Huang et al. EURASIP Journal on Wireless Communications and Networking (2017) 2017:201
DOI 10.1186/s13638-017-0985-1

EURASIP Journal on Wireless Communications and Networking

RESEARCH Open Access

Distributed resource allocation with imperfect spectrum sensing information and channel uncertainty in cognitive femtocell networks

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Abstract

To meet the ever-increasing demands of mobile traffic, femtocells are considered as one of the promising solutions. In this paper, we study a sensing-based resource allocation scenario in cognitive femtocell networks and present an efficient distributed imperfect-spectrum-sensing-based resource allocation (DIRA) algorithm while considering the channel uncertainty to maximize the total data rate of cognitive femtocell networks by jointly optimizing both subchannel assignment and power allocation taking into account the influence of the sensing accuracy. However, the general optimization problem turns out to be a mixed integer programming problem. In order to make it tractable, the original optimization problem is divided into two sub-optimization problems,namely,optimal subchannel allocation and optimal power allocation. Specifically, the proposed distributed fairness-based subchannel allocation (DFSA) algorithm guarantees fairness by introducing channel condition difference and satisfaction degree as the indicators of subchannel allocation. Additionally, optimal power allocation with the consideration of imperfect spectrum sensing and interference uncertainty is performed using the proposed chance-constrained power optimization (CPO) algorithm. Bernstein's approximation is conducted to make the chance constraint tractable. Simulation results illustrate that the distributed imperfect-spectrum-sensing-based resource allocation (DIRA) algorithm can provide considerable fairness among femtocells and at the same time maximize the total data rate of the cognitive femtocell network.

Keywords: Cognitive femtocell, Resource allocation, Imperfect spectrum sensing, Interference uncertainty

1 Introduction

To accommodate with this ever-increasing demand for mobile data transmission, the mobile network operators (MNOs) is facing with urgent requirement of seeking for new technologies to enhance the capacity by 1000 times [1]. In this context, small cell deployment has been viewed as one of the most effective and cost-efficient solutions [2]. Small cell is an umbrella term for low-powered radio access nodes with a range of 10 m up to several hundred meters. It can be generally categorized into femtocells, picocells, or microcells according to their coverage range in ascending order. As several studies show more than 70% of data traffic occurs indoors [3], this has led to increasing interest in femtocells, which is known as home base station. The low-power, short-range, easy plug-and-play, and self-organization features of femtocells benefit both the users and operators.

Embedding femtocells in traditional cellular system helps offloading the overloaded traffic in macrocells, expanding coverage, and boosting network capacity. However, the scarcity of the available wireless spectrum resource becomes a challenging issue in the development of wireless communication technologies [4], which urges MNOs to optimally utilize the bandwidth in order to obtain the maximization of network capacity. In this case, dedicated-channel deployment of femtocells is no longer preferable from the operator's perspective because of radio resource shortage and implementation difficulty. Compared with dedicated-channel deployment,

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co-channel deployment is more attractive due to easy implementation and more efficient utilization of wireless spectrum. However, co-channel deployment might introduce severe cross-tier interference between macrocells and femtocells where they share the same spectrum [5], especially when they are close to each other [6], and co-tier interference between co-channel deployed femtocells in dense deployment scenario. Therefore, interference mitigation in such two-tier networks needs to be tackled. Moreover, the random deployment, restricted/closed access of femtocells, and no coordination between two tiers turn this problem into a hard nut to crack [7]. Facing with the above challenges, cognitive radio has emerged as a potential technology which enables radio devices to monitor the radio environment and dynamically adjust transmission parameters according to the sensing result. A femtocell base station (FBS) equipped with cognitive function will be capable of identifying and choosing the suitable subchannel that provides the least harmful influence to others.

1.1 Related works

A considerable amount of literature is available for resource allocation and interference management in cognitive femtocell system. In [8], a hybrid overlay/underlay spectrum access mechanism was proposed to improve the overall system performance of cognitive femtocell networks. The subchannel allocation problem was formulated as a coalition formation game, and a modified recursive core algorithm was proposed to achieve stable and efficient allocation. In [9], a distributed joint power control method came up with a proper solution to manage the interference in two-tier femtocell networks.

Usually, subchannel allocation and power control are jointly considered in literatures. In [10], a resource allocation scheme was proposed to maximize the total data rate of cognitive small cells without causing intolerable interference to macrocell users, in which femtocell user equipments (FUEs) could estimate the interference channel through the pilot signals broadcasted by macrocell base station (MBS). Moreover, the proposed algorithm could ensure the fairness with only a tiny reduction in throughput performance. In [11], a decentralized approach for dynamic subchannel and power allocation was considered. Reinforcement learning-based algorithm was applied to solve the uncoordinated spectrum sharing problems. The spectrum allocation was based on reinforcement learning algorithm, while the power allocation applied convex optimization algorithm which was decided by each femtocell independently. Besides, game theory is also a feasible solution which has been widely investigated in existing researches to jointly investigate the subchannel and power allocation in femtocell networks. In [12], a cooperative Nash bargaining game model was developed to study the subchannel allocation and power control problem jointly for cognitive small cell networks with the consideration of cross-tier interference mitigation, outage limitation, imperfect CSI, and fairness.

Due to the inherent hardware limitations and variable wireless environment, spectrum sensing errors are inevitable, causing interference to macrocell user equipments (MUEs) in cognitive femtocell networks [13]. Hence, owing to the imperfectness of the spectrum sensing, the traditional resource allocation algorithms might experience performance decrease. In [14], authors used the particle swarm optimization (PSO) algorithm to solve a joint uplink resource allocation problem for cognitive networks under the consideration of imperfect spectrum sensing. In [15], a multi-objective optimization problem that jointly considered the femtocell throughput maximization and transmit power minimization was formulated, subject to interference constraints on both femtocell and macrocell including the co-channel interference and adjacent channel interference constraints under spectrum sensing error probabilities. In [16], pricing and power allocation strategies were studied in a two-tier femtocell network with the aim of maximizing energy efficiency, where both perfect and imperfect spectrum sensing cases were considered.

Besides, the information uncertainty is also significantly important for the variable wireless environment, including channel state information uncertainty [17], network access state uncertainty [18], and background noise uncertainty [19]. Particularly, those kind of uncertainty problems can be presented as a probabilistic problem by relaxing the constraint into an equivalent chance constraint [20–22]. Based on the aforementioned solutions, jointly considered subchannel allocation and power allocation are a proper way to solve resource allocation problems. The objectives of resource allocation mainly focus on interference management [9, 10, 12, 13, 19–22], capacity enhancement [8, 10, 13, 15, 20–22], power efficiency improvement [15, 16, 19], and fairness [10, 12]. However, to the best of our knowledge, resource allocation for cognitive femtocell network jointly considering interference management, fairness, imperfect spectrum sensing, and interference uncertainty has not been studied in previous works.

1.2 Contributions

Traditionally, the distribution of femtocells is modeled by a hexagonal grid. However, in two-tier femtocell networks, topological randomness causes the grid-based model too idealized for both macrocells and femtocells, especially when most of femtocells are installed by their subscribed users. Recently, a new analytical method has gathered considerable attention, named stochastic geometry. It can not only capture the topological randomness but also provide tractable analytical results [23]. Hence, we model the cognitive femtocell network using stochastic geometric tools [24]. In this paper, resource allocation problem including subchannel scheduling and power allocation was jointly investigated for cognitive femtocell networks, where the overall objective is to maximize the total data rate, taking into account the influence of the imperfect spectrum sensing and channel uncertainly. The main contributions of the paper are the following:

- Firstly, the distribution of femtocells is modeled by a hexagonal grid in the existing literature. However, the topological randomness causes the grid-based model too idealized for both macrocells and femtocells, especially when most of femtocells are installed by their subscribed users. In this paper, the cognitive femtocell network topology is based on stochastic geometry; macrocell and femtocell base stations are modeled as two independent Voronoi tessellations, where MBSs and FBSs follow Poisson point process (PPP) distribution in their own tessellation.
- Secondly, the reliability of the spectrum sensing is taken into consideration which will affect the interference constraint. Moreover, the femto-to-macro interference constraint under channel uncertainty can be cast as the chance constraint. Bernstein's approximation is conducted to make the chance constraint tractable.
- Thirdly, in order to maximize the total data rate of femtocells under the consideration of imperfect spectrum sensing and channel uncertainty while balancing the fairness of networks, a distributed fairness-based subchannel allocation (DFSA) algorithm is implemented which uses channel condition difference as an indicator of subchannel allocation, taking the satisfaction degree of each FBS into consideration. To further increase the total capacity of femtocell network, transmit power of femtocells on each subchannel will be adapted subject to interference constraints and minimum quality of service (QoS) requirements using the chance-constrained power optimization (CPO) algorithm.

The layout of this paper is outlined as follows. Section 2 describes the network model. Section 3 formulates our optimization task with imperfect spectrum sensing and interference uncertainty. In Section 4, the DIRA algorithm is proposed. Section 5 evaluates the performance of our proposed algorithm, and numerical results are presented with discussions. Section 6 concludes this paper.

2 System model

2.1 Network topology

In this paper, a cognitive femtocell network where macrocells are overlaid with multiple cognitive femtocells is considered. We model this two-tier femtocell network by stochastic geometry tools. Figure 1 shows an example of a two-tier network scenario; the coverage area of each cell depends on its location and other cells' locations. Assuming that the MBSs and FBSs are distributed via an independent Poisson point process (PPP) of density d_m and d_f, respectively. The two-tier network model can

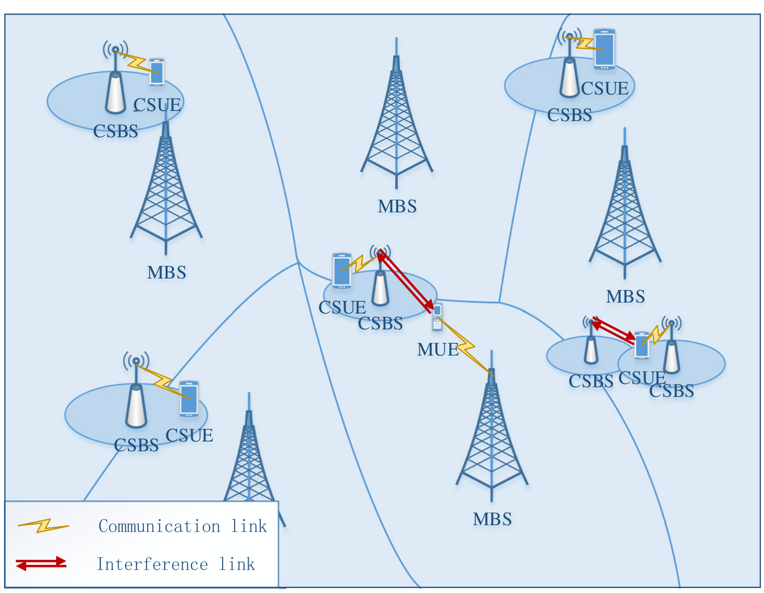

Fig. 1 A two-tier network scenario

be abstracted as two Voronoi tessellations as shown in Fig. 2. By construction, each user located in the intersection of two cells will associate with either the MBS or the FBS of the Voronoi cell covering that user. We denote the sets of MBSs and FBSs as $\mathcal{M} = [1, ..., M]$ and $\mathcal{F} = [1, ..., F]$. In each femtocell, there are K FUEs. An orthogonal frequency-division multiple access (OFDMA) downlink system is considered, where the total bandwidth of B_w is divided into N subchannels.

2.2 Channel model

Cognitive FBSs opportunistically access the licensed subchannels belong to the macrocells. Through periodic spectrum sensing performed by the cognitive FBS, subchannels can be identified as busy or idle. In this paper, we apply overlay spectrum sharing mode between MBSs and FBSs, which means subchannels which are determined as idle can be utilized by the FBS.

We assume that the knowledge of interference is uncertain for each FBS due to the uncertainty of channel gain. Let $g_{f,k,n}$ be the channel gain of the kth FUE in femtocell $f \in \mathcal{F}$ on subchannel $n \in \mathcal{N}$. Hence, the received signal-to-interfernece-plus-noise ratio (SINR) of FUE k in femtocell f on subchannel n is given as

$$\mathrm{SINR}_{f,k,n} = \frac{p^F_{f,k,n} g_{f,k,n}}{I + N_0} \tag{1}$$

where $p^F_{f,k,n}$ is the transmit power of FBS f to FUE k on subchannel n; I is the received interference from other FBSs, where $I = \sum_{e=1,e\neq f}^{F} p^F_{e,k,n} g_{e,k,n}$; and N_0 is the noise power. The data rate of FUE k in femtocell f on subchannel n is presented as

$$r_{f,k,n} = B_w/N \log_2 \left(1 + \mathrm{SINR}_{f,k,n}\right) \tag{2}$$

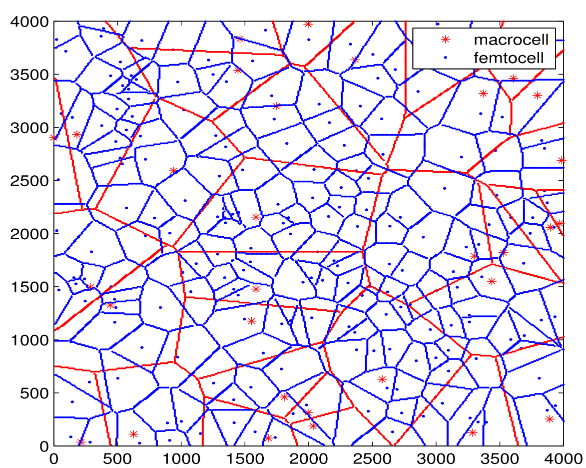

Fig. 2 The network is modeled as a superposition of two independent Voronoi tessellations

Assume that the estimated channel gain $g_{f,k,n}$ can be acquired accurately through traditional channel estimation techniques. However, the channel gain $g_{f,n}$ from FBS f to the MUE on subchannel n is difficult to estimate due to the lack of cooperation between MBSs and FBSs. Thus, we model $g_{f,n}$ as

$$g_{f,n} = \bar{g}_{f,n} + \tilde{g}_{f,n} \tag{3}$$

where $\bar{g}_{f,n}$ denotes the estimated channel gain on subchannel n obtained by averaging $g_{f,k,n}$ and $\tilde{g}_{f,n}$ represents the uncertain part of the channel gain.

3 Problem formulation with imperfect spectrum sensing

In this section, firstly, we discuss the cross-tier interference caused by femtocells, which consists of co-channel interference caused by imperfect spectrum sensing and out-of-band interference introduced by sidelobe power leakage of orthogonal frequency division modulation (OFDM) signals. Secondly, an optimization framework which aims to maximizing the total data rate of the cognitive femtocell network is formulated with the consideration of imperfect spectrum sensing and interference uncertainty.

3.1 Interference from imperfect spectrum sensing and out-of-band emission

To ensure the QoS performance of MUEs, the interference caused by opportunistic access of FBSs in the licensed channel should be controlled. The interference introduced to the macro-tier consists of two parts: (i) co-channel interference as a result of imperfect spectrum sensing and (ii) cognitive radio out-of-band (OOB) emission.

Due to the inherent hardware limitations and variable wireless environment, the spectrum usage can be falsely detected by cognitive femtocells. The result of spectrum sensing on subchannel n is a binary event denoted by $S^n_0(\tilde{x}_n = 0)$ and $S^n_1(\tilde{x}_n = 1)$, where $\tilde{x}_n = 0$ or 1 identifies that subchannel n is sensed to be idle or busy. Similarly, the actual state of the subchannel n can be denoted as $H^n_0(x_n = 0)$ and $H^n_1(x_n = 1)$, where x_n represents the actual status of subchannel n, with $x_n = 0$ or 1 indicating that the subchannel n is vacant or occupied. Generally, there are four types of spectrum sensing probabilities: (i) idle channel detection probability: $p^n_{nd} = Pr\{S^n_0|H^n_0\}$; (ii) false alarm probability: $p^n_f = Pr\{S^n_1|H^n_0\}$; (iii) miss detection probability, $p^n_m = Pr\{S^n_0|H^n_1\}$; and (iv) detection probability, $p^n_d = Pr\{S^n_1|H^n_1\}$. Among the above sensing cases, false alarm and miss detection are considered as sensing errors. Since FBSs make access decisions based on the results of spectrum sensing, leading to four possible cases of spectrum sensing, which are given in Table 1, where the sensing results of femtocells are considered as priori probabilities.

Table 1 Possibilities of spectrum sensing results for MUEs

	Sensing state	Actual state	Probability
Case 1	S_0^n	H_0^n	$P_{1,n} = Pr\{H_0^n \mid S_0^n\}$
Case 2	S_1^n	H_0^n	$P_{2,n} = Pr\{H_0^n \mid S_1^n\}$
Case 3	S_0^n	H_1^n	$P_{3,n} = Pr\{H_1^n \mid S_0^n\}$
Case 4	S_1^n	H_1^n	$P_{4,n} = Pr\{H_1^n \mid S_1^n\}$

Among the possible cases shown in Table 1, subchannel n is occupied by the MBSs only in cases 3 and 4. In case 3, the spectrum sensing result of subchannel n is vacant; however, the actual state of n is busy, resulting in great cross-tier interference from FBS to MUE on subchannel n since FBS has no idea that this subchannel is not vacant as detected. In case 4, although the detection made by FBS is correct, MUE on subchannel n can still suffer from OOB emissions due to sidelobe power leakage.

We can calculate $P_{3,n}$ and $P_{4,n}$ using Bayes' theorem. Thus, we have

$$\begin{aligned} P_{3,n} &= Pr\{H_1^n|S_0^n\} \\ &= \frac{Pr\{S_0^n|H_1^n\}Pr\{H_1^n\}}{Pr\{S_0^n\}} \\ &= \frac{Pr\{S_0^n|H_1^n\}Pr\{H_1^n\}}{Pr\{S_0^nH_0^n\}+Pr\{S_0^nH_1^n\}} \\ &= \frac{Pr\{S_0^n|H_1^n\}Pr\{H_1^n\}}{Pr\{S_0^n|H_0^n\}Pr\{H_0^n\}+Pr\{S_0^n|H_1^n\}Pr\{H_1^n\}} \\ &= \frac{p_m^n p_s^n}{p_{nd}^n(1-p_s^n)+p_m^n p_s^n} \end{aligned} \tag{4}$$

$$\begin{aligned} P_{4,n} &= Pr\{H_1^n|S_1^n\} \\ &= \frac{Pr\{S_1^n|H_1^n\}Pr\{H_1^n\}}{Pr\{S_1^n\}} \\ &= \frac{Pr\{S_1^n|H_1^n\}Pr\{H_1^n\}}{Pr\{S_1^nH_0^n\}+Pr\{S_1^nH_1^n\}} \\ &= \frac{Pr\{S_1^n|H_1^n\}Pr\{H_1^n\}}{Pr\{S_1^n|H_0^n\}Pr\{H_0^n\}+Pr\{S_1^n|H_1^n\}Pr\{H_1^n\}} \\ &= \frac{p_d^n p_s^n}{p_f^n\left(1-p_s^n\right)+p_d^n p_s^n} \end{aligned} \tag{5}$$

where p_s^n represents the occupation probability of MBS on subchannel n.

In addition to the effect of imperfect spectrum sensing, OOB emission can also cause interference to MUEs. The amount of OOB interference introduced to MUEs on subchannel j by the femtocell transmission on subchannel n, with unit transmit power, is given as

$$I_{f,k,n,j}^s = \int_{(j-1)B_w/N-(n-1/2)B_w/N}^{jB_w/N-(n-1/2)B_w/N} \varphi(f) g_{f,n} df \tag{6}$$

where $\varphi(f) = T_s\left[\frac{\sin(\pi f T_s)}{\pi f T_s}\right]^2$ represents the power spectral density (PSD) of an OFDM signal; T_s is the duration of an OFDM signal.

Based on the above analysis, to jointly consider the effect of imperfect spectrum sensing and OOB emission in the interference constraint, we formulate the cross-tier interference from FBS to MUE s with unit transmit power as

$$I_{f,k,n}^s = \sum_{j\in\mathcal{N}_v} P_{3,n} I_{f,k,n,j}^s + \sum_{j\in\mathcal{N}_o} P_{4,n} I_{f,k,n,j}^s \tag{7}$$

where $\mathcal{N}_v$ represents the set of vacant subchannel in $\mathcal{N}$ and $\mathcal{N}_o$ is the set of subchannel occupied by MBSs. Considering the uncertainty of interference, we introduce $\epsilon \in (0,1)$ to guarantee the interference constraint in probability. Hence, the interference constraint can be written as a chance constraint as follows:

$$Pr\left\{\sum_{f=1}^{F}\sum_{k=1}^{K}\sum_{n=1}^{N} I_{f,k,n}^s \le I_{\text{thre},s}\right\} \ge 1-\epsilon \qquad \forall s \tag{8}$$

where $I_{\text{thre},s}$ is the interference limitation from FBSs to MUE s.

3.2 General optimization framework

In this paper, our major target is to maximize the total data rate of the femtocell network under the constraints of minimum QoS requirement and cross-tier interference, taking into account the influence of sensing accuracy as well as channel uncertainty. Thus, the general optimization problem can be formulated as follows:

$$\begin{aligned} &\max_{\tau_{f,k,n}, p_{f,k,n}^F} \sum_{f=1}^{F}\sum_{k=1}^{K}\sum_{n=1}^{N} \tau_{f,k,n} R_{f,k,n}^F \\ &\text{s.t.} \quad \text{C1:} \quad p_{f,k,n}^F \ge 0 \qquad \forall f,k,n \\ &\qquad \text{C2:} \quad \sum_{k=1}^{K}\sum_{n=1}^{N} p_{f,k,n}^F \le p_{\max}^F \qquad \forall f \\ &\qquad \text{C3:} \quad \sum_{n=1}^{N} \tau_{f,k,n} R_{f,k,n}^F \ge R_{f,k}^0 \qquad \forall f,k \\ &\qquad \text{C4:} \quad \tau_{f,k,n} \in \{0,1\} \qquad \forall f,k,n \\ &\qquad \text{C5:} \quad \sum_{k=1}^{K} \tau_{f,k,n} \le 1 \qquad \forall f,n \\ &\qquad \text{C6:} \quad Pr\left\{\sum_{f=1}^{F}\sum_{k=1}^{K}\sum_{n=1}^{N} p_{f,k,n}^F I_{f,k,n}^s \le I_{\text{thre},s}\right\} \ge 1-\epsilon \qquad \forall s \end{aligned} \tag{9}$$

where $\tau_{f,k,n} = 1$ or 0 indicates whether subchannel n is allocated to FUE k in femtocell f or not, $p_{\max}^F$ is the maximum transmit power of a FBS, and $R^0_{f,k}$ is the minimum QoS requirement of FUE k in femtcoell f.

In our optimization problem, C1–C2 are power constraints indicating the transmit power of a FBS should not exceed the maximum transmit power $p^F_{\max}$. C3 is the minimum QoS requirement for FUEs. C4–C5 are constraints of subchannel allocation, representing that a subchannel n can not be assigned to two different FUEs in the same femtocell. C6 is the cross-tier interference constraint represented by chance constraint, taking both imperfect spectrum sensing and interference uncertainty into consideration. Via C6, the cross-tier interference from FBSs to MUE s will be limited below the threshold $I_{\text{thre},s}$ with a probability not less than $1-\epsilon$. In practice, it is difficult to acquire channel information accurately; thus, it is more reasonable to present the interference constraint as a chance constraint. According to the above analysis, C4 and C5 are integer constraints, which, as a result, lead the optimization problem to be a mixed chance-constrained integer programming problem.

4 Distributed imperfect-spectrum-sensing-based resource allocation

The optimization problem in (9) is a mixed chance-constrained integer programming problem which is computationally complex to address. To make it tractable, we divide the original problem into two sub-optimization problems and solve them in two steps. Firstly, in order to remove the integer constraints of the optimization problem, we address the subchannel allocation problem using DFSA algorithm with the consideration of fairness among FBSs. In this way, the mixed chance-constrained integer programming problem is simplified as a chance-constrained programming problem without integer variables. Secondly, optimal power allocation to the subchannels is achieved by CPO algorithm. Bernstein's approximation is applied to transform the chance constraint into a convex constraint, and the Lagrangian dual algorithm is used to solve the convex power optimization problem.

4.1 Subchannel allocation by DFSA algorithm

We propose a distributed fairness-based subchannel allocation algorithm to assign the subchannels with unit power allocation. The aim of the DFSA scheme is to achieve a fair subchannel allocation, while maintaining considerable data rate of the whole network.

Through spectrum sensing, cognitive femtocells can identify the available subchannels and estimate their channel conditions. For FBS f, the channel condition $c_{f,n}$ of subchannel n is considered as the average estimated channel gain from FBS f to all FUEs associated with this FBS, given by

$$c_{f,n} = \frac{\sum_{k=1}^{K} g_{f,k,n}}{K} \tag{10}$$

As a result, the channel condition table (CCT) is created based on this channel condition. FBS f sorts its sensing results, namely, the available subchannels in a descending order with respect to the $c_{f,n}$, as shown in Table 2. By calculating the channel condition difference between certain subchannel and the next subchannel ranked in CCT, we can obtain the channel condition difference table (CCDT), as shown in Table 3.

The DFSA algorithm is based on CCDT instead of CCT. In the CCT-based subchannel allocation scheme, FBS f may suffer large quality degradation if FBS l accesses f's first rank subchannel, although there are not too much differences between the second rank and first rank subchannel of FBS l, leaving FBS f with no choice but to access the second rank subchannel with relatively worse channel condition. However, our DFSA algorithm is based on CCDT; the FBS with larger first rank difference value may have the prior chance to access its first rank subchannel, which decrease the risk of suffering performance degradation due to the loss of the preferred subchannel. By taking channel condition difference as the indicator of subchannel allocation, the fairness between FBSs based on the average data rate can be guaranteed.

Finally, a subchannel requirement table (SRT) is formulated, which consists of the subchannel requirement of a FBS, as shown in Table 4, where N^f_A is the number of total available subchannels of FBS f under spectrum sensing result and N^f_D is the number of subchannels desired by FBS f. The actual subchannel access will take place once the whole subchannel selection procedure is finished; otherwise, there exists an available subchannel list to be accessed by FBS f. N^f_{to} is the number of subchannels for FBSs to access, which is initialized as 0 at the beginning of the algorithm. counter^f represents the number of subchannels still required by FBS f, given by

$$\text{counter}^f = N^f_D - N^f_{to} \tag{11}$$

which is initialized as N^f_D and should be updated during the allocation procedure. N^f_R is the number of remaining

Table 2 Channel condition table (CCT) for FBS f

Rank	Subchannel ID	Channel condition
1	n	$c^f_1 = c_{f,n}$
2	m	$c^f_2 = c_{f,m}$
...	...	...

Table 3 Channel condition difference table (CCDT) for FBS f

Rank	Subchannel ID	Channel condition difference
1	n	$d^f_{1,2} = c^f_1 - c^f_2$
2	m	$d^f_{2,3} = c^f_2 - c^f_3$
...	...	...

available subchannels of FBS f, which is initialized as N^f_A and also changes during the allocation procedure. SD^f is the satisfaction degree of FBS f, given by

$$\mathrm{SD}^f = \frac{N^f_{to}}{N^f_D} \tag{12}$$

Furthermore, with the consideration of satisfaction degree, the fairness in the number of subchannels allocated to FBSs can be guaranteed.

Once the DFSA algorithm starts, FBSs share the CCDT and SRT with other FBSs in the same macrocell via broadcating. As a result, FBSs in the same macrocell have the knowledge of CCDTs and SRTs of each other. With this information, each FBS can perform the subchannel allocation process individually without causing collisions on the same subchannel.

In the proposed DFSA algorithm, only one subchannel can be selected by a certain FBS each time. The detail procedure of DFSA algorithm is shown in algorithm 1. The CCT, CCDT, and SRT of FBS f is denoted by CCT_f, CCDT_f, and SRT_f, respectively. Besides, the table update procedure is shown in algorithm 2.

4.2 Power allocation for femtocells by chance-constrained power optimization

Once the subchannel allocation is obtained based on DFSA algorithm, the integer variable $\tau_{f,k,n}$ in the original optimization problem in (9) can be substituted by the subchannel allocation result. To further maximize the total data rate of cognitive femtocell network, we optimize the transmit power under the constraints of power, QoS requirement, and cross-tire interference. Thus, the optimization problem can be written as

$$\begin{aligned}
&\max_{p^F_{f,k,n}} \sum_{f=1}^{F}\sum_{k=1}^{K}\sum_{n=1}^{N} \tau^*_{f,k,n} R^F_{f,k,n}\\
&\text{s.t.}\quad \text{C1:}\quad p^F_{f,k,n} \geq 0 \qquad \forall f,k,n\\
&\qquad \text{C2:}\quad \sum_{k=1}^{K}\sum_{n=1}^{N} p^F_{f,k,n} \leq p^F_{\max} \qquad \forall f\\
&\qquad \text{C3:}\quad \sum_{n=1}^{N} \tau^*_{f,k,n} R^F_{f,k,n} \geq R^0_{f,k} \qquad \forall f,k\\
&\qquad \text{C6:}\quad Pr\left\{\sum_{f=1}^{F}\sum_{k=1}^{K}\sum_{n=1}^{N} p^F_{f,k,n} I^s_{f,k,n} \leq I_{\text{thre},s}\right\} \geq 1-\epsilon \qquad \forall s
\end{aligned} \tag{13}$$

where $\tau^*_{f,k,n}$ is the subchannel allocation achieved by employing the DFSA algorithm in the previous subsection.

However, the optimization problem in (13) is still intractable due to the non-convex chance constraint in C6. To achieve a convex feasible set of C6, a convex approximation of C6 should be performed, which can be achieved by using the Bernstein approximation method [25, 27].

4.2.1 *Bernstein's approximation of the chance constraint*

Consider a chance constraint written as a form of

$$Pr\left\{f_0(\mathbf{p}) + \sum_{n=1}^{N} \zeta_n f_n(\mathbf{p}) < 0\right\} \geq 1-\epsilon \tag{14}$$

Assuming that

1) $\mathbf{p}$ is a vector of decision parameters.
2) $\{\zeta_n\}$ is the set of random variables, with marginal distribution denoted by $\{\pi_n\}$.
3) $\{\pi_n\}$ belongs to a given family of probability distribution with bounded support of $[-1, 1]$, which means ζ_n varies in the range of $[-1, 1]$.

Let us set

$$\Lambda_n(y) = \max_{\pi_n} \ln\left(\int \exp(xy) \mathrm{d}\pi_n(x)\right) \tag{15}$$

It is shown that the function

$$\Psi(t,\mathbf{p}) = f_0(\mathbf{p}) + t\sum_{n=1}^{N} \Lambda_n\left(t^{-1} f_n(\mathbf{p})\right) + t\ln\left(\frac{1}{\epsilon}\right) \tag{16}$$

Table 4 Subchannel requirement table (SRT) for FBS f

Number of total available subchannels	N^f_A
Number of desired subchannels	N^f_D
Number of subchannels to access	N^f_{to}
Number of subchannels still needed	$\mathrm{counter}^f$
Number of remained available subchannels	N^f_R
Satisfaction degree	SD^f

Algorithm 1 Distributed fairness-based subchannel allocation algorithm(DFSA)

```
Let $\Phi = \mathcal{F}$.
for FBSs $f \in \Phi$ do
    Assume that FBS $w \in \mathcal{W}(\mathcal{W} \subseteq \Phi)$ meets $N_A^w \leq N_D^w$, $|\mathcal{W}| = W$.
    if $W > 1$ then
        Select FBS $f^* \in \mathcal{W}$ with the lowest $SD$.
    else if $W = 1$ then
        The only FBS $w$ in $\mathcal{W}$ will be selected FBS $f^*$.
    else if $W = 0$ then
        Compare the first rank subchannels in $W$ CCTs. Select FBS $f^* \in \mathcal{F}$ with the largest first rank channel condition difference.
    end if
    Mark the first rank subchannel $n^*$ in $CCDT_{f^*}$ as a selected subchannel for FBS $f^*$. Subchannel $n^*$ will be allocated to FUE $k^*$ with the lowest data rate in FBS $f^*$. Let $\tau_{f^*,k^*,n^*} = 1$.
    Update the CCT, CCDT and SRT for FBS $f^*$ and its neighbors.
    if $counter^{f^*} \neq 0$ and $N_R^{f^*} \neq 0$ then
        Delete $f^*$ from $\Phi$.
    end if
end for
```

Algorithm 2 CCDT and SRT Table update procedure

```
The subchannel $n$ is selected for FBS $f$.
for FBS $f$ do
    1) Delete subchannel $n$ from $CCDT_f$ and re-calculate the differences;
    2) Update parameters in $SRT_f$: $N_{to}^f = N_{to}^f + 1$, $N_R^f = N_R^f - 1$, $counter^f = counter^f - 1$, and re-calculate $SD^f$.
end for
for FBSs $l (l \in \mathbf{F}/f)$, $l$ is a neighbor of $f$ do
    if Subchannel $n$ is one of the available subchannel for FBS $l$ then
        1) Delete subchannel $n$ from $CCDT_l$ and re-calculate the differences;
        2) Update parameters in $SRT_l$: $N_R^f = N_R^f - 1$.
    end if
end for
```

is convex in $(t > 0, \mathbf{p})$ [27]. The Bernstein approximation of (14) can be formulated as

$$\inf_{t>0}\left[f_0(\mathbf{p}) + t\sum_{n=1}^{N}\Lambda_n\left(t^{-1}f_n(\mathbf{p})\right) + t\ln\left(\frac{1}{\epsilon}\right)\right] \leq 0 \tag{17}$$

which is a safe convex approximation of the chance constraint, that is, $\mathbf{p}$ satisfies the chance constraint if it satisifies Eq. (17). This approximation is tractable if $\{\Lambda_n(y)\}$ can be evaluated efficiently [25]. Consider a case of $\Lambda_n(y)$ when

$$\Lambda_n(y) \leq \max\left\{\mu_n^- y, \mu_n^+ y\right\} + \frac{\sigma_n^2}{2}y^2, \qquad n = 1, ..., N \tag{18}$$

where both $-1 \leq \mu_n^- \leq \mu_n^+ \leq 1$ and $\sigma_n \geq 0$ are constants. By choosing appropriate μ_n^-, μ_n^+, and σ_n and replacing Λ_n in (17) with its upper bound given in (18), Eq. (17) can be bounded by

$$\begin{aligned}
&\inf_{t>0}\left[f_0(\mathbf{p}) + t\sum_{n=1}^{N}\left(\max\left\{\mu_n^- t^{-1}f_n(\mathbf{p}), \mu_n^+ t^{-1}f_n(\mathbf{p})\right\}\right.\right.\\
&\qquad\left.\left.+\frac{\sigma_n^2}{2}\left(t^{-1}f_n(\mathbf{p})\right)^2\right) + t\ln\left(\frac{1}{\epsilon}\right)\right]\\
&= f_0(\mathbf{p}) + \sum_{n=1}^{N}\max\left\{\mu_n^- f_n(\mathbf{p}), \mu_n^+ f_n(\mathbf{p})\right\} + \frac{1}{2t}\sum_{n=1}^{N}\sigma_n^2 f_n^2(\mathbf{p})\\
&\qquad + t\ln\left(\frac{1}{\epsilon}\right) \leq 0
\end{aligned} \tag{19}$$

Invoking the arithmetic-geometric inequality for (19), the convex constraint can be written as

$$\begin{aligned}
&f_0(\mathbf{p}) + \sum_{n=1}^{N}\max\left\{\mu_n^- f_n(\mathbf{p}), \mu_n^+ f_n(\mathbf{p})\right\} + \sqrt{2\ln\left(\frac{1}{\epsilon}\right)}\\
&\qquad\times\left(\sum_{n=1}^{N}\sigma_n^2 f_n^2(\mathbf{p})\right)^{\frac{1}{2}} \leq 0
\end{aligned} \tag{20}$$

which is a safe conservative approximation of Eq. (14). Ben-Tal and Nemirovski [25] give some examples of the value of μ_n^-, μ_n^+, and $\sigma_n \geq 0$ based on some prior knowledge (e.g., support, unimodality, and symmetry) of the distributions.

Suppose $I_{f,k,n}^s$ follows a given distribution with bounded support of $\left[a_{f,k,n}^s, b_{f,k,n}^s\right]$. Introduce constants $\alpha_{f,k,n}^s \triangleq \frac{1}{2}\left(b_{f,k,n}^s - a_{f,k,n}^s\right)$ and $\beta_{f,k,n}^s \triangleq \frac{1}{2}\left(b_{f,k,n}^s + a_{f,k,n}^s\right)$ to make a normalization of support of $[-1, 1]$; that is, $\alpha_{f,k,n}^s \zeta_n^s + \beta_{f,k,n}^s \in [\alpha_{f,k,n}^s, \beta_{f,k,n}^s]$, where $\zeta_n^s \triangleq \frac{I_{f,k,n}^s - \beta_{f,k,n}^s}{\alpha_{f,k,n}^s}$. Let

$$f_0(\mathbf{p}) = -I_{f,k,n}^s + \sum_{f=1}^{F}\sum_{k=1}^{K}\sum_{n=1}^{N}\beta_{f,k,n}^s p_{f,k,n}^F \tag{21}$$

$$f_n(\mathbf{p}) = \sum_{f=1}^{F}\sum_{k=1}^{K}\alpha_{f,k,n}^s p_{f,k,n}^F \tag{22}$$

Then, C6 in (13) is equivalent to (14). Substitute $f_0(\mathbf{p})$ and $f_n(\mathbf{p})$ into (20), noting that $p^F_{f,k,n} \geq 0$, then we obtain

$$- I_{\text{thre},s} + \sum_{f=1}^{F}\sum_{k=1}^{K}\sum_{n=1}^{N} \beta^s_{f,k,n} p^F_{f,k,n} + \sum_{f=1}^{F}\sum_{k=1}^{K}\sum_{n=1}^{N} \mu^+_n \alpha^s_{f,k,n} p^F_{f,k,n} + \sqrt{2\ln\left(\frac{1}{\epsilon}\right)} \left(\sum_{n=1}^{N}\left(\sum_{f=1}^{F}\sum_{k=1}^{K} \sigma^s_n \alpha^s_{f,k,n} p^F_{f,k,n}\right)^2\right)^{\frac{1}{2}} \leq 0 \tag{23}$$

According to the mean inequality theorem, arithmetic mean cannot be greater than quadratic mean, which means $\sqrt{\sum_{n=1}^{N} X_n^2} \geq \frac{\sum_{n=1}^{N} X_n}{\sqrt{N}}$. Thus, we can further approximate the above inequality as

$$-I_{\text{thre},s} + \sum_{f=1}^{F}\sum_{k=1}^{K}\sum_{n=1}^{N} \left(\gamma^s_{f,k,n} + \sqrt{2\frac{1}{N}\ln\left(\frac{1}{\epsilon}\right)} \sigma^s_n \alpha^s_{f,k,n}\right) p^F_{f,k,n} \leq 0 \tag{24}$$

where $\gamma^s_{f,k,n} \triangleq \mu^+_n \alpha^s_{f,k,n} + \beta^s_{f,k,n}$. Therefore, the chance constraint C6 in Eq. (13) is transformed into a convex constraint.

4.2.2 Optimal power allocation for femtocells

Based on the above analysis, the chance-constrained programming problem in (13) can be transformed into a convex optimization problem. Thus, we can obtain optimal power allocation by solving the following convex optimization problem:

$$\begin{aligned} &\max_{p^F_{f,k,n}} \sum_{f=1}^{F}\sum_{k=1}^{K}\sum_{n=1}^{N} \tau^*_{f,k,n} R^F_{f,k,n} \\ &\text{s.t.} \quad \text{C1}-\text{C3} \\ &\quad \text{C7}: - I_{\text{thre},s} + \sum_{f=1}^{F}\sum_{k=1}^{K}\sum_{n=1}^{N} \left(\gamma^s_{f,k,n} + \sqrt{2\frac{1}{N}\ln\left(\frac{1}{\epsilon}\right)} \sigma^s_n \alpha^s_{f,k,n}\right) p^F_{f,k,n} \leq 0 \qquad \forall s \end{aligned} \tag{25}$$

which can be solved by applying the Lagrangian dual decomposition method. By introducing dual variables $\boldsymbol{\lambda}$, $\boldsymbol{\nu}$, and $\boldsymbol{\delta}$, the Lagrangian function is given by

$$\begin{aligned} &\mathcal{L}\left(\left\{p^F_{f,k,n}\right\}, \boldsymbol{\lambda}, \boldsymbol{\nu}, \boldsymbol{\delta}\right) \\ &= \sum_{f=1}^{F}\sum_{k=1}^{K}\sum_{n=1}^{N} \tau^*_{f,k,n} R^F_{f,k,n} - \sum_{f=1}^{F}\sum_{k=1}^{K} \lambda_{f,k}\left(P_{\max} - \sum_{n=1}^{N} p^F_{f,k,n}\right) \\ &\quad - \sum_{f=1}^{F}\sum_{k=1}^{K} \nu_{f,k}\left(\sum_{n=1}^{N} \tau^*_{f,k,n} R^F_{f,k,n} - R^0_{f,k}\right) \\ &\quad - \delta\left(I_{\text{thre},s} - \sum_{f=1}^{F}\sum_{k=1}^{K}\sum_{n=1}^{N}\left(\gamma^s_{f,k,n} + \sqrt{2\frac{1}{N}\ln\left(\frac{1}{\epsilon}\right)} \sigma^s_n \alpha^s_{f,k,n}\right) p^F_{f,k,n}\right) \end{aligned} \tag{26}$$

where $\boldsymbol{\lambda}$, $\boldsymbol{\nu}$, and $\boldsymbol{\delta}$ are the Lagrange multiplier vectors for C2, C3, and C7, respectively. In OFDMA-based cognitive femtocell system, a subchannel can be assigned to only one FUE in the same FBS; thus, the subchannel set allocated to each FUE in the same FBS is independent of each other. Hence, the Lagrangian dual problem can be decomposed into a master problem and $F \times N$ subproblems, which can be solved iteratively. Accordingly, we have

$$\begin{aligned} &\mathcal{L}\left(\left\{p^F_{f,k,n}\right\}, \boldsymbol{\lambda}, \boldsymbol{\nu}, \boldsymbol{\delta}\right) \\ &= \sum_{f=1}^{F}\sum_{n=1}^{N} \mathcal{L}_{f,n}\left(\left\{p^F_{f,k,n}, \boldsymbol{\lambda}, \boldsymbol{\nu}, \boldsymbol{\delta}\right\}\right) \\ &\quad - \sum_{f=1}^{F}\sum_{k=1}^{K} \lambda_{f,k} P_{\max} + \sum_{f=1}^{F}\sum_{k=1}^{K} \nu_{f,k} R^0_{f,k} - \delta I_{\text{thre},s} \end{aligned} \tag{27}$$

where

$$\begin{aligned} &\mathcal{L}_{f,n}\left(\left\{p^F_{f,k,n}\right\}, \boldsymbol{\lambda}, \boldsymbol{\nu}, \boldsymbol{\delta}\right) \\ &= \sum_{k=1}^{K} \tau^*_{f,k,n} R^F_{f,k,n} + \sum_{k=1}^{K} \lambda_{f,k} p^F_{f,k,n} - \sum_{k=1}^{K} \nu_{f,k} \tau^*_{f,k,n} R^F_{f,k,n} \\ &\quad + \sum_{k=1}^{K} \delta\left(\gamma^s_{f,k,n} + \sqrt{2\frac{1}{N}\ln\left(\frac{1}{\epsilon}\right)} \sigma^s_n \alpha^s_{f,k,n}\right) p^F_{f,k,n} \end{aligned} \tag{28}$$

The Karush-Kuhn-Tucker (KKT) conditions of (26) can be expressed as

$$\frac{\partial \mathcal{L}_{f,n}\left(\left\{p^F_{f,k,n}\right\}, \boldsymbol{\lambda}, \boldsymbol{\nu}, \boldsymbol{\delta}\right)}{\partial p^F_{f,k,n}} = \sum_{k=1}^{K}\left(\Omega_{f,k,n} - \Theta_{f,k,n}\right) = 0 \tag{29}$$

$$\lambda_{f,k}\left(P_{\max} - \sum_{n=1}^{N} p^F_{f,k,n}\right) = 0 \tag{30}$$

$$\nu_{f,k}\left(\sum_{n=1}^{N} \tau^*_{f,k,n} R^F_{f,k,n} - R^0_{f,k}\right) = 0 \tag{31}$$

$$\delta\left(I_{\text{thre},s} - \sum_{f=1}^{F}\sum_{k=1}^{K}\sum_{n=1}^{N}\left(\gamma^s_{f,k,n} + \sqrt{2\frac{1}{N}\ln\left(\frac{1}{\epsilon}\right)} \sigma^s_n \alpha^s_{f,k,n}\right) p^F_{f,k,n}\right) = 0 \tag{32}$$

where

$$\Omega_{f,k,n} = \frac{B_w/N\left(1 - \nu_{f,k}\right) \tau^*_{f,k,n} g_{f,k,n}}{\ln 2\left(I + p^F_{f,k,n} g_{f,k,n} + N_0\right)} \tag{33}$$

$$\Theta_{f.k.n} = \lambda_{f,k} + \delta\left(\gamma^s_{f,k,n} + \sqrt{2\frac{1}{N}\ln\left(\frac{1}{\epsilon}\right)} \sigma^s_n \alpha^s_{f,k,n}\right) \tag{34}$$

The optimal power allocation for FBS f to FUE k on subchannel n can be expressed as

$$p^*_{f,k,n} = \left[\frac{B_w/N\left(1-\nu_{f,k}\right)\tau^*_{f,k,n}}{\Theta_{f.k.n}\ln 2} - \frac{I}{g_{f,k,n}} - \frac{N_0}{g_{f,k,n}} \right]^+ \tag{35}$$

where $[x]^+ \triangleq \max(0,x)$; thus, the boundary constraint C1 in (25) is contained in Eq. (35) .

The subgradient search algorithm is used to calculate the non-negative Lagrange multipliers $\lambda_{f,k}$, $\nu_{f,k}$, and δ, given by

$$\lambda^{(t+1)}_{f,k} = \left[\lambda^{(t)}_{f,k} + \varepsilon^{(t)}_1\left(P_{\max} - \sum_{n=1}^{N} p^*_{f,k,n}\right)\right]^+ \tag{36}$$

$$\nu^{(t+1)}_{f,k} = \left[\nu^{(t)}_{f,k} + \varepsilon^{(t)}_2\left(\sum_{n=1}^{N}\tau^*_{f,k,n}R^F_{f,k,n} - R^0_{f,k}\right)\right]^+ \tag{37}$$

$$\delta^{(t+1)} = \left[\delta^{(t)} + \varepsilon^{(t)}_3\left(I_{\text{thre},s} - \sum_{f=1}^{F}\sum_{k=1}^{K}\sum_{n=1}^{N}\left(\gamma^s_{f,k,n} + \sqrt{2\frac{1}{N}\ln\left(\frac{1}{\epsilon}\right)}\sigma^s_n\alpha^s_{f,k,n}\right)p^*_{f,k,n}\right)\right]^+ \tag{38}$$

where $\varepsilon^{(t)}_1$, $\varepsilon^{(t)}_2$, and $\varepsilon^{(t)}_3$ are step sizes and t is the iteration number.

5 Simulation results and discussions

In this section, we evaluate the performance of the proposed DIRA scheme. In the simulation, we consider the downlink of a cognitive femtocell network in which femtocells are overlaid with macrocells. We assume that there are multiple macrocells and femtocells distributed in a manner of PPP with density d_m and d_f in a 4000× 4000 m scenario. MUEs and FUEs are distributed randomly in the scenario. The number of FUEs associated with an FBS is 4. The channel gains are modeled as i.i.d. exponential random variables. Shadowing effect is modeled as a log-normal variable with standard deviation 6 dB. The false alarm probability p^n_f and miss detection probability p^n_m are uniformly distributed over (0.05, 0.1) and (0.01, 0.05). The occupation probability of MBSs on subchannel n is uniformly distributed over (0, 1). The Bernstein approximation parameters μ^-_n, μ^+_n, and σ_n are chosen from [25]. The simulation parameters are given in Table 5.

In the simulation, a comparison between the proposed CCDT-based DFSA algorithm and the CCT-based subchannel allocation algorithm has been adopted to evaluate the fairness performance of the algorithm. Figure 3 shows the satisfaction degree fairness for different density of FBS, where the satisfaction degree fairness is represented by the variance of satisfaction degree. As shown in the figure, compared with the CCT-based algorithm, the proposed CCDT-based DFSA algorithm achieves lower satisfaction degree variance, which illustrates that the proposed algorithm outperforms the CCT-based algorithm in respect of satisfaction degree fairness.

Figure 4 shows the data rate fairness comparison of the proposed algorithm and the CCT-based algorithm. As we can observe from the figure, the proposed algorithm achieves a lower variance of average FUE data rate of FBSs than the CCT-based algorithm. The outstanding fairness performance for average data rate of the proposed algorithm is achieved by applying the channel condition difference as an allocation indicator. According to Figs. 3 and 4, it becomes harder to guarantee fairness among different FBSs with increasing density of FBSs. This is because more and more FBSs compete to access the same subchannel, resulting in difficulty for maintaining fairness.

Table 5 System simulation parameters

Parameter	Value
Density of MBS distribution d_m	2 BS/km^2
MBS transmit power	43 dBm
FBS transmit power	20 dBm
Carrier frequency	2 GHz
Total bandwidth	10 MHz
Number of subchannels N	50
Thermal noise PSD	−174 dBm/Hz
Shadowing standard deviation	6 dB
Pathloss from FBS to FUE (dB)[26]	$38.6 + 20\log_{10}(d) + 0.7d$

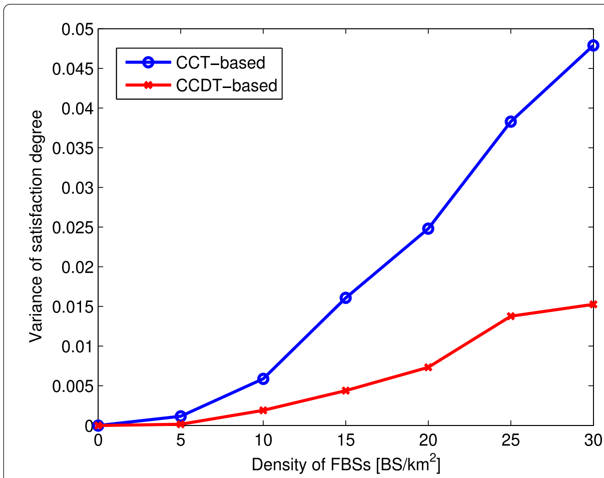

Fig. 3 Variance of satisfation degree for different density of FBSs

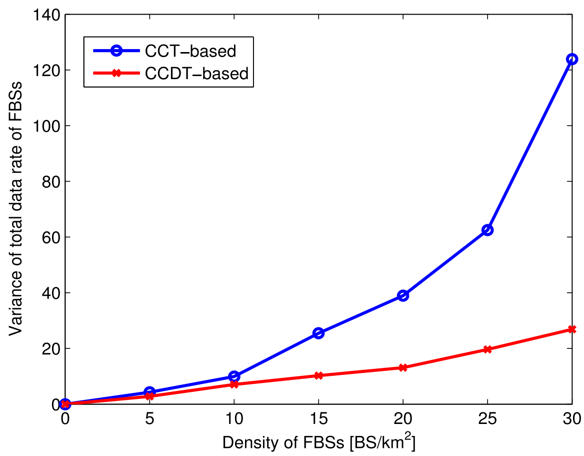

Fig. 4 Variance of average data rate of FUEs of FBSs for different density of FBSs

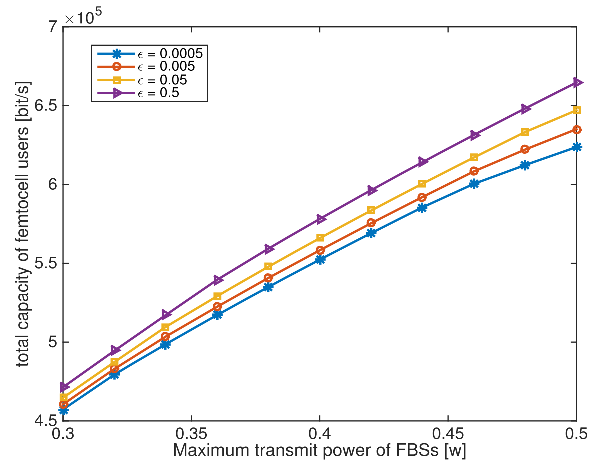

Fig. 6 Total data rate of FBSs vs. $P_{\max}$ for different ϵ

The total data rate of the FBSs for different density of FBSs is shown in Fig. 5, where the data rate achieved by two algorithms is almost the same. Through this figure, obviously, the proposed algorithm can improve the fairness performance without influencing the data rate.

We investigate the effect of ϵ in the total data rate of FBSs, where $d_f = 12$ BS/km^2, $I_{\text{thre},s} = -110$ dBm, $R^0_{f,k} = 5$ kbps. As shown in Fig. 6, the total data rate of FBSs increases when the ϵ gets larger. Obviously, a larger ϵ relaxed the interference constraints to FBSs, which results in higher tolerance for cross-tier interference from FBSs to MUEs. It illustrates that the higher data rate of FBSs can be achieved at the cost of increasing interference to MUEs.

Furthermore, we compare the total cross-tier interference from FBSs to MUEs for perfect and imperfect spectrum sensing cases, where $d_f = 12$ BS/km^2, $I_{\text{thre},s} = -110$ dBm, and $R^0_{f,k} = 5$ kbps. As shown in Fig. 7, the cross-tier interference is an increasing function of $P_{\max}$. The cross-tier interference from FBSs to MUEs in imperfect spectrum sensing case is lower than that in perfect spectrum sensing case. This is because the imperfect spectrum sensing case overestimates the cross-tier interference with the consideration of sensing errors, such as false alarm and miss detection.

Figure 8 shows the total data rate of FBSs is an increasing function of maximum transmit power of FBS. As a comparison, the random subchannel allocation (RSA) and the equal power allocation (EPA) schemes are also evaluated in the simulation. In the RSA scheme, FBSs opportunistically access vacant subchannels in CSMA/CA manner. The equal power allocation allocates equal power for each subchannel, while the proposed algorithm performs an optimal power allocation with the consideration of imperfect spectrum sensing, cross-tier interference, and

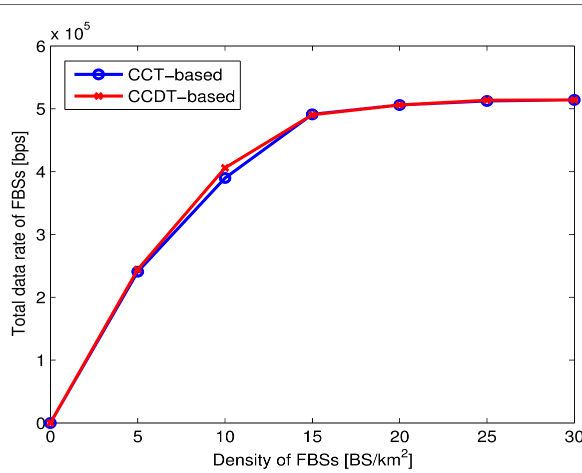

Fig. 5 Total data rate of FBSs for different density of FBSs

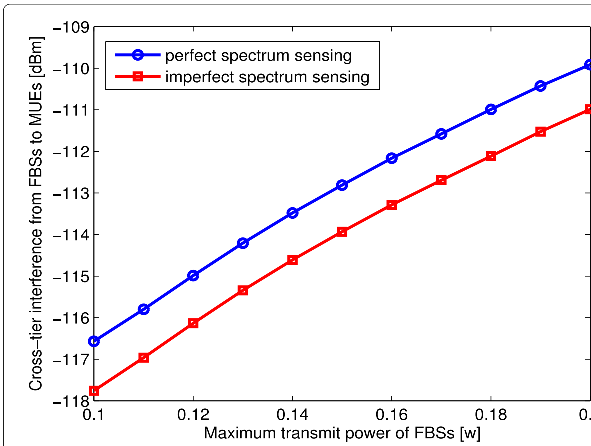

Fig. 7 Cross-tier interference from FBSs to MUEs

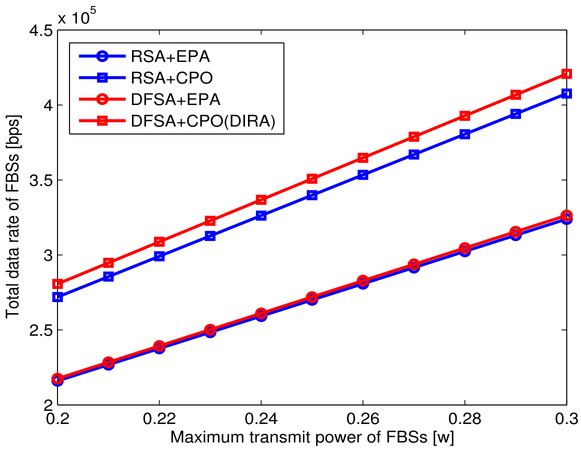

Fig. 8 Total data rate of FBSs vs. $P_{\max}$s

QoS requirements. Figure 8 compares the following kinds of combination of subchannel allocation scheme and power allocation scheme: (i) RSA+EPA; (ii) RSA+CPO; (iii) DFSA+EPA; and (iv) DFSA+CPO(DIRA). In the simulation, $d_f = 12$ BS/km^2, $I_{\text{thre},s} = -110$ dBm, and $R^0_{f,k} = 5$ kbps. It can be seen that the total data rate grows with the increase of $P_{\max}$, and the proposed DIRA scheme significantly increases the total data rate of FBSs.

6 Conclusions

In this paper, we investigated the resource allocation problem in cognitive femtocell networks. A joint subchannel and power allocation algorithm was proposed to maximize the total data rate of femtocells with the consideration of fairness and imperfect spectrum sensing. Particularly, a CCDT-based subchannel allocation algorithm DFSA was developed to allocate subchannels to FBSs while guaranteeing the fairness among femtocells. We introduced spectrum sensing error probabilities to capture the imperfect spectrum sensing influence and combined them with OOB emission to formulate the cross-tier interference constraint. Furthermore, due to the interference uncertainty, we formed the interference constraint as the chance constraint and implied Bernstein's approximation to make it tractable. Finally, the optimal power allocation problem was solved by the Lagrangian dual method. Simulation results verified that our proposed algorithm can achieve fair subchannel allocation and significant data rate improvement.

Acknowledgements
This work is supported by the National Natural Science Foundation of China (NSFC) (61401053) and Changjiang Scholars and Innovative Research Team in University (IRT1299).

Authors' contributions
XH designed the study, conception, and main algorithms. LS drafted the article and designed the simulations and theoretical certifications. CZ worked with the data analysis and prepared the manuscript. QC reviewed and edited the manuscript. DZ worked with the data analysis and encoding. All authors have made substantive intellectual contributions to this study and approved the manuscript.

Competing interests
The authors declare that they have no competing interests.

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Received: 25 July 2017 Accepted: 9 November 2017
Published online: 28 November 2017

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