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# Measuring density and Young's modulus of a log through the vibration test without measuring its weight

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## Abstract

This study examines a simple estimation method for the measurement of the mass of an on-site log through a vibration test. In this method, rather than the log itself, a cut end portion is weighed. For this purpose, the vibration method with additional mass (VAM) was applied to Sitka spruce (*Picea sitchensis* Carr.) circular truncated cones (model log) and Japanese cedar (*Cryptomeria japonica* D. Don) logs. Longitudinal vibration tests were performed on the circular truncated cones with/without an additional mass. Furthermore, the cut end portions of the circular truncated cones and logs were used as the virtual additional mass in the VAM. From the results of the vibration test using specimens with/without the concentrated mass, it is possible to estimate the mass of a circular truncated cone with 10% error by the VAM. The cut end portion of a circular truncated cone could be used as the virtual mass in the VAM. From the experimental and theoretical results, to maintain high estimation accuracy, the specimen length must not be too short as shown in our previous study for a specimen with constant cross-sectional shape. The cut end portion of the logs could be used for the virtual mass of the VAM.

**Keywords:** Circular truncated cone, End cutting, Log, Vibration method with additional mass, Weighing

## Introduction

When logs are bought in log markets, stable qualities of the logs, in other words, appropriate classifying based on log shapes such as diameter, length and warp is required. Since timber quality is affected by its Young's modulus, classifying logs based on the Young's modulus enables to enrich lineups of log markets. Evaluating the Young's modulus of a log before sawmilling allows it to be sold at a high value. However, the Young's modulus of logs cannot be estimated from the evaluation based on the shape.

In classifying lines at log markets and large-scale sawmilling factories, the shape of logs on a conveyor is measured and the logs are consequently classified. Although there is a classifying process having a scale and a tapping system to measure the mass and Young's modulus

of the logs, the process reduces the production efficiency. Because a log is large and heavy, handling it is difficult, and determining the output of a scale is time consuming. Hence, a method to measure the mass of a log in short time is desirable.

Because logs have an extra length of about 100 mm with respect to the nominal length, use of the extra length portion was examined in this study. It is possible that the density of the cut extra length portion is regarded as the density of a whole log. However, the measurement of the accurate volume of the cut end portion (the extra length portion) will be difficult because the shape of the butt end of a log is irregular due to swelled butt, notches and backcuts in felling. On the other hand, an inexpensive scale with high accuracy that is easy and safe to handle can be used for the cut end portion that is much smaller than the main body of a log. Weighing the cut end portion does not cause a serious reduction of the production efficiency.

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A method for measuring the mass, density, and Young's modulus of a specimen without needing to weigh the specimen was developed based on a frequency equation that incorporates the effect of an additional mass attached to a wooden bar [1–6]. This method is referred to as the vibration method with additional mass (VAM) in this study. This method utilizes decreases in the resonance frequency by attaching additional mass. The ratio of the resonance frequency with the additional mass to that without it is used in the frequency equation incorporating the effect of the concentrated mass attached to a specimen and its position. The mass ratio (concentrated mass/specimen) is subsequently calculated using the frequency equation.

To analyze the potential application of the VAM for practical purposes, a series of parameters were investigated, including the suitable mass ratio (additional mass/specimen) [7], the connection between the additional mass and specimen [7], the crossers' position for the piled lumber [8], the specimen moisture content [9], and the bending vibration generation method [10]. The VAM could perhaps be used to assess the deterioration of the cross beams of timber guardrails [11] and the mass of the piled lumber [12]. The estimation accuracy of the VAM decreased with an increase in the effect of shear and rotatory inertia, and it could be corrected [13].

Our previous study [14] showed that a cut end portion could be used as the virtual mass in the VAM. In this study, we investigate whether the use of a cut end portion in the VAM can be applied to a log. First, the VAM was applied to the circular truncated cone used as the model of a log. Next, a cut end portion was used for the virtual additional mass in the VAM using a circular truncated cone and log.

**Theory**

**Longitudinal vibration for a circular truncated cone with a concentrated mass**

A circular truncated cone of radii of  $r_1$  and  $r_2$  ( $r_1 \geq r_2$ ,  $k$  is the ratio of  $r_2$  to  $r_1$ ,  $k=r_2/r_1$ ) and length  $l$  with concentrated mass  $M$  placed at  $(x_1, x_2)=(al, 0)$  shown in Fig. 1 is considered. Herein, longitudinal vibration under the free-free condition is considered.

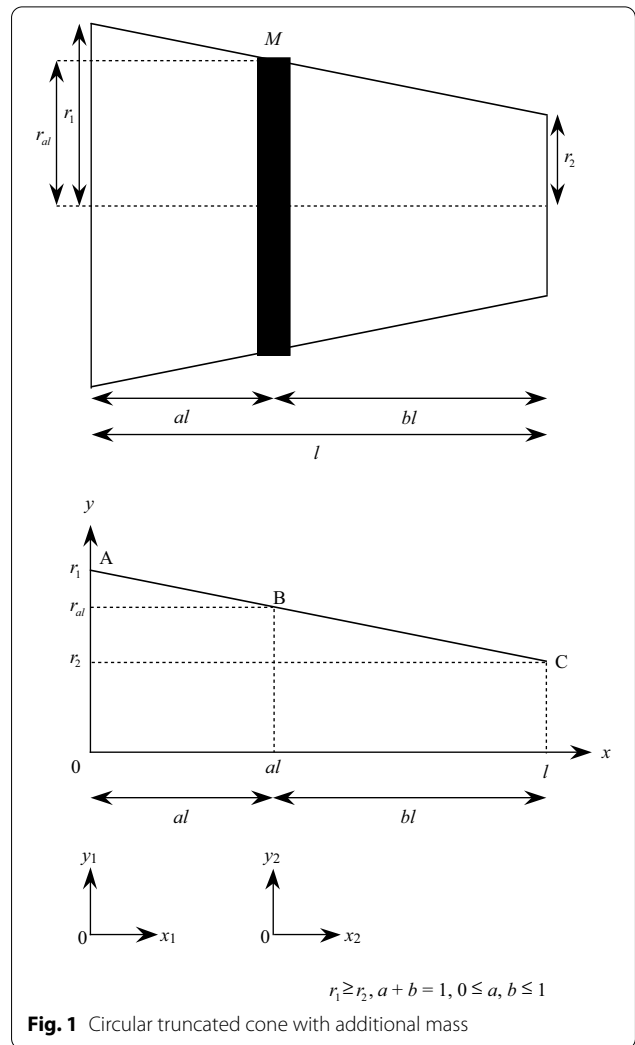
A circular truncated cone is prepared by rotating segments AB and BC about the  $x$  axis as shown in Fig. 1. The equations for the segments AB and BC are:

Segment AB:

$$y_1 = p_1x_1 + q_1(0 \leq x_1 \leq al). \tag{1}$$

$$p_1 = \frac{(k-1)r_1}{l}. \tag{2}$$

$$q_1 = r_1. \tag{3}$$



**Fig. 1** Circular truncated cone with additional mass

Segment BC:

$$y_2 = p_2x_2 + q_2(0 \leq x_2 \leq bl). \tag{4}$$

$$p_2 = p_1. \tag{5}$$

$$q_2 = r_1\{(k-1)a + 1\}. \tag{6}$$

The longitudinal displacements  $u_1(x_1, t)$  and  $u_2(x_2, t)$  are represented as

$$u_1(x_1, t) = v_1(x_1)\sin\omega_n t. \tag{7}$$

$$v_1(x_1) = \frac{C_{11}}{p_1x_1 + q_1} \sin\left(\sqrt{\frac{\rho}{E}}\omega_n x_1 + C_{21}\right), \tag{8}$$

$$u_2(x_2, t) = v_2(x_2)\sin\omega_n t, \tag{9}$$

$$v_2(x_2) = \frac{C_{12}}{p_2x_2 + q_2} \sin\left(\sqrt{\frac{\rho}{E}}\omega_n x_2 + C_{22}\right), \quad (10)$$

where  $C_{11}$ ,  $C_{21}$ ,  $C_{12}$ , and  $C_{22}$  are constants,  $t$ ,  $\rho$ ,  $E$ , and  $\omega$  (equal to  $2\pi f$ , where  $f$  is the resonance frequency) are time, density, Young’s modulus, and angular frequency, and the subscript  $n$  is the mode number.

Because the axial force does not exist at each extremity under a free-free condition:

$$\begin{cases} x_1 = 0 : \frac{\partial u_1}{\partial x_1} = 0 \\ x_2 = bl : \frac{\partial u_2}{\partial x_2} = 0 \end{cases} \quad (11)$$

Because both parts of the bar are connected and the difference in the axial force in each bar is equal to the inertia force exerted by the concentrated mass at  $(x_1, x_2) = (al, 0)$

$$\begin{cases} u_1 = u_2 \\ ES(x_2) \frac{\partial u_2}{\partial x_2} - ES(x_1) \frac{\partial u_1}{\partial x_1} = M \frac{\partial^2 u_1}{\partial t^2} = M \frac{\partial^2 u_2}{\partial t^2} \end{cases} \quad (12)$$

where  $S$  is cross-sectional area.

From Eqs. (7)–(12)

$$\left\{ (k-1)^2 + km_n^2 \right\} \tan m_n - m_n (k-1)^2 + \frac{1}{3} \mu m_n \frac{k^2 + k + 1}{\{(k-1)a + 1\}^2} \frac{\{m_n + (k-1)\tan am_n\} \{km_n + (k-1)\tan bm_n\}}{1 - \tan am_n \tan bm_n} = 0 \quad (13)$$

where  $\mu$  is the ratio of the concentrated mass to the mass of the circular truncated cone and is defined as

$$\mu = \frac{M}{\frac{1}{3}\pi(r_1^2 + r_1r_2 + r_2^2)l\rho} = \frac{3M}{\pi(k^2 + k + 1)r_1^2l\rho} \quad (14)$$

Equation (13) is the frequency equation for a circular truncated cone with a concentrated mass.

i. If  $\mu = 0$ , then Eq. (13) is as follows [15]:

$$\left\{ (k-1)^2 + km_n^2 \right\} \tan m_n - m_n (k-1)^2 = 0 \quad (15)$$

ii. If  $k = 1$ , then Eq. (13) is as follows [3]:

$$\sin m_n + \mu m_n \cos am_n \cos bm_n = 0. \quad (16)$$

### Longitudinal vibration testing method without weighing the specimen

The resonance frequency, represented by  $f_{n0}$  (0: value without the additional mass), is expressed as follows:

$$f_{n0} = \frac{m_{n0}}{2\pi l} \sqrt{\frac{E}{\rho}} \quad (17)$$

where  $m_{n0}$  is a constant depending on  $k$  for a circular truncated cone from Eq. (15).

The resonance frequency is decreased experimentally by attaching the additional mass, whereas, the dimensions, density, and Young’s modulus are not altered. Hence, it can be said that  $m_{n0}$  changes to  $m_n$ . The resonance frequency after attaching the additional mass is expressed as follows:

$$f_n = \frac{m_n}{2\pi l} \sqrt{\frac{E}{\rho}} \quad (18)$$

From Eqs. (17) and (18)

$$m_n = \frac{f_n}{f_{n0}} m_{n0} \quad (19)$$

The measured resonance frequencies  $f_{n0}$  and  $f_n$  are substituted in Eq. (19) to calculate  $m_n$ , and the calculated  $m_n$

is substituted in Eq. (13) to calculate  $\mu$ . A specimen mass and density can be obtained by substituting the calculated  $\mu$ , the concentrated mass, and the dimensions of the bar in Eq. (14). The Young’s modulus can be calculated by substituting the estimated density, resonance frequency without the concentrated mass, and dimensions of the bar in Eq. (17) [1–6].

The above steps represent the calculation procedure for the VAM. The mass of the specimen is not required for the calculations.

### Materials and methods

#### Specimens

Air-dried, 1000-mm length, circular truncated cones (larger diameter, smaller diameter) = (100 mm, 90 mm), (100 mm, 80 mm), (100 mm, 70 mm), (100 mm, 60 mm), (100 mm, 50 mm), (100 mm, 40 mm), and (100 mm,

30 mm) were produced from Sitka spruce (*Picea sitchensis* Carr.) with a cross section of 110 mm square and a length of 1500 mm without a pith. The circular truncated cone is a model of a log and differs from an actual log in that the circular truncated cone has the slope grain. The larger and smaller diameter ends are referred as to butt end and top end, respectively, in this study. Two specimens were made for each dimension. The specimens were conditioned at 20 °C and 65% relative humidity. The vibration tests for the Sitka spruce circular truncated cones were conducted under the same conditions.

In addition, six Japanese cedar (*Cryptomeria japonica* D. Don) logs with bark having a length of 3000 mm and the larger diameter of about 190–260 mm and the smaller diameter of about 160–210 mm planted in the Forestry and Forest Products Research Institute (Tsukuba, Japan) were used as specimens. Their tree ages were 16, 32, and 41 years. Japanese cedar logs were used within two days after felling. The vibration tests for the Japanese cedar logs were conducted in a sawmill building without controlling temperature and relative humidity. The cut end portions of the logs were weighed before the vibration tests and oven-dried at 105 °C.

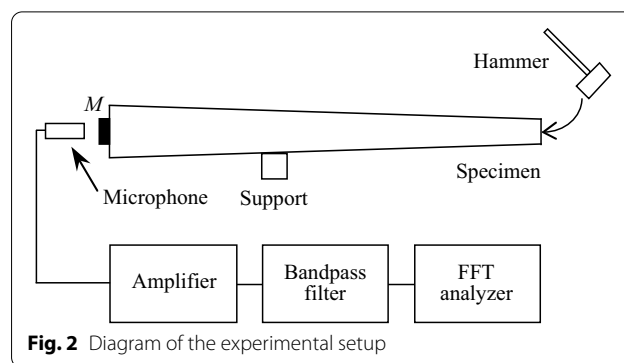
### Vibration tests

Free-free longitudinal vibration tests were conducted for the following three purposes.

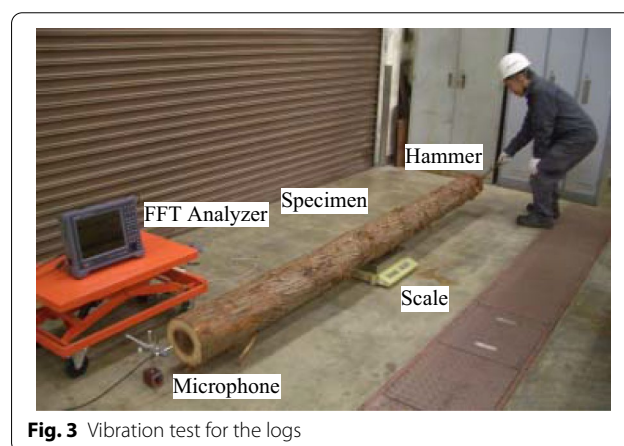
First, to experimentally prove the theory of the VAM for the circular truncated cones mentioned above, the free-free longitudinal vibration tests were conducted on the circular truncated cones with the length of 1000 mm with/without the concentrated mass. The concentrated mass was attached to the butt end ( $x=0$ ) or the top end ( $x=l$ ). Iron plates with dimensions 25 mm × 15 mm × 2 mm, mass 5.4 g (M1) and that with the dimensions of 25 mm × 25 mm × 2 mm, mass 10.0 g (M2) were used for the concentrated masses. The concentrated masses M3, M4, M5, and M6 were made by laminating 2, 3, 4, and 5 M2 iron plates using the double-sided tapes in the thickness direction, respectively.

Second, to experimentally examine the use of cut end portions in the VAM, the free-free longitudinal vibration tests were conducted on the circular truncated cones. One specimen was cut from the butt end and the other was cut from the top end. All the specimens were cut at the distances of 10 mm until the length was 700 mm. The longitudinal vibration tests were performed, while the specimens were shortened stepwise.

Third, to experimentally examine the application of the VAM to the logs, the free-free longitudinal vibration tests were conducted. Three logs were cut from the butt end and the other three were cut from the top end of a specimen. All the logs were cut at the distances of 50 mm



**Fig. 2** Diagram of the experimental setup



**Fig. 3** Vibration test for the logs

until the length was 2850 mm. The longitudinal vibration tests were performed, while the specimen was shortened stepwise.

The free-free longitudinal vibration tests were performed using the following procedure: the test bar was placed on a support near its center point. Next, longitudinal vibration was generated by tapping the RT-plane of one end in the L-direction using a hammer with an iron-head, and the first-mode vibration of the specimen was detected using a microphone (PRECISION SOUND LEVEL METER 2003, NODE Co., Ltd, Tokyo, Japan) placed at the other end. The signal was processed using a Fast Fourier Transform (FFT) digital signal analyzer (Multi-Purpose FFT Analyzer CF-5220, Ono-Sokki, Co., Ltd., Yokohama, Japan) to observe the high-resolution resonance frequencies. The diagram of the experimental setup and the photograph of the vibration test for the logs are shown in Figs. 2 and 3, respectively.

### Results and discussion

The mean (standard deviation) density and Young's modulus of the initial specimens before cutting were 480 (23) kg/m<sup>3</sup>, 15.45 (0.98) GPa (Sitka spruce), and 878 (121) kg/

m<sup>3</sup>, 5.13 (1.61) GPa (Japanese cedar), respectively. The mean (standard deviation), maximum, minimum moisture content of the cut end portion of Japanese cedar log were 172 (34), 238, and 133%, respectively.

The estimation accuracy of the VAM was expressed using the mass ratio (estimated specimen mass through the VAM/measured specimen mass)  $M_{VAM}/M_0$  and the estimation accuracy of VAM was considered to be sufficiently high when  $0.9 \leq M_{VAM}/M_0 \leq 1.1$  in this study. Since the density and Young's modulus are proportional to mass and density, respectively, the ratio of the estimated density through the VAM to the measured density, and the ratio of the estimated Young's modulus through the VAM to the Young's modulus measured through the normal vibration test without the concentrated mass are equal to  $M_{VAM}/M_0$ .

Table 1 shows the accuracy of the VAM for the circular truncated cones using the decrease in the resonance frequency by attaching the concentrated mass. The range of  $\mu$  was from 0.000156 to 0.0347, and  $M_{VAM}/M_0$  was from 0.9 to 1.1 in several  $\mu$  values. Hence, it was possible to estimate the masses of the circular truncated cones with 10% error by the VAM. The estimation accuracy of VAM for the smaller concentrated mass was higher than that for the larger concentrated mass regardless of  $k$ . This tendency is explained in the same way as the uniform cross-section specimen using the relationship between  $m_n$  and

$\mu$  calculated from the frequency equation incorporating the effect of the concentrated mass attached to a specimen and its position [3, 5].  $m_n$  decreases monotonically with respect to  $\mu$ , and the relationship between  $m_n$  and  $\mu$  is convex downward as shown in Fig. 4. Hence, the high estimation accuracy of VAM is likely to be obtained for the larger absolute value of the differential coefficient of the  $m_n - \mu$  relationship i.e. for the smaller  $\mu$  because the effect

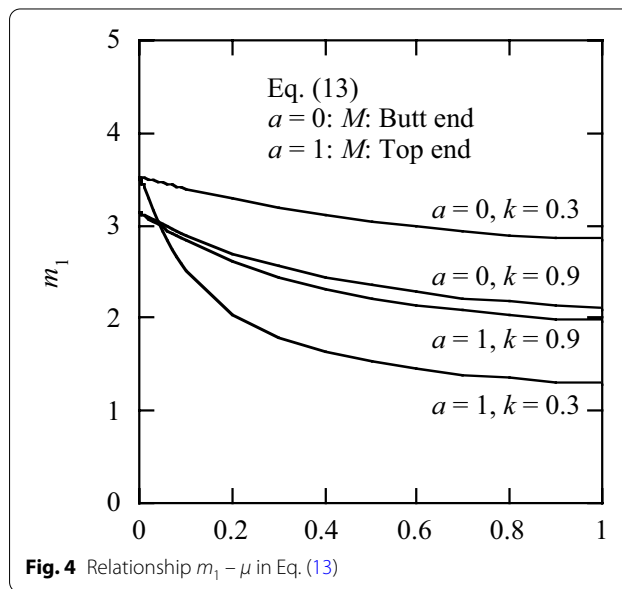


Fig. 4 Relationship  $m_1 - \mu$  in Eq. (13)

**Table 1 Estimation accuracy of the vibration method with additional mass (VAM)  $M_{VAM}/M_0$  for the circular truncated cones when the concentrated mass was placed at the butt end or the top end**

Diameter of the top end [mm]	Attaching concentrated mass to the butt end							Attaching concentrated mass to the top end						
	Concentrated mass							Concentrated mass						
	M1	M2	M3	M4	M5	M6	Total	M1	M2	M3	M4	M5	M6	Total
30	1.28	1.05	1.24	0.96	0.94	0.97	5	0.68	0.73	0.60	0.61	0.42	0.55	3
	1.18	1.12	1.04	0.87	0.76	0.63		1.19	1.11	1.02	0.94	0.86	0.96	
40	1.01	0.86	0.82	0.78	0.69	0.75	2	0.93	0.86	0.85	0.79	0.93	0.74	4
	1.33	0.86	0.90	0.83	0.68	0.66		0.94	0.91	0.86	0.83	0.73	0.69	
50	1.07	1.22	0.99	0.86	0.74	1.48	2	0.90	0.80	0.79	0.75	0.65	0.82	3
	0.70	0.89	0.82	0.69	0.70	0.69		0.89	0.93	0.83	0.68	0.96	0.73	
60	0.96	0.83	0.75	0.81	0.80	0.82	3	1.02	0.86	0.88	0.74	0.63	0.61	5
	1.08	1.29	1.11	0.84	0.74	0.94		0.92	0.91	0.97	0.92	0.66	0.81	
70	1.06	0.95	0.84	0.82	0.76	0.72	4	1.03	0.87	0.89	0.76	0.76	0.69	3
	0.81	0.90	0.96	0.85	0.75	0.73		0.79	0.96	1.07	0.88	0.74	0.77	
80	0.44	0.49	0.92	0.84	1.08	1.38	4	0.64	0.67	0.88	0.95	0.63	0.60	2
	0.96	0.96	0.87	0.80	0.70	0.69		0.97	0.83	0.84	0.74	0.81	0.64	
90	0.95	0.86	0.98	0.84	0.70	0.73	5	0.98	0.90	0.85	0.81	0.71	0.69	3
	1.64	0.90	0.96	0.94	0.86	0.70		1.49	0.97	0.81	0.86	0.66	1.12	
Total	7	5	7	2	2	2		8	6	3	3	2	1	

$M_{VAM}$ : Estimated mass by the VAM,  $M_0$ : Measured mass, Upper row of the same diameter: Specimen 1, Lower row of the same diameter: Specimen 2, Total: Total number of occurrences of  $0.9 \leq M_{VAM}/M_0 \leq 1.1$



of any errors in  $m_n$  caused by measuring the resonance frequency on  $\mu$  can be ignored. There was not a clear tendency between the estimation accuracy of VAM and  $k$ .

Figures 5 and 6 demonstrate the estimation accuracy of the VAM when the butt end portion or the top end portion of the circular truncated cones was cut. The initial properties (dimensions, mass and resonance frequency)

$$\mu_v = \frac{\frac{\pi}{3}bl(r_{al}^2 + r_{al}r_2 + r_2^2)}{\frac{\pi}{3}al(r_1^2 + r_1r_{al} + r_{al}^2)} = \frac{1-a}{a} \frac{\{(k-1)a+1\}^2 + \{(k-1)a+1\}k + k^2}{\{(k-1)a+1\}^2 + (k-1)a + 2} \tag{24}$$

for the length of 1000 mm and properties after cutting correspond to the values with/without the concentrated mass in the section “Longitudinal vibration testing method without weighing the specimen”, respectively. For accuracy,  $\mu$  was calculated by

$$\mu_M = \frac{M_i - M_{cut}}{M_{cut}} \tag{20}$$

where  $M_i$  and  $M_{cut}$  are the initial specimen mass, and specimen mass after end cutting, respectively.

$m_{n0}$  was obtained by substituting  $k$  in Eq. (15). Equation (15) was solved using a software [16].

$M_{VAM}/M_0$  decreased when the specimens were shortened. Several results deviated from the  $M_{VAM}/M_0$ —length curve at the beginning of the shortening of the specimen because the change in the resonance frequency was small. These tendencies were similar to those of the specimens with constant cross-sectional shapes [14].

The estimation accuracy reduced with the decreasing specimen length (Figs. 5 and 6). The effect of the decrease in the specimen length on the accuracy of estimation was explored theoretically.

The specimens can be divided into two parts: the end portion and the main body. The ratio of the mass (end portion/main body) is obtained. Dimensions of the main body and end portion are shown in Table 2.

When the range of  $0 \leq x \leq al$  in Fig. 1 is the end portion, i.e. when the butt end is cut, the radius ratio (top end/butt end) of the main body is

$$k_{EC} = \frac{r_2}{r_{al}} = \frac{k}{(1-k)b+k} \tag{21}$$

where the subscript EC means “end cut”.

The volume ratio (end portion/main body) is

$$\mu_v = \frac{\frac{\pi}{3}al(r_1^2 + r_1r_{al} + r_{al}^2)}{\frac{\pi}{3}bl(r_{al}^2 + r_{al}r_2 + r_2^2)} = \frac{1-b}{b} \frac{\{(1-k)b+k\}^2 + (1-k)b+k+1}{\{(1-k)b+k\}^2 + \{(1-k)b+k\}k+k^2} \tag{22}$$

When the range of  $al \leq x \leq l$  in Fig. 1 is the end portion, i.e. when the top end is cut, the radius ratio (top end/butt end) of the main body is

$$k_{EC} = \frac{r_{al}}{r_1} = (k-1)a+1. \tag{23}$$

The volume ratio (end portion/main body) is

Substituting the initial values before cutting the end (resonance frequency:  $f_{ni}$ , constant:  $m_{ni}$ , and length:  $l_i$ ) for Eq. (18):

$$f_{ni} = \frac{m_{ni}}{2\pi l_i} \sqrt{\frac{E}{\rho}} \tag{25}$$

Substituting the values after cutting the end (resonance frequency:  $f_{ncut}$ , constant:  $m_{ncut}$ , and length:  $l_{cut}$ ) for Eq. (17),

$$f_{ncut} = \frac{m_{ncut}}{2\pi l_{cut}} \sqrt{\frac{E}{\rho}} \tag{26}$$

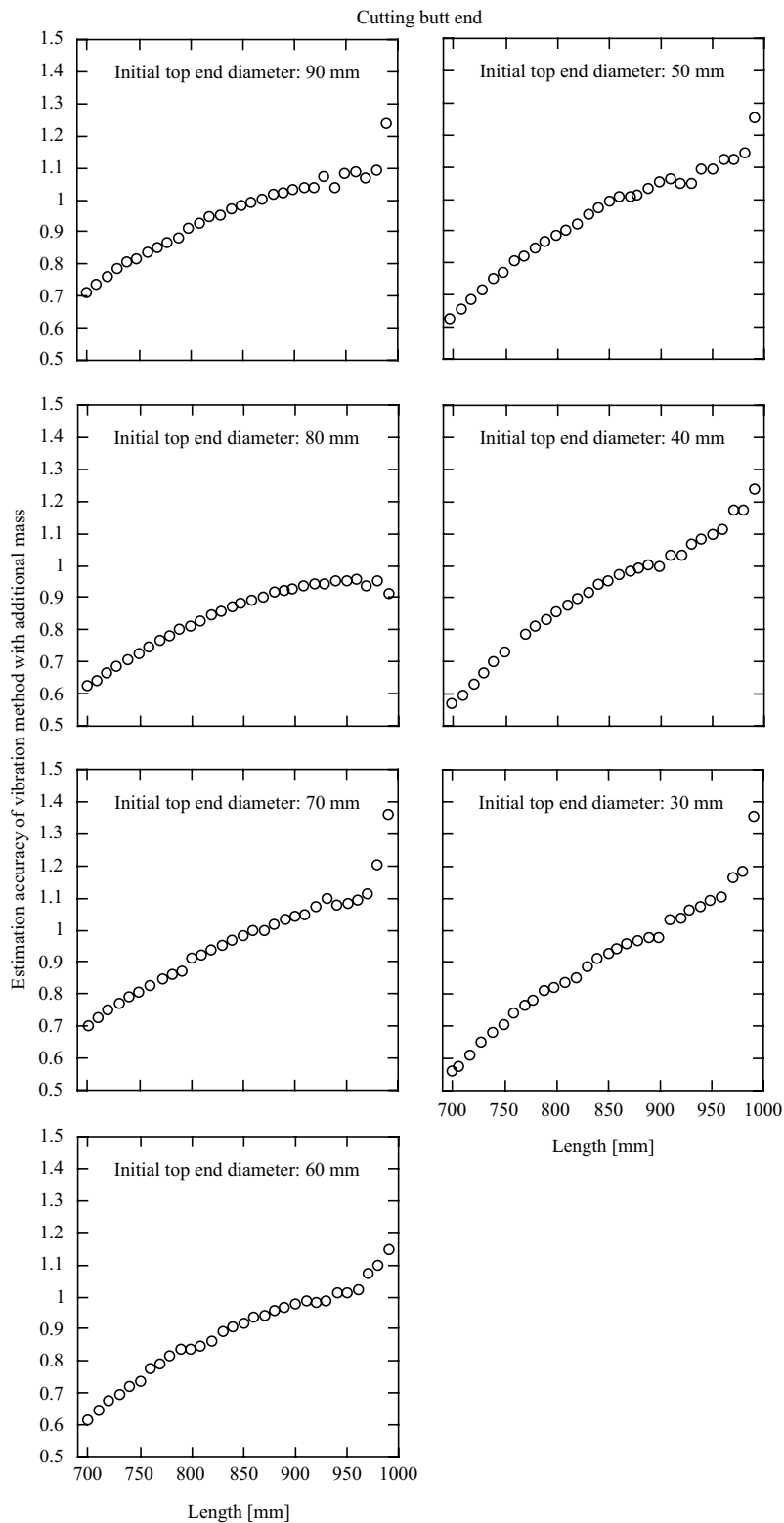
Substituting  $f_{ni}$  and  $f_{ncut}$  for Eq. (19),  $m_n$  to be substituted for Eq. (13) is

$$m_n = \frac{f_{ni}}{f_{ncut}} m_{ncut} = \frac{l_{cut}}{l_i} m_{ni} \tag{27}$$

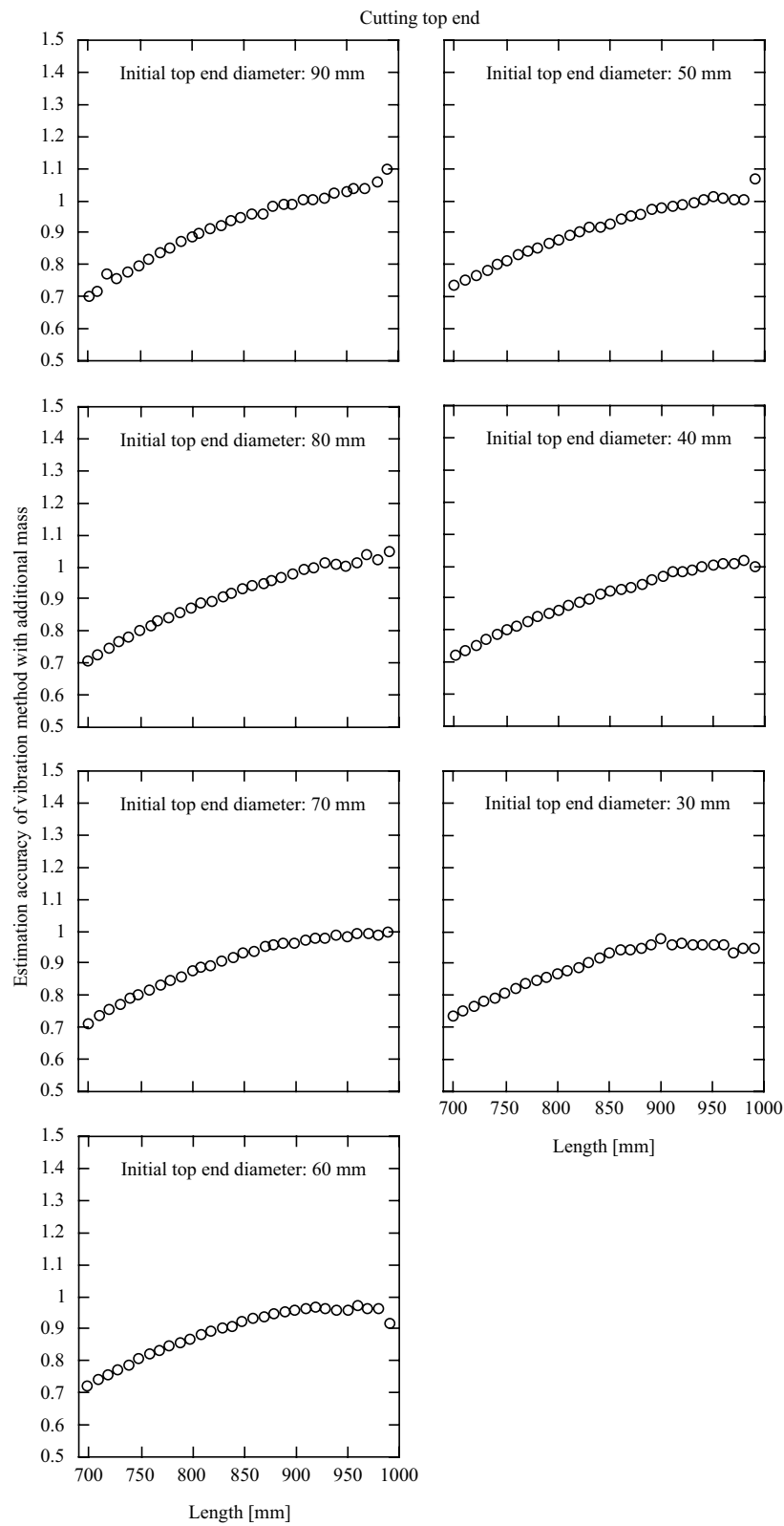
If the  $\mu$  calculated by substituting  $m_n$  from Eq. (27) for Eq. (13) is written as  $\mu_{cut}$ , the accuracy of the VAM is

$$\frac{M_{VAM}}{M_0} = \frac{\mu_v}{\mu_{cut}} \tag{28}$$

The ratio  $M_{VAM}/M_0$  is plotted against the specimen length ratio (after/before end cutting) (Figs. 7 and 8). The experimental estimations had high overlap with the theoretical estimations. To maintain high estimation accuracy, a specimen length must not be too short as shown in our previous study for a specimen with the constant cross-sectional shape [14]. From the theoretical analysis, the estimation accuracy was more than 0.90 when the



**Fig. 5** Experimental estimation accuracy of the vibration method with additional mass (VAM)  $M_{VAM}/M_0$  ( $M_{VAM}$ : Estimated mass by the VAM,  $M_0$ : Mesasured mass) for the circular truncated cones when the cut butt end portion was used for the virtual additional mass



**Fig. 6** Experimental estimation accuracy of the vibration method with additional mass (VAM)  $M_{VAM}/M_0$  ( $M_{VAM}$ : Estimated mass by the VAM,  $M_0$ : Mesasured mass) for the circular truncated cones when the cut top end portion was used for the virtual additional mass



**Table 2 Dimensions of the main body and the end portion of the circular truncated cones**

Cutting butt end (End portion: $0 \leq x \leq al$ )		
	Main body	End portion
Butt end radius	$r_{al}$	$r_1$
Top end radius	$r_2$	$r_{al}$
Length	$bl$	$al$
Cutting top end (End portion: $al \leq x \leq l$ )		
	Main body	End portion
Butt end radius	$r_1$	$r_{al}$
Top end radius	$r_{al}$	$r_2$
Length	$al$	$bl$

$$r_{al} = \{(k - 1)a + 1\}r_1 = \{(1 - k)b + k\}r_1$$

specimen length ratio was more than 0.855 for the butt end cut and 0.826 for the top end cut.

Table 3 shows the mass of the Japanese cedar logs estimated by the VAM. The calculation procedure is the same as that for the circular truncated cones. It can be said that  $M_{VAM}/M_0$  was 0.9 or more. Hence, it is possible that the mass of a log can be simply estimated using a cut end portion with the sufficient accuracy. Since the estimation accuracy was sufficiently high when the cutting length was from 50 to 150 mm, the high estimation accuracy will be obtained by cutting the aforementioned extra length.

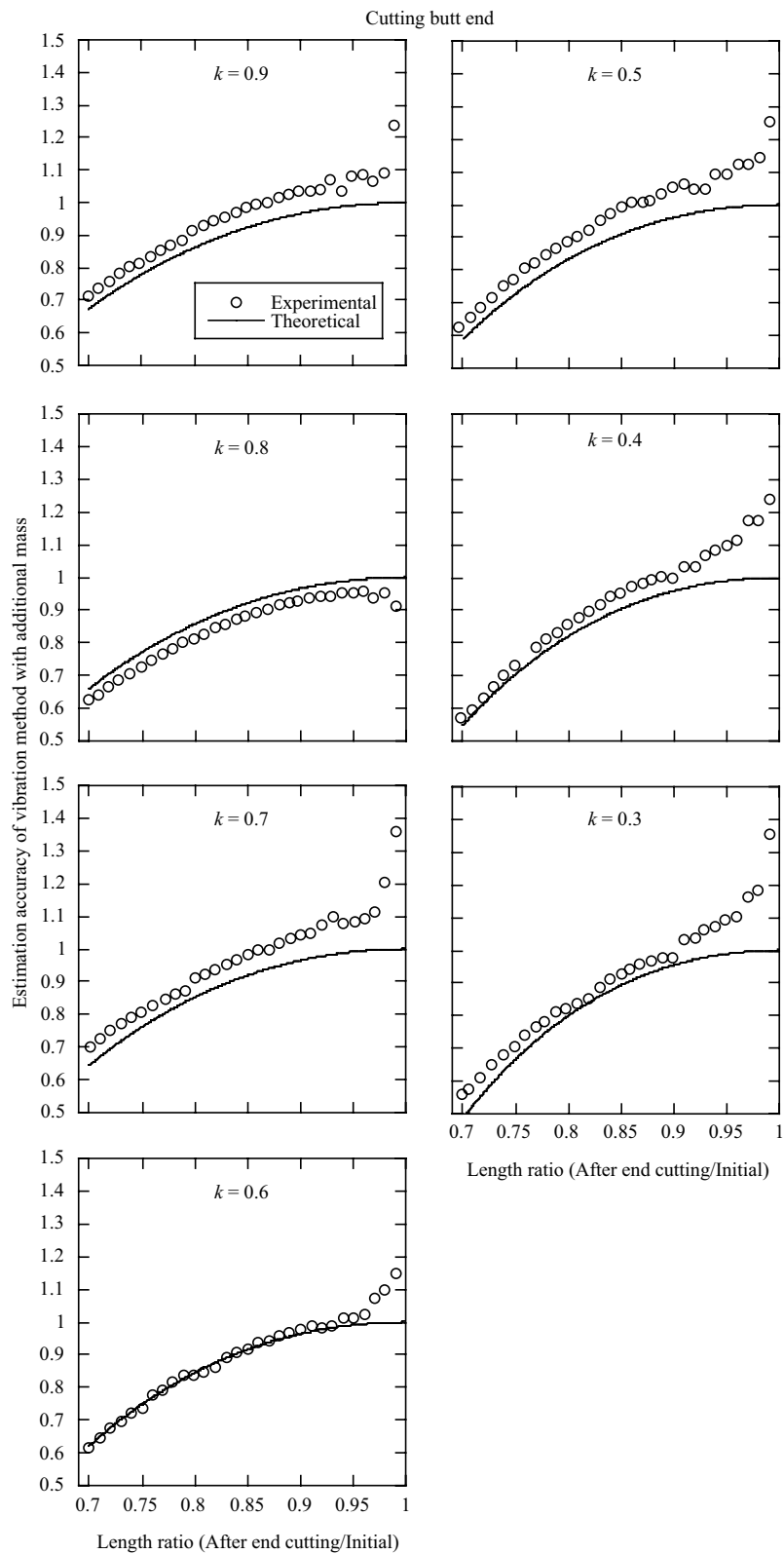
We consider the application of the VAM to on-site logs. Since  $m_{n0}$  depends on  $k$ , it is necessary to contain a process to calculate an  $m_{n0}$  or an  $m_{n0}-k$  table in the

VAM. Although the logs were placed on a support near their center point, i.e., the nodal position of the first mode of the longitudinal vibration, the logs will be supported by at least two points on-site. Hence, the number of supporting points (the resonance mode of the longitudinal vibration to be used) and materials to be used for the supporting points need to be examined in the future.

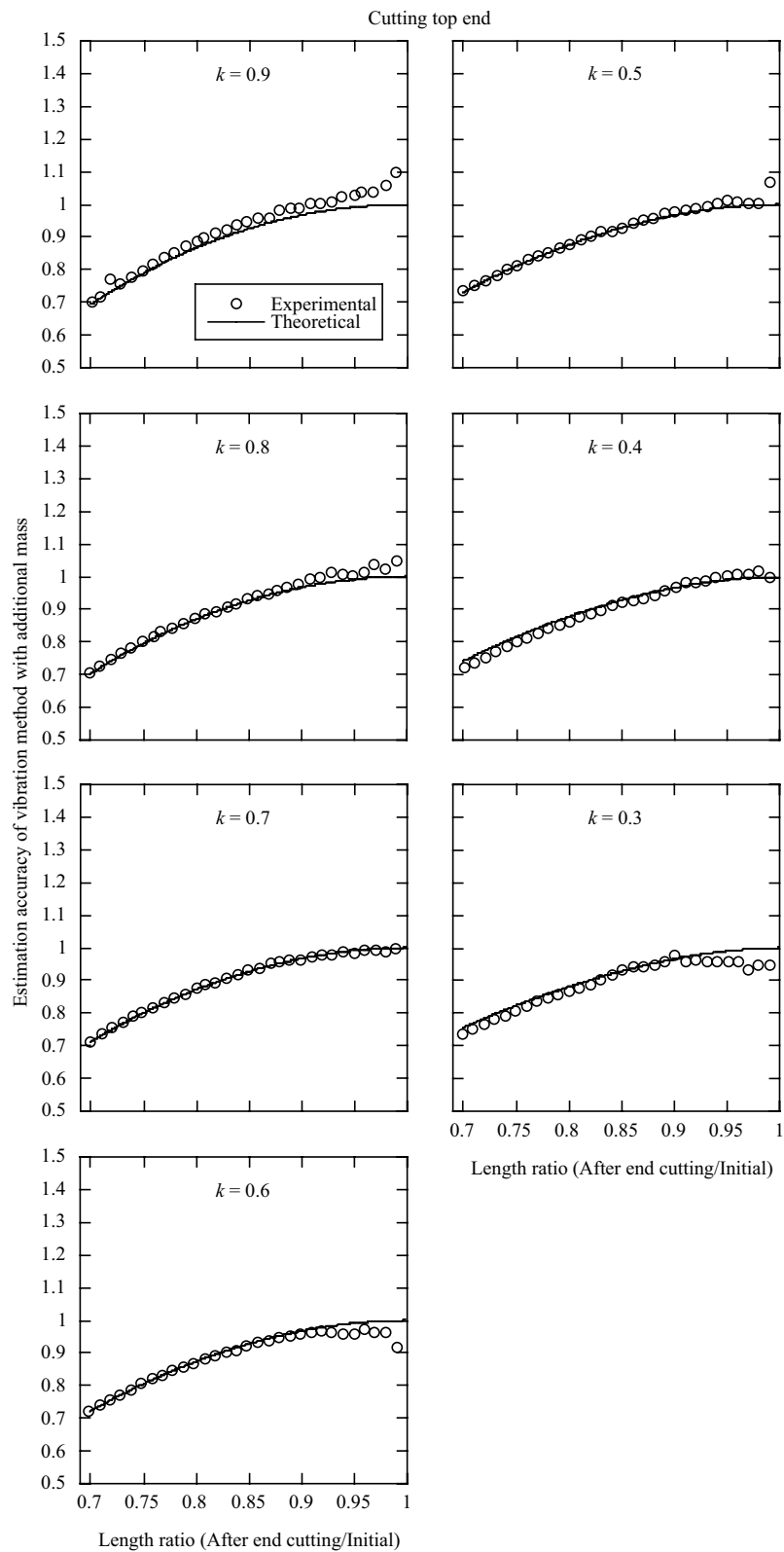
### Conclusions

Longitudinal vibrations were undertaken and the VAM was applied to the circular truncated cones and logs. The following results were obtained.

1. From the results of the vibration test using specimens with/without the concentrated mass, it was possible to estimate the mass of a circular truncated cone with 10% error by the VAM.
2. The cut butt end and the cut top end portions of a circular truncated cone could be used as the virtual mass in the VAM.
3. From the experimental results and theoretical results, in order to maintain high estimation accuracy, a specimen length must not be too short as shown in our previous study for a specimen with the constant cross-sectional shape.
4. The cut butt end and the cut top end portions of a log could be used as the virtual mass in the VAM. Hence, it is possible to simply measure the mass of log without weighing the log itself on site through a vibration test.



**Fig. 7** Theoretical estimation accuracy of the vibration method with additional mass (VAM)  $M_{VAM}/M_0$  ( $M_{VAM}$ : Estimated mass by the VAM,  $M_0$ : Mesasured mass) for the circular truncated cone when the cut butt end portion was used for the virtual additional mass



**Fig. 8** Theoretical estimation accuracy of the vibration method with additional mass (VAM)  $M_{VAM}/M_0$  ( $M_{VAM}$ : Estimated mass by the VAM,  $M_0$ : Mesasured mass) for the circular truncated cone when the cut top end portion was used for the virtual additional mass

**Table 3 Estimation accuracy of the vibration method with additional mass (VAM) for logs when the butt end or top end were cut**

Specimen	Circumference of end (mm)		Length (mm)	Measured Mass $M_0$ (g)	Frequency (Hz)	$m_{n0}$	Estimated mass by the VAM $M_{VAM}$ (g)	$M_{VAM}/M_0$
	Butt	Top						
Cutting the butt end								
1	809	658	3009.0	108,540	440.700	3.15511	Initial	Initial
	802	658	2948.0	105,440	452.700	3.15400	93,247	0.88
	790	658	2891.0	102,660	463.650	3.15219	92,871	0.90
	778	658	2832.0	100,040	474.950	3.15050	90,102	0.90
2	681	570	3082.5	84,340	372.200	3.15163	Initial	Initial
	678	570	3020.0	82,540	379.550	3.15114	76,620	0.93
	676	570	2963.5	80,740	387.000	3.15082	75,790	0.94
	674	570	2899.5	78,920	395.200	3.15050	72,893	0.92
3	612	505	3014.5	49,500	371.400	3.14962	Initial	Initial
	605	505	2964.5	48,220	380.550	3.14851	44,727	0.93
	600	505	2903.0	46,840	392.050	3.14775	41,039	0.88
	579	505	2841.5	45,520	404.550	3.14501	39,000	0.86
Cutting the top end								
4	611	506	3004.0	74,000	482.200	3.15286	Initial	Initial
	611	514	2961.0	73,180	488.800	3.15107	71,585	0.98
	611	513	2906.0	71,980	499.100	3.15129	69,060	0.96
	611	517	2850.0	70,960	506.950	3.15045	70,285	0.99
5	697	569	3022.0	95,500	384.750	3.15463	Initial	Initial
	697	566	2975.0	94,400	390.150	3.15532	97,210	1.03
	697	564	2922.0	92,960	398.000	3.15579	91,954	0.99
	697	550	2852.0	91,820	404.050	3.15933	93,970	1.02
6	611	488	3016.0	58,660	339.550	3.15758	Initial	Initial
	611	504	2963.5	57,780	346.550	3.15334	52,106	0.90
	611	478	2907.5	56,900	353.400	3.16064	55,803	0.98
	611	490	2849.0	56,060	360.300	3.15701	53,490	0.95

**Abbreviations**

VAM: Vibration method with additional mass; L: Longitudinal; R: Radial; T: Tangential; FFT: Fast Fourier Transform.

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**Authors' contributions**

All authors designed the experiments. YK performed the experiments, analyzed the data, and was a major contributor in writing the manuscript. All authors contributed to interpretation and discussed results. All authors read and approved the final manuscript.

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**Competing interests**

The authors declare that they have no competing interests.

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