eq. (4) gives the coverage area of the sensor node and the plane target, where the coverage radius of the node is R.

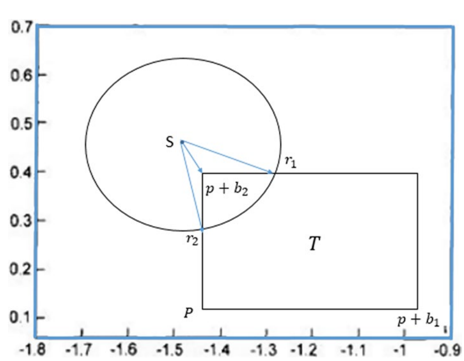

Fig. 2 Schematic representation of the intersection between single node and plane target

$$x = \frac{p \wedge B}{B} + \tau_1 b_1 + \tau_2 b_2 : \|x - s\| \leq R \tag{4}$$

Here || || represents the modulus of the vector. The four endpoints of the target can be expressed as p, $p + b_1$, $p + b_2$ and $p + b_1 + b_2$,where the vector b_1 and b_2 is vertical. As a result, if the node coverage area intersects the plane target, it must intersect or be tangent to the edge of the plane target as shown in Fig. 2. Set the intersection nodes as r_1 and r_2, respectively, then let the vector $k_1 = r_1 - S$ and the vector $k_2 = r_2 - S$. Obviously, $\|k_1\| = \|k_2\| = R$.

Suppose the angle between k_1 and k_2 is θ, then the area of the sector $Sr_1r_2 = \frac{\theta}{2\pi}\pi R^2 = \frac{\theta R^2}{2}$, where the angle θ is a vector angle. In [17] it is found that $\theta = \tan^{-1}\frac{k_1 \wedge k_2}{k_1.k_2}$. Let $a = p + b_2$, then the area of the triangle Sar_1 is $A_{\mathrm{Sar}_1} = \frac{\|a-s\|R\sin\alpha}{2}$ where, $\alpha = \tan^{-1}\frac{k_1 \wedge (a-s)}{k_1.(a-s)}$. The same $A_{\mathrm{Sar}_2} = \frac{\|a-s\|R\sin\beta}{2}$ where $\beta = \tan^{-1}\frac{(a-s)\wedge k2}{(a-s).k2}$, and the area of the plane target is$A_T = B = b_1 b_2$. So the coverage rate of the node to the plane target at this time $r = \frac{A_S}{A_T} = \frac{\frac{\theta R^2}{2} - (A_{\mathrm{Sar}_1} + A_{\mathrm{Sar}_2})}{B}$.

4 Maximum clearance path for planar target

The maximum clearance path is a path between a source and a destination that minimizes the distance between every site on the path and the sensors. To escape detection, a network intruder will seek to cross the sensor field while staying as far away from the sensors as feasible. In this situation, the detector's worst path is the maximal clearance path. The maximal breach path can be determined in the Voronoi diagram, according to Meguerdichian [19]. Gau and Peng [5] and Duttagupta et al. [38] also examined the deployment problem in order to enhance the maximal breach path.

In order to find the clearance path through a sensor network, the problem will be formalized as follows:

Given: A wireless sensor network that is deployed previously via random or deterministic deployment where each sensor location in the network is known.

Definition for Breach weight: For a path P that connects locations I and F in a given sensor network, the smallest Euclidean distance between the path P and any sensor in the network.

Problem: Finding maximum clearance path through the network connecting the locations I and F.

Note: The target crossing the network through the obtained path is plane target, not regarded as a point target.

From the concept of the coverage rate of the plane target, the choice of the path with the largest gap can be determined according to the coverage rate. Figure 3 shows a schematic diagram of the coverage of the sensor network nodes on the path. The solid line indicates the travel path of the target, and the two dashed lines indicate the area that the planar target must pass through when it travels. It can be clearly judged from Fig. 3 that the area covered by the node on the target travel path, according to formula (4). The area covered by each node can be calculated, by writing formula (4) in integral form, the coverage area can be expressed as:

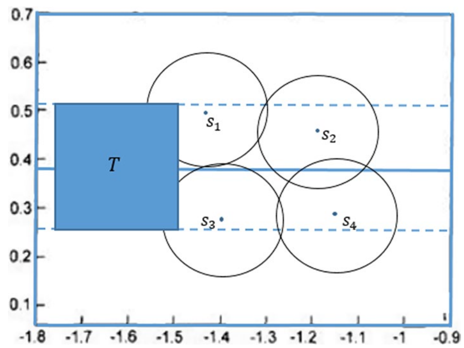

Fig. 3 Plane target passing sensor network

$$A = \int_{\|x-s\|\leq R} \left(\left(\frac{P \wedge B}{B} \right) + \tau_1 b_1 + \tau_2 b_2 \right) \mathrm{d}x \tag{5}$$

where τ_1 and τ_2 are, respectively, $\tau_1 = \left(x.b_1^{-1}\right), \tau_2 = (x.b_2^{-1})$.

Therefore, the coverage weight of the sensor network nodes on the plane target on path will be:

$$w = \frac{A}{A_T} = \frac{\int_{\|x-s\|\leq R} \left(\left(\frac{P \wedge B}{B} \right) + \tau_1 b_1 + \tau_2 b_2 \right) \mathrm{d}x}{\|B\|} \tag{6}$$

The line segments of the Voronoi diagram generated by the positions of the sensors in S must contain at least one maximal clearance path. According to generated Voronoi diagram, the distance of closest sites is maximized. By definition, any point p on the path P that deviates from Voronoi line segments will be closer to one sensor in S at least.

5 Experimental methods

This section mainly introduces the presented algorithm used to find the maximum breach path p in the network based on Clifford algebra and plane target representation. Section 5.1 introduces the structure of the algorithm.

5.1 Presented algorithm:

To find the maximum breach path p in the network. The search algorithm of the maximum gap path P is as follows [39]:

1. The Voronoi diagram D of each node position of the sensor network S is established in multiple spaces.
2. According to the parameter vector of the plane target, calculate the coverage weight of each node to the plane target on each edge of the Voronoi diagram D, as the weight assigned to each edge of the Voronoi diagram D to construct an undirected weighted graph G.
3. Using the binary search method and breadth-first search to calculate the maximum gap path P, from the weight of each edge.

Algorithm 1:

Generate bounded voronoi diagram for S with vertex set U and line segment set L
Initialize weighted undirected graph G (V, E)
FOR each vertex $u_i \in$ U
  Create duplicate vertex v_i in V
FOR each l_i $(u_j, u_k) \in$ L
  Create edge e_i (v_j, v_k) in E
Weight (e_i) = min distance from sensor $s_i \in$ S for $1 \leq i \leq S$
min_weight = min edge weight in G
max_weight = max edge weight in G
range = (max_weight - min_weight) / 2
breach_weight = min_weight + range – plane_target
WHILE (range > binary_search_tolerance)
Initialize graph $G^{'}(V^{'}, E^{'})$
FOR each v_i $\in$ V
  Create vertex v_i in $G^{'}$
FOR each $e_i \in$ E
  IF Weight $(e_i) \geq$ breach_weight
    Insert edge e_i in $G^{'}$
  IF BFS ($G^{'}$, I, F) is successful
breach_weight = breach_weight + range– plane_target
ELSE breach_weight = breach_weight + range
END IF

The difference between the presented algorithm and other algorithms is that the target used will be a plane target not regarded as a point target. This will enhance the coverage of the network due to in the case of point target, the obtained path shows that the target is not covered, however, it may be detected by at least one sensor node. So, the dimensions of the target cannot be neglected.

6 Results and discussion

A randomly configuration for 10 nodes deployed in a certain region to represent a sensor network, and the algorithm to find the maximum breach path is used to construct the breach path of a plane target crossing the sensor network as shown in Fig. 4, where the clearance path which is the minimum Euclidean distance from any point at the given path P connecting areas I and F is shown for the plane target through a 10 random distributed sensor network. Figure 5 shows a comparison between the maximum breach weight of point target and plane target through different numbers of nodes. Clearly, the maximum breach path in case of plane target is largest and the coverage quality of the sensor network is improved when the number of nodes increases, since the breach weight decreases with the increase of the number of nodes. As a result, the ability of monitoring the target is improved.

6.1 Advantage of this algorithm is:

1. The results of the worst-case path are found, that will guide the deployment of network nodes to enhance the coverage of the overall network.
2. It can be applied for sensor network path planning, target tracking and several other applications.
3. Sensor network coverage is enhanced.

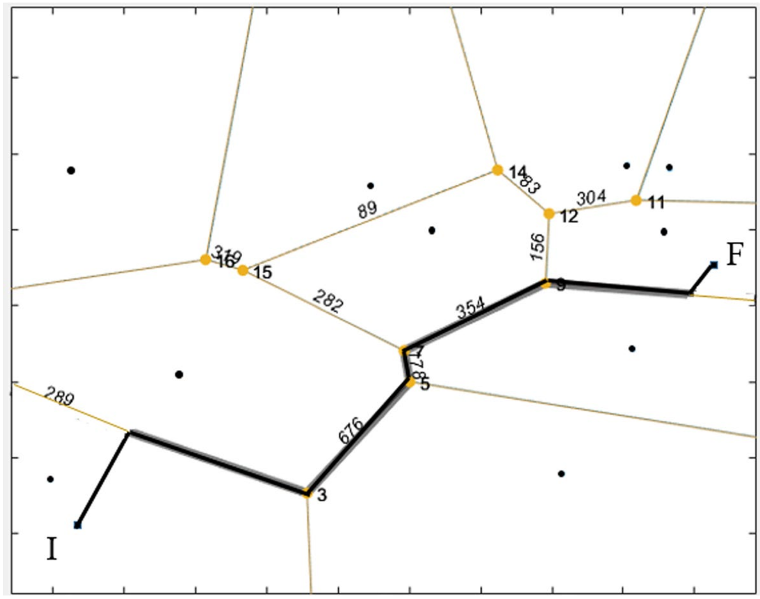

Fig. 4 Maximum breach path of the plane target

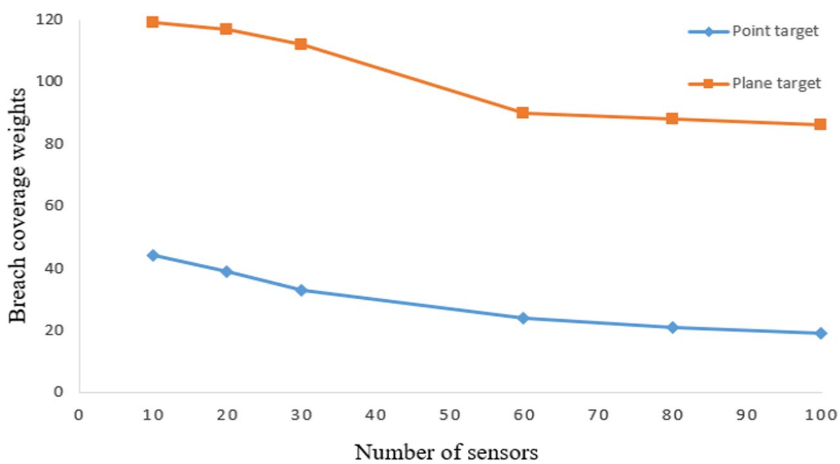

Fig. 5 Comparison between point and plane targets clearance path weights while crossing sensor network

6.2 Disadvantage of this algorithm is:

1. The locations of the sensor nodes must be known in previously.
2. Possible obstacles that may face the target, environment and noise are not considered.

As shown in Fig. 5, the coverage quality of the sensor network is improved when the number of nodes increases, since the breach weight decreases with the increase of the number of nodes. As a result, the ability of monitoring the target is improved. So, the probability of the target to be not discovered is reduced. In the case of the same number of sensor nodes, the breach weight for the plane target is greater than the breach weight of the point target. This is because during the calculation of the maximum breach weight for the plane target, not only the coverage area for the sensor node is considered but also the area of the plane target is considered.

Figure 6 represents average enhancement for breach weight coverage by applying up to four additional sensors in the network to study the breach coverage improvement. Note that for each successful installed sensor, the algorithm was repeated to calculate the new breach weight. Average enhancement over 100 random deployed sensors is presented. It is clear that coverage was improved by about 10% when just one more sensor is deployed. In the next figures, we will be able to notice the differences between other methods for calculating the breach path and the method of plane target which gives this algorithm a better results (Fig. 7).

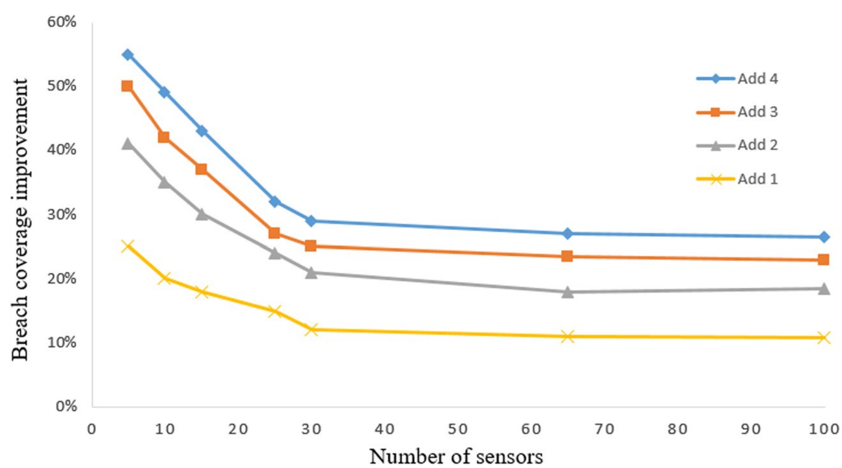

Fig. 6 Breach coverage improvement by applying four additional sensors

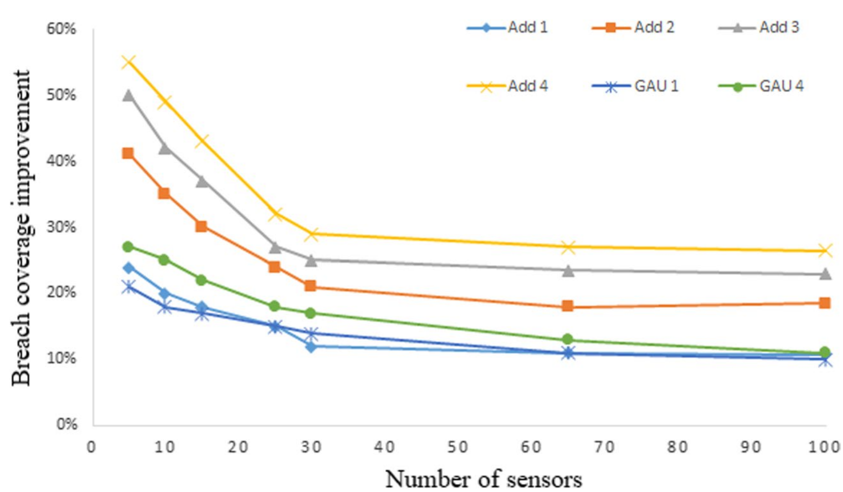

Fig. 7 Breach coverage improvement compared with Gau [21]

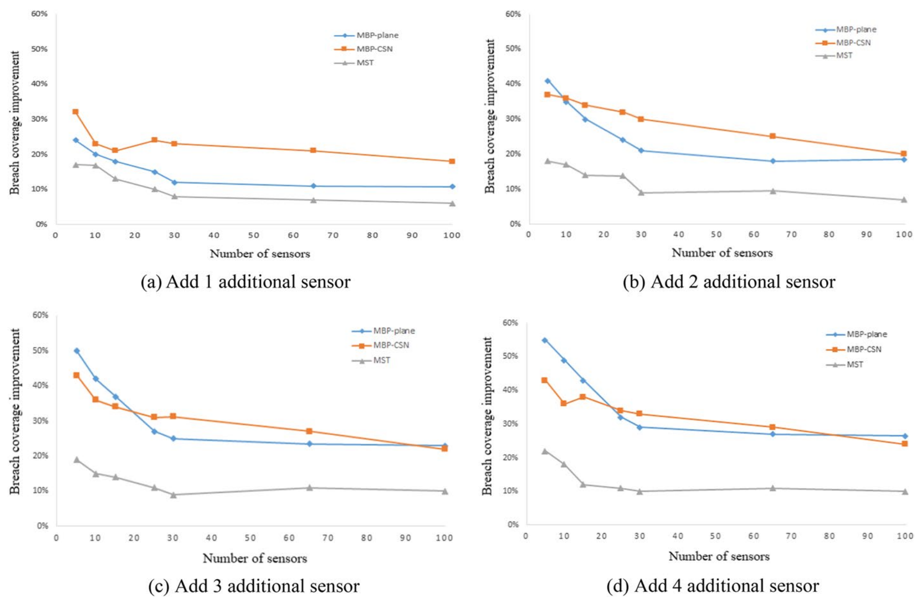

Fig. 8 Breach coverage improvement compared with MBP-CSN [22] and MST [23]

We focus on the performance of the three algorithms under the alteration of the more significant parameter, the number of sensors n, based on the parameter influence experiment. The three algorithms' performance on improving breach value decreases with the enlargement of network scale.

As indicated in Fig. 8 for the different cases of increasing additional sensors on the count of extra nodes. In some circumstances, our plane target approach beats the MBP-CSN (Maximal Breach Path for Camera Sensor Network) and MST (Minimum Spanning Tree) algorithms in terms of breach improvement ratio. The performance difference between them is greatest when four extra sensors are added, demonstrating the benefit of utilizing the plane target algorithm.

7 Conclusion

A plane target coverage analysis method in sensor networks based on Clifford algebra is proposed in this paper. Where, the target is considered as a two-dimensional surface rather than representing the target as point in previous work which gave us the opportunity to verify the benefits of this algorithm in calculating the breach path for a sensor network and improving the network quality. A formula representing the plane target and the relationship between sensor nodes and the plane target is given using Clifford algebra. The maximum breach path algorithm for a plane target is proposed. A comparison between the weights of the plane target and point target was done. The effectiveness of the algorithm was verified through experiments. Since, in the case of the same number of sensor nodes, the breach weight for the plane target is greater than the breach weight of the point target. This is because during the calculation of the maximum breach weight for the plane target, not only the coverage area for the sensor node is considered but also the area of the plane target is considered. This method can be applied to higher dimensional to monitor targets in sensor networks. In addition, due to that only omnidirectional sensor networks are used in this article, non-omnidirectional sensor networks can be included in future such as video sensor networks. That will be the next case that needs to be studied.

Abbreviations

WSN	Wireless sensor network
MBP	Maximal breach path
CA	Clifford algebra
MBP-CSN	Maximal breach path for camera sensor network
MST	Minimum spanning tree

Author contributions

AM proposed and developed the new idea of the paper and drafted it. HN and WE have substantially revised it. AM and AI conducted the data analysis and text combing. HN is responsible for supervision. All authors approved the submitted version. All authors read and approved the final manuscript.

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Availability of data and materials

Data sharing is not applicable to this article as no datasets were generated or analyzed during the current study.

Declarations

Competing interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Publisher's Note

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