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# A novel adaptive beamforming scheme for array signal data processing

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## Abstract

When the desired signal data exists in the array received data or the steering vector has a mismatch problem, the current traditional adaptive beamformers will suffer from the effect of the desired signal cancelation phenomenon, resulting in a sharp decline in performance. To address the occurrence of desired signal cancelation, an improved matrix projection-based efficient beamforming method is proposed. Firstly, based on spatial partitioning (SP) technology, a significant projection matrix for interference-plus-noise space (INS) is constructed. Secondly, using the constructed key projection matrix, the sample data covariance matrix is projected into the INS to achieve the goal of suppressing the desired signal data information. Finally, the weight data vector is calculated by Capon beamformer. The proposed algorithm does not require an iterative search for the optimal solution, which has the advantage of a small amount of calculation. Simulation experiments have verified that the proposed method has significant advantages in suppressing the desired data signals. Especially when the desired data signal has large power, the signal-to-interference-plus-noise ratio (SINR) of the proposed algorithm is better than that of the compared algorithms under the conditions of random directionality errors or local scattering errors between the desired signal and interference.

**Keywords:** Data processing, Array signal processing, Interference-plus-noise space (INS), Desired signal cancelation

## 1 Introduction

Beamforming is a technology that adjusts the amplitude and phase of signals transmitted or received by each array element in an array, so that the array can observe the target signals from a specific direction while attenuating the signal response in other directions [1–3]. It has been widely employed in many fields, such as sonar, ultrasonic imaging, Fifth Generation Mobile Communication Technology (5G), intelligent transportation [4–7]. When there is no model mismatch, Minimum Variance Distortionless Response (MVDR) beamforming is the optimal adaptive beamforming under the criterion of maximum output signal-to-interference-noise ratio (SINR). However, there are many steering vector mismatch problems caused by non-ideal factors in the actual situation, which will lead to a serious degradation of the performance of the beamformer [8, 9]. Therefore, how to improve the

robustness of adaptive beamforming algorithms in non-ideal scenarios is a major research direction [10–12].

In literature [13], a robust beamforming algorithm based on the worst-case optimization (WCO) idea is proposed, which is robust to various multiple mismatch situations. For the steering vector error caused by the steering deviation, the steering vector cyclic iterative search beamforming method, which is defined as sequential quadratic programming (SQP) [14, 15], iteratively searches for the optimal steering vector from a preset initial value in a characteristic subspace composed of a predefined observation sector, overcoming the steering vector error through optimizing the selection of orthogonal vectors of steering vectors to achieve the maximum output SINR. The literature [16] is based on the literature [14] and studies robust beamforming with as little as possible prior (LP) information. Specifically, based on steering vector estimation, a robust beamforming algorithm is proposed, making the proposed algorithm unnecessary for other prior information except for the predefined observation sectors. In literature [17], an efficient beamforming method based on sparse multi-input multi-output (MIMO) array and spatial filter bank (SFB) is proposed. In literature [18], an adaptation beamforming called continual learning-based beamforming neural network (CL-BNN) method is addressed, the advantage of which is its satisfying performance in a time varying environment without large computation. In literature [19], an efficient beamforming method called alternating direction method of multipliers (ADMM)-based is proposed for dealing with all kinds of complicated situation, which obtains better result with low complexity. An anti- array mutual coupling algorithm [20] for covariance matrix reconstruction is proposed, which only requires the information of the desired direction of the signals without requiring other prior information to achieve a good anti-mutual coupling effect.

In this paper, an improved efficient matrix projection-based beamforming algorithm is proposed, which is different from the traditional projection beamforming technology. Firstly, the projection matrix of the INS is established by adopting the method of spatial division aforementioned in literature [16] and combined with the range of known desired signals. Then, the data covariance matrix is projected into the INS to suppress the information of the desired signals in the data covariance matrix. Finally, the matrix obtained after the projection and the corrected desired steering vector is substituted into the Capon beamformer to obtain a weighted vector. Among these steps, the “matrix projection” mentioned in the second step refers to projecting the steering vectors of each signal constituting the new reconstruction matrix into the target space, respectively. Furthermore, it is theoretically proven that the matrix projection can be easily implemented through the data covariance matrix. Compared with the currently recognized robust adaptive beamforming algorithms, the proposed algorithm has advantages in performance, which is also able to suppress the influence of observation errors, and converge faster with the number of sampled snapshots.

## 2 Array data model

Assume a uniform linear array (ULA) consisting of  $M$  isotropic sensors, with an array element spacing of half a wavelength, and the received signals being a far-field narrow-band source signal. The desired signal and interference are uncorrelated. At moment  $k$ , the data received by the array can be represented as

$$\mathbf{x}(k) = \mathbf{s}(k) + \mathbf{i}(k) + \mathbf{n}(k) \tag{1}$$

Among them,  $\mathbf{s}(k) = s(k)\mathbf{d}(\theta_s)$ ,  $\mathbf{i}(k)$ , and  $\mathbf{n}(k)$  stands for statistically independent the expected signals, interference, and noise, respectively.  $s(k)$  is the source signal, and  $\mathbf{d}(\theta_s)$  is the steering vector. Therefore, the steering vector ULA-based is described as follows

$$\mathbf{d}(\theta) = [1 \ e^{j\pi \sin \theta} \ \dots \ e^{j\pi(M-1) \sin \theta}]^T \tag{2}$$

Furthermore, the received signal covariance matrix is derived as follows

$$\mathbf{R} = \sigma_s^2 \mathbf{d}(\theta_s) \mathbf{d}^H(\theta_s) + \sum_{i=1}^Q \sigma_i^2 \mathbf{d}(\theta_i) \mathbf{d}^H(\theta_i) + \sigma_n^2 \mathbf{I} \tag{3}$$

Among them,  $(\sigma_s^2, \{\sigma_i^2\}_{i=1}^Q)$  delegates the energy of the received  $Q + 1$  uncorrelated source signals,  $(\theta_s, \{\theta_i\}_{i=1}^Q)$  stands for the direction-of-arrival (DOA) of these source signals,  $\theta_0$  represents the desired source signal, and  $\{\theta_i\}_{i=1}^Q$  indicates  $Q$  interference information.  $\sigma_n^2 \mathbf{I}$  is the noise covariance matrix, and  $\mathbf{I}$  is a well-known identity matrix.

In practical applications, the theoretical covariance matrix cannot be realized. Therefore, the data sampling covariance matrix is used for calculation in the simulation experiments. And define the data covariance matrix obtained from the  $N$  samples of data as bellow

$$\mathbf{R}_x = \frac{1}{N} \sum_{n=1}^N \mathbf{x}_n(k) \mathbf{x}_n^H(k) \tag{4}$$

The adaptive beamforming can be further expressed as follows

$$y(k) = \mathbf{w}^H \mathbf{x}(k) \tag{5}$$

where  $\mathbf{w} = [w_1, \dots, w_M]^T \in \mathbb{C}^M$  denotes the complex weighted vector of the sensor array.

Therefore, the optimal output SINR of beamforming can be calculated by using the following formula

$$\text{SINR} = \frac{\sigma_s^2 |\mathbf{w}^H \mathbf{d}(\theta_0)|^2}{\mathbf{w}^H \mathbf{R}_{i+n} \mathbf{w}} \tag{6}$$

where  $\mathbf{R}_{i+n}$  stands for the covariance matrix of the noise-plus-interference.

Further, the maximization problem of SINR can be transformed into the following optimization scheme

$$\begin{aligned} & \underset{\mathbf{w}}{\text{minimize}} && \mathbf{w}^H \mathbf{R}_{i+n} \mathbf{w} \\ & \text{subject to} && \mathbf{w}^H \mathbf{d}(\theta_0) = 1 \end{aligned} \tag{7}$$

The solution to the above problem is the MVDR beamformer, also known as the Capon beamformer. The specific formula is as follows

$$\mathbf{w}_{opt} = \frac{\mathbf{R}_{i+n}^{-1} \mathbf{d}(\theta_s)}{\mathbf{d}^H(\theta_s) \mathbf{R}_{i+n}^{-1} \mathbf{d}(\theta_s)} \quad (8)$$

As can be found from Eq. (8) that the essence of the Capon beamformer is to only pass the desired steering vector without distortion, while these signals in other directions are regarded as interference, and its output should be suppressed, resulting in deep null in the output directional diagram at the position of strong interference points. In practice, the desired source signals will be suppressed as interference when there are undesirable conditions such as sensor array error and observation error, which is known as desired signal cancellation [21]. Research shows that the greater the steering vector mismatch or the stronger the desired signal, the weaker the interference suppression ability of the Capon beamformer, the more serious the cancellation of the desired signal, and the worse the beamforming performance. Therefore, if the energy of the desired signal direction in the data covariance matrix can be suppressed, the performance of the Capon beamformer will be effectively improved.

### 3 The proposed beamforming method

Based on the spatial partitioning method [16], combined with the known region of the incoming signal, the projection matrix that divides the signal space and the interference-plus-noise space can be constructed. By further analyzing the structure of the theoretical covariance matrix, Eq. (3) can be repressed as follows

$$\mathbf{R} = \int_{\theta \in [-\frac{\pi}{2}, \frac{\pi}{2})} \sigma^2(\theta) \mathbf{d}(\theta) \mathbf{d}^H(\theta) d\theta \quad (9)$$

Description the covariance matrix projection expression as follows

$$\begin{aligned} \hat{\mathbf{R}} &= \hat{\mathbf{P}}_{U_1} \cdot \mathbf{R} \cdot \hat{\mathbf{P}}_{U_1}^H \\ &= \int \sigma^2(\theta) \cdot \left( \hat{\mathbf{P}}_{U_1} \mathbf{d}(\theta) \right) \cdot \left( \hat{\mathbf{P}}_{U_1} \mathbf{d}(\theta) \right)^H d\theta \end{aligned} \quad (10)$$

Among them,  $\hat{\mathbf{P}}_{U_1}$  is composed by the region of the desired source signals. And Eq. (10) completes the projection transformation from the covariance matrix to the INS. It is known that in the projected matrix, the desired signal information will be suppressed. In the following simulation experiments, the theoretical covariance matrix will be replaced by the data covariance matrix to complete the projection operation.

Theoretical and experimental data indicate that the matrix obtained in Eq. (10) still requires diagonal processing to achieve normal noise levels, with the loading amount being the average power of the noise. The specific expression is as follows

$$\hat{\mathbf{R}} = \hat{\mathbf{R}} + \hat{\sigma}_n^2 \mathbf{I} \quad (11)$$

Among them,  $\hat{\sigma}_n^2$  is the estimated value of noise power, which can be equivalently replaced by 10 times the minimum eigenvalue of the data covariance matrix.

By substituting  $\hat{\mathbf{R}}$  into the Capon beamformer, the weighted vector of the sensor array element can be achieved

$$\mathbf{w}_{proj} = \frac{\hat{\mathbf{R}}\mathbf{d}(\theta_0)}{\mathbf{d}^H(\theta_0)\hat{\mathbf{R}}^{-1}\mathbf{d}(\theta_0)} \quad (12)$$

From the above analysis, it can be concluded that the calculated amount of the proposed beamforming algorithm is mainly concentrated on the inverse operation of the matrix in Eq. (12), so the complexity of the proposed algorithm is  $O(M^3)$ , which is the same as that of the Capon beamforming algorithm. In addition, the proposed algorithm does not require iterative search for the optimal solution operation, which greatly saves computing resources and is easier to implement in practical computers and hardware.

#### 4 Results and discussion

In this section, the simulation conditions are set to use a ULA composed of 30 array elements, with a spacing of half a wavelength between the elements. Let the noise be independent additive Gaussian white noise with zero mean and standard deviation. The directions of the two interference waves are, respectively,  $-20^\circ$  and  $-50^\circ$ , and signal-to-noise ratio (SNR) is both 30 dB. Assume the desired source signals and the interference signals are not correlated with each other, and the detected direction of the desired signal is  $5^\circ$ . All incident signals conform to the model of plane wave. In all simulation experiments, each data result is averaged from 500 Monte Carlo simulations.

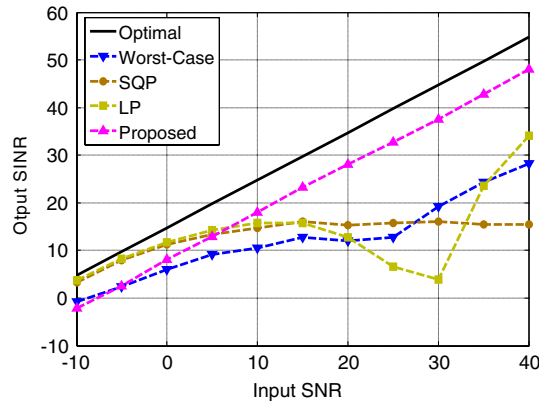
In each simulation experiment, the proposed matrix projection beamforming algorithm is compared with the worst-case beamformer [13], the SQP beamformer [14], and the LP beamformer [16], among which the uncertainty set parameter used in the worst-case beamformer is  $\varepsilon = 0.3M$  in the literature [13]. The main eigenvalues of matrix  $\mathbf{C}$  of  $K = M - 1$  are used in the proposed algorithm and taking  $\Theta = [0^\circ, 10^\circ]$ , and 10 times of the minimum eigenvalue of  $\mathbf{R}$  are selected as the diagonal loading amount  $\hat{\sigma}_n^2$ . In each figure, the optimal output SINR is given, which is calculated from covariance matrix Eq. (6) of the true desired signal arrival direction and interference noise.

##### Experiment 1: The Random Directivity Error of the Desired Signal and the Interference

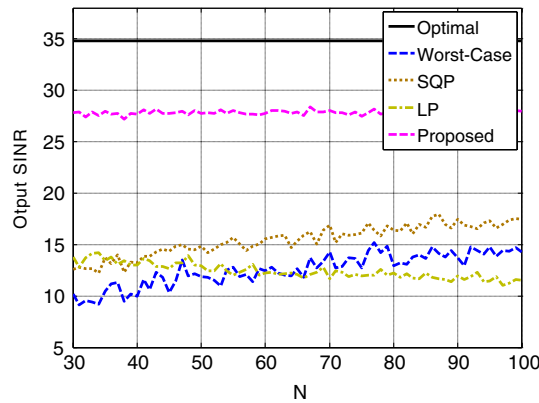
In the first experiment, assume the random directivity error is uniformly distributed in  $[-4^\circ, 4^\circ]$ , i.e., the true arrival direction of the desired signal is the arbitrary value in  $[1^\circ, 9^\circ]$  in each Monte Carlo simulation, the arrival directions of two interferences are the arbitrary values in  $[-24^\circ, -16^\circ]$  and  $[-54^\circ, -46^\circ]$ , respectively, but the arrival direction values are not changed in different sampling snapshots in each Monte Carlo simulation.

The change relationship between the output SINR of beamformer and input signal to SNR of the desired signal is shown in Fig. 1. The number of sample snapshots used in this simulation is set to 60. It can be found that the output SINR of the proposed algorithm is lower than that of the other three comparison algorithms when the input SNR is negative. However, the performance of the proposed algorithm is significantly higher than that of the comparison algorithm when the input SNR is above 10 dB, but the difference of the optimal SINR is about 6 dB.

Figure 2 shows the conditions of output SINR of the four types of beamformers varying with the number of sampled snapshots. In this simulation, the desired signal input SNR is 20 dB. As shown in the figure, the convergence rate of the proposed algorithm is obviously higher than that of the algorithms compared.



**Fig. 1** Output SINR versus input SNR. (Experiment 1)



**Fig. 2** Output SINR versus input snapshots. (Experiment 1)

**Experiment 2: Non-correlated Local Scattering Jamming Existed**

The distributed signal or non-correlated local scattering signal is caused by multipath scattering of local scattering sources, which are widely used in the applications such as radar, sonar, space radio, and wireless communication. In this simulation experiment, we assume that the desired signal is the non-correlated scattering signal with time-varying spatial characteristics, and the steering vector is expressed as follows

$$\mathbf{a}(k) = s_0(k)\mathbf{d}(\theta_s) + \sum_{p=1}^4 s_p(k)\mathbf{d}(\theta_p) \tag{13}$$

where  $s_0(k)$  and  $s_p(k)$  ( $p = 1, 2, 3, 4$ ) are independent and identically distributed zero-mean complex Gaussian random variables, and the variance is 1. The value of  $\theta_p$  ( $p = 1, 2, 3, 4$ ) follows Gaussian distribution with the mean value is  $\theta_s = 5^\circ$ , and the variance is  $4^\circ$  in each Monte Carlo simulation, but the value of  $\theta_p$  ( $p = 1, 2, 3, 4$ ) does not change in different sampling snapshots in the same Monte Carlo simulation, and the value of  $s_0(k)$  and  $s_p(k)$  ( $p = 1, 2, 3, 4$ ) will be different in different sampling snapshots. Therefore, the norm  $\|\mathbf{a}(k)\|$  of the steering vector of the desired signal is time varying. It can be found from non-correlated scattering signal model, the signal covariance matrix

$\mathbf{R}$  is no longer a rank-one matrix, and the output SINR is no longer given by Eq. (6), which should be given by the definition formula Eq. (14).

$$\text{SINR} = \frac{\mathbf{w}^H \mathbf{R}_s \mathbf{w}}{\mathbf{w}^H \mathbf{R}_{i+n} \mathbf{w}} \tag{14}$$

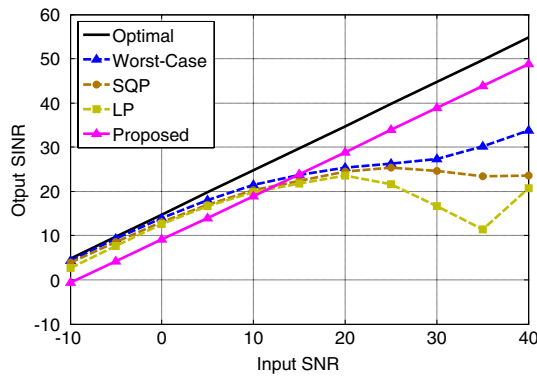
The optimal weight of the output maximizing SINR [22] can be achieved

$$\mathbf{w}_{opt} = \mathbf{P} \left\{ \mathbf{R}_{i+n}^{-1} \mathbf{R}_s \right\} \tag{15}$$

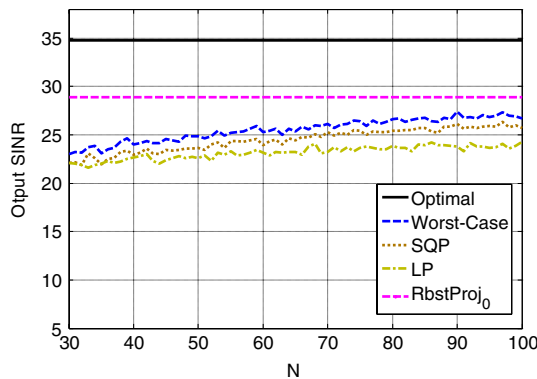
where  $\mathbf{P}\{\cdot\}$  denotes the principal eigenvalue of the matrix.

Figure 3 shows the change relationship between the output SINR of the four types of beamformers and the input SNR of the desired signal. The number of sampling snapshots is set to 60. It can be seen that when the input SNR is above 15 dB, the performance of the proposed algorithm is obviously higher than that of the comparison algorithms, but the difference of the optimal SINR is about 6 dB.

Figure 4 shows the conditions of output SINR of the four types of beamformers varying with the number of sampled snapshots. In this simulation, the desired signal input SNR is 20 dB. It can be found that the convergence rate of the proposed algorithm is obviously higher than that of the algorithms compared.



**Fig. 3** Output SINR versus input SNR. (Experiment 2)



**Fig. 4** Output SINR versus input snapshots. (Experiment 2)

## 5 Conclusions

A novel fast and efficient beamforming technique that relies on matrix projection is addressed in this paper. Specifically, firstly, under the premise of knowing the area covered by the desired source signals, the spatial partitioning technique is used to construct a projection matrix composed of INS. Secondly, the data covariance matrix is projected into the INS to effectively restrain the desired source signals, greatly improving the problem of desired signal cancelation in the case of excessive input SNR and steering vector mismatch. One of the advantages of the proposed algorithm is that its computational complexity is  $O(M^3)$  without iterative operations, which is the same as that of Capon beamformer. Therefore, it is able to facilitate computer deployment and hardware implementation. The simulation results prove that when the input SNR is high, the performance of the proposed beamforming algorithm is superior to that of the comparison algorithms in the presence of mismatch issues.

### Abbreviations

SP	Spatial partitioning
INS	Interference-plus-noise space
SINR	Signal-to-interference-plus-noise ratio
5G	5Th Generation Mobile Communication Technology
MVDR	Minimum variance distortionless response
WCO	Worst-case optimization
SQP	Sequential quadratic programming
LP	As little as possible prior
MIMO	Multi-input multi-output
SFB	Spatial filter bank
CL-BNN	Continual learning-based beamforming neural network
ADMM	Alternating direction method of multipliers
ULA	Uniform linear array
DOA	Direction-of-arrival
SNR	Signal-to-noise ratio

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### Author contributions

HPS proposed the core idea and discussed the performance. HPS and GHY wrote the manuscript. HJH and XHJ proposed some significant advice and improved the manuscript. All authors read and approved the final manuscript.

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### Availability of data and materials

The datasets used and/or analyzed during the current study are available from the corresponding author on reasonable request.

### Declarations

#### Ethics approval and consent to participate

This article does not contain any studies with human participants or animals performed by any of the authors.

#### Consent for publication

Not applicable.

#### Competing interests

The authors declare that they have no competing interests.



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