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# Development of Fixture Layout Optimization for Thin-Walled Parts: A Review

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**Abstract**

An increasing number of researchers have researched fixture layout optimization for thin-walled part assembly during the past decades. However, few papers systematically review these researches. By analyzing existing literature, this paper summarizes the process of fixture layout optimization and the methods applied. The process of optimization is made up of optimization objective setting, assembly variation/deformation modeling, and fixture layout optimization. This paper makes a review of the fixture layout for thin-walled parts according to these three steps. First, two different kinds of optimization objectives are introduced. Researchers usually consider in-plane variations or out-of-plane deformations when designing objectives. Then, modeling methods for assembly variation and deformation are divided into two categories: Mechanism-based and data-based methods. Several common methods are discussed respectively. After that, optimization algorithms are reviewed systematically. There are two kinds of optimization algorithms: Traditional nonlinear programming and heuristic algorithms. Finally, discussions on the current situation are provided. The research direction of fixture layout optimization in the future is discussed from three aspects: Objective setting, improving modeling accuracy and optimization algorithms. Also, a new research point for fixture layout optimization is discussed. This paper systematically reviews the research on fixture layout optimization for thin-walled parts, and provides a reference for future research in this field.

**Keywords** Thin-walled parts, Assembly quality, Fixture layout optimization, Modeling methods, Optimization algorithms

## 1 Introduction

With the characteristics of lightweight and easy forming, thin-walled parts are widely used in automobiles, ships, aircraft, and many manufacturing industries [1]. According to the different research objects, researchers also call thin-walled parts, sheet metal, or compliant parts. In these fields, thin-walled parts with different shapes and sizes are usually assembled to construct outer shells, such as body in white (BIW), ship hull, and fuselage, which provides necessary space for passengers and cargoes. The BIW and hull are made of sheet metal, while the fuselage may be made of composite sheets. However, no matter what their materials are, they all have common characteristics that their thickness is much less than their length and width.

In the assembly process of thin-walled parts, the fixture layout plays a critical role in the assembly quality improvement [2–4]. The assembly processes of BIW, ship hull, and fuselage are typical multi-stage assembly processes. Any dimensional variation of thin-walled parts or deformation from the designed shape at single-stage assembly can easily stack up to significant dimensional misalignments and then impact the assembly quality of the final products. A reasonable fixture layout can not only locate the position and posture of thin-walled parts but also restrict the deformation induced by gravity or other forces in the assembly process. Therefore, an

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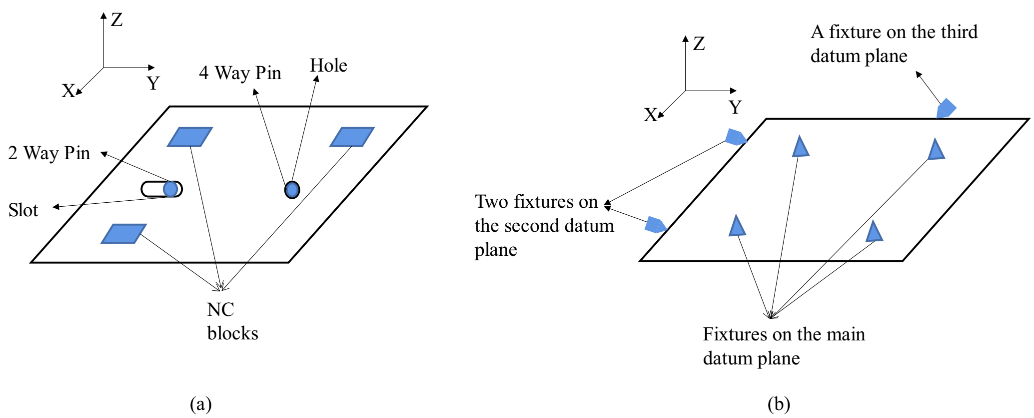


**Figure 1** Fixture layout for thin-walled parts under different assumptions: (**a**) A typical "3-2-1" fixture layout under rigid assumption, (**b**) A typical "N-2-1" fixture layout under compliant assumption

increasing number of scholars have researched fixture layout optimization for thin-walled parts during the past decades.

Fixture layout optimization for thin-walled parts can be classified into optimization under the rigid and compliant assumption. Under the rigid assumption, the part deformation induced by gravity or other forces is ignored. However, the assembly variation caused by the part manufacturing variations and fixture errors [5] will greatly threaten the assembly quality of the products in mass production. Thus, the researchers mainly focus on the robust design of fixture layouts to reduce the assembly variation. Generally, the "3-2-1" locating principle is used when the thin-walled parts are considered as rigid parts. Figure 1(a) shows a "3-2-1" fixture layout. A thin-walled part is held by three NC blocks and two pins. With the "3-2-1" locating principle, all 6 degrees of freedom can be restricted and the part can be completely located. In this locating frame, the robust design of fixture layouts mainly aims to reduce the variation in the X-Z plane (shown in Figure 1(a)), called as in-plane variation, by optimizing the positions of NC blocks, the positions of pins, and the slot orientations.

Under the compliant assumption, the researchers mainly consider the out-of-plane deformation. The low ratio of thickness to length or width leads to low out-of-plane stiffness for the thin-walled parts. Thus, the large dimensional thin-walled parts, such as compliant parts of hull or fuselage, are easy to deform under the action of gravity in the assembly process. The out-of-plane deformation will result in an assembly gap between the two parts to be assembled. Because thin-walled parts are generally assembled by welding or riveting, the assembly gap will greatly affect the quality of welding and riveting. In addition, the out-of-plane deformation in the previous assembly stage will also affect the assembly quality in the later stage. In order to reduce the out-of-plane deformation for the thin-walled parts, Cai et al. [6] proposed the "N-2-1" (N>3) locating principle. Figure 1(b) shows a typical "N-2-1" fixture layout. More than three fixtures are located on the main datum plane to reduce the out-of-plane deformation. In this locating frame, the fixture layout optimization mainly optimizes the positions and the number of "N" locators.

Figure 2 shows the development of fixture layout optimization for thin-walled parts. It counts the quantity of academic achievements in the field since 1991 based on the search results of the web of science. As the figure illustrates, there have been an increasing number of researches in this field. Especially after 2016, the number of published results has increased significantly. Thus, scholars' research on fixture layout optimization is deepening. It also shows the development of modeling methods for the relationship between the assembly quality and the fixture layout and optimization methods used in fixture layout optimization. As for modeling methods, many of them are based upon mechanism. Among them, the finite element method (FEM) was first applied to fixture layout optimization. In the middle and late 1990s, the Jacobian matrix method and state space method were successively used for modeling. With the development of computer technology, data-based modeling methods appeared around 2000. The early data-based method was mainly response surface methodology (RSM), and then researchers tried other regression models. With the development of artificial intelligence technology, artificial neural network (ANN) method comes into being. In terms of optimization algorithms, the algorithms used

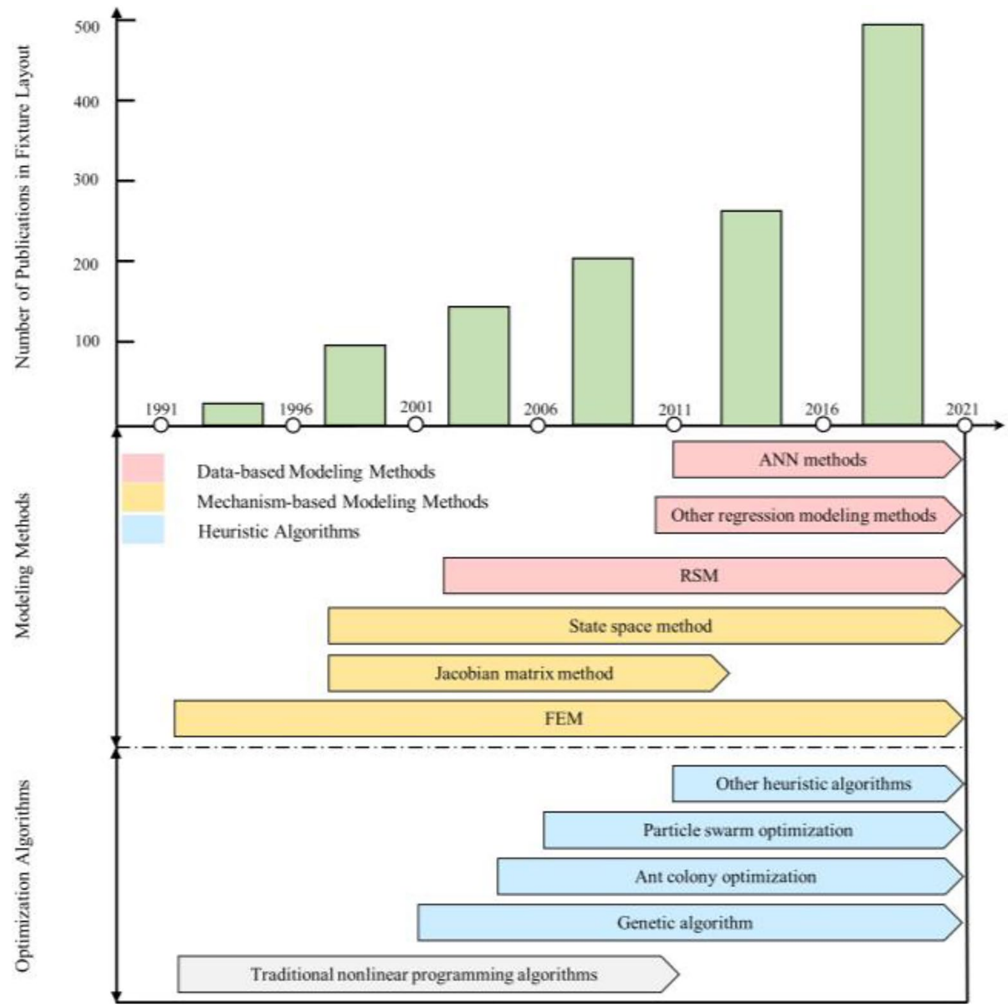


**Figure 2** Development of fixture layout optimization for thin-walled parts

in the early researches are the traditional nonlinear programming algorithms. Over time, these traditional nonlinear programming algorithms are almost replaced by various heuristic algorithms. Representative heuristic algorithms include ant colony optimization (ACO), particle swarm optimization (PSO), and genetic algorithm (GA). After 2010, many other heuristic algorithms have been applied.

Researchers have done a lot of researches in the past few decades. Based on past researches, we find that fixture layout optimization can usually be accomplished with the following three steps:

(1) Problem formulation. In this step, researchers transform the practice assembly problem into a mathematical expression and construct an optimization problem including optimization objectives, constraints, and decision variables.
(2) Modeling. Researchers use different methods to build the relationship model between optimization objectives and the fixture layout.
(3) Searching for an optimum solution. In this step, several optimization strategies will be applied to find an optimized fixture layout.

Based on these three steps, pieces of literature related to developments of the fixture layout optimization for thin-walled parts are reviewed. The following is how the rest of this paper is organized. Different optimization objectives in thin-walled part assembly are discussed in Section 2. Section 3 summarizes different modeling methodologies for the relationship between optimization objectives and fixture layout. In Section 4, several commonly used optimization algorithms used in fixture layout for the thin-walled parts are introduced. In Section 5, we discuss the limitations of current researches and future challenges. At last, this paper is summarized in Section 6.

## 2 Optimization Objectives of Fixture Layout Optimization in Thin-Walled Part Assembly

For the sake of optimizing the fixture layout for thin-walled parts, actual engineering problems have to be transformed into mathematical problems. An optimization problem always contains three elements: optimization objectives, constraints, and decision variables. In fixture layout optimization, the decision variables are usually fixture position coordinates or node numbers. Constraints are set according to the actual situation, usually including non-interference between fixtures, selection only within a certain range, etc. The overall goal of optimizing fixture layout is to improve quality. As the in-plane variations and out-of-plane deformations are independent of each other [7], researchers usually consider either of them when setting specific objectives, which will be discussed in this section. Then the comparative analysis of different optimization objectives will be conducted.

### 2.1 Considering In-Plane Variations

When considering in-plane variations, the thin-walled parts are usually assumed to be rigid, which means that they will not deform under force. Considering the in-plane variations, researchers have done a lot of studies on the robust design of fixture layouts for the thin-walled parts. The robust design means minimizing part variations caused by source variations including part manufacturing variations and fixture errors [5]. In 1999, Rikard Söderberg et al. [8] highlighted the importance of robust locating scheme design in their paper. Furthermore, Camelio et al. [9] gave an example to verify the influence of fixture layout on thin-walled part assembly variation in 2004.

Minimization of the in-plane variations is one kind of optimization objective. Variations of the key product characteristics (KPCs) are adopted to represent the in-plane variations. Cai [5] proposed a novel method to achieve this objective. Only the locations of pins and slots were optimized. Different from considering the case of single-station in this article, Masoumi et al. [10] considered the minimization of in-plane variations in the case of multi-station. The objective was to minimize the sum of squared standard in-plane deviations for KPCs. They optimized the assembly sequence and the fixture layout of each station to achieve the goal.

Minimization of the sensitivity of the part to source variations is the other kind of optimization objective. Kim and Ding [11] conducted fixture layout design for parts assembled in multiple stations. They used the E-optimality criterion to minimize the upper sensitivity bound of the fixture layout. The decision variables were the positions of principal locating points. Li et al. [12] was aimed at minimizing the square root of the condition number of the sensitivity matrix they obtained, but the E-optimality criterion has been adopted more frequently. Tian et al. [13] chosed minimizing the sensitivity of final variation to fixture errors as the objective. The slot orientations as well as the positions of pins were selected to achieve the optimization objective. Huang et al. [14] modeled an automotive floor pan assembly process and conducted the fixture layout design. A case of a three-station assembly process was studied by Xie et al. [15]. From these studies, we can find that the fixture layout optimization considering in-plane variations is mainly to improve robustness. For thin-walled parts, the out-of-plane deformation is another critical problem in the assembly process. Therefore, many scholars have considered out-of-plane deformations when designing optimization objectives.

### 2.2 Considering Out-of-Plane Deformations

When considering out-of-plane deformations, the thin-walled parts are usually assumed to be compliant, which are usually called compliant parts. The thin-walled parts will deform due to their low out-of-plane stiffness. The assembly gap caused by out-of-plane deformations will have an adverse impact on the assembly process and reduce the assembly quality. In addition, the deformation in the previous assembly process will also be transferred to the next station, resulting in the accumulation of deviations. Therefore, the reduction of part deformation is the main optimization objective.

Minimization of the deformation of KPCs is one common optimization objective. Generally, the deformation perpendicular to the surface is taken into consideration while deformations in other directions are not considered. In the earliest works, researchers would like to minimize the total deformation of thin-walled parts [6]. They meshed the plate and expressed the total deformation by adding together the squares of deformations which are perpendicular to the surface at the grid nodes. The formulation is as Eq. (1):

$$F(X) = \sum_{i=1}^{m} w_i(X)^2, \tag{1}$$

where $w_i(X)$ represents the deformations perpendicular to the part surface at the $i$th node. The loading that caused deformations can be resistance spot welding force or any other forces applied onto the surface of the parts. This method can effectively control the deformations perpendicular to the surface, but ignores the influence of the deformations in other directions.

Therefore, some other researchers use strain energy to express deformation [16–19]. When a force is applied to the body and causes deformation, the work done by the force is stored in the body. This is strain energy. Deformations in all directions can be taken into consideration with the help of strain energy. The total strain energy is calculated as Eq. (2):

$$U = \frac{1}{2}\int_V \{\boldsymbol{\sigma}\}^{\mathrm{T}}\{\boldsymbol{\varepsilon}\}\mathrm{d}V, \tag{2}$$

where $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$ are the stress and strain vectors, $U$ is the strain energy, and $V$ is the body's volume. Ahmad et al. [16] chose to minimize the total strain energy as the optimization objective. Then, Refs. [17, 18] proposed different methods with this optimization objective. A method for designing discrete fixture layout under multiple loads was presented by Bi et al. [19]. They linked the strain energy to the fixture layout and wanted to reduce it. The above two are conventional optimization objectives.

There are also some innovative optimization objectives according to different practical needs. De Meter [20] intended to reduce the maximum displacement-to-tolerance ratio of KPCs. Minimization of the rigid body displacements owing to elastic deformation of loaded fixture–part contacts was considered by Li and Melkote [21, 22]. A method presented by Du et al. [23] aimed to reduce dimensional gaps along with the interface of two parts. This objective considered that the gaps would affect the seam welding quality. Considering the expense of part production and assembly quality of parts, Aderiani et al. [24] aimed to minimize the expense of part production and satisfy the requirement of assembly at the same time. The above content introduces the common optimization objectives in fixture layout optimization. Designing appropriate objectives is the basis of fixture layout optimization. Therefore, determining the optimization objectives according to the actual needs is the first step of optimization.

### 2.3 Comparative Analysis of Different Optimization Objectives

Based on the previous introduction, we compare and analyze the optimization objectives mentioned above (shown in Table 1). There are four main expressions of optimization objectives considering in-plane variations. Two of them directly aim to minimize the statistics of variations of KPCs, while the other two aim to minimize the sensitivity of source deviation. Their common point is to improve the robustness of fixture layout. The optimization objectives considering out-of-plane deformations are changeable. Researchers have different choices according to different practical needs. Ref. [6] chosed to minimize the sum of squares of deformations at nodes, Refs. [16–19] chosed to minimize strain energy, and Ref. [20] chosed to minimize the ratio of profile error to tolerance. All of them directly control the deformation of parts. In addition to these optimization objectives that directly take deformations as the criterion, some researchers have considered other effects caused by deformations. Refs. [21, 22] considered that deformations would cause positioning deviations, so they hoped to minimize the impact of elastic deformations on positioning accuracy. Based on the engineering practice of ship plane welding, Ref. [23] took the minimization of the assembly gap between two parts caused by deformations as the optimization goal, and improved the welding quality. Considering the cost of controlling deformations, Ref. [24] aimed to reduce the cost on the premise of meeting the dimensional requirements. Considering the indirect influence of out-of-plane deformations, these references do not directly control the deformation, but put forward diversified optimization objectives from the perspective of practical demands.

### 2.4 Epilog

This section lays emphasis on the selection of optimization objectives in two different situations, considering in-plane variations or considering out-of-plane deformations. When considering in-plane variations, researchers focus on robust design of fixture layout. On the other hand, when considering out-of-plane deformations, expressions of optimization objectives have more forms. Researchers considered the influence of out-of-plane deformations either directly or indirectly. This section introduces several common optimization objectives and compares them. For fixture layout optimization, determining appropriate optimization objective is the first step. Subsequently, research on how the fixture layout influences the objective will be conducted.

## 3 Modeling Methods of Assembly Variation or Deformation for Fixture Layout Optimization

After determining the optimization objectives, we should model the relationship between optimization objectives and fixture layout. There are many different modeling methods and they consist of two categories: Mechanism-based and data-based modeling approaches, which will be discussed in Section 3.1 and Section 3.2, respectively. After that, we compare the advantages and limitations of these modeling methods.

### 3.1 Mechanism-Based Modeling Methods

Mechanism-based modeling of thin-walled part assembly is to get the causal relationship between assembly variation and variation source, find out the rules reflecting the

**Table 1** Comparison of different optimization objectives

| | Objectives | Physical meaning | Engineering significance | References |
|---|---|---|---|---|
| Considering in-plane variations | Minimize $F(X)=\frac{1}{\sqrt{2N_{KPC}}}\sqrt{\sum_{i=1}^{N_{KPC}}[\sigma^2(\delta x_0)_i+\sigma^2(\delta y_0)_i]}$<br>$\sigma(\delta x_0)_i,\sigma(\delta y_0)_i$ : The $i$ th KPC's variations in the $x$ and $y$ directions<br>$N_{KPC}$: Number of KPCs | Minimizing the pooled standard deviation of resultant errors at all KPCs | Minimizing variations caused by source variations and improving robustness | Cai [5] |
| | Minimize $F(X)=\sum_{i=1}^{m}STD_x^2(i)+STD_z^2(i)$<br>$STD_x, STD_y$ : Standard deviations in the $X$ and $Z$ directions | Minimizing the summation of squared standard in-plane deviations | | Masoumi et al. [10] |
| | Minimize $S_{\max}=\lambda_{\max}(D^{\mathrm{T}}D)$<br>$D$ : a sensitivity index reflects the relationship between the final variation and source variations | Minimizing the square of the 2-norm of sensitivity matrix | Minimizing the sensitivity of the part to source variations and improving robustness | Kim and Ding [11], Tian et al. [13], Huang et al. [14], Xie et al. [15] |
| | Minimize $F(X)=\sqrt{\mathrm{cond}(S^{\mathrm{T}}S)}$<br>$S$ : a sensitivity index reflects the relationship between the final variation and source variations | Minimizing the square root of the condition number of the sensitivity matrix | | Li et al. [12] |
| Considering out-of-plane deformations | Minimize $F(X)=\sum_{i=1}^{m}w_i(X)^2$<br>$w_i(X)$ : the deformation perpendicular to the part surface at the $i$ th node | Minimizing the sum of squares of nodal deformations | Minimizing deformations caused by forces and reducing assembly errors caused by deformations | Cai et al. [6] |
| | Minimize $F=\sum_{i=1}^{n}u_i$<br>$u_i$: strain energy of the $i$ th finite element | Minimizing the sum of strain energy of finite elements | | Ahmad et al. [16–18], Bi et al. [19] |
| | Minimize $F=\max\left\{\frac{e^i}{T^i}\text{ for } i=1,\cdots,M\right\}$<br>$e^i$ : profile error of the $i$ th point<br>$T^i$: profile tolerance of the $i$ th point | Minimizing the maximum ratio of error to tolerance | | De Meter [20] |
| | Minimize $F=\left(\sum_{i=1}^{N}\Delta_i\right)^{\mathrm{T}}\left(\sum_{i=1}^{N}\Delta_i\right)$<br>$\Delta_i$ : the rigid body motion at the $i$ th fixturing point | Minimizing the total rigid body motion | Minimizing positioning errors caused by elastic deformations and improving positioning accuracy | Li and Melkote [21, 22] |
| | Minimize $H(x)=\sum_{i=1}^{m_0}\varphi_i(x)/m_0$<br>$\varphi_i(x)$ : the dimensional gap at node $i$ along the interface between the compliant parts to be assembled<br>$m_0$ : the number of the nodes along the assembly interface between two parts | Minimizing the average dimensional gap along the interface between the compliant parts to be assembled | Reducing the assembly gap between two parts and improving the weld quality | Du et al. [23] |
| | Minimize $f=\frac{C}{C_{\max}}+\sum_{k=1}^{K}p(C_{puk})$<br>$C$: total cost of production<br>$C_{pu}$ : upper process capability index<br>$p(C_{puk})$ : a penalty function relative to $C_{pu}$ of the $k$th KPC | Minimizing the expense of production | Reducing the expense while meeting quality requirement | Aderiani et al. [24] |

internal mechanism, and then establish the mathematical model of the rules. Three common mechanism-based modeling methods in fixture layout optimization are the Jacobian matrix method, state space method, and FEM. We will introduce them respectively in the following.

#### 3.1.1 Jacobian Matrix Method

Jacobian matrix is composed of all partial derivatives of a vector function. Usually, people use Jacobian matrix for coordinate transformation.

If $f$ is a vector function of $n$ equations with $n$ variables, then the Jacobian matrix is obtained by taking the first-order partial derivatives of $f$. That is, consider the set of vector functions such as Eq. (3):

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} f_1(x_1, x_2, x_3, \ldots, x_n) \\ f_2(x_1, x_2, x_3, \ldots, x_n) \\ f_3(x_1, x_2, x_3, \ldots, x_n) \\ \vdots \\ f_n(x_1, x_2, x_3, \ldots, x_n) \end{bmatrix}. \tag{3}$$

Then, the Jacobian matrix is as shown in Eq. (4):

$$J(x_1, x_2, x_3, \ldots, x_n) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} & & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} & & \frac{\partial f_3}{\partial x_n} \\ & \vdots & & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \frac{\partial f_n}{\partial x_3} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix}. \tag{4}$$

The Jacobian matrix is used for the robust fixture layout design for rigid parts. Cai et al. [25] proposed using the Jacobian matrix modeling method to optimize fixture layout. Although what they studied were 3D parts, the modeling method is significant for thin-walled parts. Then, Cai [5] used this method to help robust layout design for thin-walled parts. Through these two papers, we can clearly know how the Jacobian matrix works. Three locators are needed to locate the part deterministically as shown in Figure 3. The variations at any KPCs can be expressed as a vector in Eq. (5) [5]:

$$\boldsymbol{\delta q}_0 = -\boldsymbol{J}^{-1}\boldsymbol{\Phi}_R \boldsymbol{\delta R}, \tag{5}$$

where the Jacobian is expressed as Eqs. (6) and (7):

$$\boldsymbol{J} = \begin{bmatrix} -n_{1x} & -n_{1y} & n_{1y}x_1 - n_{1x}y_1 \\ -n_{2x} & -n_{2y} & n_{2y}x_2 - n_{2x}y_2 \\ -n_{3x} & -n_{3y} & n_{3y}x_3 - n_{3x}y_3 \end{bmatrix}, \tag{6}$$

$$\boldsymbol{\Phi}_R = \begin{bmatrix} n_{1x} & n_{1y} & 0 & 0 & 0 & 0 \\ 0 & 0 & n_{2x} & n_{2y} & 0 & 0 \\ 0 & 0 & 0 & 0 & n_{3x} & n_{3y} \end{bmatrix}, \tag{7}$$

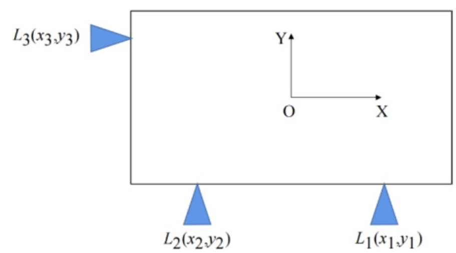


**Figure 3** A deterministically located part

and $\boldsymbol{\delta R} \equiv \left[\, \delta x_1 \;\delta y_1 \;\; \delta x_2 \;\delta y_2 \;\; \delta x_3 \;\delta y_3 \,\right]$ represents the deviations of fixture points. $\boldsymbol{n_i} \equiv [\, n_{ix} \;\, n_{iy} \,]^{\mathrm{T}}$, $i = 1, 2, 3$ represents the vector perpendicular to surface of each point. So that the variation $\boldsymbol{\delta q}_0$ is associated with the source variations $\boldsymbol{\delta R}$. Then we select fixture positions to reduce the $\boldsymbol{\delta q}_0$.

Xing et al. [26] and Lu et al. [27] conducted their research with the Jacobian matrix method respectively. Their research objects were both thin-walled parts and the number of locators was greater than three. The optimization process consisted of two stages. Firstly, they considered the robustness and geometry stability to determine three fixture locations on the datum surface. This step was taken under the rigid assumption. Then they used other methods to attain the best location for the fourth fixture. In the first stage, the Jacobian matrix was used to construct the model. They constructed the Jacobian matrix according to the method described above, and found three suitable nodes to place the fixture. This method greatly reduced the complexity of optimization and improved efficiency. Recently, Jacobian matrix has been used to analyze part positioning errors caused by fixture layout. Liu et al. [28] used Jacobian matrix to analyze the influence of pin position deviation on part position and orientation. Tang et al. [29] used Jacobian matrix to help build a linearized model, which is then derived to convey the relationship between the errors of locating points and errors. Nevertheless, Jacobian matrix method is suitable for solving single-station assembly. For multi-station assembly, the common modeling method is the state space method, which will be introduced next.

#### 3.1.2 State Space Method

State space modeling method was proposed by Jin and Shi [30]. This method is suitable for multi-station assembly. In a multi-station assembly process, the positions of the fixtures will be reused on different stations. The reusing of fixture locating holes will lead to variation propagation from station to station. This means that variations could arise at every station, and they will be propagated between each station. In the end, it will accumulate on the final assembly.

Figure 4 shows an N-station assembly line. The vector $\boldsymbol{x}_k$ means the variation accumulated to station $k$, which is composed of translations and rotations of the parts. $\boldsymbol{u}_k$ represents the fixture deviation and $\boldsymbol{w}_k$ represents other un-modeled deviations. In addition, considering that the measured value and the ideal value cannot be the same, the noise item is added as $\boldsymbol{v}_k$. The state space model can be expressed as Eq. (8) [11, 13–15]:

$$\begin{gathered} \boldsymbol{x}_k = \boldsymbol{A}_{k-1}\boldsymbol{x}_{k-1} + \boldsymbol{B}_k\boldsymbol{u}_k + \boldsymbol{w}_k, \\ \boldsymbol{y}_k = \boldsymbol{C}_k\boldsymbol{x}_k + \boldsymbol{v}_k, \\ k = 1, \ldots, N. \end{gathered} \tag{8}$$

$\boldsymbol{A}_{k-1}$ means the re-orientation influence in station $k$ caused by station $k-1$ and $\boldsymbol{B}_k$ describes the influence of fixture layout at station $k$. $\boldsymbol{C}_k$ represents KPCs at station $k$. The variation at station $k$ is represented by $\boldsymbol{y}_k$. $\boldsymbol{A}_{k-1}$, $\boldsymbol{B}_k$, and $\boldsymbol{C}_k$ are all described in detail in Refs. [30, 31].

Based on the above method, Zhang and Shi [32, 33] conducted variation modeling for compliant composite part assembly. First, they discussed variation modeling for parts assembled in single station. The fixture position error and manufacturing error were considered. Then a variation propagation model was developed with the intention to carry out variation analysis. This model took many factors into account, including relocation-induced error, fixture position error, and part manufacturing error. They linked the final variation to source variations with the state space model. Their model was proved by a two-station assembly process of three composite laminated plates.

As the state space method can show the relationship between fixture deviations and final part variations, it was applied to illustrate the complex propagation of fixture adjustments from station to station by Chaipradabgiat et al. [34]. The objective was to reduce total production expense. A case of assembly in multiple stations proved that the approach was practicable. Kim and Ding [11] applied this method to a four-station assembly process for an automotive side-frame. They only considered the variations caused by the fixtures, which is expressed by $\widehat{\boldsymbol{y}}$. The sum of product deviations, $\widehat{\boldsymbol{y}}^{\mathrm{T}}\widehat{\boldsymbol{y}}$, was used to evaluate whether the product dimension is qualified. Their goal was to get a fixture layout which made the part quality insensitive to fixture errors, so they defined a sensitivity index as:

$$S \equiv \frac{\hat{\boldsymbol{y}}^{\mathrm{T}}\hat{\boldsymbol{y}}}{\boldsymbol{u}^{\mathrm{T}}\boldsymbol{u}} = \frac{\boldsymbol{u}^{\mathrm{T}}\boldsymbol{D}^{\mathrm{T}}\boldsymbol{D}\boldsymbol{u}}{\boldsymbol{u}^{\mathrm{T}}\boldsymbol{u}}. \tag{9}$$

Apparently, $\boldsymbol{D}^{\mathrm{T}}\boldsymbol{D}$ plays an important role. Therefore, the measure of $\boldsymbol{D}^{\mathrm{T}}\boldsymbol{D}$ was used to define the sensitivity index. After comparing several optimality criteria, they chose E-optimality, which is the square of the 2-norm of design matrix $\boldsymbol{D}$, to represent the upper sensitivity bound of the fixture layout. Then they adopted an efficient algorithm to find a suitable fixture layout. Their method is helpful to enhance the robustness of the fixture layout. However, they did not take the influence of the slot orientations into consideration. Tian et al. [13] presented a method which could design pins' positions and slot orientations. In this study, the assembly process of the automotive inner panel was utilized to explain how to design a robust fixture layout. Considering that slot orientations can be any value between 0° and 180° and they are not always the same with the angle between the global and local coordinate systems, Tian et al. modified the matrix which transformed the fixture error to the part locating error. This modeling method was proved to be effective in experiments. After this study, the state space modeling method was used in the fixture optimization by several other researchers [10, 14, 15, 35]. These researchers have made the state space modeling method more than just stay in the theoretical research stage. State space method is widely used in variation modeling of multi station assembly. This method focuses on variations rather than deformations. FEM introduced next mainly focuses on the deformation of parts.

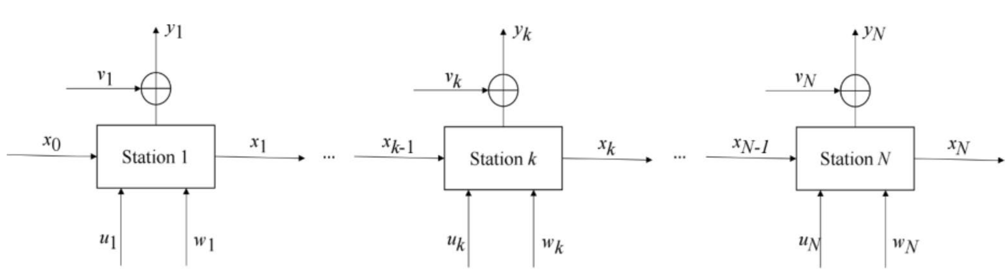


**Figure 4** A generic $N$-station assembly system

#### 3.1.3 Finite Element Method

FEM is a very accurate way to calculate the part deformation. It divides the thin-walled part into element grids to form the model of the actual part. Each element has a simple shape (such as square and triangle). Then, constraints and loads are added to make the finite element model close to reality as much as possible. In this way, we can get the stiffness matrix of each element. The unknown quantity on each element is the displacement of each node. These nodes are the connection points of the elements. We can combine the stiffness matrices of these individual elements to form the total stiffness matrix of the whole thin-walled part. Then we give the known force and boundary conditions to obtain the unknown displacement. The stress in each element can be calculated from the change of displacement on the node.

Due to its good performance in analyzing deformation, FEM is usually used for fixture layout optimization. It was applied to the fixture design and synthesis by Haynes and Lee [36]. Then the FEM was utilized by DeVries and Menassa [37]. They described how the FEM played a role in computing the deflections of the part. Then they proved this method with three different loading cases. Zhong and Hu [38] presented a method to express the part geometric variation. The static variation was calculated using the finite element analysis (FEA) software. The method could be generalized to the case with N fixtures.

When using FEM, linear elastic deformation assumption is adopted, so the global stiffness equation for the thin-walled part can be written as:

$$\boldsymbol{K}\boldsymbol{u} = \boldsymbol{f}, \tag{10}$$

where $\boldsymbol{f}$ means force vector, $\boldsymbol{u}$ means displacement vector, and $\boldsymbol{K}$ means the global stiffness matrix. Researchers usually calculate and analyze displacement directly in the FEA software without deriving the stiffness matrix. In other words, the calculation process is a black box. We input constraints and forces and get deformation results without considering the intermediate process or the modifications of the stiffness matrix. The FEA software ANSYS is widely used. Chen et al. [39] calculated the deformation of machined surfaces of parts with the help of ANSYS. Researchers wrote parameters including the clamping force and the fixture positions to a text file. These parameters can then be read and calculated by ANSYS. Then the results turned into fitness values in the GA procedure. The method that combines ANSYS and MATLAB is popular. Liao et al. [40] and Vishnupriyan et al. [41] adopted this method to find the best locations of fixtures or optimize fixture layout and clamping forces simultaneously. In the optimization process, ANSYS environment for assembly variation computation and MATLAB environment for optimization procedure exchange data directly. Kumar and Paulraj [42] presented the optimization of the locations of clamps and locators using GA with ANSYS parametric design language (APDL). Wu et al. [43] used ANSYS to reduce the deflections of the flexible blade. Meanwhile, ABAQUS is another finite element software. Hajimiri et al. [44] calculated the part deformation under clamping and other forces using ABAQUS software. Xiong et al. [45] and Yang et al. [46] fixed the parameters of the FEA model with Python.

In papers mentioned above, researchers only thought about static or quasi-static forces. They did not take the dynamic response of the part into account. Considering the dynamic forces, Dou et al. [47] employed APDL to compute the objective function to find an optimal fixture layout. For the composite fuselage, Wen et al. [48] realized the control of the fuselage shape through FEA. They used ANSYS to build the finite element model, calibrated the model through the actual measurement results, and then optimized the actuators' positions to realize shape control.

In addition to calculating the deformation directly in the FEA software, some researchers also chose to derive and modify the stiffness matrix to calculate the deformation. Such behavior can be summarized as building a finite element solver. Du et al. [23] adopted direct stiffness method to obtain the deformation of each node. They modified the stiffness matrix according to modification rules proposed by Wu et al. [49], and then obtained the deformation according to Hooke's law. Liu and Hu [50] adopted the method of influence coefficients (MIC) to deal with stiffness matrix. Aderiani et al. [51] also utilized MIC to optimize parameters including the location and number of clamps, slot orientation, and type and location of hole and slot simultaneously. In order to reduce the amount of calculation by FEM, Sayeed et al. [52, 53] proposed a linear mixed-integer programming formulation. They utilized a statically reduced finite element model and used indicator variables to find the connection between the boundary conditions of the finite element model and the fixture positions. The computational efficiency was improved with the use of these variables because researchers did not have to reduce and inverse the stiffness matrix repeatedly.

FEA is the most common modeling method. Through FEA, the deformation of the part can be known accurately. However, dividing the part into multiple small elements also means that the amount of calculation increases, thus the calculation time increases and the efficiency of fixture layout optimization decreases. To enhance optimization efficiency, data-based modeling methods are applied.

### 3.2 Data-Based Modeling Methods

In addition to mechanism-based modeling methods, there are data-based modeling methods. The data are mainly obtained by FEA. There are two typical kinds of data-based methods: regression modeling methods and ANN methods. Regression modeling is a traditional method to find the relationship among variables. Multiple groups of data are obtained through experiments, and the type of regression model is determined according to the data distribution. Then the model parameters are estimated, and finally the mathematical relationship between the dependent and independent variables is obtained. The ANN is a new modeling method with the development of machine learning, it is made up of an input layer, one or more hidden layers and an output layer. The ANN method can represent the complex relationship between multiple inputs and outputs. In this section, we will introduce and analyze the regression modeling methods and ANN methods.

#### *3.2.1 Regression Modeling Methods*

Different kinds of regression methods have been used to solve fixture layout optimization problems, such as RSM, support vector regression (SVR), and partial least squares regression (PLSR).

RSM is a method which combines mathematics and statistics. For the problems where there are many design parameters, RSM is an effective tool [54]. In the response model, the expected response can be linked to the independent variables with Eq. (11):

$$y = f(x_1, x_2, x_3, \ldots, x_n) \pm \epsilon, \tag{11}$$

where $y$ is the expected response. In fixture layout optimization problem, $y$ is generally the quantity related to the optimization objective, such as the maximum deformation or the variation of the part. $f$ means the response function. $x_1, x_2, x_3, \ldots, x_n$ are the position where the fixtures are located. $\epsilon$ means the fitting error. What's more, $f$ appears as a surface when plotted. A two-stage RSM was developed by Li et al. [55] according to the data from FEA to optimize the fixture layout. An enhanced polynomial RSM was presented to improve the model's precision. The method was applied to the robust fixture design for thin-walled parts which were assembled with resistance spot welding [56]. Sundararaman et al. [54] used RSM to model part deformation. They tested the model they proposed and the results of the model matched the simulated data very well. Then they continued to use this method to reflect the relationship between fixture layouts and maximum deformation of the part [57]. Xia et al. [58] built up models based on RSM and 3DCS. RSM was used in the welding process of high-speed train body sidewall as well [59]. RSM may have overfitting problems. With the aim to improve generalization ability of the model, SVR is proposed.

Support vector machine (SVM) was first proposed by Vapnik [60]. Its regression version, named SVR, introduces Vapnik's $\varepsilon$-insensitive loss function into SVM. SVR can solve nonlinear regression problems conveniently. The training data can be mapped to a higher dimensional feature space by SVR. Eq. (12) links the input to the output:

$$\hat{F}(X) = W\varphi(X) + b, \tag{12}$$

where $\varphi(X)$ represents the feature which is mapped from $X$ nonlinearly; $W$ and $b$ are the adjustable coefficients. $X$ is the normalized fixture position and it is the input value. $\widehat{F}(X)$ is the output value, usually the quantity related to the deformation of the part. The way to adjust the coefficients was described in Refs. [61–63]. A SVR-based approach was developed by Su et al. [61] to figure out the influences of the clamps and the temperature on the surface shape error of the production. Yang et al. [62] established the relationship models between overall deformation and fixture layout and between maximum deformation and fixture layout respectively. The effect of SVR relies on the kernel function, so researchers should select parameters cautiously when constructing regression model.

Kriging is also used to model the relationship between optimization objectives and fixture layout. It occurred in geo statistics for the first time while Sacks et al. [64] popularized its use. The basic form of the Kriging model is described as:

$$y(X) = F(\beta, X) + z(X), \tag{13}$$

where $y(X)$ expresses the deterministic response, which is generally represented by the part deformation. $X$ expresses input variables. In fixture layout optimization problem, it means the position of fixtures. $F(\beta, X)$ means a linear regression of $\beta$, and $z(X)$ means the fitting error. Concrete expressions of $F(\beta, X)$, and $z(X)$ can be seen in Ref. [64]. Yang et al. [65] built the kriging surrogate model to express the relationship between fixture layouts and thin-walled part deformations. Through FEA and Latin hypercube sampling, they obtained the training data and test data. After that, they constructed Kriging model and BPNN respectively. By comparing the errors between the predicted and real value, it was found that the Kriging model has higher accuracy. Therefore, they constructed the objective function through the Kriging model. To make the model more accurate, Yue et al. [66] considered unquantified uncertainty, modeling uncertainty, part uncertainty, and actuator uncertainty. They built up the model to control the shape of the fuselage. Kriging method considers the relationship among known data, so it has high accuracy.

When there are few sampling data or under the condition of serious multiple correlations of independent variables, PLSR can be used. Bi et al. [19] established the multivariate exponential regression model regarding the strain energy as response variables and the fixture layout parameters as predictor variables. PLSR was used to help establish the fundamental relationship considering the multicollinearity among design parameters. The PLSR method was introduced in Ref. [67] in detail.

In addition, grey prediction model is another kind of model used to model the relationship between optimization objectives and fixture layout. As a popular method, the grey model has been applied in many fields since it was presented by Deng [68]. Yang et al. [69] constructed the grey model to link the maximum deformations of the parts to the number of fixtures.

The advantage of the regression modeling method is that the results can be obtained without a large amount of data, which reduces the work required to collect data compared with the ANN method. Also, through the analysis of the regression model, we can know how the design parameters affect the deformation. However, when more fixture locating parameters are involved, the regression model can hardly reflect the complex relationship between fixture locating parameters and responses. Thus, many researchers applied ANNs in the modeling for the fixture layout optimization.

#### *3.2.2 Artificial Neural Network Methods*

Since machine learning is developing rapidly, a great number of ANNs are used in the modeling for the fixture layout optimization. ANNs are inspired by the human brain. The typical architecture of a neural network is shown in Figure 5. It is necessary for networks to have input and output layers, and at least one hidden layer. In fixture layout optimization process, common input data includes fixture position coordinates, fixture position nodes, clamping force and other parameters. Input data is received by the cells of the input layer. The processing of data is done by the cells of the hidden layer. Then the processed data is given by output cells. Usually, researchers take the statistics of deformation at each node of the part as the output. In order to make the network models available, the models need to be trained and tested. Basically, data of training dataset and test dataset come from FEA and designed sampling. Through learning from representative datasets, ANNs link the output(s) to the fixture layout parameters.

With the continuous development of technology, more ANN models have been created, among which the representative one is the feed-forward neural network model. There are two commonly used networks. They are radial basis function neural network (RBFNN) and back-propagation neural network (BPNN).

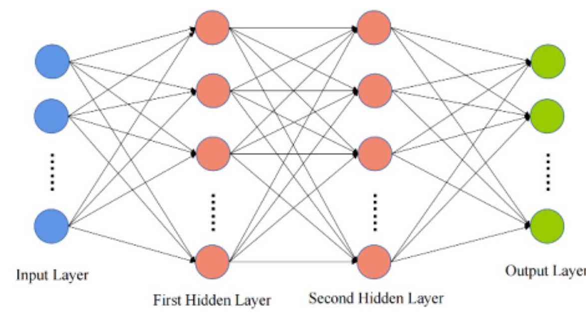


**Figure 5** Architecture of a typical neural network

Back propagation is an important step in neural network training. With back propagation, the weight of neural network is adjusted according to the error rate obtained in the previous iteration. By properly adjusting the weight, the generalization ability of the model is improved. This method is helpful to calculate the gradient of a loss function relative to all the weights in the network. Selvakumar et al. [70] proposed an ANN-based method. They utilized a trained BPNN to estimate the maximum deformations for different fixture layouts. Rex and Ravindran [71] established an ANN–based model to estimate the part deformation for possible fixture layouts. Qin et al. [72, 73] used BPNN to obtain the clamping deformation from multiple fixturing parameters. Ramachandran et al. [74] used BPNN to approximate the elastic deformation in order to find an optimal fixture layout for the engine mount bracket.

Compared with BPNN, RBFNN has faster convergence speed, and it is easier to obtain the best approximation than BPNN. Therefore, in recent years, some researchers chose to build RBFNN to reflect the relation between part deformation and fixture layout parameters. Wang et al. [75, 76] proposed an RBFNN model to help optimization of fixture layout. Ma et al. [77] combined GA with RBFNN to find an optimal fixture layout.

With the support of a large amount of data, ANNs have a better modeling effect and can better reflect the actual situation. They can reflect the complex relationship between deformation and multiple fixture design parameters. However, we cannot know how the design parameters affect the deformation because the relationship function obtained by the ANN is a black box.

### 3.3 Comparative Analysis of Different Modeling Methods

Based on Section 3.1 and Section 3.2, we compare and analyze the mentioned modeling methods, as shown in

**Table 2** Comparison of different modeling methods

| | Methods | | Scope of application | Advantages | | Limitations | References |
|---|---|---|---|---|---|---|---|
| Mechanism-based modeling methods | Jacobian matrix method | | Single-station process<br>Robust design<br>Rigid assumption | Simple calculation<br>Direct reflect the influence of fixture deviations on part variations | | Applicable to rigid assumptions<br>Limited for complex problems | Cai et al. [25], Cai [5], Xing et al. [26] and Lu et al. [27] |
| | State space method | | Multi-station process<br>Variation propagation<br>Considering source variations | Suitable for multi-station assembly<br>Considering various source variations | | Cannot be used for complex problems<br>Complicated derivation and calculation | Masoumi et al. [10], Kim and Ding [11], Tian et al. [13], Huang et al. [14], Xie et al. [15], Jin and Shi [30], Ding et al. [31], Zhang and Shi [32, 33], Chaipradabgiat et al. [34], Tyagi et al. [35] |
| | FEM | | Considering deformations caused by force<br>Compliant assumption | Intuitively reflect the deformations at nodes<br>Easy to understand its principle | | Large amount of calculation<br>Calculation accuracy depends on FEA software | Du et al. [23], Haynes and Lee [36], DeVries and Menassa [37], Zhong and Hu [38], Chen et al. [39], Liao et al. [40], Vishnupriyan et al. [41], Kumar and Paulraj [42], Wu et al. [43], Hajimiri et al. [44], Xiong et al. [45], Yang et al. [46], Dou et al. [47], Wen et al. [48], Wu et al. [49], Liu and Hu [50], Aderiani et al. [51], Sayeed et al. [52, 53] |
| Data-based modeling methods | Regression modeling methods | RSM | Large amount of calculation<br>Multiple input variables | Simple calculation | Reduce the amount of calculation<br>Require a small amount of data | Model accuracy depends on data and parameters<br>Requirements for experimental design and data sampling<br>Time for parameter optimization | Sundararaman et al. [54, 57], Li et al. [55, 56], Xia et al. [58], Yu et al. [59] |
| | | SVR | | Good generalization ability | | | Su et al. [61], Yang et al. [62] |
| | | Kriging | | Stable accuracy | | | Yang et al. [65], Yue et al. [66] |
| | | PLSR | | Suitable for multiple outputs | | | Bi et al. [19] |
| | | Grey model | | Low requirements for data | | | Yang et al. [69] |
| | ANN methods | BPNN | Large amount of calculation<br>Multiple input variables | Relatively simple structure | Reflecting the complex relationship between inputs and outputs<br>More accurate | Need a large amount of data<br>Not interpretable<br>Long time to obtain the model | Selvakumar et al. [70], Rex and Ravindran [71], Qin et al. [72, 73], Ramachandran et al. [74] |
| | | RBFNN | | Fast convergence<br>Global approximation | | | Wang et al. [75, 76], Ma et al. [77] |

Table 2. There are two kinds of modeling approaches: Mechanism-based and data-based modeling approaches. The commonly used mechanism-based modeling methods include Jacobian matrix method, state space method and FEM. Their application scenarios are different. For single-station assembly, under the rigid assumption, only the variations caused by fixture errors are considered. To enhance the robustness of fixture layout, the Jacobian matrix method is applied to directly reflect the influence of fixture errors through simple calculation. However, Jacobian matrix method is only suitable for rigid assumption. It cannot be used to solve complex problems. The state space method is suitable for multi-station assembly. With this method, researchers can build a variation propagation model which can show how the source variations of each station affect the final variation. When modeling with state space method, various variations including part manufacturing error and fixture deviation can be considered. One of its advantages is that it is suitable for multi-station assembly. In addition, due to the consideration of multiple source variations, the result is closer to the real situation. However, its derivation process is complicated and is not suitable for solving complex problems. FEM is a major method in fixture layout optimization. Under the compliant assumption, the force and deformations of parts are considered. Its advantages are that its principle is simple and easy to understand, and it can directly represent the deformations at the mesh nodes. Its limitation is that when too many meshes are divided, its computation will increase greatly.

When the size of parts and number of fixtures is large, using mechanism-based modeling methods will make the amount of calculation increase. Therefore, data-based modeling methods are widely used. Regression model methods and ANN methods are typical data-based modeling methods. The regression models mentioned above include RSM, SVR, Kriging, PLSR and grey model. Each model has its own advantages. In general, the advantage of using regression model is that the relationship between fixture layout and part deformation can be obtained through a small amount of data, which greatly reduces the amount of calculation. However, the model's precision relies on the selection of data and coefficients of the model. Therefore, to establish a model with high accuracy, researchers need to carry out reasonable experimental design and scientific data sampling for the thin-walled parts' assembly in the early stage. To enhance the model's precision, algorithms are needed for optimizing the parameters. This work is time-consuming and difficult.

Since the computer technology has developed a lot, ANN methods, such as BPNN and RBFNN, are used in fixture layout optimization. The advantage of ANN methods is that they can express the complex relationship between the multi-inputs (fixture layout) and multi-outputs (assembly variation or deformation). However, the limitation is that training an ANN with high accuracy requires a large amount of data, which means we need collect enough assembly variation or deformation data and corresponding fixture layout. What's more, due to the complexity of its structure, an ANN needs long training time.

Through comparative analysis, we found the scope of application of each modeling method, their advantages and limitations. When choosing the modeling method, we should consider the optimization objectives and design variables of the research problem. Moreover, we should consider the accuracy requirements and time cost, and select the appropriate method.

### 3.4 Epilog

This section introduces several modeling methods commonly used in fixture layout optimization, which consist of mechanism-based and data-based modeling approaches. Among the former, Jacobian matrix and state space method are suitable for single-station and multi-station, respectively. The FEM is suitable for the out-of-plane deformation analysis. Because the calculation of FEM is complex and it is hard to show the influence of several design parameters on the results, data-based modeling methods came into being. The data are from sampling and FEA. Regression modeling methods and ANNs are two different data-based modeling methods. At last, we compare and analyze several modeling methods. Through modeling, we get the relationship between fixture layout parameters and optimization objectives, which lays a foundation for subsequent optimization using algorithms.

## 4 Algorithms for Fixture Layout Optimization

For fixture layout optimization, the last step is to select the appropriate optimization algorithm to find an optimum layout strategy. Optimization algorithms basically consist of two categories: Traditional nonlinear programming and heuristic algorithms. Traditional nonlinear programming algorithms mainly use mathematical programming techniques to describe the quantitative relationship of logistics systems. Different from traditional nonlinear programming algorithms, heuristic algorithms are based on intuition or experience. A heuristic algorithm gives a feasible solution to the optimization problem with limited computing time and space. But the feasible solution may not be the same as the optimal solution. In addition, there are other methods that can solve the optimization problems [78–81], but they are specific to the problems and are not the mainstream methods in the fixture layout. In this section, we will make a detailed analysis of the application of traditional nonlinear programming and heuristic algorithms in the fixture layout of thin-walled part assembly and make a comparative analysis of them.

### 4.1 Traditional Nonlinear Programming Algorithms

Fixture layout optimization problems are usually regarded as nonlinear programming problems. There are one or several nonlinear functions in the objective function or constraints. The basic form of nonlinear programming is as follows:

$$\begin{aligned}
&\min F(X),\\
&\text{s.t.}, AX \le b\\
&(\mathit{Linear\ inequality\ constraints}),\\
&Aeq \cdot X = beq\\
&(\mathit{Linear\ equality\ constraint}),\\
&C(X) \le 0\\
&(\mathit{Nonlinear\ inequality\ constraints}),\\
&Ceq(X) = 0\\
&(\mathit{Nonlinear\ equality\ constraints}),\\
&VLB \le X \le VUB\\
&(\mathit{Bounded\ constraint}),
\end{aligned} \tag{14}$$

where $F(X)$ is the optimization objective, and $X$ means the decision variable. In fixture layout optimization, decision variables are usually related to the fixture position.

There are many methods to solve nonlinear programming problems. Generally, there are two categories. One is to reduce the objective function in the feasible region. The other is to construct the augmented objective function to make constrained problem unconstrained. The representative of the first kind is the feasible direction method. The initial feasible solution then changed into a better feasible solution. Li and Melkote [21] used Zoutendijk's method of feasible direction to reduce displacements caused by part locating error. In this study, in addition to fixture position, clamping force and rigid body motion were taken as decision variables because they are uniquely determined by fixture layout.

Typical examples of the second kind are sequential quadratic programming (SQP) and Lagrange method. DeVries and Menassa [37] applied the Broyden-Fletcher-Goldfarb-Shanno method to design the optimal positions of fixtures. The SQP technique is also employed to help optimization. An optimization software named VMCON, employs SQP technique [6]. Li and Melkote [22] improved the SQP algorithm. They designed an iterative synthesis algorithm. Firstly, the algorithm searched for the optimal clamping force and layout under the initial number of fixtures. It calculated the positioning error, and observed whether it met the tolerance requirements. If not, the algorithm would increase the number of fixtures. Lagrange method is another method to eliminate constraints. Its basic idea is to introduce Lagrange multipliers to transform constraints into variables. Li et al. [12] used the Lagrange conditional extremum method. In their research, they constrained the location of selectable points for fixtures and the square root of the maximum eigenvalue of the sensitivity matrix. Then these constraints were eliminated by introducing Lagrange operator. Considering that the large-scale workpieces are commonly used in practical engineering, Zhang et al. [82] came up with a method on the basis of augmented Lagrange method. They formulated a multi-constrained fixture layout optimization problem. The constraints consisted of two parts: equality constraints and inequality constraints. The equality constraints were obtained by kinematic analysis and kinetic analysis, and the inequality constraints were obtained by the coulomb law and surface quality requirements. Then, the proposed method was used to transform the problem into an unconstrained one.

Traditional algorithms can only effectively solve small and medium-sized problems. However, the amount of calculation generally increases exponentially with the increase of the problem scale, such as the increase of fixture number and the dimension of the thin-walled parts, which will lead to the exponential explosion problem. Considering the rapid development of artificial intelligence technology, the traditional algorithms only suitable for small-scale calculation are gradually replaced by heuristic algorithms.

### 4.2 Heuristic Algorithms

For the large size thin-walled parts, such as ship hull and fuselage, the number of "N" locators will increase dramatically. When we optimize the fixture layout for these parts, the scale of problem exponentially increases, the amount of calculation and storage space required to solve the optimal solution of these problems will grow very fast, which makes it almost impossible to obtain the optimal solution through various traditional algorithms under the existing computing power. In this case, heuristic algorithms came into being. Heuristic algorithms are suitable for solving large-scale problems and are widely used in layout optimization [83–85]. In this section, we will discuss some commonly used methods, such as GA and PSO.

#### *4.2.1 Genetic Algorithm*

The GA simulates the natural evolution of biological creatures to search for an optimal solution. Several fundamental operations are consisting of the GA mechanism, such as reproduction, crossover, and mutation [86]. In fixture layout optimization problems, the numbers of finite element grid nodes are usually used to represent the position of the fixtures. At this time, integer coding or binary coding is used to represent the variables. In addition, some researchers prefer directly using coordinates to represent the fixture position. In this case, real number coding is generally used to represent variables. After determining the coding mode, multiple groups of fixture layouts are initialized, and the fitness value is calculated. Then, new solutions are generated through reproduction, crossover and mutation. The process continues until the maximum number of iterations or other termination conditions are reached. More details about reproduction, crossover and mutation are introduced in Ref. [87]. After several iterations, the solution satisfying the conditions is obtained, and then the feasible solution is decoded into fixture layout parameters.

Krishnakumar and Melkote [88] proposed a GA-based method. They compared GA with traditional nonlinear programming methods and discussed the proposed method's advantages. GA can be used to solve problems without a clear mathematical expression that link the objective function to the decision variables. Vallapuzha et al. [87, 89] compared several fixture layout optimization methods from three aspects of solution quality, solution repeatability, and computation time. They found that GA had the best overall performance. Rex et al. [90] used a discrete GA with mixed integer-discrete variables to optimize the fixture layout. Researchers have applied GA to fixture layout optimization of different kinds of parts, such as flexible aerospace parts [45], car dashboards [91, 92], and near-net-shaped jet engine blades [93, 94]. With the hope to make the algorithm perform better, Cheng et al. [95] improved the GA. They discussed the genetic and ants manipulations respectively, and connected these two parts to enhance the performance of their method in fixture layout optimization. To improve the computational efficiency, Xing et al. [96, 97] proposed the filtering methods based on manufacturing constraints so that there would be a smaller candidate pool for locating points. The improved GA can also solve the multi-objective optimization problem. Yang et al. [62] used the elitist nondominated sorting GA to conduct the optimization with multiple objectives. The two optimization objectives were to minimize the overall deformation and maximize the deformation. In addition to the above optimization problems considering deformations, GA can also be used for robust fixture design. Tian et al. [13] presented a methodology on the basis of GA for the robust fixture layout design.

The above literature takes the fixture position as the decision variable. In addition to the fixture position, some researchers also considered the simultaneous optimization of other design parameters. Kulankara et al. [98] optimized clamping force and fixture layout with GA. They proposed an iterative algorithm by changing the clamping force and fixture layout alternatively. Another fixture design parameter is the number of fixtures. Liao [99] proposed a GA-based methodology to optimize the number and positions of fixtures. Chen et al. [39] developed a GA-based method. Their aim was to reduce deformation and make the deformation distribution as uniform as possible. Hajimiri et al. [44] improved the GA optimization method. They introduced fixturing regions and fixturing sequence into design variables.

All the above studies show that GA is a widely used optimization algorithm in the fixture layout optimization. Because GA uses binary coding to represent variables, it is more suitable for solving discrete problems. In fixture layout optimization, sometimes the optimized parameters are continuous variables like fixture position coordinates. Therefore, a heuristic algorithm more suitable for solving continuous problems is needed.

#### *4.2.2 Particle Swarm Optimization*

Different from the GA suitable for discrete problems, PSO is suitable for solving continuous problems. PSO was first proposed to solve optimization problems by Kennedy and Eberhart [100]. It uses a swarm of particles to conduct a search. Each particle represents a possible solution for the fixture layout and it updates its velocity and position in order to obtain the best particle. More details can be seen in Ref. [100]. In fixture layout optimization problems, the fixture position is generally represented by grid node index. When integer coding is used, each dimension of a particle represents the position of one fixture. If there are $N$ fixtures on a part, the particles will have $N$ dimensions. Some researchers use binary coding. In this case, a set of binary numbers represents the number of nodes where a fixture is located. Suppose there are $N$ fixtures waiting for positioning, then the dimension of a particle is $N \times m$, where $m$ represents the number of bits of a set of binary numbers. Sometimes, coordinates are used to represent the fixture positions. At this time, real number coding is adopted. The dimension of particles is related to the number of fixtures and the number of coordinates required for the positioning of a fixture. As PSO can be helpful to find a suitable position in a multidimensional space, it is propitious to the locator positions optimization and has attracted many scholars' attention.

Researchers utilized PSO to optimize positions of fixtures. Dou et al. [101] used a particle library to make the optimization more efficient. Then they improved the PSO algorithm by embedding the mutation operator to help PSO achieve better global optimization results [47]. By balancing global optimization capability and convergence speed, the improved PSO was better than GA and unimproved PSO when solving the fixture layout optimization problems. Zhou et al. [102] applied PSO algorithm to the locator layout of the door of a car. Sundararaman et al. [57, 103] used RSM to connect the positions of locators and clamps to the maximum part deformation. GA and PSO were applied to optimize the developed model. Researchers compared different algorithms' performances. The result showed that the approach which integrated RSM and PSO performed better. Xing et al. [104] improved the PSO algorithm to carry out the optimization. The traditional discrete binary PSO algorithm was improved in the aspects of individual optimization function, cross-boundary particle processing, and reduction of searching area for locating layout optimization problems. A body floor assembly case was studied to explain the method developed by them. The shortcomings of PSO are that the convergence speed is too fast and the local search ability is weak. In order to get a better solution, many other heuristic algorithms are applied to assist fixture layout optimization.

#### 4.2.3 Other Heuristic Algorithms

Since artificial intelligence is developing, a great number of heuristic algorithms have been developed. ACO, social radiation algorithm (SRA), simulated annealing (SA) algorithm, and other heuristic algorithms including artificial bee colony algorithm [74], bat algorithm [76] and cuckoo search algorithm [46, 65] have been used to solve fixture layout optimization problems. Next, we introduce some applications of ACO, SRA and SA in the fixture layout optimization.

ACO simulates the foraging behavior of ants [105]. Prabhaharan et al. [106] used GA and ACO separately to reduce the form and dimensional errors of parts. Experiments showed that ACO reports faster and more accurate solutions. Padmanaban and Prabhaharan [107] employed ACO and GA so that the dynamic response of the part can be minimized. Padmanaban et al. [108] came up with an approach on the basis of ACO. The result showed that the ACO-based continuous method was superior to the discrete method. In order to adapt to different types of problems, researchers have improved the ACO. An augmented ACO was developed to optimize fixture layouts for rigid parts [15]. A case of a three-station assembly process was studied and results displayed that the augmented ACO performed better than the basic ACO. Khodabandeh et al. [109] came up with an ACO-based approach for a multi-objective problem. Their aim was to minimize deformation and number of fixtures simultaneously. Their method optimized the positions of fixtures and the number of clamps at the same time.

SRA is inspired by the development of human society. Population of individuals with different capabilities and radiations are generated randomly. The individual can update its capability and radiation. Xing et al. [110] used SRA to optimize the locators' positions and compared the algorithm with GA. The results of the case of side frame assembly showed that the SRA was more efficient. They used a non-domination sorting SRA to design a fixture layout which synchronously satisfied the quality requirements [111].

SA algorithm is another popular heuristic algorithm for optimization. SA simulates the annealing procedure of the metal working [112]. Du et al. [23] applied the SA algorithm to design an optimal fixture layout to reduce the assembly gap in the ship assembly process. Pan et al. [113] optimized the clamping positions with the adaptive SA, multi-island GA, and PSO algorithm. Their purpose was to minimize the deformation. The result showed that the adaptive SA performed better.

The application of heuristic algorithm makes it possible to solve large-scale optimization problems. But there is a common shortcoming of heuristic algorithm that it is easy to be trapped in local optimization zone. Therefore, when using heuristic algorithm, we need to carefully adjust the parameters to improve the performance of the algorithm.

### 4.3 Comparative Analysis of Different Optimization Algorithms Used in Fixture Layout

Based on the previous content, we compare and analyze different optimization algorithms. In Table 3, we list the feasible direction method, SQP, Lagrange multiplier method and three commonly used heuristic algorithms, including GA, PSO and ACO. Firstly, we analyze the traditional nonlinear programming algorithms. One of the advantages of the feasible direction method is that the fixture position found in each iteration meets the constraints. In addition, better fixture layout will usually be obtained using feasible direction method. SQP and Lagrange multiplier methods are more commonly used. SQP not only has global convergence, but also has super linear convergence rate. It is an effective algorithm to solve simple fixture layout optimization problems. Faster convergence rate means that the suitable fixture position can be found faster. The convergence rate of Lagrange multiplier method is affected by the selection of penalty function. It also has global convergence. In addition, by introducing Lagrange multipliers, constraints are transformed into variables so that the constrained problem becomes an unconstrained problem. Fixture layout

**Table 3** Comparison of different optimization algorithms

| | Methods | Application | Advantages | Limitations | References |
|---|---|---|---|---|---|
| Traditional nonlinear programming algorithms | Feasible direction method | Small and medium-sized problems | Keep the feasibility of the solution<br>Local optima can usually be found | Only applicable to small and medium-sized problems<br>The result depends on the initial solution | Li and Melkote [21] |
| | Sequential quadratic programming | | Fast convergence<br>High computational efficiency<br>Global convergence | | Li and Melkote [22], DeVries and Menassa [37] |
| | Lagrange multiplier | | Global convergence<br>Simplify the problem | | Li et al. [12], Zhang et al. [82] |
| Heuristic algorithms | GA | Large scale problem | Global search capability<br>Scalable and easy to combine with other algorithms | Slow convergence<br>Complex encoding and decoding<br>Results are affected by the parameters of algorithms | Tian et al. [13], Chen et al. [39], Vishnupriyan et al. [41], Hajimiri et al. [44], Yang et al. [62], Krishnakumar and Melkote [88], Vallapuzha et al. [87, 89], Chen et al. [91], Liu et al. [92], Zhang et al. [93],Wu et al. [94], Zeshan Ahmad et al. [114], Cheng et al. [95], Xing et al. [96, 97], Kulankara et al. [98], Liao [99] |
| | PSO | | Fast convergence<br>Few parameters need to be adjusted | Poor local search ability<br>Easy to fall into local optimization<br>Results are affected by the parameters of algorithms | Dou et al. [47], Dou et al. [101], Zhou et al. [102], Sundararaman et al. [57, 103], Xing et al. [104] |
| | ACO | | Easy to combine with a variety of heuristic algorithms<br>Good robustness | Slow convergence<br>Easy to fall into local optimization<br>Results are affected by the parameters of algorithms | Xie et al. [15], Prabhaharan et al. [106], Padmanaban and Prabhaharan [107], Padmanaban et al. [108], Khodabandeh et al. [109] |

optimization problem generally contains multiple constraints. Through this method, the optimization process is greatly simplified.

The common problem of traditional nonlinear programming algorithms is that they are only suitable for solving small and medium-sized problems. However, most fixture layout optimization problems are large-scale problems. When the size of thin-walled parts becomes large or the number of fixtures is large, the calculation will become very complex when using traditional algorithms. In addition, this kind of algorithm is highly dependent on the initial solution, and the final solution has a close connection with the selection of the initial solution. In fixture layout optimization, the initial fixture positions are usually selected at random. In this way, the stability of the final solution will not be guaranteed. These are the two main limitations of this kind of algorithm.

Because of the limitations of traditional nonlinear programming algorithms, heuristic algorithms are widely applied to optimize the fixture layout. There are many kinds of heuristic algorithms. At present, GA, PSO and ACO are widely used. The advantage of GA is that it has good global search ability and can be combined with other algorithms easily to enhance its performance. However, the convergence rate of GA is slow. In addition, when using GA, it is often necessary to encode and decode the fixture layout parameters, which is very cumbersome. Another commonly used heuristic algorithm is PSO. The variables of PSO are usually the sequence number of grid nodes. Different from GA, PSO has faster convergence speed and fewer parameters to be adjusted, so it is more convenient to use. However, too fast convergence speed will make PSO easier to be trapped in local optimization zone. In addition, the local search ability of PSO is poor and thus the search accuracy is not high. In the fixture layout optimization problem, this means that the fixture layout found out is not optimal. ACO is also a commonly used algorithm in fixture optimization problem. Its advantages are that it can be combined with other algorithms conveniently and has good robustness. Therefore, the optimized fixture layout is rarely affected by the initial fixture layouts. Its limitations are that it has slow convergence speed. Thus, using ACO to find the optimized fixture layout will take more time. What's more, it will be trapped in local optimization zone easily, which means that there is still difference between the found fixture layout and the optimal fixture layout.

To sum up, optimization algorithms applied in the fixture layout optimization can be divided into traditional nonlinear programming algorithms and heuristic algorithms. Traditional nonlinear programming algorithms are suitable for solving small and medium-sized problems while the actual fixture layout optimization problems are often large-scale. Fixture layout optimization usually involves multiple fixtures and many positions to be selected, so it is usually a large-scale problem, which means the heuristic algorithms are widely used. It is easy for heuristic algorithms to be trapped in local optimization zone. Also, the quality of the obtained solution relies on the parameters of the algorithms. Therefore, researchers need to innovate and improve the algorithms all the time.

### 4.4 Epilog

This section introduces the optimization methods commonly used in fixture layout optimization. The traditional nonlinear programming methods are suitable for small and medium-sized optimization. With the development of artificial intelligence, heuristic algorithms have become the common optimization method for the fixture layout optimization. We introduce the application of several heuristic algorithms in fixture layout optimization, including GA, PSO, ACO and so on. Finally, we compare and analyze the commonly used optimization algorithms.

## 5 Discussion: Future Challenges of Fixture Layout Optimization for Thin-Walled Parts

Because of the progressive technology and the deepening of the research on fixture layout optimization, more advanced technologies can be applied to assist optimization. In this section, we discuss the limitations and the future challenges for the fixture layout optimization from three aspects: The selection of optimization objectives of fixture layout, modeling methods of assembly variation or deformation, and optimization methods applied in fixture optimization. New research points are also put forward to provide a reference for future research on fixture layout optimization. The future research trend is shown in Figure 6. The details will be discussed as follows.

### 5.1 Selection of Optimization Objectives in Fixture Layout Optimization

Through the above literature review, we find that the researches on fixture layout optimization for thin-walled parts consist of two categories: considering in-plane variations and out-of-plane deformations. The existing research work has some limitations. When selecting optimization objectives, researchers usually select only a single optimization objective. In most cases, minimizing the deformation of the plate is regarded as the only optimization goal. Few researchers have studied multi-objective optimization problems. However, there are many inconsistent objectives in the optimization process, for instance, minimizing the overall deformation and maximum deformation of parts at the same time. In this way, although the optimized fixture layout meets the

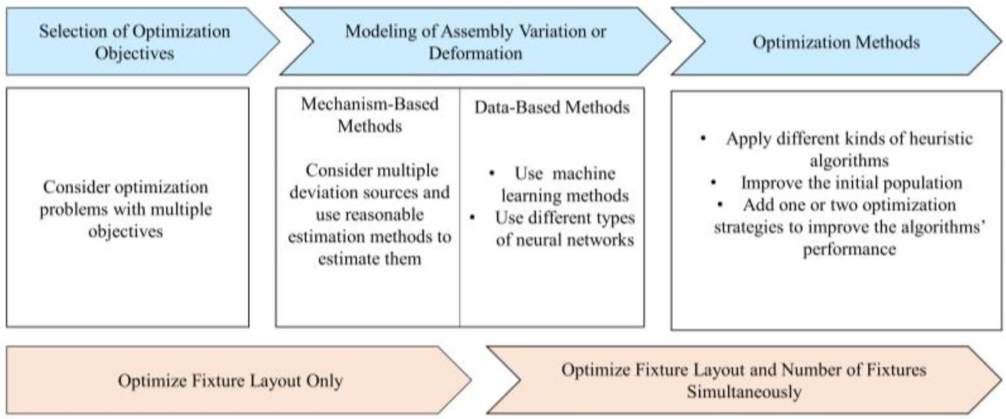


**Figure 6** Future research trend of fixture layout optimization

requirements researchers raised, it may not improve the overall assembly quality of parts.

In the future research, we can consider the design of multi-objective optimization problem. Researchers can consider two conflicting objectives simultaneously. For example, some researchers tried to minimize the average deformation and maximum deformation at the same time. The optimization algorithm can be used to find the Pareto frontier, and then the optimal layout can be determined. Designing appropriate multiple objectives can make the results of theoretical research more suitable for practical needs.

### 5.2 Modeling Methods and Ways to Improve Modeling Accuracy

In this review, there are two kinds of modeling methods: mechanism-based and data-based modeling approaches. When there are many fixture layout parameters as inputs, it is difficult for mechanism-based modeling methods to find the relationship between fixture layout parameters and part deformation. So, they are not convenient for large-scale problems. When mechanism-based modeling methods are used to solve the problems with few variables, it is important to consider multiple variation sources, such as part manufacturing errors and fixture positioning errors. Using appropriate statistical methods to estimate these errors can effectively improve the modeling accuracy. As there are usually multiple stations in thin-walled part assembly process, the variation arise at one station will influence the part deformation. Therefore, to improve the modeling accuracy, it is important to consider the variation brought by previous stations and study the variation propagation law. The material properties of thin-walled parts will also affect the part deformation, so they should be considered when modeling. For example, as the mechanical properties of composite materials are different from those of metal materials, the modeling processes are different when using mechanism-based modeling methods. Many mechanism-based modeling approaches are used under the assumption of linear elastic deformation. However, when the deformation is large, it may not be linear elastic. Therefore, when the deformation is large, it is necessary to consider the nonlinear deformation to build a high-precision model using mechanism-based methods.

With the continuous development of computer technology, we can find that data-based modeling methods are widely applied for fixture layout optimization in recent years. The data-based modeling methods have the advantages of easy calculation and optimization. When there are multiple fixture layout parameters as input, data-based modeling methods can build a simple relationship between part deformation and multiple parameters.

The data-based modeling process is generally as follows. Firstly, a set of data including fixture layouts and corresponding part deformation are obtained through FEA and data sampling. Then, the model is obtained by interpolation or ANN method to predict the part deformation with different fixture layouts. Common interpolation methods include cubic spline interpolation, inverse distance weight interpolation, Kriging interpolation and so on. Among them, Kriging interpolation performs well in the modeling of fixture layout optimization problem. In fact, Kriging method has been used in assembly process. Yue et al. [115] used grouped Latin hypercube sampling for data acquisition, and then used Kriging interpolation method to build a model to predict the deformation of parts. The model they built laid a foundation for deformation control in the assembly process. Some machine learning methods are also used to help build models. Du et al. [116] developed a sparse learning model and the parameter estimation algorithm to help find optimal position of actuators. Then they used a new sparse-learning model to reduce the gap between two pieces of composite fuselage [117]. Qazani et al. [118] combined machine learning methods with many different heuristic algorithms to predict the maximum workpiece deformation with different fixture layout. The method proposed in these papers was used in the fuselage shape control, but it is also instructive for fixture layout optimization. Machine learning methods and Kriging model have good performance in helping to build data-based models and they are the future research directions in fixture layout optimization.

With the rise of intelligent manufacturing, the data in the manufacturing process can be obtained easily by the sensors, which also provides convenience for the construction of data-based models. Therefore, data-based modeling methods will be more widely used. However, how to obtain an accurate model from these data is always a great difficulty. In the current researches, researchers used Latin hypercube sampling to select a set of FEA results for training the model. This method improves the accuracy of the model by improving the data set. This is effective, but the effect is limited. Therefore, in the future research, researchers need to consider how to improve the modeling accuracy. There are many ways to help make the model more accurate. With the development of machine learning, researchers can choose a variety of machine learning algorithms to help establishing the model. In addition, the parameters in the model can be adjusted by grid search, randomized search and other methods. Wang et al. [119] studied the calibration of model parameters. They introduced the concept of sensible (calibration) variables and developed a novel method to identify and determine appropriate values for the sensible variables. This method improved the accuracy of parameters. Yue et al. [120] proposed two new active learning algorithms for the Gaussian process with uncertainties to help build models. The proposed approach was proved to be effective in suppressing the impact of uncertainties. These methods are of great significance to the fixture layout optimization. Ensemble learning is also a good method to make the model more accurate and available. We can build a better model by improving and absorbing experience from the traditional methods. When the relationship between fixture layout parameters and part deformation or other outputs is complex, ANNs can help modeling. At present, few researches have been conducted about the influence of fixture layout on part deformation using ANNs. There are many types of neural networks, and their application in fixture layout optimization is also a direction worthy of research.

### 5.3 Methods of Enhancing the Performance of Optimization Algorithms

In regard of optimization algorithms for the fixture optimization, they are made up of two kinds: the traditional nonlinear methods and the heuristic algorithms. The solution obtained by the traditional nonlinear methods is easily influenced by the initial feasible fixture layout, and these methods are only used to solve small and medium-sized problems. In fact, many fixture layout optimization problems are large-scale problems. If the number of fixtures or the size of the part is large, or the meshes are thin, fixture layout optimization problems will become large-scale problems. Therefore, heuristic algorithms are commonly used. In fact, heuristic algorithms have many applications in the optimization of facility layout [83] and wind farm layout [85], which are worthy of reference. When applying heuristic algorithms, researchers often encounter the following problems. Multiple setting parameters of the algorithms need to be manually debugged and matched according to the specific fixture layout optimization problem, which affects the solution efficiency. What's more, the heuristic algorithms will fall into local optimization easily, resulting in a final fixture layout which cannot effectively help reduce the part deformation. In addition, the convergence speed is also slow, especially when there are many candidate solutions.

To improve the performance of optimization methods, researchers can apply many other heuristic algorithms to optimize fixture layout, and compare them with traditional heuristic algorithms such as GA and PSO from the perspective of convergence speed and solution quality. With the development of computer technology, a great number of heuristic algorithms have been proposed, such as sine–cosine crow search algorithm [121], salp swarm algorithm [122], and equilibrium optimizer [123]. Compared with GA, PSO and other classical algorithms, these new algorithms have their own advantages. When solving the fixture layout optimization problem, researchers can try to find the most suitable algorithm according to the specific problem.

Another major direction of future research is to improve heuristic algorithms. The algorithms can be improved from two aspects. One is to improve the initial population. In many cases, the initial positions of fixtures are generated randomly, which leads to great uncertainty of the initial fixture layout. If the initial population selected randomly is not good, the algorithm is likely to be stuck at locally optimal value. Through pre-screening and other methods, the positions which are more suitable for the fixtures can be selected according to the engineering experience, so as to improve the ability of the fixture layout to restrain deformation. By improving the quality of initial fixture layouts, the performance of the algorithm can be improved. The second is to add one or two optimization strategies in the iterative process to improve the algorithm's search ability. The performance of algorithms is closely related to their own parameters. Appropriate optimization strategies can help find better parameters to improve the performance of algorithms. By adding optimization strategies, we can make up for the shortcomings of the original algorithm while retaining the advantages of the original algorithm and improve the performance of optimization methods.

### 5.4 Future Work

The current fixture layout optimization for thin-walled parts is mostly to optimize the positions of fixtures when the number of fixtures is determined. The number of fixtures is determined by engineering experience. In this way, the manufacturing cost may increase due to the excessive number of fixtures. At the same time, too many fixtures will also introduce more fixture positioning errors, which will make it difficult to accurately predict the deformation of the thin-walled parts. Therefore, it is necessary to consider reducing the number of fixtures while optimizing the fixture locations.

At present, some researchers have realized the importance of reducing the number of fixtures. But most of them use the trial-and-error method to reduce the number of fixtures. This method is to remove a fixture at random after optimizing the fixture layout, and then optimize the fixture layout again when the number of fixtures is reduced by one. Repeat the above process until the algorithm cannot find the required fixture layout within the specified time. This way is very time-consuming. Therefore, how to quickly optimize the number and locations of fixtures at the same time is a challenge in the future. One idea is to take minimizing the number of fixtures as the optimization goal and take the requirements of part deformation as the constraints. After optimization, the solution will not only meet the requirements of part assembly accuracy, but also reduce the number of fixtures.

## 6 Conclusions

Fixture layout is a key factor that influences the assembly quality of thin-walled parts. Therefore, many researchers have studied the fixture layout optimization for thin-walled parts. By classifying, comparing, and analyzing the literatures, this paper systematically introduces the development of fixture layout optimization for thin-walled parts. After reviewing the researches, this paper provides suggestions for future research on fixture layout optimization for thin-walled parts. Conclusions are as follows:

(1) Selection of optimization objectives. The optimization objectives can be divided into two categories: Considering in-plane variations and out-of-plane deformations. Designing multiple optimization objectives according to the actual engineering needs is a major trend in this field.
(2) Modeling methods of assembly variation or deformation. There are two kinds of modeling methods: Mechanism-based and data-based modeling methods. Mechanism-based modeling methods have accurate results, but the amount of calculation is often large, which makes them not suitable for solving large-scale problems. As for data-based modeling methods, they are easy to calculate, so they are widely used. How to build a high-precision model is the focus and difficulty for future research.
(3) Optimization algorithms. Optimization algorithms consist of traditional nonlinear algorithms and heuristic algorithms. Traditional nonlinear algorithms are suitable for solving small and medium-sized problems. Heuristic algorithms are suitable for solving large-scale problems. They are widely used in the fixture layout optimization of large dimensional thin-walled parts with "N-2-1" fixture frame. However, heuristic algorithms are easy to be trapped in the zone of local optimization, and their performance is related to the parameters and initial populations. It is worthy of further research for scholars to think about how to improve the performance of the algorithms.
(4) Current researches neglect the influence of the number of fixtures on the manufacturing cost. How to optimize the number and position of fixtures at the same time is a direction that can be studied in the future.

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**Data availability**
Data in this paper can be provided upon reasonable requirement.

## Declarations

### Competing Interests
The authors declare no competing financial interests.

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