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Coding to reduce the interference to carrier ratio of OFDM signals

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Abstract

In this paper, we propose a simple approach to reduce the sensitivity of OFDM system to the carrier frequency offset (CFO). The main idea is to choose a well-known channel code and to rotate each coordinate by a fixed phase shift such that the maximum inter-carrier interference (ICI) taken over all sub-carriers is minimized. This approach is based on a geometric interpretation of the peak interference to carrier ratio (PICR) of OFDM signals. Simulation results show that a reduction in PICR of 7 dB can be easily achieved. Furthermore, we addressed the fundamental limit of the proposed technique by providing both an exact and an approximate lower bound of PICR of phase-shifted binary codes. The bounds are applied to some codes: non-redundant binary code, BCH codes, and Reed-Muller codes. Simulation results demonstrated that phase-shift designs approach lower bounds and are resilient to CFO change.

Keywords: OFDM, PIRC, Coding

1 Introduction

1.1 Motivations

Orthogonal frequency-division multiplexing (OFDM) is a multicarrier modulation technique that has been implemented for many high-speed wireless and wireline applications. Indeed, the orthogonal properties among subcarriers of OFDM lead to high spectral efficiency and excellent ability to cope with multipath fading environment. Since the spectra of the subcarriers are overlapping, an accurate frequency-synchronization technique is needed. Due to oscillator inaccuracies and non-ideal transmitter/receiver synchronization, the orthogonal properties are easily broken down and result in errors due to frequency offsets. It is well-known that the carrier frequency offset (CFO) reduces the useful signal power and creates inter-carrier interference (ICI) [1, 2].

1.2 Related works

Two main approaches have been proposed in the literature to mitigate the ICI problem caused by the CFO. One approach is to estimate and remove the frequency offset [3–6]. In this approach, the frequency-offset estimation is generally performed in two steps: *coarse frequency-offset estimation*, which estimates the partial frequency offset that is a multiple of the subcarrier spacing and *fine frequency-offset estimation*, which estimates the remaining part of the offset. There exist several CFO estimation techniques, e.g., training based methods [7–12], and semi-blind or blind methods [3, 13]. Another approach is to use coding[1] to reduce the sensitivity of the OFDM system to frequency offsets [14–17]. The coding approach is generally adopted after the removal of coarse CFO and hence assumes *fine* residual CFO. A large range of coding techniques has been developed in recent years and can be classified as follows (see Table 1):

- *Self-cancellation*: A simple and effective method—known as the ICI self-cancelation scheme—was proposed by Zhao and Haggman in [18] and later generalized in [15, 19], where polynomial coding in the frequency domain is used to mitigate the effect of frequency offset. In this method, copies of the same data symbol are modulated on r adjacent sub-carriers using specific weights. This method can reduce the ICI at the price of lowering the transmission rate by a factor r. To further improve the ICI self-cancellation performance, many researchers proposed more efficient mapping (optimized weighting coefficients) for the redundant data symbol modulated over adjacent or non-adjacent subcarriers [20–27]. The weights are designed such that, at the receiver, the ICI at each subcarrier is reduced when a frequency

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Table 1 Coding techniques to reduce OFDM systems sensitivity to frequency shifts

Coding technique	Complexity	Transmission-rate reduction
Self-cancellation [15, 18–27]	Slight increase	≥2
Windowing [30, 31]	Moderate increase	≥2
Two-path transmission [32–36]	Slight increase	2
Correlative coding [37, 38]	Moderate increase	None
This paper	Slight Increase	None

offset is present. Note that ICI self-cancellation techniques can be regarded as a coding scheme, where only the codewords with low ICI are used, this key idea was first proposed in [28, 29].

- *Windowing*: Seyedi and Saulnier [30] showed that the polynomial coding in the frequency domain—used by ICI self-cancellation schemes—is equivalent to using windowing in time domain. They hence used windowing at both the transmitter and the receiver to generalize and improve self-cancellation schemes. Recently, Real and Almenar [31] used windowing at the transmitter only allowing a reduction of the receiver complexity without any performance degradation. The windowing methods reduce ICI sensitivity significantly but they lower the transmission rate and increase the complexity of the transceiver.
- *Two-path transmission*: The two-path transmission method halves the spectral efficiency similar to ICI self-cancellation techniques in [18]. However, unlike ICI self-cancellation method, two-path transmission method transmits the data copies in two concatenated OFDM blocks. The first path represents the standard OFDM signal and the second one is formed by a conjugate of the first path [32, 33]. This approach was generalized in [34–36] by introducing phase rotation and space-time coding.
- *Correlative-coding*: Contrary to all previous coding techniques, that reduce the sensitivity to CFO at the price of reducing the transmission rate, the correlative coding method do not reduce the data rate [37, 38]. This method is based on frequency-domain Partial-response Coding (PRC). PRC were originally proposed in the time domain to reduce the sensitivity of single-carrier systems to time offset.
  The PRC does not sacrifice spectral efficiency but needs a maximum-likelihood sequence estimator (MLSE) thus increasing the receiver complexity. An interesting combination of correlative-coding and self-cancellation was proposed in [39].

1.3 Our contributions

The intend of this paper is to reduce the sensitivity of OFDM systems to CFO without reducing the transmission rate. Our technique is inspired by the following key observation: all current wireless OFDM systems include forward-error-correction or channel codes to reduce the probability of error. Indeed, the forward-error-correction or channel codes have been studied and used for controlling errors in data transmission for the last half century (since Shannon 1948 landmark paper). Sathananthan and Tellambura related ICI reduction to channel coding and showed the capability of any forward-error correction code to reduce the errors caused by the ICI in [14]. We propose here to extend the role of those codes in OFDM systems. The main idea is to rotate each coordinate of the channel code by a fixed phase-shift such that the maximum inter-carrier interference taken over all sub-carriers is minimized. Clearly, the phase-shifted version of the code has the same error correcting capability, rate, decoding complexity as the original code. But it has lower maximum ICI and this reduction comes with no reduction of the spectral efficiency of the system. We proposed a simple technique—phase-shift—to reduce the sensitivity of OFDM system to CFO in [40]. This paper is an extension of this work—it provides an analytical evaluation of the proposed technique.

The key departure of our research from prior works on CFO effect reduction is that (a) we provide a geometric interpretation of the peak interference to carrier ratio (PICR) of OFDM signals, (b) we propose a simple phase-shift approach to reduce the sensitivity of OFDM to fine frequency offset, (c) we analyze the fundamental limit of the proposed technique and obtain a lower bound of the PICR of phase-shifted binary codes which we evaluate for non-redundant binary code, BCH codes, and Reed-Muller codes, and (d) we numerically evaluate phase-shift designs and prove their resilience to CFO change.

2 Statement of the problem

2.1 Notations and preliminaries

An OFDM signal is the sum of many independent signals modulated onto sub-channels of equal bandwidth. The OFDM transmitter applies the inverse discrete Fourier transform (IDFT) operation and sends the symbol sequence to the RF chain. The transmitted signal at time t may be represented as the real part of the complex envelope

$$S_{\mathbf{c}}(t) = \sum_{n=0}^{N-1} c_n \exp(j2\pi(f_0 + nf_s)t) \quad \text{for} \quad 0 \le t \le T_s, \tag{1}$$

where $j = \sqrt{-1}$, f_0 is the first subcarrier frequency and f_s is the bandwidth of each subcarrier. The OFDM-symbol duration T_s is equal to $\frac{1}{f_s}$ to ensure the orthogonality among the subcarriers. The vector $\mathbf{c} = [c_0, c_1, \ldots, c_{N-1}]$ of length N is the modulating data sequence. We refer to $\mathbf{c}$ as codeword of code $\mathcal{C}$ that maps blocks of k input bits into blocks of N symbols. The element of the vector $\mathbf{c}$, c_n, are typically taken from a constellation $\mathcal{Q}$. In this paper, we assume an equal energy constellation $\mathcal{Q}$ (i.e., all the points in $\mathcal{Q}$ have the same energy such as MPSK). We assume that $S_{\mathbf{c}}(t)$ is transmitted on an additive white Gaussian noise (AWGN) channel. The receiver receives the signal $\Re(S_{\mathbf{c}}(t))$ perturbed by noise and performs the inverse operations: the RF chain at the receiver down-converts, processes the received data. The receiver then applies a discrete Fourier transform (DFT) to generate estimate of $\mathbf{c}$. Finally it extracts the block of input bits by applying a suitable error-correction algorithm. The received signal sample for the kth subcarrier after DFT can be written as

$$\begin{aligned} y_k &= c_k b_0 + \sum_{l=0, l\neq k}^{N-1} b_{l-k} c_l + n_k \\ &= c_k b_0 + I_k + n_k \text{ for } k = 0, 1, \ldots, N-1, \end{aligned} \tag{2}$$

where n_k is a complex Gaussian noise sample. The first term in the right-hand side of (2) represents the desired signal. The ICI term, attributable to the CFO, on the kth subcarrier is expressed by I_k. The ICI coefficients b_k are given by [29]

$$\begin{aligned} b_k &= \frac{\sin \pi (k+\varepsilon)}{N \sin \frac{\pi}{N}(k+\varepsilon)} \exp\left[j\pi \left(1 - \frac{1}{N}\right)(k+\varepsilon)\right], \\ &= \frac{\sin \pi \varepsilon}{N \sin \frac{\pi}{N}(k+\varepsilon)} \exp\left[j\pi \left(1 - \frac{1}{N}\right)\varepsilon - j\pi \frac{k}{N}\right], \end{aligned} \tag{3}$$

where ε is the normalized *fine* frequency offset defined as a ratio between the fine frequency offset and the subcarrier spacing (i.e. $|\varepsilon| \leq \frac{1}{2}$). We note that I_k is a function of both $\mathbf{c}$ and ε. In the following, we will list the main properties of the the ICI coefficients b_k. Those properties give insights into the ICI interference I_k and will be used in the rest of the paper.

1. Without frequency error ($\varepsilon = 0$), b_k reduces to the unit impulse sequence $b_k = \delta[k]$.
2. The coefficient b_0 depends on ε but is independent of k. In other words, all subcarriers experience the same degree of attenuation and rotation of the wanted component.
3. $b_k = b_{N+k}$, so if we consider an N modulo numbering $-k$ maps to $N - k$. This property is easy to prove and is used in [20] to propose a repetition code that reduces the ICI.
4. All the coefficients b_k for $k \neq 0$ slowly vary with k, $b_{k+1} \simeq b_k$, the slow varying nature of b_k is the key motivation for self ICI cancelation [19]. This property will be used here to derive a more practical (approximate) lower bound of PICR of phase-shifted binary codes.
5. Under the assumption of small normalized frequency offset ($|\varepsilon| \ll 1$), the coefficients b_k became
$$b_k \simeq \frac{\sin \pi \varepsilon}{N \sin \frac{\pi k}{N}} \exp\left[j\pi \left(1 - \frac{1}{N}\right)\varepsilon - j\pi \frac{k}{N}\right].$$
6. The average power of the inter-carrier interference is derived as [1]
$$E\left[|I_k|^2\right] = \sum_{l=1}^{N-1} |b_l|^2 = 1 - |b_0|^2, \tag{4}$$
where $E\,[.]$ denotes the expected value over the distribution of data.

3 Peak interference to carrier ratio evaluation

In this section, we provide a geometric interpretation of the maximum ICI taken over all sub-carriers that explicitly incorporates the distance between the codewords $\mathbf{c}$ and some specific points. Based on this interpretation, we will propose a simple phase-shifted approach to reduce the sensitivity of OFDM systems to CFO.

3.1 PICR definition

Sathanannthan and Tellambura [28, 29] defined the peak interference to carrier ratio (PICR) as the maximum interference-to-signal ratio for any subcarrier:

$$\mathrm{PICR}(\mathbf{c}, \varepsilon) = \max_{0 \leq k \leq N-1} \frac{|I_k|^2}{|b_0 c_k|^2}. \tag{5}$$

It specifies the worst-case ICI on any subcarrier. Large I_k value cause high bit errors in subcarriers. Therefore, to reduce ICI effects, PICR should be minimized [29]. Here we also define the PICR of a code $\mathcal{C}$ as the maximum PICR over all codewords in $\mathcal{C}$

$$\mathrm{PICR}(\mathcal{C}, \varepsilon) = \max\left\{\mathrm{PICR}(\mathbf{c}, \varepsilon) \quad | \quad \forall \mathbf{c} \in \mathcal{C}\right\}. \tag{6}$$

Note that the expression of PICR is similar to the *peak-to-mean envelope power ratio* (PMEPR) that is considered as measure of the fluctuation of the OFDM signal[2] $\mathrm{PMEPR}(\mathbf{c}) = \max_{0 \leq t \leq 1} \frac{|S_{\mathbf{c}}(t)|^2}{\|\mathbf{c}\|^2}$.

3.2 PICR geometric representation

We rewrite $\mathrm{PICR}(\mathcal{C}, \varepsilon)$ as

$$\mathrm{PICR}(\mathcal{C}, \varepsilon) = \max_{0 \leq k \leq N-1} \mathcal{J}(k, \varepsilon), \quad \text{where} \quad \mathcal{J}(k, \varepsilon) = \max_{\mathbf{c} \in \mathcal{C}} \frac{|I_k|^2}{|b_0 c_k|^2}. \tag{7}$$

We refer to $\mathcal{J}(k, \varepsilon)$ as the maximum interference-to-signal ratio of code $\mathcal{C}$ on subcarrier k and focus on its computation. The next theorem provides a geometric interpretation of the PICR per subcarrier [40].

Theorem 3.1 *Let $\mathcal{C}$ be a code of length N defined over unit energy constellation Q. For each k, the value of $\mathcal{J}(k, \varepsilon)$*

is determined by the distance between the codewords $\mathbf{c} \in \mathcal{C}$ *and the points* $\mathbf{w_k}$ *and* $j\mathbf{w_k}$*, where*

$$\mathbf{w_k} = \left[b^*_{-k}, \ldots, b^*_{-1}, 0, b^*_1, \ldots, b^*_{N-1-k}\right]. \quad (8)$$

If $\mathcal{C}$ *is a cyclic code then*

$$PICR(\mathcal{C}, \varepsilon) = \mathcal{J}(0, \varepsilon). \quad (9)$$

The value of $\mathcal{J}(0, \varepsilon)$ *is determined by the distance between the codewords* $\mathbf{c} \in \mathcal{C}$ *and the two points* $\mathbf{w_0}$ *and* $j\mathbf{w_0}$.

Proof For any vectors $\mathbf{x} = (x_0, x_1, \ldots, x_{N-1})$ and $\mathbf{y} = (y_0, y_1, \ldots, y_{N-1})$, let $\mathbf{x}.\mathbf{y} = \sum_{i=0}^{N-1} x_i y_i^*$ denote the inner product of $\mathbf{x}$ and $\mathbf{y}$. Using the identities

$$\begin{aligned} ||\mathbf{x} - \mathbf{y}||^2 &= ||\mathbf{x}||^2 + ||\mathbf{y}||^2 - 2\Re(\mathbf{x}.\mathbf{y}), \\ ||\mathbf{x} - j\mathbf{y}||^2 &= ||\mathbf{x}||^2 + ||\mathbf{y}||^2 - 2\Im(\mathbf{x}.\mathbf{y}), \end{aligned}$$

we derive the following equalities

$$\begin{aligned} ||\mathbf{c} - \mathbf{w_k}||^2 &= ||\mathbf{c}||^2 + ||\mathbf{w_k}||^2 - 2\Re(I_k), \\ ||\mathbf{c} - j\mathbf{w_k}||^2 &= ||\mathbf{c}||^2 + ||\mathbf{w_k}||^2 - 2\Im(I_k). \end{aligned}$$

where $\mathbf{c} = [c_0, \ldots, c_{N-1}]$. Without loss of generality, we assume that the elements of Q have unit amplitude ($|c_n|^2 = 1,\ 0 \le n \le N-1$) then $||\mathbf{c}||^2 = N$.[3]

Using properties 3 and 6 of the ICI coefficients, we get $||\mathbf{w_k}||^2 = \sum_{l=-k, l\neq 0}^{N-1-k} |b_l|^2 = \sum_{l=1}^{N-1} |b_l|^2 = 1 - |b_0|^2$. Hence, we rewrite $\mathcal{J}(k, \varepsilon)$ as follows:

$$\begin{aligned} \mathcal{J}(k,\varepsilon) &= \max_{\mathbf{c}\in\mathcal{C}} \frac{|I_k|^2}{|b_0|^2} = \max_{\mathbf{c}\in\mathcal{C}} \frac{1}{|b_0|^2}\left[(\Re(I_k))^2 + (\Im(I_k))^2\right], \\ &= \max_{\mathbf{c}\in\mathcal{C}} \frac{\left(N+1-|b_0|^2-||\mathbf{c}-\mathbf{w_k}||^2\right)^2 + \left(N+1-|b_0|^2-||\mathbf{c}-j\mathbf{w_k}||^2\right)^2}{4|b_0|^2}. \end{aligned} \quad (10)$$

Let $\mathbf{c}^+$ denote the codeword for which $\mathcal{J}(k, \varepsilon)$ is attained, $\mathbf{c}^+$ maximizes

$$\begin{aligned} &\left(N + 1 - |b_0|^2 - ||\mathbf{c} - \mathbf{w_k}||^2\right)^2 \\ &+ \left(N + 1 - |b_0|^2 - ||\mathbf{c} - j\mathbf{w_k}||^2\right)^2, \end{aligned}$$

among all the codewords. So for each k, the value of $\mathcal{J}(k, \varepsilon)$ is determined by the distance between the codewords $\mathbf{c} \in \mathcal{C}$ and the points $\mathbf{w_k}$ and $j\mathbf{w_k}$.

The proof of the second part of Theorem 3.1 is based on the definition of cyclic code. For every codeword $\mathbf{c} = [c_0, \ldots, c_{N-1}] \in$ cyclic code $\mathcal{C}$, the word $[c_{-k}, \ldots, c_{N-k-1}]$ obtained by a cyclic shift of k components is again a codeword. In addition, based on property 3 of the coefficients b_n, $\mathbf{w_k}$ is obtained by k left cyclic shift of the element of $\mathbf{w_0}$. If we assume that the PICR($\mathcal{C}, \varepsilon$) is reached for the subcarrier $k \neq 0$ and code $\mathbf{c}^{k+}$, then we have just to shift the codeword $\mathbf{c}^{k+}$ by k to find a codeword $\mathbf{c}^{0+}$ and it is easy to prove that the interference generated for the subcarrier 0 and code $\mathbf{c}^{0+}$ is equals to PICR($\mathcal{C}, \varepsilon$). Hence, the PICR($\mathcal{C}, \varepsilon$) is also reached for the subcarrier $k = 0$. □

The geometric interpretation of Theorem 3.1 is simple but important because it explicitly states that the value of PICR($\mathcal{C}, \varepsilon$) is determined by the distance between the codewords $\mathbf{c} \in \mathcal{C}$ and the points $\mathbf{w_k}$ and $j\mathbf{w_k}$, $k = 0, \ldots, N-1$. Furthermore for cyclic code, the value of PICR is only function of the distance between the codewords and the two points $\mathbf{w_0}$ and $j\mathbf{w_0}$. This representation leads to an understanding of the value of PICR($\mathcal{C}, \varepsilon$) and how to reduce it through phase-shift approach which is described in the following Section 4.

4 PICR reduction

4.1 Phase-shift technique: illustration

Theorem 3.1 provides a geometric interpretation of the value of PICR. Based on this interpretation, we propose a simple approach that was originally proposed by Jones and Wilkinson [41, 42] and further developed by Tarokh, Jafarkhani and Paterson [43, 44] to reduce PMEPR. The idea is to take a code with good BER performance and to rotate each coordinate of the code by a fixed phase-shift such that the maximum PICR evaluated for all codewords is minimized. For the purpose of illustration, we assume a unit amplitude constellation; thus all the codewords are points on the N-dimensional complex sphere of radius $\sqrt{N}$. The phase-shifts may reduce the PICR by reducing/increasing the distance between the codewords and the points $\mathbf{w_k}$ and $j\mathbf{w_k}$, $k = 0, \ldots, N-1$. For cyclic code, we just focus on the distance between the codewords and the points $\mathbf{w_0}$ and $j\mathbf{w_0}$ only. Our design will shift the codewords without changing the distance between them as illustrated in Fig. 1. So it leaves the error-correcting properties of the code unchanged. In addition, we can use the standard decoding algorithm of the original code after the received signals are rotated back.

The reduction of interference for OFDM can now be tackled as follows: For a given code $\mathcal{C}$, the reducing phase-shift problem is to find offset $\mathbf{v} = [\nu_0, \ldots, \nu_{N-1}] \in \mathbb{Z}^N_{2^h}$ such that PICR($\mathcal{C}_\nu, \epsilon$) is minimized, where $\mathcal{C}_\nu = \{e^{j\pi \mathbf{v}/2^h}\mathbf{c}\ |\mathbf{c} \in \mathcal{C}\}$, h is an integer that set the resolution of the offset. Notice that, when h tends to infinity, we can consider PICR($\mathcal{C}_\nu, \epsilon$) as a continuous multi-variable function of $\phi_0 = 2\pi\nu_0/2^h, \ldots, \phi_{N-1} = 2\pi\nu_{N-1}/2^h$. Because of the continuity, the global minimum of this function over $[0, 2\pi] \times [0, 2\pi] \times \cdots \times [0, 2\pi]$ exists. But this function may have various local minima and saddle points over this compact set as well. There are various methods for minimization of PICR($\mathcal{C}_\nu, \epsilon$) in the literature, but they can only guarantee convergence to a local minima. As representative of these methods, we have used the gradient method in our simulations. It is important to note that this approach can be applied to binary and non-binary codes. Then, we used the lower bound, derived in the following section, to evaluate the optimality of our design.

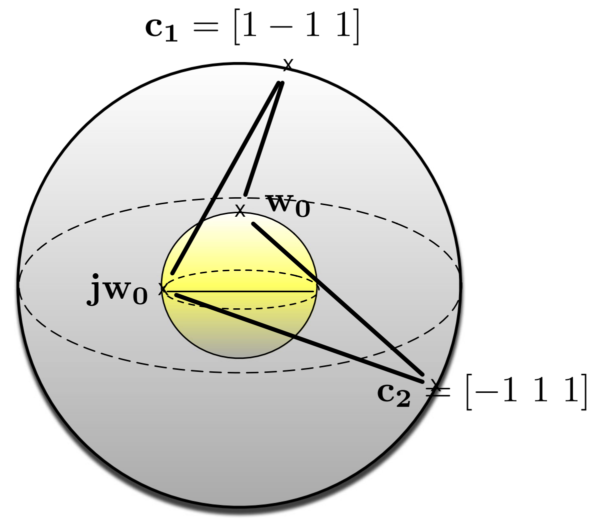

Fig. 1 Illustration of the geometric interpretation of PICR. We plot two codewords on the sphere $N = 3$. By rotating the codewords over the sphere (through phase-shift) we can change the distance between the codewords and the two points $\mathbf{w_0}$ and $j\mathbf{w_0}$

4.2 Phase-shift technique: range

This section is dedicated to prove that phase-shift designs are robust to CFO change. The optimum offset $\mathbf{v}$ minimizes the PICR for a range of frequency offset (not for a specific frequency offset value). To do so we need the following proposition.

Proposition 4.1 *Let $\mathcal{C}$ be a code of length N defined over an equal energy constellation Q. Let ε_1 and ε_2 be small normalized frequency offsets ($|\varepsilon_1| \ll 1$ and $|\varepsilon_2| \ll 1$). If the offset $\mathbf{v} \in \mathbb{Z}_{2^h}^N$ minimizes PICR$(\mathcal{C}_v, \varepsilon_1)$ that it minimizes PICR$(\mathcal{C}_v, \epsilon_2)$ too.*

Proof The proof is based on the fifth property of the coefficients b_n. We rewrite the PICR$(\mathcal{C}, \varepsilon_2)$ as

$$\text{PICR}(\mathcal{C}, \varepsilon_2) = \max_{0 \le k \le N-1, \mathbf{c} \in \mathcal{C}} \frac{|I_k(\varepsilon_2)|^2}{|b_0(\varepsilon_2) c_k|^2}$$

where $I_k(\varepsilon_2)$ is the inter-carrier interference on the kth subcarrier and $b_0(\varepsilon_2)$ is the ICI coefficient on the desired signal evaluated for normalized CFO of ε_2 as in Eq. (2). Using the property 5 of the ICI coefficients, we have

$$\begin{aligned}\text{PICR}(\mathcal{C}, \varepsilon_2) &= \max_{0 \le k \le N-1, \mathbf{c} \in \mathcal{C}} \frac{|I_k(\varepsilon_2)|^2}{|b_0(\varepsilon_2) c_k|^2}, \\ &\simeq \frac{|\sin(\pi \varepsilon_2) b_0(\varepsilon_2)|^2}{|\sin(\pi \varepsilon_1) b_0(\varepsilon_1)|^2} \max_{0 \le k \le N-1, \mathbf{c} \in \mathcal{C}} \frac{|I_k(\varepsilon_1)|^2}{|b_0(\varepsilon_1) c_k|^2}, \\ &= |\frac{\sin(\pi \varepsilon_2) b_0(\varepsilon_2)}{\sin(\pi \varepsilon_1) b_0(\varepsilon_1)}|^2 \ \text{PICR}(\mathcal{C}, \varepsilon_1).\end{aligned}$$

□

Hence, the offset $\mathbf{v} \in \mathbb{Z}_{2^h}^N$, that minimizes PICR$(\mathcal{C}_v, \varepsilon_1)$, minimizes PICR$(\mathcal{C}_v, \epsilon_2)$ too. Therefore, there is no need to search for an offset $\mathbf{v}$ specific to each carrier frequency offset. The simulation results will show in Section 6 that the optimum set of shifts, derived for $\varepsilon = 0.1$, reduce the inter-carrier interference for a *wide* range of CFO ($0 < \varepsilon < 0.5$).

5 Minimum PICR for phase-shifted binary code

Motivated by the previous design of phase-shifts, we would like to know how much PICR reduction can be achieved. In particular, in our framework, similar to that of Schmidt's [45], we consider the following question. For a given binary code $\mathcal{B}$, let $\mathcal{B}_v$ be the phase-shifted version of $\mathcal{B}$, what is

$$\min_{\mathcal{B}_v \in E_h(B)} \text{PICR}(\mathcal{B}_v, \epsilon).$$

We provide a lower bound and an approximated lower bound for this limit. The bounds are used to evaluate the optimality of our design.

5.1 Main results

Our lower bound in the spirit of Schmidt's [45] will be expressed in terms of the covering radius of the binary code $\mathcal{B}$ defined below.

Definition *The covering radius of a code $\mathcal{B} \subseteq \{1, -1\}^N$ is defined to be*

$$\rho(\mathcal{B}) \triangleq \max_{x \in \mathbb{Z}_2^N} \min_{\mathbf{b}^+ \in \mathcal{B}^+} wt_H(b + x).$$

where each codeword $\mathbf{b}^+ \in \mathcal{B}^+$ is associated with a codeword $\mathbf{b} \in \mathcal{B}$ (where $b_i^+ = \frac{b_i+1}{2}$), $wt_H(b)$ is the Hamming weight of a binary vector b. We now state our main result.

Theorem 5.1 *Given a binary code $B \subseteq \{1, -1\}^N$ with covering radius $\rho(\mathcal{B})$, then if $h > 1$*

$$\min_{C \in E_h(B)} \text{PICR}(\mathcal{C}, \epsilon) \ge$$

$$\frac{1}{8|b_0|^2} \left[2N - 2|b_0|^2 - 2 \sum_{i=1}^{N-1} (1 - |b_i|)^2 - \lambda \left(N + 2\rho(\mathcal{B}) \right) \right]^{+^2},$$

where $\lambda = b_{\max} 2^{2h-2} \sin^2\left(\frac{\pi}{2^h}\right)$, $b_{\max} = \max_{i \neq 0} |b_i|$ and $[x]^+ = \max(x, 0)$.
If $|\epsilon| \ll 1$ then

$$\min_{C \in E_h(B)} \text{PICR}(\mathcal{C}, \epsilon) \gtrsim$$

$$\frac{1}{8|b_0|^2} \left[2N - 2|b_0|^2 - 2 \sum_{i=1}^{N-1} (1 - |b_i|)^2 - \lambda_{\text{avg}} \left(N + 2\rho(\mathcal{B}) \right) \right]^{+^2},$$

where $\lambda_{\text{avg}} = b_{\text{avg}} 2^{2h-2} \sin^2\left(\frac{\pi}{2^h}\right)$ *and* $b_{\text{avg}} = mean_{1\le i\le N-1}|b_i|$.

The proof of Theorem 5.1 is provided in Appendix. The preceding lower bound is a decreased function of h. The corollary below states the asymptotic lower bound for $h \to \infty$.

Corollary 5.2 *Given a binary code* $B \subseteq \{1,-1\}^N$ *with covering radius* $\rho(\mathcal{B})$*, then if* $h > 1$

$$\min_{C\in E_\infty(B)} \text{PICR}(\mathcal{C},\epsilon) \geq$$

$$\frac{1}{8|b_0|^2}\left[2N-2|b_0|^2-2\sum_{i=1}^{N-1}(1-|b_i|)^2-b_{\max}\frac{\pi^2}{4}(N+2\rho(\mathcal{B}))\right]^{+2}.$$

If $|\epsilon| \ll 1$ *then*

$$\min_{C\in E_\infty(B)} \text{PICR}(\mathcal{C},\epsilon) \gtrsim$$

$$\frac{1}{8|b_0|^2}\left[2N-2|b_0|^2-2\sum_{i=1}^{N-1}(1-|b_i|)^2-b_{\text{avg}}\frac{\pi^2}{4}(N+2\rho(\mathcal{B}))\right]^{+2}.$$

Proof Similarly to [45], we used $\lim_{h\to\infty} 2^{2h-2}\sin^2\left(\frac{\pi}{2^h}\right) = \frac{\pi^2}{4}$. □

5.2 Examples

5.2.1 Non-redundant BPSK signaling

The code $\{1,-1\}^N$ is a binary code without redundancy, $\rho(\{1,-1\}^N) = 0$, Theorem 5.1 and Corollary 5.2 imply the following.

Corollary 5.3 *For* $h > 1$*, the PICR of a* 2^h*-ary PSK code equivalent to* $\{1,-1\}^N$ *is at least*

$$\min_{C\in E_h(\mathbb{Z}_2^N)} \text{PICR}(\mathcal{C},\epsilon) \gtrsim$$

$$\frac{1}{8|b_0|^2}\left[2N-2|b_0|^2-2\sum_{i=1}^{N-1}(1-|b_i|)^2-\lambda_{\text{avg}} N\right]^{+2},$$

and

$$\min_{C\in E_\infty(\mathbb{Z}_2^N)} \text{PICR}(\mathcal{C},\epsilon) \gtrsim$$

$$\frac{1}{8|b_0|^2}\left[2N-2|b_0|^2-2\sum_{i=1}^{N-1}(1-|b_i|)^2-b_{\text{avg}}\frac{\pi^2}{4}N\right]^{+2}.$$

For $N = 16$ and $N = 64$ the PICR approximated lower bounds are 0.10 and 0.22, respectively. Since the PICR for non-redundant binary code are 0.19 ($N = 16$) and 0.50 ($N = 64$), the maximum PICR reduction of the proposed phase-shift designs is 3 dB for non-redundant code.

5.2.2 Reed-Muller codes

In the following we inspect codes that are equivalent to first order Reed-Muller codes of length 2^m, $RM(1,m)$.

Corollary 5.4 *For* $h > 1$ *and* $N = 2^m$*, the PICR of a* 2^h*-ary PSK code equivalent to RM(1,m) is at least*

$$\min_{C\in E_h(RM(1,m))} \text{PICR}(\mathcal{C},\epsilon) \gtrsim \frac{1}{8|b_0|^2}\left[2N-2|b_0|^2-2\sum_{i=1}^{N-1}(1-|b_i|)^2 -\lambda_{\text{avg}}\left(N+(2^m-2^{m/2})\right)\right]^{+2},$$

and when $h = \infty$

$$\min_{C\in E_\infty(RM(1,m))} \text{PICR}(\mathcal{C},\epsilon) \gtrsim \frac{1}{8|b_0|^2}\left[2N-2|b_0|^2-2\sum_{i=1}^{N-1}(1-|b_i|)^2 -b_{\text{avg}}\frac{\pi^2}{4}\left(N+(2^m-2^{m/2})\right)\right]^{+2}.$$

Proof It was proved in [46] that

$$2^{m-1}-2^{(m-1)/2} \leq \rho(RM(1,m)) \leq 2^{m-1}-2^{m/2-1}.$$

□

5.2.3 Binary primitive BCH codes

Next we present the lower bounds on the PICR of codes that equivalent to binary primitive t-error-correcting BCH codes, $\mathcal{B}(t,m)$.

Corollary 5.5 *For* $h > 1$ *and* $N = 2^m - 1$*, the PICR of a* 2^h*-ary PSK code equivalent to t-error-correcting BCH codes,* $\mathcal{B}(t,m)$ *is at least*

$$\min_{C\in E_h(\mathcal{B}(t,m))} \text{PICR}(\mathcal{C},\epsilon) \gtrsim$$

$$\frac{1}{8|b_0|^2}\left[2N-2|b_0|^2-2\sum_{i=1}^{N-1}(1-|b_i|)^2-\lambda_{\text{avg}}(N+(4t-2))\right]^{+2},$$

and when $h = \infty$

$$\min_{C\in E_\infty(\mathcal{B}(t,m))} \text{PICR}(\mathcal{C},\epsilon) \gtrsim$$

$$\frac{1}{8|b_0|^2}\left[2N-2|b_0|^2-2\sum_{i=1}^{N-1}(1-|b_i|)^2-b_{\text{avg}}\frac{\pi^2}{4}(N+(4t-2))\right]^{+2}.$$

Proof The authors of [47] proved that there exists an m_o depending on t such that $\rho(\mathcal{B}(t,m)) = 2t-1$ for all $m \geq m_o$. □

6 Numerical results

The numerical results presented in this section aim to evaluate the ICI reduction performance of the proposed phase-shift approach. We considered a normalized frequency offset of $\varepsilon = 0.1$ and two well-known codes extended Bose-Chaudhuri-Hocqenghem (BCH) and Reed

Table 2 Optimum phase shifts for BCH(64,10) in radians

$\phi_0 = 2.4683$	$\phi_1 = 1.7502$	$\phi_2 = 4.4962$	$\phi_3 = 4.0882$	$\phi_4 = 0.1253$	$\phi_5 = 2.9330$	$\phi_6 = 5.3401$	$\phi_7 = 2.2695$
$\phi_8 = 4.2003$	$\phi_9 = 4.0068$	$\phi_{10} = 6.2288$	$\phi_{11} = 5.6747$	$\phi_{12} = 5.3153$	$\phi_{13} = 1.0958$	$\phi_{14} = 4.5622$	$\phi_{15} = 6.0023$
$\phi_{16} = 3.2120$	$\phi_{17} = 1.5937$	$\phi_{18} = 4.9119$	$\phi_{19} = 3.7611$	$\phi_{20} = 4.1274$	$\phi_{21} = 5.8449$	$\phi_{22} = 5.5075$	$\phi_{23} = 5.4723$
$\phi_{24} = 4.0787$	$\phi_{25} = 4.3756$	$\phi_{26} = 4.4991$	$\phi_{27} = 3.1733$	$\phi_{28} = 3.6295$	$\phi_{29} = 2.6383$	$\phi_{30} = 5.2078$	$\phi_{31} = 5.7397$
$\phi_{32} = 3.4149$	$\phi_{33} = 0.9170$	$\phi_{34} = 0.8856$	$\phi_{35} = 5.4017$	$\phi_{36} = 5.2185$	$\phi_{37} = 2.2959$	$\phi_{38} = 0.6368$	$\phi_{39} = 1.1671$
$\phi_{40} = 3.1718$	$\phi_{41} = 2.9789$	$\phi_{42} = 5.0463$	$\phi_{43} = 5.1208$	$\phi_{44} = 0.9384$	$\phi_{45} = 3.8918$	$\phi_{46} = 2.3587$	$\phi_{47} = 2.3106$
$\phi_{48} = 2.7175$	$\phi_{49} = 3.2082$	$\phi_{50} = 0.9428$	$\phi_{51} = 1.0826$	$\phi_{52} = 5.4491$	$\phi_{53} = 5.7840$	$\phi_{54} = 4.6229$	$\phi_{55} = 1.2202$
$\phi_{56} = 3.5798$	$\phi_{57} = 0.1929$	$\phi_{58} = 2.7888$	$\phi_{59} = 5.5207$	$\phi_{60} = 1.8680$	$\phi_{61} = 4.5057$	$\phi_{62} = 0.4838$	$\phi_{63} = 3.2663$

Muller (RM). We note that Pyndiah [48] has proposed a simple soft-input soft-output (approximated MAP) decoder for BCH codes, based on Chase algorithm, that gives excellent BER performance. RM codes achieve capacity on erasure channels and reduce PMEPR [49, 50].

For the BPSK case, we have optimized the phase-shifts by finding a local minima of PICR$(\mathcal{C}, \varepsilon)$. Once the optimal phase-shifts were computed, they were rounded to 8-PSK, and 16-PSK phase-shifts. The PICR was then calculated for the rounded phase shifted. The optimal set of shifts for BCH(64,10) are given in Table 2. The reduction in PICR is 2.56 dB, it decreases to 1.99 and 1.39 dB for 16-PSK and 8-PSK, respectively. The optimal set of shifts for RM(1,6) are given in Table 3. The reduction in PICR is 7.10 dB; it decreases to 6.28 and 5.86 dB for 16-PSK and 8-PSK, respectively. The numerical results show that a fixed phase-shift applied to code coordinates reduces the ICI, and that most of the gain can be obtained by using 8-PSK and 16-PSK type phase-shifts. The phase-shift design reduce the PICR from 0.255 (BCH) and 0.519 (RM) to 0.14 and 0.098, respectively. These values approaches the lower bound in Corollary 5.4 and Corollary 5.5.

Furthermore, we use the same set of phase-shifts with various CFO ($0 < \varepsilon < 0.5$). The PICR results shown in Figs. 2 and 3 prove that the optimum set of shifts, derived for $\varepsilon = 0.1$, reduce the inter-carrier interference for a wide range of CFO. For the purpose of comparison, we also plot the PICR with a self-interference cancellation that reduces the transmission-rate by half [18]. In fairness, we implemented the self-cancellation technique over the non-redundant data sequences (no channel coding). The simulation results prove that our simple phase-shift designs are very competitive - especially when applied to codes with high large covering radius like RM codes. To illustrate the impact of our phase-shift technique on the bit error rate (BER), we consider RM(1,6) code and BPSK modulation. The BER vs. signal to noise ratio (SNR) performance with and without phase-shifts are shown in Fig. 4. Note that the phase-shift technique improves the performance at BER 10^{-6} by 10 dB for normalized frequency offset $\epsilon = 0.1$.

7 Conclusions

In this paper, we proposed an interference reduction scheme to mitigate the effects of CFO in OFDM system. We provided a geometrical interpretation of PICR$(\mathcal{C}, \varepsilon)$, the PICR of $\mathcal{C}$. To reduce PICR, we focused on a simple phase-shift approach. The idea is to take a code with good BER performance and to rotate each coordinate of the code by a fixed phase-shift such that the maximum PICR taken over all codewords is minimized. This shift leaves unchanged the error-correcting properties of the code. In addition, we can use the standard decoding algorithm of the original code after back rotation of the received signals. Simulation results show that a reduction of 7 dB can freely be obtained.

We also study the fundamental limit of the proposed technique by providing an approximated lower bound of the PICR. The bound is applied to some codes: non-redundant binary code, BCH codes, and Reed-Muller

Table 3 Optimum phase shifts for RM(1,6) in radians

$\phi_0 = 1.5620$	$\phi_1 = 4.5390$	$\phi_2 = 3.5491$	$\phi_3 = 0.3789$	$\phi_4 = 2.9522$	$\phi_5 = 4.9135$	$\phi_6 = 1.9776$	$\phi_7 = 3.7244$
$\phi_8 = 3.7291$	$\phi_9 = 5.2304$	$\phi_{10} = 3.8296$	$\phi_{11} = 2.8186$	$\phi_{12} = 5.5612$	$\phi_{13} = 4.4629$	$\phi_{14} = 3.4743$	$\phi_{15} = 6.1346$
$\phi_{16} = 4.7435$	$\phi_{17} = 1.3264$	$\phi_{18} = 3.1416$	$\phi_{19} = 0.1649$	$\phi_{20} = 4.9018$	$\phi_{21} = 2.6437$	$\phi_{22} = 5.8063$	$\phi_{23} = 0.9283$
$\phi_{24} = 4.0979$	$\phi_{25} = 3.9276$	$\phi_{26} = 5.6677$	$\phi_{27} = 5.7450$	$\phi_{28} = 3.8378$	$\phi_{29} = 1.3179$	$\phi_{30} = 4.9345$	$\phi_{31} = 5.4689$
$\phi_{32} = 0.0817$	$\phi_{33} = 3.2805$	$\phi_{34} = 2.8142$	$\phi_{35} = 5.0262$	$\phi_{36} = 5.0639$	$\phi_{37} = 1.0568$	$\phi_{38} = 0.8457$	$\phi_{39} = 2.9594$
$\phi_{40} = 5.7340$	$\phi_{41} = 5.5270$	$\phi_{42} = 3.2478$	$\phi_{43} = 3.5318$	$\phi_{44} = 1.1401$	$\phi_{45} = 1.7986$	$\phi_{46} = 2.3697$	$\phi_{47} = 0.3248$
$\phi_{48} = 4.5588$	$\phi_{49} = 1.6792$	$\phi_{50} = 0.0914$	$\phi_{51} = 6.2392$	$\phi_{52} = 1.8721$	$\phi_{53} = 1.1146$	$\phi_{54} = 3.4850$	$\phi_{55} = 6.2436$
$\phi_{56} = 1.3990$	$\phi_{57} = 5.3068$	$\phi_{58} = 0.0079$	$\phi_{59} = 6.0174$	$\phi_{60} = 4.3187$	$\phi_{61} = 4.3175$	$\phi_{62} = 6.1484$	$\phi_{63} = 5.8044$

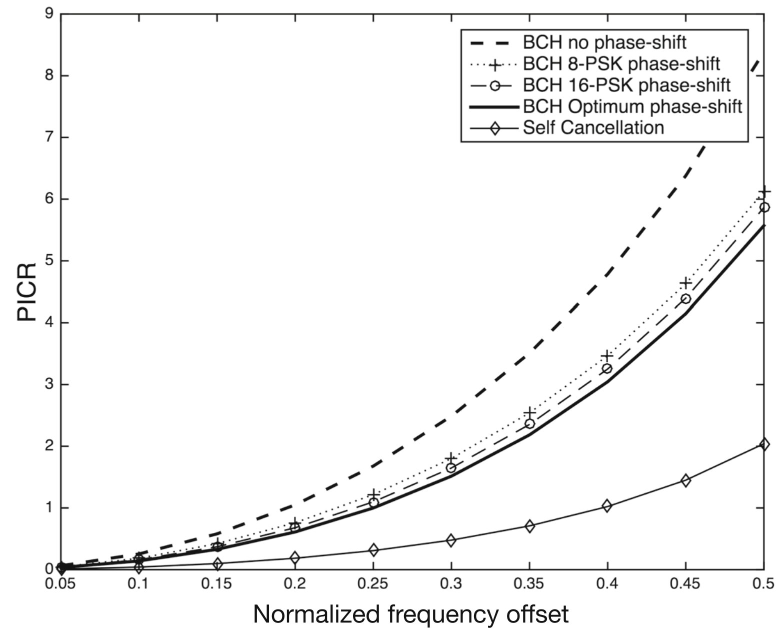

Fig. 2 PICR of 2^h-ary PSK code equivalent to BCH(64,10)

codes. It is demonstrated that our phase-shift designs approach our bound.

Endnotes

[1] One more approach uses signal processing, such as time and frequency equalization, to reduce the sensitivity of OFDM systems to frequency offset. Although efficient, signal processing techniques usually increase the complexity of the transceiver (see [16] and references therein).

[2] The authors of [29] have noted that despite the similarities between PICR and PMEPR, the PICR problem differs from PMEPR issue in several ways.

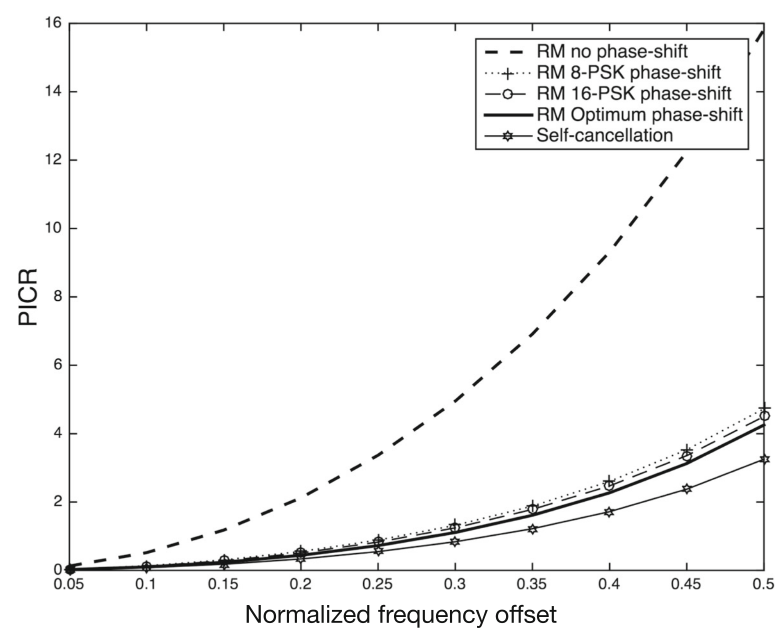

Fig. 3 PICR of 2^h-ary PSK code equivalent to RM(1,6)

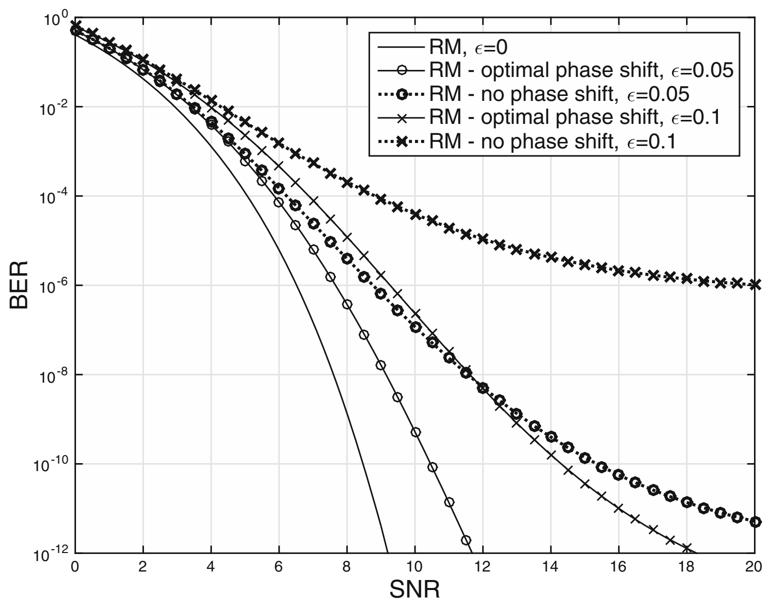

Fig. 4 BER vs. SNR of RM(1,6) code with and without phase-shifts

[3] In the case of non-uniform energy constellation, like QAM, $\mathcal{J}(k,\varepsilon)$ will be determined for each energy level and then minimized over all the levels.

Appendix
Proof of Theorem 5.1

The proof follows along the lines of [45]. Based on Eq. (7), we have

$$\mathrm{PICR}(\mathcal{C},\varepsilon) \geq \mathcal{J}(0,\varepsilon) \tag{11}$$

Using the following inequality

$$\begin{aligned} x^2+y^2 &= \frac{1}{2}\left[(x+y)^2+(x-y)^2\right] \\ &\geq \frac{1}{2}(x+y)^2, \quad x,y \in R \end{aligned}$$

and Eq. (10), we get

$$\mathrm{PICR}(\mathbf{c},\varepsilon) \geq \frac{1}{8|b_0|^2}\left[2N+2-2|b_0|^2-||\mathbf{c}-\mathbf{w_0}||^2-||\mathbf{c}-j\mathbf{w_0}||^2\right]^2 \tag{12}$$

We need some preliminaries in order to prove Theorem 5.1. If we assume that elements of $\mathcal{C}$ have unit energy, It is straightforward to establish that

$$||\mathbf{c}-\mathbf{w_0}||^2 = 1+\sum_{i=1}^{N-1}(1-|b_i|)^2+4\sum_{i=1}^{N-1}|b_i|\sin^2\left(\frac{\angle b_i-\angle c_i}{2}\right) \tag{13}$$

and

$$||\mathbf{c}-j\mathbf{w_0}||^2 = 1+\sum_{i=1}^{N-1}(1-|b_i|)^2+4\sum_{i=1}^{N-1}|b_i|\sin^2\left(\frac{\angle b_i-\angle c_i}{2}+\frac{\pi}{4}\right) \tag{14}$$

where $|x|$ and $\angle x$ are the amplitude and the angle of x, respectively. Let $\mathbb{Z}_{2^h}$ denote the ring of integers modulo 2^h. We define $\overline{c}_i$ and $\overline{b}_i \in \mathbb{Z}_{2^h}$ as: $\frac{2\pi\overline{c}_i}{2^h} \simeq \angle c_i$ and $\frac{2\pi\overline{b}_i}{2^h} \simeq \angle b_i$. Note that when h tends to the infinity the phases can be approximated with arbitrary high probability.

Schmidt showed in [45] that

$$\sin^2\left(w\frac{\pi}{2^h}\right) \leq \frac{2^h}{4}\sin^2\left(\frac{\pi}{2^h}\right)w$$

for $h>1$ and $w=0,1,\ldots,2^{h-1}$.

Based on this result, we get

$$\begin{aligned} ||\mathbf{c}-\mathbf{w_0}||^2 \leq 1+\sum_{i=1}^{N-1}(1-|b_i|)^2+2^h\sin^2\left(\frac{\pi}{2^h}\right) \\ \sum_{i=1}^{N-1}|b_i|\min\left(\overline{b_i}-\overline{c_i},2^h-\overline{b_i}+\overline{c_i}\right), \end{aligned} \tag{15}$$

and

$$\begin{aligned} ||\mathbf{c}-j\mathbf{w_0}||^2 \leq 1+\sum_{i=1}^{N-1}(1-|b_i|)^2+2^h\sin^2\left(\frac{\pi}{2^h}\right) \\ \sum_{i=1}^{N-1}|b_i|\min\left(\overline{b_i}-\overline{c_i}-2^{h-2},2^h-\overline{b_i}+\overline{c_i}+2^{h-2}\right). \end{aligned} \tag{16}$$

We now use the previous results to derive a lower bound of the PICR:

$$\mathrm{PICR}(\mathbf{c},\varepsilon) \geq \frac{1}{8|b_0|^2}\left[2N-2|b_0|^2-2\sum_{i=1}^{N-1}(1-|b_i|)^2 -2^h|b_{max}|\sin^2(\frac{\pi}{2^h})\left(wt_L\left(\overline{b}-\overline{c}\right)+wt_L\left(\overline{b}-\overline{c}-2^{h-2}\right)\right)\right]^2 \quad (17)$$

where $b_{max} = \max_{1\leq i\leq N-1}|b_i|$, $\overline{b} = [\overline{b}_0, \ldots, \overline{b}_{N-1}]$, $\overline{c} = [\overline{c}_0, \ldots, \overline{c}_{N-1}]$ and $wt_L(\mathbf{x})$ is Lee weight of the vector $\mathbf{x} = [x_0, x_1, \ldots, x_{N-1}]$ defined as

$$wt_L(x) \triangleq \sum_{i=0}^{N-1}\min\{x_i, 2^h - x_i\}. \quad (18)$$

Note that this lower bound is not always tight. Therefore, since all the coefficients b_k slowly vary with k, $b_{k+1} \simeq b_k$ (if $|\epsilon| \ll 1$), we derived this approximated lower bound

$$\mathrm{PICR}(\mathbf{c},\varepsilon) \gtrsim \frac{1}{8|b_0|^2}\left[2N-2|b_0|^2-2\sum_{i=1}^{N-1}(1-|b_i|)^2 -2^h|b_{\text{avg}}|\sin^2\left(\frac{\pi}{2^h}\right)\left(wt_L\left(\overline{b}-\overline{c}\right)+wt_L\left(\overline{b}-\overline{c}-2^{h-2}\right)\right)\right]^2 \quad (19)$$

where $b_{\text{avg}} = \text{mean}_{1\leq i\leq N-1}|b_i|$. The resulting values are very good approximations to the PICR lower bound yet analytically tractable. We use Definition 12 and Lemma 13 in [45] to derive

$$wt_L\left(\overline{b}-\overline{c}\right)+wt_L\left(\overline{b}-\overline{c}-2^{h-2}\right) = 2^{h-2}N+2^{h-1}\rho(\mathcal{B}), \quad (20)$$

where $\rho(\mathcal{B})$ is covering radius of the binary code $\mathcal{B} \subseteq \{1,-1\}^N$. We replace Eq. (20) in Eqs. (17) and (19) to complete the proof.

Competing interests
The authors declare that they have no competing interests.

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