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The economic lot-sizing problem with remanufacturing: analysis and an improved algorithm

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Abstract

We address the economic lot sizing problem with product returns and remanufacturing. For this problem we provide a new theoretical result about the form of the optimal solutions that can be considered a generalization of the well-known zero-inventory property. Based on this result we suggest an optimized version of an existing Tabu Search procedure. The original and the optimized version of the procedure are evaluated on a recent benchmark set of large instances of 52 periods. The numerical experiment carried out shows that both variants of the procedure outperform the solving procedure suggested in the literature in over 90 % of the tested cases and in about tenth of computation time in the worst case.

Keywords: Economic lot-sizing problem; Remanufacturing; Inventory control; Optimization

Background

In the economic lot-sizing problem with product returns and remanufacturing (ELSR) the objective is to determine the quantities to produce and remanufacture at each period in order to meet the demand requirements of a single product on time, minimizing all the costs involved. Eventually, disposal option for returns is considered. This kind of problem has attracted growing attention over the last years from both the academic as well as the industry side [1–5]. Governmental, social pressures and economic opportunities have motivated many firms to become involved with the return of used products for recovery. Among the industrial options for recovering, the remanufacturing activity can be defined as the recovery process of returned products after which it is warranted that the remanufactured products offer the same quality and functionality that those newly manufactured [6, 7]. Products that are remanufactured include automotive parts, engines, tires, aviation equipment, cameras, medical instruments, furniture, toner cartridges, copiers, computers, and telecommunications equipment. Remanufacturing offers benefits for all of the parties involved. From the consumer's point-of-view, remanufactured products assume the same quality of new products and are sometimes offered at an inferior market price. For the manufacturer, remanufacturing provides cost savings in energy consumption, raw materials, and labour. Finally, the environment benefits from the more efficient use of raw materials and energy employed during the production phase. In addition, remanufacturing tends to reduce the total number of products put in the market and later disposed, i.e. long-life products. For overviews about remanufacturing, the reader is referred to Ijomah [6], Hormozi [8], Sundin [9] and Gutowski et al. [10].

Richter and Sombrutzki [11] and Richter and Weber [12] consider the ELSR for the particular case in which the number of returns in the first period is sufficient to satisfy the total demand over the planning horizon. Golany et al. [13] provide a Network Flow formulation for the ELSR and an exact algorithm of $O(T^3)$ time for the case of linear cost functions. They also show that the ELSR is NP-hard for the case of general concave cost functions. Yang et al. [14] show the same result of complexity for the case of stationary concave cost functions and suggest a heuristic procedure of $O(T^4)$ time for the ELSR. van den Heuvel [15] shows that ELSR is NP-hard for the case of set-up and unit costs for the activities and unit costs for holding inventory, even in the time-invariant case, i.e., the same values for every period. Teunter et al. [16] consider two ELSR variants with joint and separate set-up costs for the production and remanufacturing, respectively. For the case of joint set-up costs they provide an exact algorithm of $O(T^4)$. For the case of separate set-up costs, they adapt and compare three well-known heuristics, including the Silver-Meal based heuristic. Later, Schulz [17] extends and improves the work of Teunter et al. [16] about the Silver-Meal based heuristic for the ELSR, and Retel-Helmrich et al. [18] provide and compare different mathematical formulations for the ELSR with both separate and joint set-up costs. They also show that both ELSR variants are NP-hard in general. In Piñeyro and Viera [19] we suggest and compare several inventory policies for the ELSR using a divide-and-conquer approach, including a Tabu Search-based on procedure. Piñeyro and Viera [20] consider the problem of determining the remanufacturing quantities of an optimal solution of the ELSR assuming that the periods where the remanufacturing is allowed are known in advance. Li et al. [21] suggest and evaluate a more sophisticated Tabu Search based on procedure for the ELSR. They propose a block-chain method to produce high-quality initial solutions and a new LP formulation for determining the minimum cost of each block. Baki et al. [22] also exploit the block-chain structure of the ELSR solutions in order to provide a dynamic programming based procedure of polynomial time. In Sifaleras et al. [23] Variable Neighborhood Search (VNS) based on procedures are suggested and compared with others solving methods in the literature. The authors also present a benchmark set of problem instances with a large number of periods. Several authors have considered extensions of the ELSR. Pan et al. [24] address a dynamic lot-sizing problem with returns recovery and capacity constraints. Mitra [25] analyzes a two-echelon inventory system with returns. Piñeyro and Viera [26] consider the ELSR with substitution of remanufactured products by new ones but not viceversa.

In this paper we consider the ELSR formulation introduced by Teunter et al. [16] for the separate set-up scheme, i.e., separate set-up costs for producing and remanufacturing. We note that this ELSR formulation is also considered in Schulz [17], Li et al. [21], Baki et al. [22] and Sifaleras et al. [23]. Our contributions for this formulation of the ELSR are threefold. First we provide a new theoretical result about the form of the optimal solutions of the ELSR, which can be considered an extension of the well-known zero-inventory property for the classic economic lot-sizing problem without returns options (ELSP). We also note that this result is also valid for the case of time-variant costs with non-speculative motives. Secondly, we use this theoretical result for improving the Tabu Search based on procedure suggested in Piñeyro and Viera [19] for the ELSR. Finally, we evaluate the original as well as the optimized version of the procedure in a benchmark set of large instances (52 periods) introduced by Sifaleras et al. [23]. The numerical experiment carried out shows that both the original and the optimized version of the Tabu Search procedure outperform the VNS based procedure suggested in Sifaleras et al. [23] in over 90 % of the tested cases and in about tenth of computation time in the worst case.

The rest of the paper is organized as follows. Methods section provides the problem formulation and a theoretical result about the optimal solutions of the ELSR. Then we describe the improvement performed on the Tabu Search based on procedure of Piñeyro and Viera [19]. In Results and discussion section we report the numerical experiment carried out for the set of large instances introduced by Sifaleras et al. [23]. The paper ends with the Conclusions section with some guidelines for future research.

Methods

Problem formulation and analysis

We address a dynamic lot-sizing problem of a single product for which the demand requirements of each period over a finite planning horizon must be satisfied on time either by producing new items or by remanufacturing used items returned to the origin. Figure 1 shows a picture of the flows of items for the inventory system that represents the lot-sizing problem under consideration.

There are set-up costs for producing and remanufacturing, and unit costs for carrying ending positive inventory from one period to the next. As in Teunter et al. [16], we consider the following assumptions. Unit costs are assumed zero for both production and remanufacturing. Inventory costs for used items are at most equal to the inventory costs for serviceable items, i.e. low-cost returns. Cost values are time-invariant, i.e. stationary costs pattern. The objective is to determine the quantities to produce and remanufacture for each one of the periods in the planning horizon in order to meet the demand requirements on time and minimizing the sum of the involved costs. This problem can be formulated as the following Mixed Integer Linear Programming (MILP) [16, 17, 21–23]:

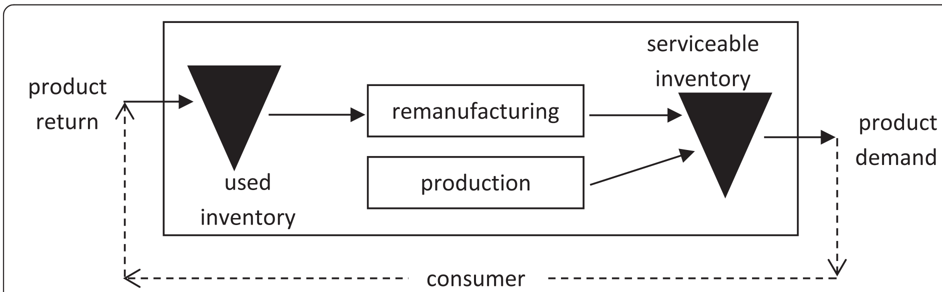

Fig. 1 Flow of items in the ELSR

$$\min\sum_{t=1}^{T}\left\{K^p\delta_t^p+K^r\delta_t^r+h^s y_t^s+h^u y_t^u\right\} \quad (1)$$

subject to:

$$y_t^s=y_{t-1}^s+p_t+r_t-D_t \quad \forall t=1,2,\dots,T \quad (2)$$

$$y_t^u=y_{t-1}^u-r_t+R_t \quad \forall t=1,2,\dots,T \quad (3)$$

$$M\delta_t^p\geq p_t \quad \forall t=1,2,\dots,T \quad (4)$$

$$M\delta_t^r\geq r_t \quad \forall t=1,2,\dots,T \quad (5)$$

$$y_0^s=y_0^u=0 \quad (6)$$

$$\delta_t^p,\delta_t^r\in\{0,1\}\ \ p_t,r_t,y_t^s,y_t^u\geq 0 \quad \forall t=1,2,\dots,T \quad (7)$$

In model (1) – (7) the parameters T, D_t and R_t denote the length of the planning horizon, demand and returns values in periods $t=1,\dots,T$, respectively; K^p (K^r), is the set-up cost for production (remanufacturing); h^s (h^u) is the unit cost of holding inventory for serviceable (used) products; M is a number at least as large as $\max\{D_{1T}, R_{1T}\}$, where D_{ij} and R_{ij} are the accumulative demand and returns between periods i and j, with $1\leq i\leq j\leq T$. The decision variables p_t and r_t denote the number of units produced and remanufactured in periods $t=1,\dots,T$, respectively; δ_t^p (δ_t^r) is a binary variable equal to 1 if production (remanufacturing) is carried out in periods $t=1,\dots,T$, or 0 otherwise; y_t^s (y_t^u) is the inventory level of serviceable (used) items for periods $t=1,\dots,T$.

A result about the optimal solutions form

In this section we present a property about the form of optimal solutions of the ELSR. It can be considered a generalization of the well-known zero-inventory property for the ELSP introduced by Wagner and Whitin [27]. Also Teunter et al. [16] generalize this property for the case of joint set-up scheme of the ELSR and Richter and Sombrutzki [11] and Richter and Weber [12] for the case of sufficiently large number of returned products in the first period. Here we demonstrate that there is an optimal solution for which it holds that for any couple of periods i and j of either positive production or positive remanufacturing, there must be at least one period with zero-inventory level of serviceable items. Note that if it not the case, we can obtain a new feasible solution by transferring at least one unit of the production (remanufacturing) quantity from period i to the next period j, thus reducing the inventory level of serviceable items and with at most the same cost.

Proposition 1. There is an optimal solution for which it holds that for any pair of periods i and j, $i<j$, there is at least one period t such that $p_i y_t^s p_j=0$ and $r_i y_t^s r_j=0$, with $i\leq t<j$.

Proof. Consider an ELSR feasible solution $s=(p,r)$ with a production plan p and a remanufacturing plan r, respectively. Without loss of generality, suppose first that there is only one pair of successive periods i and j for which $p_i y_t^s p_j>0$ for all period t in $1\leq i\leq t<j\leq T$. Then, define $\varepsilon=\min\left\{p_i,y_i^s,y_{i+1}^s,\dots,y_{j-2}^s,y_{j-1}^s\right\}>0$ and determine a new solution $s^1=(p^1,r^1)$ with $r^1=r$ and $p^1=p$ except by $p_i^1=(p_i-\varepsilon)$ and $p_j^1=\left(p_j+\varepsilon\right)$. This new solution s^1 is feasible since it satisfies that $y_t^{1s}\geq 0$ for all t in $1\leq t\leq T$. For s^1 we have that $p_i^1 y_t^{1s} p_j^1 = 0$ for some period t in $1 \le i \le t < j \le T$, since at least one of the following two equations is fulfilled: $p_i^1 = 0$ and/or $y_t^{1s} = 0,\ \ t : 1 \le i \le t < j \le T$. Note that the cost of s^1 is lower than the cost of the original solution s at least by a term equal to $h^s(j - i)$. The same reasoning is valid for the remanufacturing plan r. In this case, the new solution is less costly than the original at least by a term equal to $(h^s - h^u)(j - i)$. Note that $(h^s - h^u) \ge 0$ since we are assuming low-cost returns. Therefore, given any feasible solution $s = (p, r)$, even optimal, we can determine a new feasible solution with at most the same cost and fulfilling the property of Proposition 1.

From Proposition 1 we can establish that in order to determine an optimal solution of the ELSR we may avoid those solutions in which the serviceable-inventory level is positive between any pair of successive periods of either positive production or positive remanufacturing. We also note that the zero-serviceable-inventory property as stated in Proposition 1 is valid in the case of time-variant costs with non-speculative motives, even with positive unit costs. Non-speculative motives on the costs mean that it is profitable to produce or remanufacture as late as possible, i.e., $c_i^p + \sum_{t=i}^{(j-1)} h_t^s \ge c_j^p$ and $c_i^r + \sum_{t=i}^{(j-1)} \left(h_t^s - h_t^u\right) \ge c_j^r$ with $1 \le i \le j \le T$. The proof for this case is straightforward and follows from the proof of above, since we are transferring forward either newly manufactured o remanufactured items. We state this generalization of Proposition 1 in the next corollary.

Corollary 1. Proposition 1 is also valid in the case of time-variant costs with non-speculative motives, even with positive unit costs.

The Tabu Search based on procedure

The ELSR as formulated in (1) – (7) is a NP-hard problem [18, 22]. Therefore, it is unlikely that we can develop any efficient time procedure for determining an optimal solution of the problem. Considering this complexity result, many authors have proposed heuristic methods for obtaining high quality solutions [14, 16, 17, 21–23]. In Piñeyro and Viera [19] we suggest a Tabu Search based on procedure in order to obtain a near optimal solution to the ELSR, even with a more general cost structure and disposal option for returns. The procedure is referred in that paper as Basic Tabu Search procedure (BTS) because it uses only the rudiments of the Tabu Search metaheuristic [28]. In addition, a simple notion of neighbourhood is used and only one move is defined to explore it. Despite this fact, the numerical experiment carried out in Piñeyro and Viera [19] show the effectiveness of the BTS procedure in a wide range of demand-returns-cost patterns. In Piñeyro and Viera [20] we present a thorough analysis about the problem of determining the remanufacturing quantities of an optimal solution that can be considered a theoretical support for the good behaviour of the BTS procedure reported in Piñeyro and Viera [19].

The original BTS procedure description

The BTS procedure begins by determining an initial ELSR solution from the set of periods fixed as positive-remanufacturing periods received as input. For each period i fixed as positive-remanufacturing period, the remanufacturing quantity is determined by means of the following remanufacturing rule:

$$r_i = \min\left(y_{i-1}^u + R_i, D_{i(j-1)}\right), \quad 1 \leq i < j \leq T+1 \tag{8}$$

In (8) the period j is the next positive-remanufacturing period if it exists, or $(T+1)$ in the case that period i is the last one. Then, the corresponding optimal production plan (periods and quantities) is determined by means of a W-W algorithm type. The initial solution is marked as current solution and the exploration phase of the process begins. The set of neighbouring solutions is obtained from the current one by swapping the periods where remanufacturing occurs. For each one of the neighbouring solution, the remanufacturing quantities are determined by means of equation (8) and the corresponding optimal production plan by applying the W-W algorithm. The neighbouring solution with smaller cost is marked as the current solution for the next step. The exploration phase continues until either the number of iterations is greater than the total number of iterations allowed or the number of iterations without improvement is greater than the maximum allowed. The procedure returns the best global solution evaluated. For more details about the procedure we refer to Piñeyro and Viera [19].

An optimization process for the BTS procedure

Given the solution returned by the BTS procedure, we can check whether that solution satisfy the zero-serviceable-inventory property of Proposition 1. First we note that for any solution considered by the BTS procedure, the production plan satisfies the property stated in Proposition 1, since it is determined using a W-W algorithm [27]. Therefore, we must check the property only for the remanufacturing plan. We proceed as

```
for {i = 1; (r_i > 0 ∧ i ≤ T); i++}
  q = r_i ;
  j = i + 1;
  while (r_j = 0 ∧ j ≤ T) do j = j + 1;
  if (j ≤ T)
    for {t = i; (r_i y_t^s r_j > 0 ∧ t ≤ j − 1); t++}
      if y_t^s < q then q = y_t^s ;
    endfor
    if (q > 0)
      r_i ← (r_i − q);
      r_j ← (r_j + q);
    endif
  endif
endfor
```

Fig. 2 Pseudocode of the optimization suggested for the BTS procedure follows. For each period i of the remanufacturing plan with a strictly positive remanufacturing quantity consider the immediately next period j for which $r_i y_t^s r_j > 0$ for all period t in $1 \leq i \leq t < j \leq T$. Define $q > 0$ as the minimum quantity between the remanufacturing level of period i and the serviceable-inventory level for all periods t in $1 \leq i \leq t < j \leq T$, i.e. $q = \min\left\{r_i, y_i^s, y_{i+1}^s, \ldots, y_{j-2}^s, y_{j-1}^s\right\} > 0$. Then, determine a new feasible remanufacturing plan by subtracting the quantity q to the remanufacturing level of period i, and adding q to the remanufacturing level of period j. The new remanufacturing plan satisfies the zero-serviceable-inventory property of Proposition 1 since either the remanufacturing level of period i or the serviceable-inventory level is zero for some period t in $1 \leq i \leq t < j \leq T$. In Fig. 2 we provide the pseudocode of the optimization stage added to the end of the BTS procedure according to the description above.

We note that the optimization process of Fig. 2 is $O(T^2)$ time, thus keeping the running time of the original BTS procedure. This fact can be appreciated in the time

Table 1 Results on benchmark set of Sifaleras et al. [23]. Instances 1–27.

No	Gurobi 5.6.2	RGVNS		BTS		Optimized BTS	
	Obj. Value	Obj. Value	Error (%)	Obj. Value	Error (%)	Obj. Value	Error (%)
1	8698.80	8895.20	2.2600	8751.40	0.6047	8741.80	0.4943
2	**8781.80**	9185.40	4.6000	9078.40	3.3774	9071.20	3.2955
3	8541.60	8793.80	2.9500	**8739.20**	2.3134	8739.20	2.3134
4	8943.77	9391.20	5.0000	9294.60	3.9226	9186.60	2.7151
5	9717.00	9853.00	1.4000	9741.00	0.2470	9720.00	0.0309
6	9962.45	10240.50	2.7900	10127.50	1.6567	10072.50	1.1046
7	9598.00	9955.00	3.7200	9654.50	0.5887	9631.00	0.3438
8	9803.50	10327.00	5.3400	10046.50	2.4787	10003.50	2.0401
9	**10266.20**	10573.80	3.0000	10306.20	0.3896	10293.60	0.2669
10	10812.80	11184.80	3.4400	10947.00	1.2411	10923.00	1.0192
11	10290.76	10445.40	1.5000	10301.60	0.1053	10290.80	0.0004
12	10745.60	11027.00	2.6200	10879.00	1.2414	10851.40	0.9846
13	13200.95	13568.00	2.7800	**13405.40**	1.5488	13405.40	1.5488
14	12131.36	*12387.40*	2.1100	12438.60	2.5326	12353.80	1.8336
15	13018.36	13403.60	2.9600	13186.00	1.2877	13172.40	1.1833
16	11853.20	12230.60	3.1800	12104.40	2.1193	12044.40	1.6131
17	14236.50	14429.50	1.3600	14335.50	0.6954	14319.50	0.5830
18	13150.99	13660.00	3.8700	13360.50	1.5931	13242.00	0.6920
19	13901.98	14278.50	2.7100	13958.00	0.4030	13952.00	0.3598
20	13495.47	**13725.00**	1.7000	13913.00	3.0939	13843.00	2.5752
21	14842.38	15063.60	1.4900	14874.20	0.2144	14865.20	0.1537
22	14122.20	14595.40	3.3500	14159.60	0.2648	14130.80	0.0609
23	14561.16	14854.40	2.0100	14725.60	1.1293	14703.60	0.9782
24	13865.91	14428.20	4.0600	13983.60	0.8488	13958.60	0.6685
25	**25657.20**	26068.20	1.6000	25989.00	1.2932	25885.00	0.8879
26	**21247.40**	22016.80	3.6200	21706.40	2.1603	21540.00	1.3771
27	**24364.40**	24988.20	2.5600	24409.00	0.1831	24374.60	0.0419

consumption of CPU reported for both BTS variants in the next section about the numerical experiment.

Results and discussion

In this section we report the results of the numerical experiment carried out for the original BTS procedure of Piñeyro and Viera [19] and the optimized version according to the improvement suggested above. For the experiment we resort to the benchmark set of 108 different large instances of 52 periods developed by Sifaleras et al. [23] and downloadable from the authors web site http://users.uom.gr/~sifalera/benchmarks.html (last access: 24/07/2015). The benchmark set was designed with the aim to evaluate the robustness of the RGVNS procedure suggested in that paper. It is based on the Schulz [17] well-known benchmark set, but with instances more than four times larger. As the authors claim "this new benchmark set is more difficult and larger than those that have ever been used in the literature, for the ELSR problem". For this benchmark set of large instances, the different values for both setup costs are 200, 500 and 2000; the values for

Table 2 Results on benchmark set of Sifaleras et al. [23]. Instances 28–54

No	Gurobi 5.6.2	RGVNS		BTS		Optimized BTS	
	Obj. Value	Obj. Value	Error (%)	Obj. Value	Error (%)	Obj. Value	Error (%)
28	**20328.95**	20479.40	0.7400	20647.20	1.5655	20639.20	1.5261
29	**26561.00**	26888.00	1.2300	26828.00	1.0052	26791.00	0.8659
30	**22332.50**	22851.50	2.3200	22784.00	2.0217	22732.50	1.7911
31	**26625.50**	27326.00	2.6300	26727.00	0.3812	26676.50	0.1915
32	**23229.50**	23945.50	3.0800	23863.50	2.7293	23635.00	1.7456
33	**27872.60**	28482.20	2.1900	28047.00	0.6257	28019.00	0.5252
34	**24116.80**	25426.00	5.4300	24816.20	2.9001	24738.40	2.5775
35	**26762.40**	27823.60	3.9700	26874.20	0.4178	26808.40	0.1719
36	**24065.20**	24785.80	2.9900	24682.20	2.5639	24599.60	2.2206
37	**10622.20**	10937.40	2.9700	**10657.00**	0.3276	10657.00	0.3276
38	**12011.00**	12266.80	2.1300	12194.60	1.5286	12131.40	1.0024
39	**10652.20**	10867.00	2.0200	**10749.00**	0.9087	10749.00	0.9087
40	11741.60	12088.60	2.9600	11948.00	1.7579	11912.00	1.4513
41	12249.48	12585.00	2.7400	**12455.50**	1.6819	12455.50	1.6819
42	13844.99	**13998.50**	1.1100	14003.00	1.1413	14001.00	1.1268
43	**12309.00**	12616.00	2.4900	**12320.50**	0.0934	12320.50	0.0934
44	13626.97	13895.00	1.9700	13737.50	0.8111	13697.50	0.5176
45	13348.00	13584.40	1.7700	**13477.40**	0.9694	13477.40	0.9694
46	**15030.80**	15543.40	3.4100	15236.00	1.3652	15215.40	1.2281
47	**13635.60**	13979.00	2.5200	13785.00	1.0957	13778.60	1.0487
48	15051.80	15788.20	4.8900	15330.20	1.8496	15301.00	1.6556
49	15625.40	15982.80	2.2900	**15759.20**	0.8563	15759.20	0.8563
50	15447.80	15984.80	3.4800	15817.80	2.3952	15667.40	1.4216
51	14997.80	15409.60	2.7500	**15119.80**	0.8135	15119.80	0.8135
52	15176.60	15494.00	2.0900	15376.80	1.3191	15360.80	1.2137
53	16782.00	17172.00	2.3200	17011.50	1.3675	16957.50	1.0458
54	17102.50	17757.00	3.8300	17186.50	0.4912	17102.50	0.0000

the holding cost of used items are 0.2, 0.5 and 0.8. The holding cost for serviceable items is equal to 1. Demand values follow a normal distribution of mean 100 per period. Returns values also follow a normal distribution but three different means of 30, 50 and 70 are considered. The coefficient of variation of the normal distributions can be of 10 % and 20 % (small and large variance respectively). A total of 108 different instances of 52 periods were generated.

For both BTS procedure variants we use the following configuration values. The size of the tabu list is fixed at 1,000,000. The total number of iterations is 10,000 and the maximum without improvement is 50. The size of the tabu list and the total number of iterations are greater than those considered in Piñeyro and Viera [19] since the ELSR instances considered here are four times longer. However, we set the maximum number of iterations without improvement in 50 rather than 500 in order to maintain low computation times. The zero-remanufacturing plan is used as initial solution for all the instances, as in Piñeyro and Viera [19] and later in Sifaleras et al. [23].

Table 3 Results on benchmark set of Sifaleras et al. [23]. Instances 55–81

No	Gurobi 5.6.2	RGVNS		BTS		Optimized BTS	
	Obj. Value	Obj. Value	Error (%)	Obj. Value	Error (%)	Obj. Value	Error (%)
55	16591.44	17149.50	3.3600	16716.50	0.7538	16704.50	0.6814
56	17217.93	17544.00	1.8900	17356.50	0.8048	17313.00	0.5522
57	18047.53	18215.60	0.9300	**18132.60**	0.4714	18132.60	0.4714
58	18780.16	**19158.40**	2.0100	19234.80	2.4209	19166.60	2.0577
59	17742.76	18337.20	3.3500	18085.20	1.9300	18055.60	1.7632
60	18646.54	19212.60	3.0400	18678.20	0.1698	18646.60	0.0003
61	**27623.80**	28428.20	2.9100	27810.60	0.6762	27793.80	0.6154
62	**24892.60**	25562.40	2.6900	**25190.20**	1.1955	25190.20	1.1955
63	**26587.40**	28083.00	5.6300	26802.80	0.8102	26665.20	0.2926
64	**24252.40**	**24596.20**	1.4200	24640.00	1.5982	24622.40	1.5256
65	**29328.50**	30033.00	2.4000	29538.00	0.7143	29527.00	0.6768
66	26961.50	27544.00	2.1600	27184.00	0.8253	27091.50	0.4822
67	**28484.00**	29139.00	2.3000	28524.00	0.1404	28484.00	0.0000
68	**27019.00**	28116.00	4.0600	27313.50	1.0900	27186.50	0.6199
69	30515.40	31117.40	1.9700	30644.20	0.4221	30626.00	0.3624
70	28758.40	30014.60	4.3700	28924.40	0.5772	28850.60	0.3206
71	**29864.60**	30497.40	2.1200	29940.00	0.2525	29889.00	0.0817
72	28195.06	30267.60	7.3500	28628.00	1.5355	28573.20	1.3412
73	**14443.40**	14835.40	2.7100	**14443.40**	0.0000	14443.40	0.0000
74	**18364.00**	18517.40	0.8400	**18364.00**	0.0000	18364.00	0.0000
75	**14954.00**	15007.00	0.3500	**14954.00**	0.0000	14954.00	0.0000
76	**17857.80**	17979.20	0.6800	**17960.80**	0.5768	17960.80	0.5768
77	**18546.00**	18903.50	1.9300	**18828.00**	1.5205	18828.00	1.5205
78	**23069.50**	23266.50	0.8500	**23130.00**	0.2623	23130.00	0.2623
79	**18657.50**	18821.00	0.8800	**18760.00**	0.5494	18760.00	0.5494
80	**23329.00**	23449.50	0.5200	**23378.00**	0.2100	23378.00	0.2100
81	**20999.80**	21431.80	2.0600	**21088.60**	0.4229	21088.60	0.4229

The BTS procedure was coded in Java and the experiment was carried out in a Java Runtime Environment 1.8.0_25 on HP laptop with Intel(R) Core(TM) i5-42010 CPU, 8.00 GB of RAM, and operating system Windows 8.1 of 64 bits. We note that this computing environment is less powerful of that used in Sifaleras et al. [23].

In Table 1, Table 2, Table 3 and Table 4 we report the results for Gurobi optimizer, the VNS procedure from Sifaleras et al. [23] and for both BTS variants from our numerical experiment. As in Sifaleras et al. [23], in the 2nd column we mark in bold those instances for which Gurobi reaches the optimal solution. In the 3rd column we mark in bold those instances for which the VNS procedure achieves a minor cost solution than both BTS variants and with italic for only the original BTS procedure. Finally, in the 5th column we mark in bold those instances for which both BTS variants obtain the same solution, i.e. the optimization performed is not able to improve the solution of the original BTS. In order to compare the results obtained, the error is defined as in Sifaleras et al. [23], that is the percentage gap between the objective value of Gurobi (*g*)

Table 4 Results on benchmark set of Sifaleras et al. [23]. Instances 82–108

No	Gurobi 5.6.2	RGVNS		BTS		Optimized BTS	
	Obj. Value	Obj. Value	Error (%)	Obj. Value	Error (%)	Obj. Value	Error (%)
82	**26519.60**	27048.60	1.9900	**26856.40**	1.2700	26856.40	1.2700
83	**21114.40**	21649.40	2.5300	**21399.40**	1.3498	21399.40	1.3498
84	**26162.80**	27142.60	3.7500	**26896.20**	2.8032	26896.20	2.8032
85	**19646.00**	19834.80	0.9600	**19769.40**	0.6281	19769.40	0.6281
86	**22567.57**	23128.60	2.4900	**22612.60**	0.1995	22612.60	0.1995
87	**19880.40**	19963.60	0.4200	**19880.40**	0.0000	19880.40	0.0000
88	**22483.80**	22646.60	0.7200	**22524.80**	0.1824	22524.80	0.1824
89	23013.50	23190.00	0.7700	**23144.00**	0.5671	23144.00	0.5671
90	27076.00	27632.00	2.0500	**27309.50**	0.8624	27309.50	0.8624
91	**22706.50**	23090.50	1.6900	**22706.50**	0.0000	22706.50	0.0000
92	26754.00	**26793.00**	0.1500	**27027.50**	1.0223	27027.50	1.0223
93	25890.80	26118.80	0.8800	**26101.40**	0.8134	26101.40	0.8134
94	30229.20	30669.60	1.4600	30535.00	1.0116	30505.60	0.9143
95	**26188.80**	26935.80	2.8500	**26240.80**	0.1986	26240.80	0.1986
96	**29504.40**	30231.20	2.4600	29607.40	0.3491	29562.20	0.1959
97	**32952.36**	33512.00	1.7000	**33078.40**	0.3825	33078.40	0.3825
98	33332.20	33940.00	1.8200	**33757.80**	1.2768	33757.80	1.2768
99	**33072.40**	33586.20	1.5500	**33088.80**	0.0496	33088.80	0.0496
100	**33115.00**	*33539.20*	1.2800	33587.20	1.4259	33491.20	1.1360
101	36285.90	36581.00	0.8100	**36461.50**	0.4839	36461.50	0.4839
102	37205.50	38265.50	2.8500	37500.50	0.7929	37438.00	0.6249
103	**36173.50**	37194.00	2.8200	36912.00	2.0415	36824.00	1.7983
104	**36817.00**	37861.50	2.8400	**37230.50**	1.1231	37230.50	1.1231
105	38728.00	39020.20	0.7500	38862.20	0.3465	38857.80	0.3352
106	40310.40	41261.80	2.3600	40615.20	0.7561	40597.00	0.7110
107	38611.20	39281.00	1.7300	38639.20	0.0725	38617.60	0.0166
108	**39826.10**	40932.40	2.7800	40330.00	1.2653	40301.20	1.1929

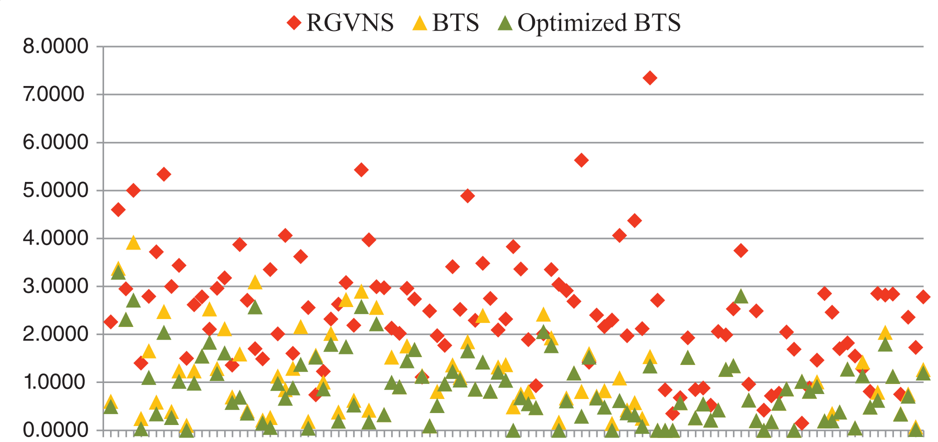

Fig. 3 Error values of the RGVNS and BTS procedures for the 108 instances

and the objective value of the solution determined by the procedure under consideration (p): $100 \times (p - g)/g$.

From Tables 1 to 4 we note that both BTS procedure variants outperform the RGVNS procedure of Sifaleras et al. [23] in most cases. The original BTS achieves better solutions in 100 of 108 instances, and the optimized version in 102 instances, that is 92.59 % and 94.44 % of the total number of instances respectively. In addition, the optimized BTS achieves better solutions than the original version in 70 instances, which results in 64.81 % of the total number of instances. The original BTS procedure is able to achieve the same objective value as Gurobi in 5 instances, all them optimal values, and the optimized variant in 2 instances more, one of them optimal. In Fig. 3 you can clearly see the good performance of both BTS procedures.

Tables 5 and 6 provide the statistics of percentage errors and computation times respectively. As stated in Sifaleras et al. [23] the Gurobi optimizer is set to 3600 s of maximum time of running and 10^{-4} of tolerance. The RGVNS ran for 30s for all instances.

In average, solutions achieved by the optimized BTS procedure are nearly three times more cost-effective than those of the VNS procedure with a running time of 1.31 seconds and in less than tenth computation time in the worst case. In addition, the extra running time required for the optimization phase is almost negligible. Although the solutions provided by Gurobi are more cost-effective in most cases, the BTS procedure variants may also offer high quality solutions and with 1870 times less computational effort on average.

Conclusions

In this paper we have tackled the economic lot-sizing problem with remanufacturing under the assumptions of time-invariant costs and low-costs for the returns. We

Table 5 Percentage cost errors

	Avg. (%)	Std. (%)	Min. (%)	Max. (%)
RGVNS	2.4402	1.2453	0.1458	7.3507
BTS	1.0530	0.8350	0.0000	3.9226
Optimized BTS	0.8784	0.7239	0.0000	3.2955

Table 6 CPU times in seconds

	Avg. (s)	Std. (s)	Min. (s)	Max. (s)
Gurobi 5.6.2	2450.55	1425.82	3.47	3600.03
RGVNS	30.00	0.00	30.00	30.00
BTS	1.30	0.49	0.72	3.03
Optimized BTS	1.31	0.47	0.74	2.88

present a new theoretical result about the form of optimal solutions that can be considered a generalization of the well-known zero-inventory property of the classic economic lot-sizing problem, i.e. without returns options. Based on this extended property we develop an improvement to the Basic Tabu Search procedure of Piñeyro and Viera [19] for the ELSR. The original and the optimized version of the BTS procedure are compared against the VNS procedure of Sifaleras et al. [23] on a benchmark set of large instances (108 instances of 52 periods). The numerical experiment performed reveals that the original BTS procedure outperforms the VNS procedure in 92.59 % and the optimized version in 94.44 % of the total number of instances, respectively. The running time required to obtain the solution by both BTS procedures is about twenty times less on average and ten in the worst case. In addition, the optimized version outperforms the original BTS procedure in 64.81 % of the total number of instances in the benchmark set. We also note that both the original and the optimized BTS are able to reach an optimal solution for some of the instances.

In future works it would be interesting to see what the consequences are on the production plan of the optimization process on the remanufacturing plan presented here. Since the remanufacturing quantities may be changed during the optimization process, this can lead in a different optimal production plan [19]. We also note that the numerical experiment carried out in this paper is for a planning horizon of 52 periods which seems to be unrealistic in practice under deterministic assumptions. Therefore, it would be interesting to evaluate the optimized BTS procedure in a benchmark set of instances with smaller number of periods, in which the parameter values of the problem can be estimated more accurately. The cases of multi-item [29] or heterogeneous quality for the returns [30] are also interesting extensions for future works about the ELSR.

Abbreviations

BTS: Basic Tabu Search.; ELSP: Economic Lot-Sizing Problem.; ELSR: Economic Lot-Sizing Problem with Remanufacturing.; MILP: Mixed Integer Linear Programming.; RGVNS: Randomised General Variable Neighborhood Search.; VNS: Variable Neighborhood Search.

Competing Interests

The authors declare that they have no competing interests.

Authors' contributions

PP developed the concepts and carried out the numerical experiment. OV participated in the validation and the correction of the different versions of the paper. All authors read and approved the final manuscript.

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