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# Power allocation for SWIPT in $K$-user interference channels using game theory

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**Abstract**

A simultaneous wireless information and power transfer system in interference channels of multi-users is considered. In this system, each transmitter sends one data stream to its targeted receiver, which causes interference to other receivers. Since all transmitter-receiver links want to maximize their own average transmission rate, a power allocation problem under the transmit power constraints and the energy-harvesting constraints is developed. To solve this problem, we propose a game theory framework. Then, we convert the game into a variational inequalities problem by establishing the connection between game theory and variational inequalities and solve the variational inequalities problem. Through theoretical analysis, the existence and uniqueness of Nash equilibrium are both guaranteed by the theory of variational inequalities. A distributed iterative alternating optimization water-filling algorithm is derived, which is proved to converge. Numerical results show that the proposed algorithm reaches fast convergence and achieves a higher sum rate than the unaided scheme.

**Keywords:** Distributed algorithm, Game theory, Interference channels, Power allocation, Simultaneous wireless information and power transfer, Variational inequality theory

## 1 Introduction

Simultaneous wireless information and power transfer (SWIPT), which transports both information and energy simultaneously by the same radio-frequency (RF) signal, has caused great concern in both academic and industrial fields and offers great convenience to wireless terminals [1–3]. According to [3], time switching (TS) and power splitting (PS) are two practical receiver designs. As a PS receiver plays a significant role in SWIPT, it divides the received signal into two signal flows, one for energy harvesting (EH) and the other for information decoding (ID) [4, 5]. Based on the position relationship between the ID receiver and the EH receiver, which is spatially separated or co-located, there are two types of SWIPT networks [6].

Recently, the study of realizing SWIPT in the interference channels (IFC) has received considerable attention. As an extra energy source in the IFC, the cross-link signals are salutary to EH [7]. However, the cross-link signals are harmful to information transmission, which brings new challenges to the transmission designs. The authors of [8] investigated an adaptive resource allocation scheme called proportional-fair power allocation (PFPA) in multiuser OFDM systems for fair share of resources and efficient operation. In [9], the authors solved a sum rate maximization problem in a two-user IFC where the two receivers can simultaneously decode information and harvest energy. The author of [10] found a necessary condition of the optimal transmission strategy considering three different scenarios according to the receiver mode in a $K$-user IFC. In [11], the authors divided an optimal robust secure beamforming and power splitting scheme to minimize the total transmit power while satisfying the constraints on the minimum amounts over IFC. The authors of [12] jointly designed the allocating transfer power and receive PS coefficient for a two-link SWIPT system in IFC. In [13], a new transmission strategy was derived to maximize the energy beamforming and minimize the leakage beamforming in a two-user MIMO IFC. In [14], the authors derived a hybrid algorithm comprised of a linear combination of maximum ratio transmission (MRT) and zero-forcing (ZF) beamforming to minimize required power in a $K$-user IFC network. To minimize the total transmission power, [15]

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proposed a joint beamforming and power splitting algorithm based on second-order cone programming (SOCP) relaxation in a $K$-user IFC network. Based on PS scheme, the work [16] considered a multi-user SWIPT interference system and studied joint transceiver design to minimize the total transmit power. Reference [17] proposed a synchronous power descending (SPD) algorithm to updates each links' transmit power and PS ratio in a IFC SWIPT system. However, to the best of our knowledge, an iterative water-filling algorithm based on a game theory to solve the power allocation problem in IFC SWIPT systems with $K$ direct links to maximize its sum rate has not yet been studied.

In this paper, we devise an iterative water-filling algorithm based on game theory to solve the formulated power allocation problem for the IFC SWIPT systems. The main contributions of this paper are listed in the following:

1. A $K$ transceiver links SWIPT system in interference channels is developed. In particular, each transmitter-receiver link is modeled as a strategy player who chooses its transmit power to maximize its individual rate.
2. A game framework is formulated to solve the proposed power allocation problem. Then, we convert the game into a variational inequality (VI) problem using VI theory and analyze the existence and uniqueness of the Nash equilibrium (NE). In addition, a pricing mechanism is introduced to keep a balance between EH and ID.
3. Different from that of reference [18], an alternating optimization (AO) method is employed because of the coupled variables and non-convexity, which converts the original problem into two sub-problems.
4. To deal with the formulated sub-problem, a distributed iterative water-filling algorithm (IWFA) is devised to optimize the power allocation problem.

## 2 Problem formulation and power allocation method

### 2.1 Problem formulation

#### *2.1.1 Game-theoretic framework formulation*

An SWIPT system in the interference channels consisting $K$ source-destination links is considered. In this system, each link includes a single-antenna source node $S$ and a single-antenna destination node $D$ equipped with a power splitter, which splits the received signal into two streams, one for EH and the other for ID. Moreover, it is assumed that perfect channel state information (CSI) is available, only one single-data stream is transmitted and all nodes operate in half-duplex mode. As illustrated in Fig. 1, each source node $S_j$ transmits data to its own receiver $D_j$ at the same time. The received signal at $D_k$ can be expressed as

$$y_k = \sqrt{p_k} h_{kk} s_k + \sum_{j \neq k, j=1}^{K} \sqrt{p_j} h_{jk} s_j + n_{ak} \tag{1}$$

where $p_k$ and $p_j$ are the transmit power of $S_k$ and $S_j$, $s_k$ and $s_j$ are the transmit symbols of $S_k$ and $S_j$ with $\mathbb{E}\{|s_k|^2\} = 1, \mathbb{E}\{|s_j|^2\} = 1$, $h_{kk}$ and $h_{jk}$ are the channel gains from $S_k$ to $D_k$ and $S_j$ to $D_j$, and $n_{ak} \sim \mathcal{CN}(0, \sigma_{ak}^2)$ is the additive white Gaussian noise (AWGN) introduced by the receiver antenna at $D_k$, and $\mathbf{K} = \{1, \ldots, K\}$. The transmit power constraint of each node can be given by

$$0 \leq p_j \leq p_j^{max} \tag{2}$$

where $p_j^{max}, j \in \mathbf{K}$ denotes the peak power of each user.

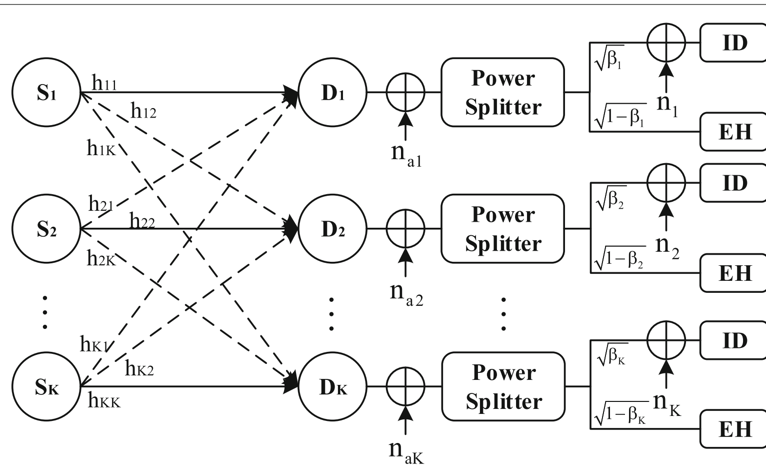


**Fig. 1** The system model for SWIPT in $K$-user interference channels. This system include $K$ source-destination links. Each link includes a single-antenna source node and a destination node equipped with a power splitter, which splits the received signal into two streams, one for EH and the other for ID. The solid lines represent the channel gain between the related node pairs, and the dotted lines represent the channel gain between other unrelated node pairs. The symbol $n_{ak}, k \in \{1, \ldots K\}$ represents the noise at nodes $D_k$, and $\beta$ is the PS ratio

A portion, $\beta_j \in (0,1)$, of the received signal is allocated for ID. Then, the signal for ID at $D_k$ can be expressed as

$$y_k^{ID} = \sqrt{\beta_k p_k} h_{kk} s_k + \sum_{j \neq k, j=1}^{K} \sqrt{\beta_j p_j} h_{jk} s_j + n_{all} \tag{3}$$

where $n_{all} = \sqrt{\beta_k} n_{ak} + n_k$ is the overall noise at $D_k$ with covariance $\sigma_{all}^2$ and $n_k$ is the AWGN originating from the power splitter with covariance $\sigma_k^2$. Meanwhile, the power of the desired signal received by $D_k$ for ID can be given as

$$\mathbb{E}\{|y_k^{ID}|^2\} = \beta_k p_k |h_{kk}|^2 + \sum_{j \neq k, j=1}^{K} \beta_j p_j |h_{jk}|^2 + \sigma_{all}^2. \tag{4}$$

Dividing the desired signal power by the interference and noise power, the signal-to-interference-noise-ratio (SINR) for the $k$th link between $S_k$ and $D_k$ can be expressed as

$$\gamma_k = \frac{\beta_k p_k |h_{kk}|^2}{\sum_{j \neq k, j=1}^{K} \beta_j p_j |h_{jk}|^2 + \sigma_{all}^2}. \tag{5}$$

Accordingly, the achievable rate of the $k$th link is

$$r_k\left(p_k, \mathbf{p_{-k}}\right) = log_2\left(1 + \gamma_k\right) \tag{6}$$

where $\mathbf{p}_{(-k)} \triangleq (p_j)_{j \neq k}$ is the set of allocating power of all users except the $k$th one.

A portion, $1 - \beta_j$, of the signal received at $D_k$ is for EH and the power collected at $D_k$ follows the constraints

$$(1 - \beta_k) \sum_{j=1}^{K} |h_{jk}|^2 p_j \geq e_k \tag{7}$$

where $e_k$ is the power threshold at $D_k$.

To maximize the achievable sum rate, a rate maximization problem with power constraint can be stated as follows

$$\max_{p} \quad \sum_{k=1}^{K} r_k(p) \tag{8a}$$

$$\text{s.t.} \quad \mathbf{p} \in \mathcal{P} \tag{8b}$$

where $\mathbf{p} = \left[p_1, p_2, ..., p_K\right]^T$ represents the power allocation strategy of all users, and $\mathcal{P} \triangleq \left\{\mathbf{p} | 0 \leq p_j \leq p_j^{max}, (1-\beta_k) \sum_{j=1}^{K} |h_{jk}|^2 p_j \geq e_k\right\}$ is the set of power constraints.

Since the non-convex problem (8) is difficult to solve, we devise a distributed framework based on game theory to solve the power allocation problem. Specifically, we consider the scenario where each user maximizes its own rate selfishly via allocating the transmit power. Then, the game $\mathcal{G}$ is formulated as

$$\begin{aligned} \mathcal{G}_1 : \max_{p_k} \quad & r_k\left(p_k, \mathbf{p}_{-k}\right) \\ \text{s.t.} \quad & p_k \in \mathcal{P}_k \end{aligned} \tag{9}$$

where $\mathcal{P}_k \triangleq \left\{p_k | 0 \leq p_k \leq p_k^{max}, (1 - \beta_k)\left(|h_{kk}|^2 p_k + \sum_{j=1, j \neq k}^{K} |h_{jk}|^2 p_j\right) \geq e_k\right\}$ is the feasible set of the $k$th user.

It can be observed from (9) that the problem is still non-convex and difficult to solve because the objective function $r_k$ is non-convex and the constraint $\mathcal{P}_k$ of the $k$th user is coupled. Therefore, we redefine the objective function as

$$f_k\left(p_k, \mathbf{p}_{-k}\right) \triangleq -r_k\left(p_k, \mathbf{p}_{-k}\right). \tag{10}$$

To let the optimization problem more decentralized and keep a balance between EH and ID, a pricing mechanism is introduced through a punishment in the payoff function. We define the pricing factor as $\alpha = \left(\alpha_j\right)_{j=1}^{K}$, where $\alpha_j$ represents the penalty of the $j$th user. From (7), we define a linear function about the power constraint of the $k$th link,

$$\begin{aligned} \varphi_k\left(p_k, \mathbf{p}_{-k}\right) &= (1 - \beta_k) \sum_{j=1}^{K} |h_{jk}|^2 p_j - e_k \\ &= A p_k + B \end{aligned} \tag{11}$$

where $A = (1 - \beta_k) |h_{kk}|^2$ and $B = (1 - \beta_k) \sum_{j=1, j \neq k}^{K} |h_{jk}|^2 p_j - e_k$. Then, we define the total penalty at the $k$th user as

$$\alpha_k \varphi_k\left(p_k, \mathbf{p}_{-k}\right) = \zeta_k p_k + \alpha_k B \tag{12}$$

where $\zeta_k \triangleq \alpha_k (1 - \beta_k) |h_{kk}|^2$.

Considering the penalty, the payoff function of the $k$th user can be given as

$$v_k\left(p_k, \mathbf{p}_{-k}; \alpha_k\right) = f_k\left(p_k, \mathbf{p}_{-k}\right) - \zeta_k p_k. \tag{13}$$

Consequently, the original game $\mathcal{G}_1$ can be formulated as a game

$$\mathcal{G}_2 : \min_{p_k} \quad v_k\left(p_k, \mathbf{p}_{-k}; \alpha_k\right) \tag{14a}$$

$$\text{s.t.} \quad p_k \in \mathcal{P}_k \tag{14b}$$

Our purpose is determining a NE point $\{\mathbf{p}^*, \alpha^*\}$ in the feasible set to minimize the objective function $v_k$ in $\mathcal{G}_2$, which satisfies the following condition

$$v_k(p_k^*, \mathbf{p}_{-k}^*; \alpha_k^*) \leq v_k\left(p_k, \mathbf{p}_{-k}^*; \alpha_k\right). \tag{15}$$

#### 2.1.2 VI problem formulation

It can be observed from (14) that the objective function and the constraints of the game $\mathcal{G}_2$ involves coupled variables, then we derived an AO method to solve this problem, which converts the original problem into two subproblems. In this section, we analyze the NE point of the game $\mathcal{G}_2$ using VI theory, optimize the power allocation problem using IWFA algorithm, and update the price vector using variable-step projection scheme.

In this subsection, we focus on finding the optimal power strategy $\mathbf{p}^*$ of the game $\mathcal{G}_2$ for the given price $\alpha$. First, we rewrite the original game $\mathcal{G}_2$ into a VI problem $VI(\mathcal{P}_k, V_k)$, which is denoted to find a power strategy $p^* \in \mathcal{P}_k$ satisfying the following condition

$$\left(p_k - p_k^*\right) V_k \left(p_k, \mathbf{p}_{-k}^*\right) \geq 0; \qquad p_k, \mathbf{p}_{-k}^* \in \mathcal{P}_k \tag{16}$$

where

$$\begin{aligned} V_k \left(p_k, \mathbf{p}_{-k}^*\right) &= \nabla_{p_k} v_k(p_k, \mathbf{p}_{-k}; \alpha_k) \\ &= -\nabla_{p_k} f_k(p_k, \mathbf{p}_{-k}) + \zeta_k \\ &= -\frac{1}{\ln 2} \left( \frac{\sum_{j=1}^{K} |h_{jk}|^2 p_j}{\beta_k |h_{kk}|^2} \right)^{-1} + \zeta_k \\ &\triangleq V + \zeta_k. \end{aligned} \tag{17}$$

is the gradient $\nabla v_k$. Then, we establish the relation between the formulated game $\mathcal{G}_2$ and the VI problem $VI\,(\mathcal{P}_k, V_k)$.

**Proposition 1** *The game $\mathcal{G}_2$ is equivalent to the problem $VI\,(\mathcal{P}_k, V_k)$.*

*Proof* A proof is given in Appendix A. □

The game $\mathcal{G}_2$ can be rewritten as $VI\,(\mathcal{P}_k, V_k)$ after the proof is completed. And the existence and uniqueness of the NE can be analyzed by studying the VI problem $VI\,(\mathcal{P}_k, V_k)$.

### 2.2 Power allocation method

#### 2.2.1 *Analysis of the NE*

The following theorem proposed in [19] is commonly used to verify the existence of the NE.

**Theorem 1** *A NE exists in a VI $(\mathcal{A}, F)$ problem if the set $\mathcal{A}$ is convex and compact; the function $F$ is continuous in its feasible set.*

After investigating the properties of the set $\mathcal{A}$ and the function $F$ of the VI problem, we have the following proposition regarding the existence of the NE.

**Proposition 2** *The game $\mathcal{G}_2$ possesses at least one NE.*

*Proof* A proof is given in Appendix B. □

Based on ([20], Eq. 13) , the $VI\,(\mathcal{A}, F)$ admits a unique solution if $F$ is strongly monotone on $\mathcal{A}$. The definition of strongly monotone is provided in the following.

**Definition 1** *Given a mapping $F : \mathcal{A} \subseteq \mathbb{R}^n \to \mathbb{R}^n$, if the set $\mathcal{A}$ is convex and there exists a constant $c > 0$ satisfying the following condition, $F$ is strongly monotone.*

$$(F(x) - F(y))(x - y) \geq c|\,x - y\,|^2, \forall x, y \in \mathcal{A}. \tag{18}$$

We analyze the uniqueness of the NE by proving a sufficient condition for the strong monotonicity of $V_k$. To prove the strong monotonicity of $V_k$, the second derivative of $v_k(p)$ can be given as

$$\nabla^2_{p_k} v_k(p) = \frac{\beta_k^2 |h_{kk}|^4}{\left(\beta_k \sum_{j=1}^{K} |h_{jk}|^2 p_j + \sigma_k^2\right)} \tag{19a}$$

$$\nabla^2_{p_k, p_j} v_k(p) = \frac{\beta_k^2 |h_{kk}|^2 |h_{jk}|^2}{\left(\beta_k \sum_{j=1}^{K} |h_{jk}|^2 p_j + \sigma_k^2\right)}. \tag{19b}$$

we have the following proposition regarding the uniqueness of the NE.

**Proposition 3** *$V_k$ is strongly monotone in its feasible set. Furthermore, the game $\mathcal{G}_2$ has the unique NE.*

*Proof* A proof is given in Appendix C. □

The above analysis about the existence and uniqueness of the NE shows that the NE of game $\mathcal{G}_2$ always exists and admits its uniqueness for the given price factor. To achieve the unique NE, we use a water-filling method based on the best response. For any fixed $\mathbf{p}_{-k}$ and $\alpha$, the NE point of the game $\mathcal{G}_2$ is the fixed-point of the water-filling mapping, which can be expressed as

$$\begin{aligned} p^*(\alpha) &= wv_k\left(\mathbf{p}_{-k}^*(\alpha); \alpha\right) \\ &\triangleq \left[ \frac{1}{\mu_k + \zeta_k} - \frac{\beta_k \sum_{j \neq k, j=1}^{K} |h_{jk}|^2 p_j + \sigma_k^2}{\beta_k |h_{kk}|^2} \right]_0^{p_k^{max}} \end{aligned} \tag{20}$$

where the majorization notion in $p^*(\alpha)$ represents $p(\alpha)$ after optimization, $[x]_a^b \triangleq min(max(x, a)b)$ with $0 \leq a \leq b$ and $\mu_k \geq 0$ is chosen to satisfy the power constraint (2).

#### 2.2.2 *Optimal α with variable-step projection scheme*

In this section, we discuss the optimization of $\alpha$ with $\mathbf{p}$ fixed. For the optimal $p_k^*(\alpha)$, we rewrite Eq. (11) as

$$\begin{aligned} \phi(\alpha) &= (1 - \beta_k)\, |h_{kk}|^2 p_k^*(\alpha) \\ &\quad + (1 - \beta_k) \sum_{j=1, j \neq k}^{K} |h_{jk}|^2 p_j - e_k. \end{aligned} \tag{21}$$

To optimize $\alpha$, we introduce a nonlinear complement problem (NCP), which is to find the price vector such that

$$NCP(\phi) : 0 \leq \alpha \perp \phi(\alpha) \geq 0. \quad (22)$$

The NCP is an equivalent form to VI problem. We use the well-known variable-step projection scheme to solve this NCP, which is described in the following algorithm.

#### *2.2.3 Distributed iterative algorithm*

Denote the allocating power and the pricing factor of the $k$th user at the $n$th iteration as $p_k^{(n)}(\alpha)$ and $\alpha^{(n)}$, respectively. To achieve the unique NE, a distributed iterative algorithm with AO is summarized as the Proposed Iterative AO Algorithm in Table 1. It is worth noting that such the AO scheme guarantees the local optimum.

It can be seen that the price $\alpha$ is computed by the variable-step projection scheme and $\epsilon_n$ is the $n$th iteration step size. As mentioned in [21], we could choose a sufficiently small value to assign to the step size $\epsilon_n$. The notion $[\cdot]^+$ represents that the value is meaningful when it is larger than zero, and let the value be zero when it is less than zero. And the proofs of convergence property about $\alpha$ and $p_k(\alpha)$ are similar to the Theorems 6 and 10 in [18].

## 3 Numerical results and discussions

It is assumed that the channels between all links are mutually independent Rayleigh fading and the free-space propagation pathloss coefficient is 2. And random channels with 100 slots are generated. The variances of the noises are $\sigma_{all}^2 = \sigma^2$, $p_k^{max} = P_{max}$, and $\beta_k = 0.5$.

We study the convergence property of IWFA in a four-link network. Figure 2 shows the allocating power of four users versus iterations considering two different initial points: $(1.1, 1.5, 1.2, 1.3)$ and $(1.4, 1.3, 0.8, 1.0)$ under the same conditions $P_{max} = 25$ and $\sigma^2 = 1$. It can be observed that all users' allocating power converge to the same points $(1.28, 1.39, 1.00, 1.41)$. After several similar attempts, we have that the IWFA quickly converges to the unique NE from different initial points.

**Table 1** Proposed iterative AO algorithm

**step 1**: **Initialize**.

Set $p^{(0)}$ as a vector satisfying $p_k^{(0)} \in \mathcal{P}$, $\alpha^{(0)} \geq 0$ and $\epsilon_n > 0$ as the $n$th iteration step size.

**step 2**: **Repeat**.

(1) Compute $p_k\left(\zeta\left(\alpha^{(n)}\right)\right)$ using water-filling method as follows with fixed $\alpha^{(n)}$:

$p_k\left(\zeta\left(\alpha^{(n)}\right)\right) = wv_k\left(\mathbf{p}_{-k}\left(\zeta\left(\alpha^{(n)}\right)\right); \zeta\left(\alpha^{(n)}\right)\right)$.

(2) Compute $\alpha$ using variable-step projection scheme as follows with fixed $p_k^*(\alpha)$:

$\alpha^{(n+1)} = \left[\alpha^{(n)} - \epsilon_n \phi(\alpha^{(n)})\right]^+$.

**step 3**: Until **convergence**.

Figure 3 provides the results of sum rate versus the interlink distance $dL$. Different conditions of $P_{max} \in \{20dB, 30dB\}$ and $\sigma^2 \in \{0.5, 1.0\}$ are simulated, respectively. There are two schemes for comparison. Scheme 1 is the unaided scheme without iteration, and scheme 2 is IWFA scheme. We consider a linear topology where every transmitter-receiver link is parallel to each other. It can be seen that the sum rate increases as the $dL$ increasing for two schemes. Then, the IWFA method outperforms to the unaided method under all conditions. In addition, the system performance is better when the AWGN is smaller and the maximum transmit power is bigger.

Figure 4 shows results of the average transmit power versus the harvest energy threshold $e_k$. Different conditions of $K \in \{3, 4\}$ and the different initial pricing $\alpha \in \{0.5, 0.6, 0.7\}$ are simulated, respectively. It can be observed that the average transmit power decreases as the harvest energy threshold $e_k$ increases for our proposed scheme, and the system requires less average transmit power with the increase of the number of links. We can also see that the average transmit power decreases as $\alpha$ increasing. In addition, the average transmit power of our proposed IWFA scheme always smaller than the unaided scheme when the number of links is the same.

Figure 5 provide results of the average bit error rate (BER) versus SINR. As we can see, our propose IWFA scheme is always better than the unaided scheme in BER performance. The reason is that the unaided is the simplest in computing complication with no iteration involved.

## 4 Conclusions

In this paper, a power allocation problem was solved for a SWIPT system in $K$-user interference channels using the framework of game theory. We rewrote the formulated game as a variational inequality problem to analyze the NE of the game. Furthermore, we provided a distributed iterative algorithm with AO scheme to solve the formulated problem and update the price factor. Numerical results demonstrated that the proposed IWFA scheme can attains more sum rate and requires less transmit power than the unaided scheme under the same conditions of $P_{max}$ and $\sigma^2$.

## Appendix A

### Proof of Proposition 1

The relationship between game and VI is usually verified using the following theorem proposed in the reference [19].

**Theorem 2** *A given game* $\mathcal{G} = \langle \mathcal{N}, \{\mathcal{A}_n\}, \{f_n(x)\} \rangle$ *is equivalent to* $VI(\mathcal{A}, F)$ *if the following two conditions hold:*

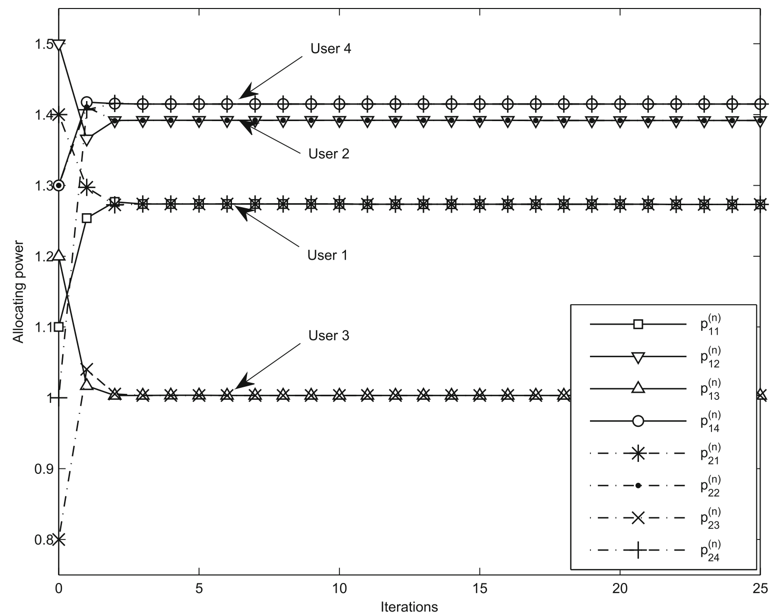


**Fig. 2** The power allocation profile versus iterations of IWFA from two different sets of initial points. Figure 2 shows the allocating power of four users versus iterations considering two different initial points. The solid lines and the dotted lines represent different allocating power results for four users under different initial points

*(i) The strategy set $\mathcal{A}_n$ is closed and convex; (ii) the payoff function is convex and continuously differential for $x \in \mathcal{A}_n$.*

Now, let us prove the relationship between the game $\mathcal{G}_2$ and the VI problem $VI(\mathcal{P}_k, V_k)$ using *Theorem 2.* First, note that the strategy set $\mathcal{P}_k$ in (14b) is convex. Then, the payoff function $\nu_k(p_k, \mathbf{p}_{-k})$ in (14a) is continuously differentiable in its feasible set. Finally, the payoff function $\nu_k(p_k, \mathbf{p}_{-k})$ is convex in its feasible set because the logarithmic function is always concave and $\zeta_k p_k$ is a

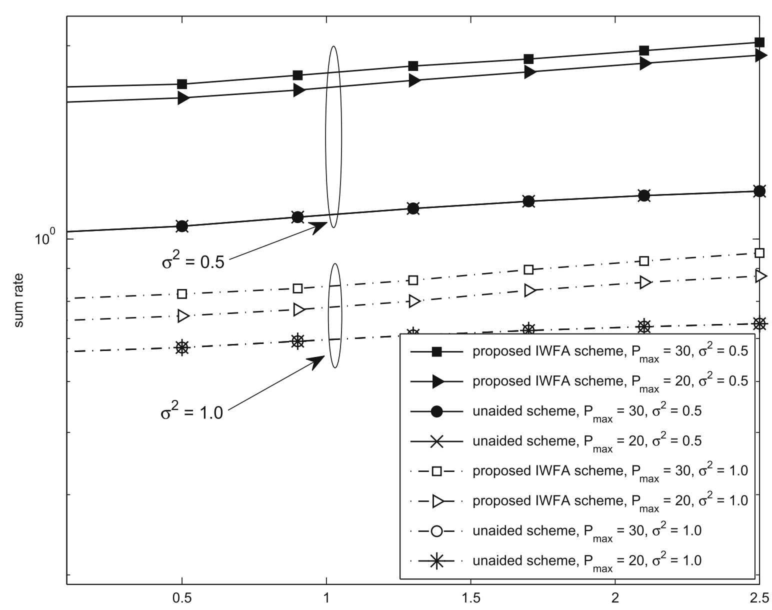


**Fig. 3** The sum rate as a function of the interlink distance *dL* in a two-link system. Figure 3 provides the results of sum rate versus the interlink distance *dL*. Each link represents the sum rate under the different interlink distance. And the different types lines represent the proposed IWFA scheme and the unaided scheme under different conditions of $P_{max} \in \{20dB, 30dB\}$ and $\sigma^2 \in \{0.5, 1.0\}$

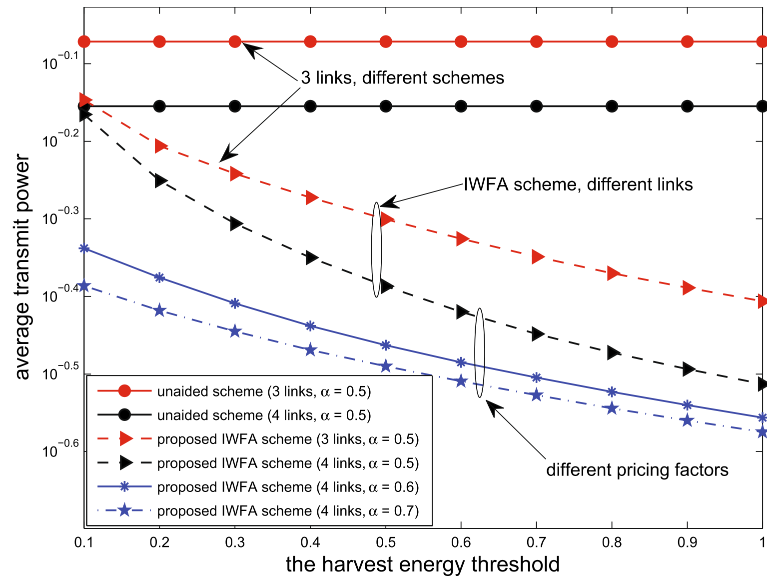


**Fig. 4** The average transmit power as a function of the harvest energy threshold. Figure 4 shows the results of the average transmit power versus the harvest energy threshold. The different types of lines represent the proposed IWFA scheme and the unaided scheme under different conditions of $K \in \{3, 4\}$ and $\alpha \in \{0.5, 0.6, 0.7\}$

linear function. Both conditions in *Theorem 2* are satisfied. Thus, the problem $VI\,(\mathcal{P}_k, V_k)$ is equivalent to the formulated game $\mathcal{G}_2$.

## Appendix B

### Proof of Proposition 2

For problem $VI\,(\mathcal{P}_k, sV_k)$, the strategy set $\mathcal{P}_k$ is convex and compact since the items $0 \leq p_k \leq p_k^{max}$ and $(1-\beta_k)\left(|h_{kk}|^2 p_k + \sum_{j=1, j\neq k}^{K} |h_{jk}|^2 p_j\right) \geq e_k$ of $\mathcal{P}_k$ are linear functions.

It is can be observed from Eq. (17) that $V_k = \nabla_{p_k} v_k(p_k, \mathbf{p}_{-k}; \alpha_k)$, we can prove the continuity of $V_k$ by computing its first derivative. And we can see that its first derivative is exists; furthermore, the utility function $V_k$ is continuous in $\mathcal{P}_k$.

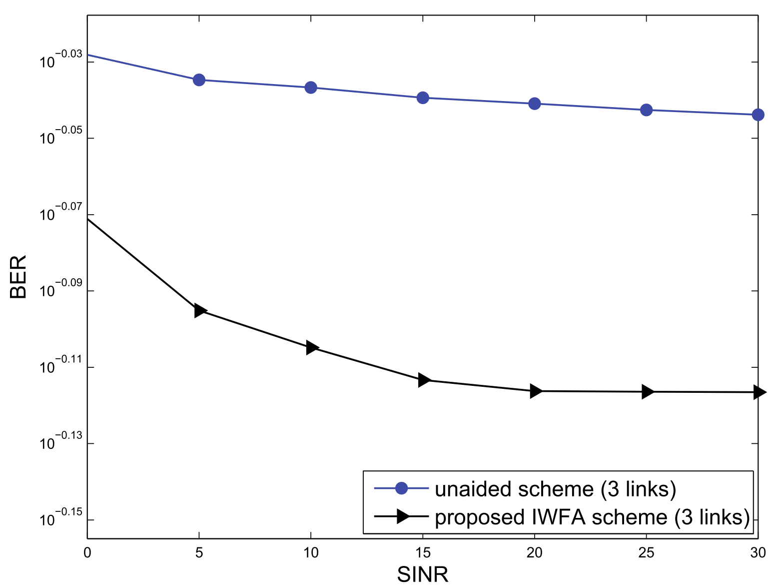


**Fig. 5** The average bit error rate (BER) versus SINR. Figure 5 shows the results of the BER of 3 users versus SINR. The different types of lines represent the proposed IWFA scheme and the unaided scheme

Thus, we know that the VI problem admits at least one solution according to *Theorem 1* and the NE existence is proved.

## Appendix C

### Proof of Proposition 3

From (17), we have that $\nabla_{p_k} v_k(p_k, \mathbf{p}_{-k}) = V_k$. Consider two different power $p_k^{(1)}$ and $p_k^{(2)}$ in the strategy set. According to the mean-value theorem, we have

$$\begin{aligned}\left(p_k^{(1)} - p_k^{(2)}\right)&\left\{\left[V_k(p^{(1)})\right] - \left[V_k(p^{(2)})\right]\right\}\\ &=\left(p_k^{(1)} - p_k^{(2)}\right)\left\{\nabla_{p_k} v_k\left(p_k^{(1)}\right) - \nabla_{p_k} v_k\left(p_k^{(2)}\right)\right\}\\ &=\left(p_k^{(1)} - p_k^{(2)}\right)\sum_{j=1}^{K}\nabla^2_{p_k,p_j} v_k(z_k)\left(p_j^{(1)} - p_j^{(2)}\right)\\ &\triangleq \mathcal{Q}(t).\end{aligned} \tag{23}$$

Define $d_k = p_k^{(1)} - p_k^{(2)}$ and $d_j = p_j^{(1)} - p_j^{(2)}$, we have

$$\begin{aligned}\mathcal{Q}(t) =& d_k \sum_{j=1}^{K}\nabla^2_{p_k,p_j} v_k(z_k) d_j\\ \geq& d_k\, \nabla^2_{p_k}\, v_k(z_k) d_k\end{aligned} \tag{24}$$

since $d_k \sum_{j\neq k}^{K} \nabla^2_{p_k,p_j} v_k(z_k) d_j > 0$. Then, we have

$$\mathcal{Q}(t) \geq d_k\, \nabla^2_{p_k}\, v_k(z_k) d_k \tag{25}$$

Then, the strongly monotonicity of $V_k$ can be proved as

$$\begin{aligned}\left(p_k^{(1)} - p_k^{(2)}\right)&\left\{\left[V_k(p^{(1)})\right] - \left[V_k\left(p^{(2)}\right)\right]\right\}\\ &\geq \sum_{k=1}^{K} d_k\, \nabla^2_{p_k}\, v_k(z_k) d_k\\ &= \sum_{k=1}^{K} {d_k}^2\, \nabla^2_{p_k}\, v_k(z_k)\\ &\geq {d_k}^2 \sum_{k=1}^{K}\nabla^2_{p_k} v_k(z_k)\\ &= c_{sm} |\, p_k^{(1)} - p_k^{(2)}\, |^2\end{aligned} \tag{26}$$

where $c_{sm} = \sum_{k=1}^{K} \nabla^2_{p_k} v_k(z_k) > 0$ is the strongly monotone constant. Therefore, the Eq. (18) always hold, which completes the proof.

#### Abbreviations

AO: Alternating optimization; AWGN: Additive white Gaussian noise; CSI: Channel state information; EH: Energy harvesting; ID: Information decoding; IFC: Interference channels; IWFA: Iterative water-filling algorithm; MRT: Maximum ratio transmission; NCP: Nonlinear complement problem; NE: Nash equilibrium; PFPA: Proportional-fair power allocation; PS: Power splitting; RF: Radio-frequency; SINR: Signal-to-interference-noise-ratio; SOCP: Second-order cone programming; SPD: Synchronous power descending; SWIPT: Simultaneous wireless information and power transfer; TS: Time switching; VI: Variational inequality; ZF: Zero-forcing

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#### Availability of data and materials

The custom code: https://github.com/BUPT7/paper-code.

#### Authors' contributions

First, we formulated a power allocation problem, designed a scheme to optimize the power allocation strategy, performed numerical simulations, and prepared the initial draft as well as the revision. Then, we modified the solution pattern, verified the mathematical derivations, checked and analyzed the simulation results, and improved the writing. All authors read and approved the final manuscript.

#### Competing interests

The authors declare that they have no competing interests.

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