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Intertemporal pricing strategies for fashion tech products with consumption externalities

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Abstract

With the rapid development of technology, more and more products tend to both meet customers' functional needs and provide stylish consumption experiences at the same time. We define them as "fashion tech" products. In practice there exist two opposite consumption externalities associated with "fashion tech" products. One, some customers are more likely to purchase the product if fewer customers can afford or have access to it to advertise their prosperity or good taste. In contrast, other customers' utility increases with the rising number of other customers. Thus the firm needs to consider such consumption externalities in their pricing decisions in order to appropriately position products and maximize profits. In such contexts, this paper optimizes intertemporal pricing strategies for fashion tech products selling to strategic customers with two kinds of externalities. We find that a markdown strategy is always optimal. In addition, it is appropriate for the firm to use slight markdowns when both the fraction of snobs and probability of stockout are small or use sharp markdowns otherwise.

Keywords: Fashion tech products, Consumption externalities, Intertemporal pricing strategies

Introduction

In recent years, a new class of products combining technology and fashion has emerged. Examples include 3D clothes design and smart watches for message notification and health monitoring. Some traditional luxury brands are actively seeking cooperation with electronics manufacturers. For instance, Dolce & Gabbana together with the audio brand Frends jointly launched "Beats by Dolce headphones", which was priced at $7,095 (Sharkey 2015). Calfskin backpacks using advanced material technology which can alter color based on user interaction or the environment they are placed in have entered the market (Arthur 2015). We define these products which combine both practicality and aesthetics as "fashion tech" products.

In general there are two opposite consumption externalities in the market of fashion tech products. At an early stage, such fashion tech products are often supplied in small quantities and priced at high levels. Hence the "Veblen effect" (Veblen 1953), the phenomenon whereby some customers buy expensive goods to demonstrate their wealth and social status, appears. Thus there exist negative consumption externalities which means that customers' utility decreases with the number of users. As time passes and users accumulate, typically when similar products have been launched, fashion tech firms focus more and more on improving the functionality of their product and user experience through technology innovation. All of these result in positive consumption externalities from the "network effect," whereby customers' utility increases with the number of users (Katz and Shapiro 1985).

Multi-period sales are common practices used by firms to maximize profits from potential customers with heterogeneous preferences. The early sales, typically from conspicuous consumption, play a role in capturing high margins and establishing the brand. The late sales, typically from positive consumption externalities, are targeted at collecting profits by enlarging the market. Thus for firms selling fashion tech products, it is very important to set an appropriate intertemporal pricing strategy and take advantages of both consumption externalities.

In this paper we focus on studying the optimal intertemporal pricing strategies of fashion tech products when there exist two consumption externalities. We mainly address the following research questions:

1. How do customers decide when to buy the product based on two opposite externalities?
2. What are the optimal intertemporal pricing strategies for firms when facing heterogeneous customers?
3. How do factors related to consumption externalities influence the firm's pricing strategies?

The remainder of this paper is organized as follows. "Literature review" section reviews the literature. "Model description" section introduces the model. "Model analysis" section analyzes three possible pricing strategies and compares the three in different cases. "Summary" section summarizes the paper and all proofs are provided in the Appendix.

Literature review

Pricing strategies for luxury goods as well as high-tech products have been studied extensively, while little research work has considered the dual attributes of such new "fashion tech" products together. Next we first review the papers concerning negative and positive consumption externalities respectively, then we review the papers with respect to commitment pricing strategies and strategic customers.

Veblen (1953) is the first to propose the concept of negative consumption externality, which describes how individuals consume highly conspicuous goods and services in order to show off their social status. Leibenstein (1950) emphasizes the significance of such social factors in consumption. Amaldoss and Jain (2005) introduce conspicuous consumption to the study of pricing decisions when facing deterministic price-dependent demands, and observe that snobs may create an upwards sloping demand curve only when there also exist followers. This models the utility of snobs as negative and the linear function of the user base size. Tereyagoglu and Veeraraghavan (2012) bridge the gap between the marketing and operational decisions of a firm when it sells to strategic customers engaged in conspicuous consumption by regarding the probability of stockout as scarcity in the newsvendor model. The interesting result is that firms may ensure a high availability of goods despite the presence of conspicuous consumption in equilibrium. Similar with Amaldoss and Jain (2005) and Tereyagoglu and Veeraraghavan (2012), we describe the customer utility with conspicuous consumption as a function of negative externalities. Different from Amaldoss and Jain (2005), we study the pricing strategies under customer choice. Different from Tereyagoglu and Veeraraghavan (2012) we focus on pricing strategies in two periods facing heterogeneous customers, excluding the production strategy.

As for positive consumption externalities, Katz and Shapiro (1985) observe the phenomenon whereby customer utility increases with the number of other customers consuming the same product. Positive externalities can be generated through a direct physical effect of the number of purchasers on the quality of the product, or the indirect effects from complementary products and post-purchase service. Ellison and Fudenberg (1999) model a customer's utility from a product with positive externalities as a function of the product's inherent value and of the number of customers using the product. When it comes to pricing strategies, Teng et al. (2010) build a two-stage dynamic game model to study the optimal prices for the monopolist producing products with positive externalities. They find that the stronger the positive externalities are, the more necessary for the firm to decrease the prices in the first period and raise the prices in the second period. Winter and Sundqvist 2010 demonstrates that although penetration pricing can be recommended for these kinds of products in some cases, it is not suitable all the time. Premium prices can be charged depending on the relative importance of intrinsic and extrinsic product values. Different from Teng et al. (2010) and Winter and Sundqvist (2010), we model the positive externalities in customer utility, instead of in total demand function. We take both positive and negative consumption externalities into account at the same time. Note that a relevant paper of Wang, Wang and Lai (2017), which deals with both pricing and production strategies, facing customers with both Veblen and network effects. However, different from their paper, excluding the firm's production strategy, we explicitly model customers as heterogeneous in their intrinsic valuations of the product.

Price commitment is one of the typical pricing schemes in marketing: When faced with strategic customers, firms commit to a price path in advance to deter the strategic waiting. Aviv and Pazgal (2008) find that when taking into account strategic consumer behavior, announced pricing policies can be advantageous to firms compared to contingent pricing schemes. Dasu and Tong (2010) study the posted pricing scheme and the contingent pricing scheme for a monopolist selling perishable products, and observe that the difference in expected revenues of these two schemes is small, and neither of them is dominant. Shum et al. (2016) compare dynamic pricing, price commitment and price matching used for products with cost uncertainty. They find that when the level of uncertainty is low, price commitment dominates the other two schemes. Since we are studying the intertemporal pricing policy for products selling to strategic customers with no uncertainty, this paper will focus on the commitment pricing strategy.

When an inter-temporal pricing strategy is introduced, it is reasonable to consider strategic customers, who rationally choose to buy at the period with highest utility. Su (2007) develops a model of dynamic pricing with endogenous intertemporal demand. At each point in time, customers may purchase the product at current prices or delay purchases and wait for sales. This paper demonstrates that heterogeneity in valuation and patience determines the structure of optimal prices and strategic customer behavior may actually benefit the seller. Su and Zhang (2008) study the impact of strategic customer behaviors on supply chain performance based on a newsvendor model facing forward-looking customers, and find that a seller's stocking level is lower than that in the classic model without strategic customers. We follow the utility function of Su and Zhang (2008), and differing from the above papers, we incorporate two externalities into the utility function: A negative externality may motivate strategic customers to purchase early while a positive externality may make them wait.

Model description

We make the following assumptions:

1. A product is produced by a monopolist firm.
2. The product is sold in two periods with prices p_1 and p_2. Without loss of generality, we assume the marginal cost $c = 0$.
3. We denote ϵ $(0 < \epsilon < 1)$ as the stockout probability and regard it as the market effects of the product. A large stockout probability means scarcity and a small stockout probability means popularity. The firm could also determine the stockout probability of the product by managing inventory. But to simplify the analysis, we only model the pricing decisions and do not model the firm's inventory decision.
4. Customers are classified into two categories. First are snobs, with fraction θ and sensitivity to stockout probability α $(\alpha > 0)$, who make conspicuous consumption purchases. These consumers typically have a higher utility for consuming a product when they expect other customers are unable to consume this product. Second are followers, with fraction $1 - \theta$ and sensitivity to stockout probability β $(\beta > 0)$. Followers typically have a higher utility as more people purchase the product.
5. Customers have intrinsic valuations for the product, v, which is independent of externality and $v \sim U[0, 1]$.
6. Customers decide whether to buy the product and when to buy it (in the first/second period) at the beginning of the first period when they are informed of the prices p_1 and p_2.
7. Customers' utility functions buying at different periods are defined in Table 1.

Note that the utility if the customer buys the product in the first period is the difference between the customers' valuations and the first-period price. The snobs and the followers are different in the extra term added to their intrinsic utility: $\alpha\epsilon$ and $-\beta\epsilon$. The expected utility in the second period is the difference between the intrinsic value and the second-period price, multiplied by the probability of non-stockout, $1 - \epsilon$. The profit function for the firm is:

Table 1 Utility function

	Snobs (θ)	Followers ($1 - \theta$)
first period	$U_1^s = v + \alpha\epsilon - p_1$	$U_1^f = v - \beta\epsilon - p_1$
second period	$U_2^s = (1 - \epsilon)(v - p_2)$	$U_2^f = (1 - \epsilon)(v - p_2)$

$$\pi = \left(q_1^s + q_1^f\right) p_1 + \left(q_2^s + q_2^f\right) p_2,$$

where q_i^j denotes the sale quantity in period i from customer category j, and can be calculated from the utility function above. We want to find optimal p_1 and p_2 to maximize the profit π.

Model analysis

We assume customers are maximizing their utility and thus they will buy in the period in which their utility is non-negative and is larger than that in another period as shown in Table 2.

In order to guarantee the feasibility of quantity q_i^j, we make additional assumptions below:

1. $\alpha + 3\beta < 1, 3\alpha + \beta < 1$;
2. $\epsilon > \max\{0, 2 + \frac{2\beta-1}{\alpha+\beta}\}$.

Three cases

Under different pricing strategies, we can analyze the sale volumes in each period and corresponding optimal prices.

Strategy 1 (Sharp Markdown: the price in the second period is much lower than the price in the first period)

The firm's optimization problem becomes:

$$\begin{aligned} \max_{p_1,p_2} \quad & \pi_1 = \left(q_1^s + q_1^f\right) p_1 + \left(q_2^s + q_2^f\right) p_2 \\ \text{s.t.} \quad & p_1 - p_2 \geq \alpha\epsilon \\ & p_1 \geq 0 \\ & p_2 \geq 0, \end{aligned}$$

where q_i^j is shown in Table 3. We denote $p_1^{S_1}, p_2^{S_1}$ as the optimal prices under Strategy 1 and derived the following result.

Lemma 1 *When* $\epsilon < \frac{4\alpha+2\beta-1}{\alpha}$,

$$\begin{cases} p_1^{S_1} = \frac{1+\alpha\epsilon}{2},\ p_2^{S_1} = \frac{1-\alpha\epsilon}{2} & 0 < \theta < \theta_1 \\ p_1^{S_1} = \frac{\alpha\epsilon\theta+\beta\epsilon\theta-\beta\epsilon+2}{4-\epsilon},\ p_2^{S_1} = \frac{-\alpha\epsilon\theta-\beta\epsilon\theta+\beta\epsilon-\epsilon+2}{4-\epsilon} & \theta_1 \leq \theta < 1 \end{cases}$$

where $\theta_1 = 1 + \frac{\alpha(2-\epsilon)-1}{2(\alpha+\beta)}$.

When $\epsilon \geq \frac{4\alpha+2\beta-1}{\alpha}$,

$$p_1^{S_1} = \frac{\alpha\epsilon\theta + \beta\epsilon\theta - \beta\epsilon + 2}{4-\epsilon},\ p_2^{S_1} = \frac{-\alpha\epsilon\theta - \beta\epsilon\theta + \beta\epsilon - \epsilon + 2}{4-\epsilon}.$$

Table 2 Customers' decisions

	Snobs (θ)	Followers ($1-\theta$)
first period	$v > \max\{p_1 - \alpha\epsilon, p_2 + \frac{p_1-p_2}{\epsilon} - \alpha\}$	$v > \max\{p_1 + \beta\epsilon, p_2 + \frac{p_1-p_2}{\epsilon} + \beta\}$
second period	$p_2 < v < p_2 + \frac{p_1-p_2}{\epsilon} - \alpha$	$p_2 < v < p_2 + \frac{p_1-p_2}{\epsilon} + \beta$

Table 3 $p_1 - p_2 \geq \alpha\epsilon$

	Snobs	Followers
first period	$q_1^s = \theta(1 - (p_2 + \frac{p_1 - p_2}{\epsilon} - \alpha))$	$q_1^f = (1-\theta)(1 - (p_2 + \frac{p_1 - p_2}{\epsilon} + \beta))$
second period	$q_2^s = \theta(\frac{p_1 - p_2}{\epsilon} - \alpha)$	$q_2^f = (1-\theta)(\frac{p_1 - p_2}{\epsilon} + \beta)$

Strategy 2 (minor pricing adjustment: the price in the second period may be higher or lower than the price in the first period, but their difference is small)

The firm's optimization problem becomes:

$$\begin{aligned}\max_{p_1, p_2} \quad & \pi_2 = \left(q_1^s + q_1^f\right) p_1 + \left(q_2^s + q_2^f\right) p_2 \\ \text{s.t.} \quad & -\beta\epsilon \leq p_1 - p_2 < \alpha\epsilon \\ & p_1 \geq 0 \\ & p_2 \geq 0,\end{aligned}$$

where q_i^j is shown in Table 4. We denote $p_1^{S_2}, p_2^{S_2}$ as the optimal prices under Strategy 2 and derived the following result.

Lemma 2 *When* $\epsilon < \frac{4\alpha + 2\beta - 1}{\alpha}$,

$$p_1^{S_2} = \frac{2\alpha\epsilon\theta + \beta\epsilon\theta - \beta\epsilon + 2}{4 - \epsilon + \theta\epsilon}, \; p_2^{S_2} = \frac{-\alpha\theta\epsilon^2 + 2\alpha\epsilon\theta + \beta\epsilon\theta + \beta\epsilon - \epsilon + 2}{4 - \epsilon + \theta\epsilon}.$$

When $\epsilon \geq \frac{4\alpha + 2\beta - 1}{\alpha}$,

$$p_1^{S_2} = \frac{1 + \alpha\epsilon}{2}, \; p_2^{S_2} = \frac{1 - \alpha\epsilon}{2}.$$

Strategy 3 (Sharp Markup: the price in the second period is much higher than the price in the first period)

The optimization problem becomes:

$$\begin{aligned}\max_{p_1, p_2} \quad & \pi_3 = \left(q_1^s + q_1^f\right) p_1 + \left(q_2^s + q_2^f\right) p_2 \\ \text{s.t.} \quad & p_1 - p_2 < -\beta\epsilon \\ & p_1 \geq 0 \\ & p_2 \geq 0,\end{aligned}$$

where q_i^j is shown in Table 5. We denote $p_1^{S_3}, p_2^{S_3}$ as the optimal prices under Strategy 3 and can derive the following result.

Lemma 3

$$p_1^{S_3} = \frac{\alpha\epsilon\theta + \beta\epsilon\theta - \beta\epsilon + 1}{2}, \; p_2^{S_3} > p_1^{S_3} + \beta\epsilon.$$

Table 4 $-\beta\epsilon \leq p_1 - p_2 < \alpha\epsilon$

	Snobs	Followers
first period	$q_1^s = \theta(1 - (p_1 - \alpha\epsilon))$	$q_1^f = (1-\theta)(1 - (p_2 + \frac{p_1 - p_2}{\epsilon} + \beta))$
second period	$q_2^s = 0$	$q_2^f = (1-\theta)(\frac{p_1 - p_2}{\epsilon} + \beta)$

Table 5 $p_1 - p_2 < -\beta\epsilon$

	Snobs	Followers
first period	$q_1^s = \theta(1 - (p_1 - \alpha\epsilon))$	$q_1^f = (1-\theta)(1 - (p_1 + \beta\epsilon))$
second period	$q_2^s = 0$	$q_2^f = 0$

Optimal strategy selection

By comparing the profits in three cases above, we can derive the optimal intertemporal pricing strategies as below.

Proposition 1 *When* $\epsilon < \frac{4\alpha+2\beta-1}{\alpha}$,

$$\begin{cases} p_1^* = \frac{2\alpha\epsilon\theta+\beta\epsilon\theta-\beta\epsilon+2}{4-\epsilon+\theta\epsilon},\ p_2^* = \frac{-\alpha\theta\epsilon^2+2\alpha\epsilon\theta+\beta\epsilon\theta+\beta\epsilon-\epsilon+2}{4-\epsilon+\theta\epsilon} & 0 < \theta < \theta^* \\ p_1^* = \frac{\alpha\epsilon\theta+\beta\epsilon\theta-\beta\epsilon+2}{4-\epsilon},\ p_2^* = \frac{-\alpha\epsilon\theta-\beta\epsilon\theta+\beta\epsilon-\epsilon+2}{4-\epsilon} & \theta^* \leq \theta < 1 \end{cases}$$

where $\theta^* = \frac{-\alpha\epsilon^2+(5\alpha+3\beta-1)\epsilon-4\alpha-4\beta+\sqrt{(\alpha^2\epsilon^3-2\alpha(3\alpha+\beta-1)\epsilon^2+(3\alpha+\beta-1)^2\epsilon-4(\alpha+\beta)^2)(\epsilon-4)}}{2\epsilon(\alpha+\beta)}$.

When $\epsilon \geq \frac{4\alpha+2\beta-1}{\alpha}$,

$$p_1^* = \frac{\alpha\epsilon\theta+\beta\epsilon\theta-\beta\epsilon+2}{4-\epsilon},\ p_2^* = \frac{-\alpha\epsilon\theta-\beta\epsilon\theta+\beta\epsilon-\epsilon+2}{4-\epsilon}.$$

Proposition 1 can be interpreted by Fig. 1. When the probability of stockout and the fraction of snobs are both small, Strategy 2 is adopted which implies that the gap between prices in the two periods is small. Take Apple Watch as an example. Series 2 was introduced in September, 2016 and the price started at $369. One year later, when Series 3 was launched, the price of Series 2 had dropped to around $299. We can observe that the change in price is relatively small, partially because the firm wants to cultivate relationships with both snobs and followers, and especially to expand the market over the long run with followers. This confirms our finding that Strategy 2 is more likely to be used when the stockout level is low and most of the potential customers are followers. In contrast, when either the probability of stockout or the fraction of snobs is large, Strategy 1 is adopted, which means the price in the first period is much higher than that in the second

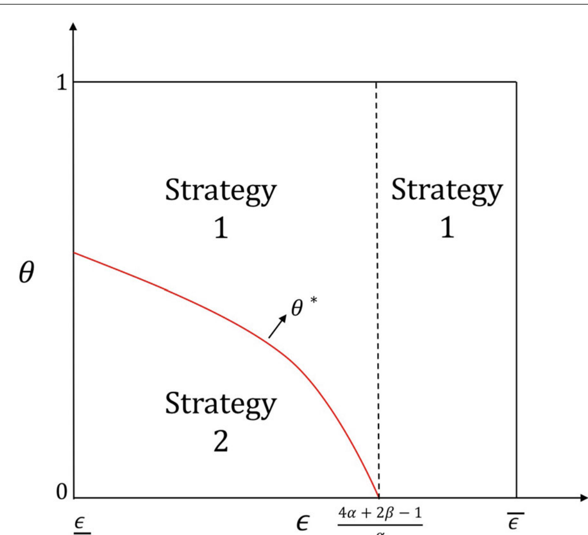

Fig. 1 Optimal strategy selection

period. A close example is "Beats by Dolce headphones" launched by Dolce & Gabbana. Those headphones are made of leather and jewelry, mainly targeting at snobs. Their initial price is as high as $7,095, about 35 times higher than the price of a common style of the headphone brand.

Remark 1 *It is always optimal to adopt a markdown strategy* $\left(p_1^* > p_2^*\right)$.

Proposition 1 indicates that Strategy 3 is never optimal. In addition, no matter whether Strategy 1 or Strategy 2 is adopted, the price in the first period is always higher than that in the second period. Thus we get Remark 1. This result implies that the pricing strategy for fashion tech products is more likely to cater to and profit from snobs rather than followers.

Remark 2 *When Strategy 1 is adopted,*

$$\frac{\partial p_1^*}{\partial \alpha} > 0,\ \frac{\partial p_1^*}{\partial \beta} < 0,\ \frac{\partial p_2^*}{\partial \alpha} < 0,\ \frac{\partial p_2^*}{\partial \beta} > 0.$$

When Strategy 2 is adopted,

$$\frac{\partial p_1^*}{\partial \alpha} > 0,\ \frac{\partial p_1^*}{\partial \beta} < 0,\ \frac{\partial p_2^*}{\partial \alpha} > 0,\ \frac{\partial p_2^*}{\partial \beta} > 0.$$

No matter whether under Strategy 1 or under Strategy 2, the optimal price in the first period increases as α increases or β decreases and the optimal price in the second period increases as β increases. These results are intuitive because when snobs are more positively sensitive to the probability of stockout (more conspicuous), the firm is able to charge a higher price in the first period. Similarly, when followers are more negatively sensitive to the probability of stockout (more conservative), the firm should set a lower price in the first period and a higher price in the second period. An interesting result is that under Strategy 1 the second-period price decreases in α while under Strategy 2 the second-period price increases in α. This can be explained as follows. When Strategy 2 is adopted, the stock-out probability and the fraction of snobs are typically small. The increase of α induces the firm to increase the optimal price in the first period to profit more from snobs. In addition, the firm cares a lot about the second-period profit from followers. So it is optimal for the firm to increase the second-period price simultaneously but at a relatively smaller pace in order to maintain an appropriate price gap under the condition of Strategy 2.

Numerical experiments

In this subsection, we numerically compare the profits under different strategies, and investigate the parameter impact on the switching between different optimal strategies.

Figure 2 compares the profits of three strategies under different levels of stockout probability. We set $\alpha = \frac{1}{5}$ and $\beta = \frac{1}{6}$ to guarantee that the feasible region for ϵ is $[\frac{2}{11}, 1]$. As shown in Fig. 2(a) (when $\epsilon = 0.8$, $\epsilon > \frac{4\alpha+2\beta-1}{\alpha}$), Strategy 1 is optimal with the highest profit. As shown in Fig. 2(b) (when $\epsilon = 0.2$, $\epsilon < \frac{4\alpha+2\beta-1}{\alpha}$), the optimal strategy will depend on the value of θ. Since $\theta^* = 0.24$, the profit under Strategy 2 is slightly higher than that of Strategy 1 when $\theta < \theta^*$, otherwise the profit under Strategy 1 is the highest. Figure 2 gives a typical decision setting for the firm producing fashion tech products with

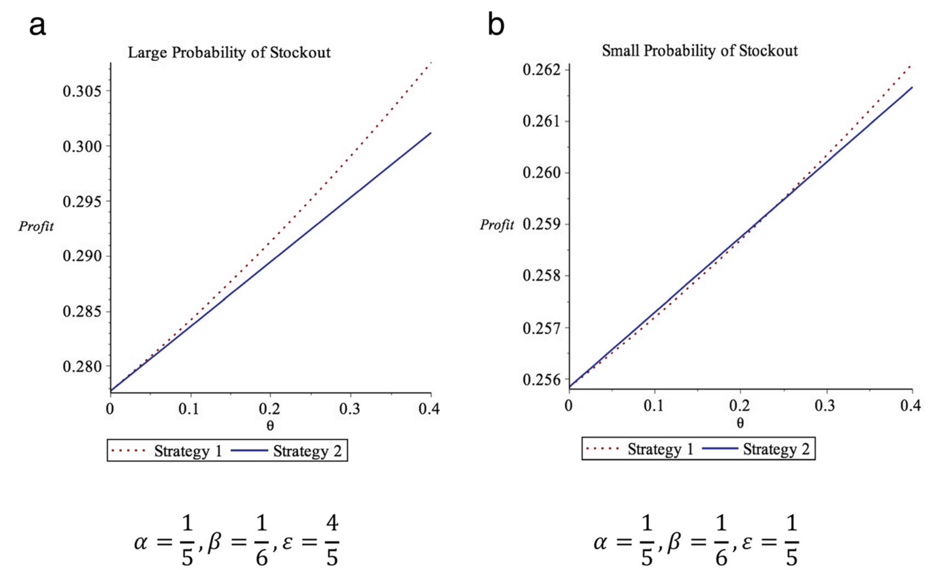

Fig. 2 Strategy selection with different levels of stockout

a given customer market (when α, β and θ are exogenously given). Strategy 1 is the most appropriate policy for high stockout levels, or low stockout levels along with a large fraction of snobs, while Strategy 2 is the most suitable when there is enough supply (low stockout probability) and many followers.

Figure 3 shows the relationship between the fraction threshold θ^* and parameters ϵ, α, β. Since θ^* decreases in ϵ (Fig. 3(a)) and increases in β (Fig. 3(c)), we find that the firm is more likely to use Strategy 1 when the probability of stockout is high and followers' sensitivity to stockout is weak. This is intuitive because when products are scarce and there are weak positive externalities, the firm should focus on profiting from snobs by raising the price in the first period. Counter-intuitively, θ^* increases in α (Fig. 3(b)); the firm is more likely to use Strategy 2 when snobs are more sensitive to stockout. The reason is as follows. Recall the definition for Strategy 2: $p_1 - p_2 < \alpha\epsilon$. As α increases, although $p_1 - p_2$ may increase, $\alpha\epsilon$ increases faster than the increase of that price gap. Hence the condition for Strategy 2 is still satisfied.

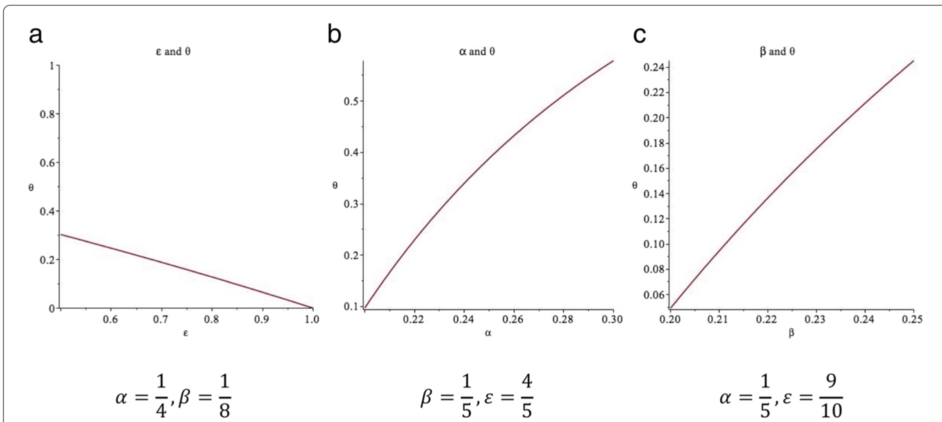

Fig. 3 Relationship between θ^* and ϵ, α, β

Summary

This paper focuses on the intertemporal pricing strategies for fashion tech products with two opposite consumption externalities. Based on a heterogeneous customer model, we derive the optimal pricing strategies for different supply policies and market structures. We find that it is always optimal to set the price in the first period higher than that in the second period. In addition, when the availability of products is high and the fraction of followers is large (in other words, stockout probability and the fraction of snobs are small), it is optimal to reduce the price slightly, otherwise a sharp markdown strategy is better. When customers' sensitivity to negative consumption externalities is stronger, the firm is more likely to set the price at a high level in the first period, but may not necessarily cut the price in the second period. Meanwhile, as the sensitivity to positive consumption externalities becomes stronger, the optimal price in the first period decreases and that in the second period increases.

There are several remaining research questions. First, we can extend our work to the strategies for more periods or with multiple objectives as the firm may consider the reputation or market share of the product over the long term. Second, we can combine pricing strategies with production strategies to take quantity constraints into account and make simultaneous decisions. Third, our work can be extended to include empirical studies identifying the relationship between the extent of consumption externalities and the intrinsic attributes of products.

Appendix

Proof of Lemma 1

Consider the QP problem

$$\begin{aligned}\min_{p_1,p_2}\quad & f_1 = -\pi_1 = -\left(q_1^s + q_1^f\right)p_1 - \left(q_2^s + q_2^f\right)p_2\\ \text{s.t.}\quad & -p_1 + p_2 \le -\alpha\epsilon\\ & -p_1 \le 0\\ & -p_2 \le 0.\end{aligned}$$

The Hessian matrix of f_1 is: $\begin{bmatrix} \frac{2}{\epsilon} & \frac{-2+\epsilon}{\epsilon} \\ \frac{-2+\epsilon}{\epsilon} & \frac{2}{\epsilon} \end{bmatrix} \succ 0$. Thus this is a (strict) convex optimization.

$$\begin{cases} \frac{\partial f_1}{\partial p_1} = 0 \\ \frac{\partial f_1}{\partial p_2} = 0 \end{cases} \Rightarrow \begin{cases} p_1^{S_1} = \frac{\alpha\epsilon\theta+\beta\epsilon\theta-\beta\epsilon+2}{4-\epsilon} \\ p_2^{S_1} = \frac{-\alpha\epsilon\theta-\beta\epsilon\theta+\beta\epsilon-\epsilon+2}{4-\epsilon} \end{cases}.$$

Then check the domain of p_1 and p_2.

$$p_1^{S_1} - p_2^{S_1} - \alpha\epsilon \ge 0 \quad \Rightarrow \quad \theta \ge 1 + \frac{\alpha(2-\epsilon)-1}{2(\alpha+\beta)}$$

When $\theta < 1 + \frac{\alpha(2-\epsilon)-1}{2(\alpha+\beta)}$, take the boundary solution $\begin{cases} p_1^{S_1} = \frac{1+\alpha\epsilon}{2} \\ p_2^{S_1} = \frac{1-\alpha\epsilon}{2} \end{cases}$.

Note that with either the global optimum or boundary solution, we both have $p_1^{S_1} > 0$ and $p_2^{S_1} > 0$ since $0 < \alpha < 1, 0 < \beta < 1, 0 < \epsilon < 1, 0 < \theta < 1$.

Denote $1 + \frac{\alpha(2-\epsilon)-1}{2(\alpha+\beta)}$ as θ_1. Then we need to compare θ_1 with 0 and 1. It turns out that $\theta_1 < 1$ (use $\alpha < \frac{1}{3}$). When $\epsilon < \frac{4\alpha+2\beta-1}{\alpha}$, $0 < \theta_1 < 1$, and when $\epsilon \ge \frac{4\alpha+2\beta-1}{\alpha}$, $\theta_1 \le 0$.

Thus we can sort out the optimal price level for Strategy 1 as Lemma 1.

The last step is to check the feasibility of q_i^j, which is $\begin{cases} 0 \le 1-(p_2+\frac{p_1-p_2}{\epsilon}-\alpha) \le 1 \\ 0 \le \frac{p_1-p_2}{\epsilon}-\alpha \le 1 \\ 0 \le 1-(p_2+\frac{p_1-p_2}{\epsilon}+\beta) \le 1 \\ 0 \le \frac{p_1-p_2}{\epsilon}+\beta \le 1 \end{cases}$, and can be simplified as $p_2+\frac{p_1-p_2}{\epsilon}+\beta \le 1$. This implies $\theta \le \frac{2\beta-1}{(\alpha+\beta)(\epsilon-2)}$. Because $\epsilon > \max\{0, 2+\frac{2\beta-1}{\alpha+\beta}\}$, $\frac{2\beta-1}{(\alpha+\beta)(\epsilon-2)} > 1$, hence $p_2+\frac{p_1-p_2}{\epsilon}+\beta \le 1$ always holds for $0<\theta<1$. □

Proof of Lemma 2

Consider the QP problem

$$\begin{aligned} \min_{p_1,p_2} \quad & f_2 = -\pi_2 = -\left(q_1^s+q_1^f\right)p_1 - \left(q_2^s+q_2^f\right)p_2 \\ \text{s.t.} \quad & -\alpha\epsilon < -p_1+p_2 \le \beta\epsilon \\ & -p_1 \le 0 \\ & -p_2 \le 0. \end{aligned}$$

The Hessian matrix of f_2 is: $\begin{bmatrix} 2\theta+\frac{2(1-\theta)}{\epsilon} & \frac{(-2+\epsilon)(1-\theta)}{\epsilon} \\ \frac{(-2+\epsilon)(1-\theta)}{\epsilon} & \frac{2(1-\theta)}{\epsilon} \end{bmatrix} \succ 0$. Thus this is a (strict) convex optimization.

$$\begin{cases} \frac{\partial f_2}{\partial p_1}=0 \\ \frac{\partial f_2}{\partial p_2}=0 \end{cases} \Rightarrow \begin{cases} p_1^{S_2} = \frac{2\alpha\epsilon\theta+\beta\epsilon\theta-\beta\epsilon+2}{4-\epsilon+\theta\epsilon} \\ p_2^{S_2} = \frac{-\alpha\theta\epsilon^2+2\alpha\epsilon\theta+\beta\epsilon\theta+\beta\epsilon-\epsilon+2}{4-\epsilon+\theta\epsilon} \end{cases}.$$

Then check the domain of p_1 and p_2.

$p_1^{S_2}-p_2^{S_2} \ge -\beta\epsilon$ always holds, and $p_1^{S_2}-p_2^{S_2}-\alpha\epsilon < 0$ yields $\epsilon < \frac{4\alpha+2\beta-1}{\alpha}$.

When $\epsilon \ge \frac{4\alpha+2\beta-1}{\alpha}$, take the boundary solution $\begin{cases} p_1^{S_2} = \frac{1+\alpha\epsilon}{2} \\ p_2^{S_2} = \frac{1-\alpha\epsilon}{2} \end{cases}$.

Note that with either the global optimum or boundary solution, we both have $p_1^{S_2} > 0$ and $p_2^{S_2} > 0$ since $0<\alpha<1, 0<\beta<1, 0<\epsilon<1, 0<\theta<1$.

Then check the feasibility of q_i^j, which is $\begin{cases} 0 \le 1-(p_1-\alpha\epsilon) \le 1 \\ 0 \le 1-(p_2+\frac{p_1-p_2}{\epsilon}+\beta) \le 1 \\ 0 \le \frac{p_1-p_2}{\epsilon}+\beta \le 1 \end{cases}$, and can be simplified as $\begin{cases} \alpha\epsilon \le p_1 \le \alpha\epsilon+1 \\ p_2+\alpha+\beta \le 1 \end{cases}$.

$p_1-\alpha\epsilon = \frac{2+(\alpha-\alpha\theta)\epsilon^2+(2\alpha\theta+\beta\theta-4\alpha-\beta)\epsilon}{\theta\epsilon-\epsilon+4} > \frac{(\alpha-\alpha\theta)\epsilon^2+(2\alpha\theta+\beta\theta)\epsilon}{\theta\epsilon-\epsilon+4}$ (using $3\,\alpha+\beta<1$) >0.

$p_1-\alpha\epsilon-1 = \frac{-\alpha\theta\epsilon^2+2\alpha\theta\epsilon+\alpha\epsilon^2+\beta\theta\epsilon-4\alpha\epsilon-\beta\epsilon-\theta\epsilon+\epsilon-2}{\theta\epsilon-\epsilon+4} < \frac{-\alpha\theta\epsilon^2-\alpha\epsilon-\theta\epsilon-1}{\theta\epsilon-\epsilon+4} < 0$.

$p_2+\alpha+\beta-1 = \frac{-\alpha\theta\epsilon^2+((2\beta+3\alpha-1)\theta-\alpha)\epsilon+4\alpha+4\beta-2}{\theta\epsilon-\epsilon+4} < \frac{-\alpha\theta\epsilon^2+(\alpha\theta-\alpha)\epsilon}{\theta\epsilon-\epsilon+4}$ (using $\alpha+\beta<\frac{1}{2}$) <0. □

Proof of Lemma 3

In this case, $q_2^s = q_2^f = 0$, and the QP becomes

$$\begin{aligned}
\min_{p_1,p_2} \quad & f_3 = -\pi_3 = -\left(q_1^s + q_1^f\right) p_1 \\
\text{s.t.} \quad & p_1 - p_2 < -\beta\epsilon \\
& -p_1 \le 0 \\
& -p_2 \le 0.
\end{aligned}$$

which is only related with p_1.

$$\frac{\mathrm{d}f_3}{\mathrm{d}p_1} = 0 \quad \Rightarrow \quad p_1^{S_3} = \frac{\alpha\epsilon\theta + \beta\epsilon\theta - \beta\epsilon + 1}{2}.$$

Obviously $p_1^{S_3} > 0$ because $0 < \alpha < 1$, $0 < \beta < 1$, $0 < \epsilon < 1$, $0 < \theta < 1$, and $p_2^{S_3} > p_1 + \beta\epsilon > 0$.

Then check the feasibility of q_i^j, which is $\begin{cases} 0 \le 1 - (p_1 - \alpha\epsilon) \le 1 \\ 0 \le 1 - (p_1 + \beta\epsilon) \le 1 \end{cases}$, and can be simplified as $\alpha\epsilon \le p_1 \le 1 - \beta\epsilon$.

$p_1 - \alpha\epsilon = \frac{1}{2}(1 + ((\theta - 1)(\alpha + \beta) - \alpha)\epsilon) > \frac{1}{2}\left(1 - (\frac{1-\theta}{2} + \alpha)\epsilon\right)$ $\left(\text{using } \alpha + \beta < \frac{1}{2}\right) > 0$.

$p_1 + \beta\epsilon - 1 = \frac{1}{2}(-1 + (\theta(\alpha + \beta) + \beta)\epsilon) < \frac{1}{2}\left(-1 + \left(\frac{\theta}{2} + \beta\right)\epsilon\right)$ $\left(\text{using } \alpha + \beta < \frac{1}{2}\right) < 0$. □

Proof of Proposition 1

When $\epsilon \ge \frac{4\alpha+2\beta-1}{\alpha}$, the optimal solution of Strategy 2 is the boundary solution of Strategy 1, thus $\pi_1 \ge \pi_2$ and we need to compare π_1 and π_3.

$\pi_1 - \pi_3 = \frac{\epsilon((\epsilon-2)(\alpha+\beta)\theta+(2-\epsilon)\beta+1)^2}{16-4\epsilon} \ge 0$. So when $\epsilon \ge \frac{4\alpha+2\beta-1}{\alpha}$, Strategy 1 is the best.

When $\epsilon < \frac{4\alpha+2\beta-1}{\alpha}$ and $0 < \theta < \theta_1$, the optimal solution of Strategy 1 is the boundary solution of Strategy 2, thus $\pi_2 \ge \pi_1$ and we need to compare π_2 and π_3.

$\pi_2 - \pi_3 = \frac{(((\alpha+\beta)\theta-\beta)\epsilon+2\beta+1)^2(1-\theta)\epsilon}{16+(4\theta-4)\epsilon} \ge 0$. So when $\epsilon < \frac{4\alpha+2\beta-1}{\alpha}$ and $0 < \theta < \theta_1$, Strategy 2 is the best.

When $\epsilon < \frac{4\alpha+2\beta-1}{\alpha}$ and $\theta_1 \le \theta < 1$, from discussion above, we know $\pi_1 \ge \pi_3$ and $\pi_2 \ge \pi_3$, so we need to compare π_2 and π_3.

Denote $\Delta\pi = \pi_1 - \pi_2$. $\Delta\pi = \frac{\epsilon\theta}{(4-\epsilon)(4-\epsilon+\epsilon\theta)}(A\theta^2 + B\theta + C)$,

where $\begin{cases} A = \epsilon(\alpha + \beta)^2 \\ B = (\alpha + \beta)(\alpha\epsilon^2 - 5\alpha\epsilon - 3\beta\epsilon + 4\alpha + 4\beta + \epsilon) \\ C = (\beta\epsilon - 2\beta - 1)(-\alpha\epsilon + 4\alpha + 2\beta - 1) \end{cases}$.

$\frac{\epsilon\theta}{(4-\epsilon)(4-\epsilon+\epsilon\theta)} > 0$ so we need to focus on $A\theta^2 + B\theta + C$. Denote it as l.

Note that $-\frac{B}{2A} < 0$, so $l(\theta)$ has at most one zero point when $\theta > 0$.

$l(\theta = 1) = (\alpha\epsilon - 2\alpha + 1)^2 > 0$ and $l(\theta = \theta_1) = -\frac{1}{4}(\epsilon - 4)(\alpha\epsilon - 2\alpha + 1)(\alpha\epsilon - 4\alpha - 2\beta + 1) < 0$.

Hence there exists $\theta_1 < \theta^* < 1$ such that $l(\theta = \theta^*) = 0$. When $\epsilon < \frac{4\alpha+2\beta-1}{\alpha}$ and $\theta_1 \le \theta < \theta^*$, Strategy 2 is the best, and when $\epsilon < \frac{4\alpha+2\beta-1}{\alpha}$ and $\theta^* \le \theta < 1$, Strategy 1 is the best. Solving the equation, we can derive:

$\theta^* = \frac{-\alpha\epsilon^2+(5\alpha+3\beta-1)\epsilon-4\alpha-4\beta+\sqrt{(\alpha^2\epsilon^3-2\alpha(3\alpha+\beta-1)\epsilon^2+(3\alpha+\beta-1)^2\epsilon-4(\alpha+\beta)^2)(\epsilon-4)}}{2\epsilon(\alpha+\beta)}$.

By organizing results above, we can thus prove Proposition 1. □

Proof of Remark 1

From Proposition 1, we can summarize that the optimal pricing strategy is either Strategy 1 or Strategy 2.

When Strategy 1 is adopted, $p_1^* - p_2^* \geq \alpha\epsilon > 0$.

When Strategy 2 is adopted, $p_1^* - p_2^* = \frac{\alpha\theta\epsilon^2 + (1-2\beta)\epsilon}{4+\theta\epsilon-\epsilon} > 0$. □

Proof of Remark 2

When Strategy 1 is adopted,

$\frac{\partial p_1^*}{\partial \alpha} = \frac{\theta\epsilon}{4-\epsilon} > 0$, $\frac{\partial p_1^*}{\partial \beta} = \frac{(\theta-1)\epsilon}{4-\epsilon} < 0$,

$\frac{\partial p_2^*}{\partial \alpha} = \frac{\theta\epsilon}{-4+\epsilon} < 0$, $\frac{\partial p_2^*}{\partial \beta} = \frac{(\theta-1)\epsilon}{-4+\epsilon} > 0$.

When Strategy 2 is adopted,

$\frac{\partial p_1^*}{\partial \alpha} = \frac{2\theta\epsilon}{4+\theta\epsilon-\epsilon} > 0$, $\frac{\partial p_1^*}{\partial \beta} = \frac{(\theta-1)\epsilon}{4+\theta\epsilon-\epsilon} < 0$,

$\frac{\partial p_2^*}{\partial \alpha} = \frac{(2\epsilon-\epsilon^2)\theta}{4+\theta\epsilon-\epsilon} > 0$, $\frac{\partial p_2^*}{\partial \beta} = \frac{(\theta+1)\epsilon}{4+\theta\epsilon-\epsilon} > 0$.

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Authors' contributions

TW carried out model setup, analyzing and drafted the manuscript. XW revised the manuscript and provided guidance on the model analysis. Both authors read and approved the final manuscript.

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Competing interests

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