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Optimal Design for Anti-Skid Control of Electric Vehicles by Fuzzy Approach

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Abstract

In this paper, a new fuzzy approach is applied to optimal design of the anti-skid control for electric vehicles. The anti-skid control is used to maintain the wheel speed when there are uncertainties. The control is able to provide an appropriate torque for wheels when the vehicle is about to skid. The friction coefficient and the moments of inertia of wheels and motor are considered as uncertain parameters. These nonlinear, bounded and time-varying uncertainties are described by fuzzy set theory. The control is deterministic and is not based on IF-THEN fuzzy rules. Then, the optimal design for this fuzzy system and control cost is proposed by fuzzy information. In this way, the uniform boundedness and uniform ultimate boundedness are guaranteed and the average fuzzy performance is minimized. Numerical simulations show that the control can prevent vehicle skidding with the minimum control cost under uncertainties.

Keywords: High-order control, Fuzzy set theory, Uncertainty, Optimal design

1 Introduction

With people's consciousness on the environment and the development of motor and battery technology, electric vehicles (EVs) are getting great attention these years [1]. The anti-skid control which is related to the safety is one of focuses in EVs research field. In Ref. [2], a fast feedback control with motor was applied for anti skid control. The braking distance was reduced by 20%. In Ref. [3], an anti-skid controller with PI regulator is designed based on back-EMF observer and dynamic model error observer. The more effective control approach should be available with the fast torque response and accurate feedback of the motors in EVs.

However, the anti skid system includes nonlinearity and uncertainties from motor, wheel and road. These parameters can not be measured precisely such as the friction coefficient. Thus, a fuzzy approach is applied to describing this system in this paper.

Many researches about EV anti skid control have been explored. Most paper use the parameter slip ratio to

design control. In Ref. [4], the sliding-mode-observer-based adaptive sliding mode control was presented for EVs. In Ref. [5], the yaw-rate control is proposed based on the slip ratio. In Ref. [6], the parameter slip ratio was used to design a sliding mode control for EVs. Then, a reference slip related to road condition is needed [7, 8]. These approaches are useful. However, more precise control may not be achieved based on slip ratio. In Ref. [9], a road condition estimation approach is proposed. This approach is based on the driving force observer, which is difficult to be applied to engine vehicle. In Ref. [10], a PI controller is proposed without speed sensor. It can reduce the speed detector device and decrease the cost. But, the robustness may be not good enough based on PI control. In Ref. [11], a sliding mode control is applied to anti skid system of EVs. This control can overcome uncertainties.

Uncertainty is inevitable in a practical system. The probability theory is the most widely used method to give a mathematical description of uncertainty and many great achievements are made based on that [12, 13]. However, it may not always be suitable to deal with uncertainty by using the frequency of occurrence [14]. In 1965, Zadeh introduced the fuzzy set theory to describe

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uncertainty by employing the degree of occurrence [15]. Recently, this theory has been developed and applied to many fields, such as fuzzy choice [16], fuzzy difference equation [17]. By contrast, there are not many explorations between fuzzy set theory and system theory. In this paper, the fuzzy dynamical system is established and the uncertainty in the system is described by fuzzy set theory which is different from the popular T-S fuzzy model or IF-THEN model [18, 19]. These two fuzzy models have been applied to many fields. In Ref. [20], the model was used to design tracking control for nonlinear dynamic systems. In Ref. [21], the time delay problem was taken into consideration. In Ref. [22], a guaranteed cost control method for nonlinear systems was proposed based on Takagi-Sugeno (T-S) fuzzy models. According to the fuzzy dynamical system based on the fuzzy set theory, the optimal design of control is solved for this fuzzy dynamical system when there are multiple tunable parameters. In Ref. [23], the fundamental frame of the fuzzy dynamical system was established. In Ref. [24], the theory was applied to the flexible joint manipulators. In Ref. [25], a fuzzy approach for optimal robust control design of an automotive electronic throttle system was proposed. However, only one parameter was considered in the optimization problem of the previous works. The originality of the paper can be summarized as follows: describing the uncertainties of the EV system by fuzzy set theory; designing a novel high-order robust control; investigating the relationships among the control parameters, system performance, and control cost.

The main contributions are threefold. First, the fuzzy anti-skid dynamical system of EVs is established. The uncertainties and disturbances are taken into consideration based on fuzzy set theory. Second, a novel higher-order control is proposed. It is deterministic and not based on IF-THEN fuzzy rule. By applying this control, we prove that the system is uniformly bounded and uniformly ultimately bounded via the Lyapunov theory [26]. Third, an index combined average fuzzy system performance and control cost is proposed with multiple parameters. By minimizing the index, the optimal design problem is solved.

2 Fuzzy Dynamical Systems with Uncertainties

Before giving the fuzzy dynamical systems, some preliminary knowledges related to fuzzy sets theory are given as follows.

Fuzzy Number. Let \mathfrak{S} be a fuzzy set in \mathbf{R} , the real number. \mathfrak{S} is called fuzzy number if: (i) \mathfrak{S} is normal, (ii) \mathfrak{S} is convex, (iii) the support of \mathfrak{S} is bounded, and (iv) all α -cuts are closed intervals in \mathbf{R} . In addition, the universe of discourse of a fuzzy number is considered to be its 0-cut.

Fuzzy Arithmetic. Let \mathfrak{S} and \mathfrak{T} to be two fuzzy numbers and $\mathfrak{S}_\alpha = [s_\alpha^-, s_\alpha^+]$, $\mathfrak{T}_\alpha = [t_\alpha^-, t_\alpha^+]$ be their α -cuts, $\alpha \in [0, 1]$. The addition, subtraction, multiplication, and division of \mathfrak{S} and \mathfrak{T} are defined as follows,

$$(\mathfrak{S} + \mathfrak{T})_\alpha = [s_\alpha^- + t_\alpha^-, s_\alpha^+ + t_\alpha^+], \tag{1}$$

$$(\mathfrak{S} - \mathfrak{T})_\alpha = [\min(s_\alpha^- - t_\alpha^-, s_\alpha^+ - t_\alpha^-), \max(s_\alpha^- - t_\alpha^-, s_\alpha^+ - t_\alpha^-)], \tag{2}$$

$$(\mathfrak{S} \cdot \mathfrak{T})_\alpha = [\min(s_\alpha^- t_\alpha^-, s_\alpha^- t_\alpha^+, s_\alpha^+ t_\alpha^-, s_\alpha^- t_\alpha^-), \max(s_\alpha^- t_\alpha^-, s_\alpha^- t_\alpha^+, s_\alpha^+ t_\alpha^-, s_\alpha^- t_\alpha^-)], \tag{3}$$

$$(\mathfrak{S}/\mathfrak{T})_\alpha = [\min(s_\alpha^-/t_\alpha^-, s_\alpha^-/t_\alpha^+, s_\alpha^+/t_\alpha^-, s_\alpha^+/t_\alpha^+), \max(s_\alpha^-/t_\alpha^-, s_\alpha^-/t_\alpha^+, s_\alpha^+/t_\alpha^-, s_\alpha^+/t_\alpha^+)], \tag{4}$$

Decomposition Theorem. Define a fuzzy set $\widehat{\mathfrak{D}}_\alpha$ in \mathcal{U} with the membership function $\varpi_{\widehat{\mathfrak{D}}_\alpha} = \alpha I_{\mathfrak{D}_\alpha}(x)$ where $I_{\mathfrak{D}_\alpha}(x) = 1$ if $x \in \mathfrak{D}_\alpha$ and $I_{\mathfrak{D}_\alpha}(x) = 0$ if $x \in \mathcal{U} - \mathfrak{D}_\alpha$. Then, the fuzzy set \mathfrak{D} is obtained as

$$\mathfrak{D} = \bigcup_{\alpha \in [0,1]} \widehat{\mathfrak{D}}_\alpha, \tag{5}$$

where \bigcup is the union of the fuzzy sets (that is, *sup* over $\alpha \in [0, 1]$). Based on these, after the operation of two fuzzy numbers via their α -cuts, one may apply the decomposition theorem to build the membership function of the resulting fuzzy number.

D-operation. Consider a fuzzy set

$$\Psi = \{(\xi, \phi(\xi)) | \xi \in L\}. \tag{6}$$

For any function $f: L \rightarrow \mathbf{R}$, the D-operation $D[f(\xi)]$ is given by

$$D[f(\xi)] = \frac{\int_L f(\xi)\phi(\xi)d\xi}{\int_L \phi(\xi)d\xi}. \tag{7}$$

$D[f(\cdot)]$ stands for taking the D-operation of function $f(\cdot)$. Consider a fuzzy number ξ , a membership function $\phi(\xi)$ and an arbitrary function $f(\xi)$. The D-operation $D[f(\xi)]$ represents the average of $f(\xi)$ over $\phi(\xi)$. In this paper, the D-operation is applied to giving a deterministic representation for the system performance [27].

In addition, for any crisp constant $\alpha \in \mathbf{R}$

$$D[\alpha f(\xi)] = \alpha D[f(\xi)]. \tag{8}$$

The proof is as follows: by Eq. (7)

$$D[\alpha f(\xi)] = \frac{\int_L \alpha f(\xi)\phi(\xi)d\xi}{\int_L \phi(\xi)d\xi} = \alpha \frac{\int_L f(\xi)\phi(\xi)d\xi}{\int_L \phi(\xi)d\xi} = \alpha D[f(\xi)]. \tag{9}$$

Now, the following uncertain system is considered:

$$\begin{aligned} \dot{x}(t) = & Ax(t) + (B + \Delta B(x(t), \sigma(t), t))(u(t) + v(t)) \\ & + Be(x(t), \sigma(t), t), \quad x(t_0) = x_0, \end{aligned} \tag{10}$$

where $t \in \mathbf{R}$ is the time, $x(t) \in \mathbf{R}^n$ is the state, x_0 is the uncertain initial state, $u(t) \in \mathbf{R}^m$ is the control, $\sigma(t) \in \mathbf{R}^p$ is an unknown time-varying parameter, $A \in \mathbf{R}^{n \times n}$ and $B \in \mathbf{R}^{n \times m}$ are known matrices, $v(t)$ is a known time-varying vector, $\Delta B(x(t), \sigma(t), t)$ and $e(x(t), \sigma(t), t)$ are unknown matrix and vector, respectively. The functions $v(t)$, $e(\cdot, t)$ and $\Delta B(\cdot, t)$ are continuous. The functions $v(\cdot)$, $e(x, \sigma, \cdot)$ and $\Delta B(x, \sigma, \cdot)$ are Lebesgue measurable.

Remark 1 In system (10), Ax , B and v are linear nominal portions, ΔB and e are nonlinear portions. v represents the disturbance which can be measured. e represents the uncertainty and disturbance which is unknown. Such expression of nonlinear system is available when matching condition is satisfied [28].

Assumption 1 The pair (A, B) are stabilizable.

Assumption 2 For each bounded entry of x_0 , namely x_{0i} and $\sigma(t)$, namely σ_i , they lie within known fuzzy sets L_{0i} and N_i .

$$L_{0i} = \{(x_{0i}, \phi_{\Theta_i}(x_{0i})) \mid x_{0i} \in \Theta_i\}, \quad i = 1, 2, \dots, l, \tag{11}$$

$$N_i = \{(\sigma_i, \phi_i(\sigma_i)) \mid \sigma_i \in \Omega_i\}, \quad i = 1, 2, \dots, n, \tag{12}$$

where $\Theta_i \in \mathbf{R}$ is the known and compact universe of discourse, $\phi_{\Theta_i} : \Theta_i \rightarrow [0, 1]$ is the membership function of L_{0i} ; $\Omega_i \in \mathbf{R}$ is the known and compact universe of discourse, $\phi_{\Omega_i} : \Omega_i \rightarrow [0, 1]$ is the membership function of N_i . The function $\sigma_i(\cdot)$ is Lebesgue measurable.

Remark 2 The system (10) is called fuzzy dynamical system because the fuzzy set theory is applied to describing the uncertainties in Eq. (10) which is shown in Assumption 2. To be more clear, this fuzzy model is totally different from the very popular Takagi–Sugeno model or other fuzzy if-then rules-based models. As for the approach of dealing with uncertainty, the most common one is the probability theory. It uses the frequency of occurrence to describe uncertainties. This kind of interpretation requires a large quantity of data. However, the data source of uncertainty can not be obtained or repeated easily in many practical systems such as earthquake data. Recently, the fuzzy set theory, as an alternative approach, is applied to describing uncertainty via the degree of occurrence. More detailed discussions on the relative advantages of fuzzy set theory compared with probability theory are shown in Ref. [29].

Consider the following Riccati equation:

$$A^T P + PA - 2PBR^{-1}B^T P + Q = 0, \tag{13}$$

where $Q > 0$ and $R > 0$. The solution $P > 0$ exists and is unique if (A, B) is stabilizable.

Assumption 3

- (i) There exist a matrix $E(x, \sigma, t)$ such that $\Delta B = BE(x, \sigma, t)$.
- (ii) There exists fuzzy numbers ρ_{E1} , ρ_{E2} , a and b such that for $(x, t) \in \mathbf{R}^n \times \mathbf{R}$

$$\begin{aligned} & \frac{1}{2} \lambda_m(E(x, \sigma, t)R^{-1} + R^{-1}E(x, \sigma, t)) \\ & \geq \rho_{E1} > -\lambda_m(R^{-1}), \end{aligned} \tag{14}$$

$$\|E(x, \sigma, t)\| \leq \rho_{E2}, \tag{15}$$

$$\|e(x, \sigma, t)\| \leq a\|x\| + b. \tag{16}$$

Remark 3 The inequalities (14), (15) imply that the uncertainty can not change the direction of the control. The inequality (16) implies that there is a threshold of the nonlinear portion. We can obtain the extreme value of ρ_{E1} since the universes of discourse Ω_i 's are known. Based on $\phi_i(\cdot)$ in Eqs. (11), (12), the fuzzy arithmetic (1)–(4) and the decomposition theorem [23, 30–32], the associated membership functions of fuzzy numbers a , b and ρ_{E1} , ρ_{E2} can be calculated.

By applying the control $u(t)$ in the following section to the uncertain system (10), it can be rendered the following deterministic performance:

- (i) *Uniform Boundedness*: For any $r > 0$ with $\|x_0\| \leq r$, there exists $d(r) > 0$ with $\|x(t)\| \leq d(r)$ for all $t \geq t_0$.
- (ii) *Uniform Ultimate Boundedness*: For any $r > 0$ with $\|x_0\| \leq r$, there exist $\bar{d}(r) > 0$ and $T(\bar{d}(r), r) \geq 0$ such that $\|x(t)\| \leq \bar{d}(r)$ as $t \geq t_0 + T(\bar{d}(r), r)$.

3 Robust Control Design for EVs Anti Skid System

For simplifying the expression, we let $\alpha = B^T P x$, $h = \lambda_m(R^{-1}) + \rho_{E1}$, $\bar{a} = a + \rho_{E2}\|R^{-1}B^T P\|$, $\delta = \frac{\bar{a}}{\delta_1} + \frac{b}{\delta_2}$. Then, we propose a class of robust controls as follows:

$$u(t) = -R^{-1}B^T P x(t) - \gamma \psi(x(t))R^{-1}B^T P x(t) - v(t), \tag{17}$$

where

$$\psi(x) = \frac{(\delta_1\|x\| + \delta_2)^{2\eta} \|\alpha(x)\|^{2\eta-2}}{(\delta_1\|x\| + \delta_2)^\eta \|\alpha(x)\|^\eta + \varepsilon}. \tag{18}$$

In this control, $\delta_1 > 0$, $\delta_2 > 0$, $\varepsilon > 0$, $\gamma > 0$ and $\eta > 1$ are scalar constants. Besides, γ and η are optimal design parameters.

Remark 4 The control (17) is deterministic and is not based on IF-THEN rules. The first and second parts are used for the nominal system without uncertainties and compensating the uncertainties, respectively.

Theorem 1 Suppose that the system (10) is subject to Assumptions 1–3, then the control (17) can guarantee Uniform Boundedness and Uniform Ultimate Boundedness for the system (10).

Proof This proof is via the Lyapunov minimax approach [33, 34]. Consider the Lyapunov function candidate

$$V(x) = x^T P x, \tag{19}$$

$$\begin{aligned} \dot{V}(x) &\leq -\lambda_m(Q) \|x\|^2 + 2\|\alpha\| \rho_{E2} \|R^{-1} B^T P\| \|x\| \\ &\quad - 2\gamma h (\delta_1 \|x\| + \delta_2)^\eta \|\alpha\|^\eta + 2\gamma h \varepsilon + 2\|\alpha\| (a \|x\| + b) \\ &= -\lambda_m(Q) \|x\|^2 + 2\gamma h \varepsilon - 2\gamma h (\delta_1 \|x\| + \delta_2)^\eta \|\alpha\|^\eta + 2(\bar{a} \|x\| + b) \|\alpha\|. \end{aligned} \tag{25}$$

where P is the solution of Riccati equation (13). Then, take the derivative of Eq. (19) with respect to time t

$$\begin{aligned} \dot{V}(x) &= 2x^T P [Ax + B(I + E)(-R^{-1} B^T P x - \gamma \psi R^{-1} B^T P x) + Be] \\ &= x^T (A^T P + PA - 2PBR^{-1} B^T P)x + 2x^T PBE(-R^{-1} B^T P x) \\ &\quad + 2x^T PB(I + E)(-\gamma \psi R^{-1} B^T P x) + 2x^T Be. \end{aligned} \tag{20}$$

By the Riccati equation (13) and Rayleigh’s principle, for the first term on the RHS (right hand side) of Eq. (20), we have

$$x^T (A^T P + PA - 2PBR^{-1} B^T P)x = -x^T Q x \leq -\lambda_m(Q) \|x\|^2. \tag{21}$$

By Eq. (15) and $\alpha = B^T P x$, for the second term on the RHS of Eq. (20), we have

$$2x^T PBE(-R^{-1} B^T P x) \leq 2\|\alpha\| \rho_{E2} \|R^{-1} B^T P\| \|x\|. \tag{22}$$

For the third term on the RHS of Eq. (20), by Eqs. (14), (18), $h = \lambda_m(R^{-1}) + \rho_{E1}$ and Rayleigh’s principle, we can obtain that

$$\begin{aligned} &- 2x^T PB\gamma\psi R^{-1} B^T P x - 2x^T PBE\gamma\psi R^{-1} B^T P x \\ &= -2(\alpha^T R^{-1} \alpha) \gamma \psi - \alpha (ER^{-1} + R^{-1} E^T) \alpha \gamma \psi \\ &\leq -2\lambda_m(R^{-1}) \|\alpha\|^2 \gamma \psi - 2\rho_{E1} \|\alpha\|^2 \gamma \psi \\ &= -2\gamma h \frac{(\delta_1 \|x\| + \delta_2)^{2\eta} \|\alpha\|^{2\eta}}{(\delta_1 \|x\| + \delta_2)^\eta \|\alpha\|^\eta + \varepsilon} \\ &\leq -2\gamma h \frac{(\delta_1 \|x\| + \delta_2)^{2\eta} \|\alpha\|^{2\eta} - \varepsilon^2}{(\delta_1 \|x\| + \delta_2)^\eta \|\alpha\|^\eta + \varepsilon} \\ &= -2\gamma h (\delta_1 \|x\| + \delta_2)^\eta \|\alpha\|^\eta + 2\gamma h \varepsilon. \end{aligned} \tag{23}$$

Regarding the last term on the RHS of Eq. (20), by Eq. (16), we have

$$2x^T PBe \leq 2\|\alpha\| (a \|x\| + b). \tag{24}$$

Thus, by combining Eqs. (21)–(24) and $\bar{a} = a + \rho_{E2} \|R^{-1} B^T P\|$, we have

Next, we define the function

$$G(\|\alpha\|) = -2\gamma h (\delta_1 \|x\| + \delta_2)^\eta \|\alpha\|^\eta + 2(\bar{a} \|x\| + b) \|\alpha\|. \tag{26}$$

Thus, the derivative of $G(\|\alpha\|)$ can be obtained

$$\frac{dG(\|\alpha\|)}{d\|\alpha\|} = -2\eta\gamma h (\delta_1 \|x\| + \delta_2)^\eta \|\alpha\|^{\eta-1} + 2(\bar{a} \|x\| + b). \tag{27}$$

Since $\eta > 1$, we can obtain the extreme point of $G(\|\alpha\|)$ by letting $\frac{\partial G(\|\alpha\|)}{\partial \|\alpha\|} = 0$.

$$\|\alpha_M\| = \left(\frac{2(\bar{a} \|x\| + b)}{2\eta\gamma h (\delta_1 \|x\| + \delta_2)^\eta} \right)^{\frac{1}{\eta-1}}. \tag{28}$$

Because $-2\gamma h (\delta_1 \|x\| + \delta_2)^\eta \|\alpha\|^\eta < 0$, if $\|\alpha\| > \|\alpha_M\|$, then $\frac{\partial G(\|\alpha\|)}{\partial \|\alpha\|} < 0$; if $\|\alpha\| < \|\alpha_M\|$, then $\frac{\partial G(\|\alpha\|)}{\partial \|\alpha\|} > 0$. Thus, there exists the maximum value of $G(\|\alpha\|)$. Then, we get

$$\begin{aligned}
 G(\|\alpha\|) &\leq G(\|\alpha_M\|) \\
 &= -2\gamma h(\delta_1\|x\| + \delta_2)^\eta \|\alpha_M\|^\eta + 2(\bar{a}\|x\| + b)\|\alpha_M\| \\
 &\leq 2(\bar{a}\|x\| + b)\|\alpha_M\| \\
 &= \left(\frac{2^\eta(\bar{a}\|x\| + b)^\eta}{2\eta\gamma h(\delta_1\|x\| + \delta_2)^\eta} \right)^{\frac{1}{\eta-1}} \\
 &= 2\eta^{-\frac{1}{\eta-1}} \gamma^{-\frac{1}{\eta-1}} h^{-\frac{1}{\eta-1}} \left(\frac{\bar{a}\|x\| + b}{\delta_1\|x\| + \delta_2} \right)^{\frac{\eta}{\eta-1}}.
 \end{aligned}
 \tag{29}$$

By Eq. (29) and $\delta = \frac{\bar{a}}{\delta_1} + \frac{b}{\delta_2}$, we have

$$\begin{aligned}
 \dot{V}(x) &\leq -\lambda_m(Q)\|x\|^2 + 2\gamma h\varepsilon + 2\eta^{-\frac{1}{\eta-1}} \gamma^{-\frac{1}{\eta-1}} h^{-\frac{1}{\eta-1}} \left(\frac{\bar{a}\|x\| + b}{\delta_1\|x\| + \delta_2} \right)^{\frac{\eta}{\eta-1}} \\
 &\leq -\lambda_m(Q)\|x\|^2 + 2\gamma h\varepsilon + 2\eta^{-\frac{1}{\eta-1}} \gamma^{-\frac{1}{\eta-1}} h^{-\frac{1}{\eta-1}} \left[\max\left(\frac{\bar{a}}{\delta_1}, \frac{b}{\delta_2} \right) \right]^{\frac{\eta}{\eta-1}} \\
 &\leq -\lambda_m(Q)\|x\|^2 + 2\gamma h\varepsilon + 2\eta^{-\frac{1}{\eta-1}} \gamma^{-\frac{1}{\eta-1}} h^{-\frac{1}{\eta-1}} \left(\frac{\bar{a}}{\delta_1} + \frac{b}{\delta_2} \right)^{\frac{\eta}{\eta-1}} \\
 &= -\lambda_m(Q)\|x\|^2 + 2\gamma\eta\varepsilon h + 2\gamma^{-\frac{1}{\eta-1}} \eta^{-\frac{1}{\eta-1}} h^{-\frac{1}{\eta-1}} \delta^{\frac{\eta}{\eta-1}}.
 \end{aligned}
 \tag{30}$$

This means that \dot{V} is negative definite for all $\|x\|$ such that

$$\|x\| > \sqrt{\frac{1}{\lambda_m(Q)}(2\gamma\eta\varepsilon h + 2\gamma^{-\frac{1}{\eta-1}} \eta^{-\frac{1}{\eta-1}} h^{-\frac{1}{\eta-1}} \delta^{\frac{\eta}{\eta-1}})}.
 \tag{31}$$

In the RHS of Eq. (31), η , γ , ε and $\lambda_m(Q)$ are all crisp. In addition, all universes of discourse are compact, δ is bounded. Therefore, we can say \dot{V} is negative definite for sufficiently large $\|x\|$. By [28], the system is uniformly bounded and uniformly ultimately bounded.

Remark 5 From Eq. (31), the parameters η and γ are able to influence the size of the ultimate boundedness region. At the same time, the control magnitude is also decided by these two parameters. Hence, there is a trade-off between the control cost and the system performance by choosing the optimal η and γ when we regard Eqs. (17) and (31) as the control cost and the system performance.

Remark 6 The uniform boundedness and uniform ultimate boundedness system performance are concluded based on the Lyapunov method with some preconditions. The preconditions are crucial, which include the novel high-order robust control (17) and the assumptions of the EV system (11), (12), (14)–(16).

4 Optimal Gain Design with Multi Parameters

In Section 3, we find that the parameters η and γ can influence the system performance and the control cost. In this section, a performance index will be presented and the minimization of the index will be shown as well. First, by Rayleigh principle

$$V = x^T P x \leq \lambda_M(P) \|x\|^2.
 \tag{32}$$

Thus,

$$-\|x\|^2 \leq -\frac{1}{\lambda_M(P)} V.
 \tag{33}$$

Combining Eqs. (30) and (33), we have

$$\dot{V}(t) \leq -\frac{\lambda_m(Q)}{\lambda_M(P)} V(t) + \gamma_\eta,
 \tag{34}$$

where $\gamma_\eta = 2\gamma\eta\varepsilon h + 2\gamma^{-\frac{1}{\eta-1}} \eta^{-\frac{1}{\eta-1}} h^{-\frac{1}{\eta-1}} \delta^{\frac{\eta}{\eta-1}}$, $V(t_0) = x_0^T P x_0$. This is a differential inequality [35], and the solution of Eq. (34) can be obtained as follows [23]

$$V(t) \leq \left(V_0 - \frac{\lambda_m(P)}{\lambda_m(Q)} \gamma_\eta \right) \exp\left[-\frac{\lambda_m(Q)}{\lambda_M(P)} (t - t_0) \right] + \frac{\lambda_M(P)}{\lambda_m(Q)} \gamma_\eta,
 \tag{35}$$

for all $t \geq t_0$. Hence, for any t_s and any $\tau \geq t_s$

$$V(\tau) \leq (V_s - \kappa\gamma_\eta) \exp\left[-\frac{1}{\kappa} (\tau - t_s) \right] + \kappa\gamma_\eta,
 \tag{36}$$

where $V_s = V(t_s) = x^T(t_s) P x(t_s)$ and $\kappa = \lambda_M(P)/\lambda_m(Q)$. Since $V(\tau) \geq \lambda_m(P)\|x\|^2$ by Rayleigh principle, the RHS of Eq. (36) can be considered as the upper bound of $\lambda_m(P)\|x\|^2$. This also leads to an upper bound of $\|x\|^2$. Next, let

$$\Phi_1(\gamma_\eta, \tau, t_s) = (V_s - \kappa\gamma_\eta) \exp\left[-\frac{1}{\kappa} (\tau - t_s) \right],
 \tag{37}$$

$$\Phi_2(\gamma_\eta) = \kappa\gamma_\eta.
 \tag{38}$$

Note that for each $\gamma_\eta, t_s, \Phi_1(\gamma_\eta, \tau, t_s) \rightarrow 0$ as $\tau \rightarrow \infty$. The functions Φ_1 and Φ_2 are regarded as the transient portion

and steady state portion of the system performance. The overall transient performance from the time t_s may be represented as $\int_{t_s}^{\infty} \Phi_1^2(\gamma, \tau, t_s) d\tau$.

$$\begin{aligned} \int_{t_s}^{\infty} \Phi_1^2(\gamma, \tau, t_s) d\tau &= (V_s - \kappa\gamma\eta)^2 \int_{t_s}^{\infty} \exp\left[-2\frac{1}{\kappa}(\tau - t_s)\right] d\tau \\ &= (V_s - \kappa\gamma\eta)^2 \left(-\frac{\kappa}{2}\right) \exp\left[\frac{2}{\kappa}(\tau - t_s)\right] \Big|_{t_s}^{\infty} \\ &= (V_s - \kappa\gamma\eta)^2 \frac{\kappa}{2}. \end{aligned} \tag{39}$$

We define the performance index as $J = D[\alpha J_p] + D[\beta J_c]$, where $J_p = \int_{t_s}^{\infty} \Phi_1^2(\gamma, \tau, t_s) d\tau + \Phi_2^2$ represents the system performance, $J_c = \gamma^2 + \frac{\eta^2}{2}$ represents the control cost, α and β are weighting factors. Since $\gamma > 0$ and $\eta > 1$, let the constant $\varepsilon = \gamma^{\frac{1}{\eta-1}} \eta^{\frac{-1}{\eta-1}} > 0$. Then, $\gamma\eta = 2(\gamma^{\frac{-1}{\eta-1}} \eta^{\frac{-1}{\eta-1}})(h + h^{\frac{-1}{\eta-1}} \delta^{\frac{\eta}{\eta-1}})$, taking the D-operation (7), we have

$$\begin{aligned} J(\gamma, \eta, t_s) &= D[\alpha J_p] + D[\beta J_c] \\ &= \alpha \left(D\left[(V_s - \kappa\gamma\eta)^2 \frac{\kappa}{2} \right] + D[\Phi_2^2(\gamma, \eta)] \right) + \beta \left(\gamma^2 + \frac{\eta^2}{2} \right) \\ &= \alpha \left(\kappa_1 - \kappa_2 \kappa_4 \gamma^{\frac{-1}{\eta-1}} \eta^{\frac{-1}{\eta-1}} + \kappa_2 \kappa_4^{\frac{-1}{\eta-1}} \kappa_5^{\frac{\eta}{\eta-1}} \gamma^{\frac{-1}{\eta-1}} \eta^{\frac{-1}{\eta-1}} + \kappa_3 \kappa_6 \gamma^{\frac{-2}{\eta-1}} \eta^{\frac{-2}{\eta-1}} \right. \\ &\quad \left. + 2\kappa_3 \kappa_4^{\frac{\eta-2}{\eta-1}} \kappa_5^{\frac{\eta}{\eta-1}} \gamma^{\frac{-2}{\eta-1}} \eta^{\frac{-2}{\eta-1}} + \kappa_3 \kappa_6^{\frac{-1}{\eta-1}} \kappa_7^{\frac{-1}{\eta-1}} \gamma^{\frac{-2}{\eta-1}} \eta^{\frac{-2}{\eta-1}} \right) + \beta \left(\gamma^2 + \frac{\eta^2}{2} \right), \end{aligned} \tag{40}$$

where

$$\begin{aligned} \kappa_1 &= \frac{\kappa}{2} D[V_s^2], \quad \kappa_2 = 2\kappa^2 D[V_s], \quad \kappa_3 = 2\kappa^3 + 4\kappa^2, \\ \kappa_4 &= D[h], \quad \kappa_5 = D[\delta], \quad \kappa_6 = D[h^2], \quad \kappa_7 = D[\delta^2]. \end{aligned} \tag{41}$$

Remark 7 Consider a fuzzy number ξ , a membership function $\phi(\xi)$ and an arbitrary function $f(\xi)$. The D-operation $D[f(\xi)]$ represents the average of $f(\xi)$ over $\phi(\xi)$ [23].

To obtain the solution of this optimal design problem, the partial differential operator to J is applied with respect to γ and η , respectively. Next, let

$$\frac{\partial J}{\partial \gamma} = 0, \quad \frac{\partial J}{\partial \eta} = 0, \tag{42}$$

where $\gamma \in (0, +\infty)$, $\eta \in (1, +\infty)$. Assuming that the solutions of Eq. (42) are (γ_i, η_i) . Substitute the solutions into the following inequality (43) and find the ones that satisfy it:

$$\left[\begin{array}{cc} \frac{\partial^2 J}{\partial \gamma^2} & \frac{\partial^2 J}{\partial \gamma \partial \eta} \\ \frac{\partial^2 J}{\partial \eta \partial \gamma} & \frac{\partial^2 J}{\partial \eta^2} \end{array} \right] \Big|_{(\gamma, \eta) = (\gamma_i, \eta_i)} > 0. \tag{43}$$

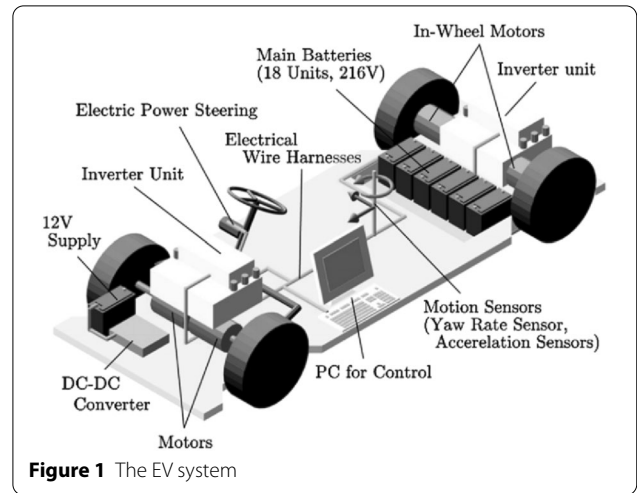


Figure 1 The EV system

Denote the solution of Eq. (42) which also satisfies Eq. (43) as (γ^*, η^*) . Then, (γ^*, η^*) is the minimum point

of the performance index J .

5 EVs Anti Skid System

The EV system is shown in Figure 1 [8]. The motor drives the wheel by the transmission to move forward. In this paper, the focus is on wheel and motor. We aim to control the motor to provide the appropriate torque to wheel when the friction between wheel and road is unknown.

Figure 2 shows the relation among motor, transmission and wheel. ω_m , T_m and J_m represent the speed, output torque and moment of inertia of the motor, respectively. ω_w , T_w and J_w have the same meanings associated with the wheel. The transmission ratio is i .

From this figure, we have

$$\omega_m(t) = i\omega_w(t), \tag{44}$$

$$T_w(t) = iT_m(t). \tag{45}$$

The model of the motor is given as follows [36]:

$$J_m \dot{\omega}_m(t) = \frac{k_t}{R} u(t) - \frac{k_t k_e}{R} \omega_m(t) - B_m \omega_m(t) - T_m(t), \tag{46}$$

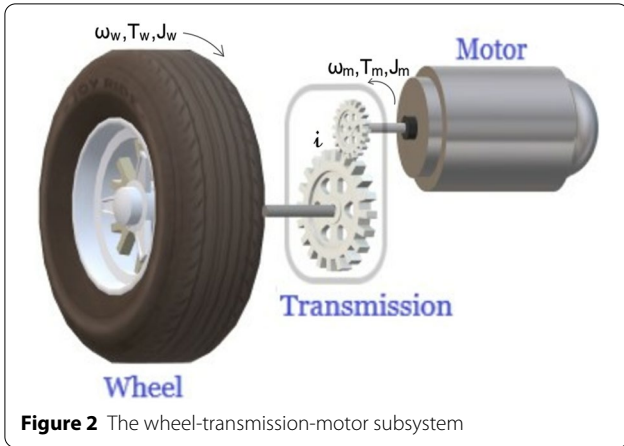


Figure 2 The wheel-transmission-motor subsystem

where k_t and k_e are the constants of the motor torque and the electromotive force, respectively, R is the resistance, B_m is the viscous damping coefficients of the motor, u is the control input voltage.

The wheel subsystem is shown in Figure 3. ω_w , T_w and J_w have been shown in Figure 2. F_n is the normal force of wheel, F_d is the driving force, r is the wheel's radius, ω_0 is the desired speed of the wheel, μ is the friction coefficient which is regarded as the uncertainty in this paper. The dynamic model of the wheel is given as follows [37, 38]:

$$J_w \dot{\omega}_w(t) = T_w(t) - F_d(t)r, \tag{47}$$

$$F_d(t) = \mu F_n. \tag{48}$$

Combining Eqs. (44)–(48), the dynamic equation of the whole system can be obtained as follows:

$$J_a \dot{\omega}_w(t) = -\left(\frac{k_t k_e}{R} i^2 - B_m i^2\right) \omega_w(t) - \mu F_n r + \frac{k_t i}{R} u(t), \tag{49}$$

where $J_a = J_m i^2 + J_w$ is the equivalent moment of the wheel and motor.

Let $x(t) = \omega_w(t) - \omega_0(t)$. Thus, $\dot{x}(t) = \dot{\omega}_w(t) - \dot{\omega}_0(t)$. The dynamic equation (49) can be rewritten as

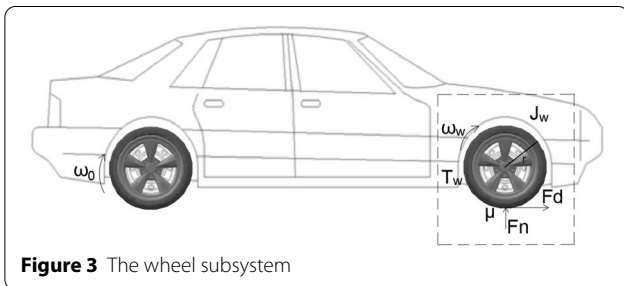


Figure 3 The wheel subsystem

$$\begin{aligned} \dot{x}(t) &= -\left(\frac{k_t k_e}{R} + B_m\right) \frac{i^2}{J_a} x(t) \\ &+ \frac{k_t i}{R J_a} \left[u(t) + \frac{R}{k_t i} (-J_a \dot{\omega}_0(t) \right. \\ &\left. - \left(\frac{k_t k_e}{R} + B_m\right) i^2 \omega_0(t) - \mu F_n r \right] \\ &= \hat{A} x(t) + \hat{B}(u(t) + \hat{v}(t)). \end{aligned} \tag{50}$$

Suppose the equivalent moment J_a [39] and the friction coefficient [40] μ are uncertainties. $J_a = \bar{J}_a + \Delta J_a(t)$, $\mu = \bar{\mu} + \Delta\mu(t)$. Then $\hat{A} = A + \Delta A$, $\hat{B} = B + \Delta B$ and $\hat{v} = v + \Delta v$ with

$$\begin{aligned} A &= -\frac{B_a}{\bar{J}_a}, \Delta A = \frac{B_a \Delta J_a}{\bar{J}_a(\bar{J}_a + \Delta J_a)}, \\ B &= \frac{k_t i}{R \bar{J}_a}, \Delta B = -\frac{k_t i \Delta J_a}{R \bar{J}_a(\bar{J}_a + \Delta J_a)}, \\ v &= -\frac{R}{k_t i} (\bar{J}_a \dot{\omega}_0 + B_a \omega_0 + \bar{\mu} F_n r), \\ \Delta v &= -\frac{R}{k_t i} (\Delta J_a \dot{\omega}_0 + \Delta \mu F_n r), \end{aligned} \tag{51}$$

where $B_a = (k_t k_e / R + B_m) i^2$. By the matching condition, $\Delta A = BD$ and $\Delta B = BE$, we have

$$D = \frac{B_a R}{k_t i} \frac{\Delta J_a}{\bar{J}_a + \Delta J_a}, E = -\frac{\Delta J_a}{\bar{J}_a + \Delta J_a}, \tag{52}$$

which satisfies the Assumption 3(i).

By Eqs. (51) and (52), the dynamic equation (50) can be rewritten in the form of Eq. (10) with

$$e = Dx + (I + E)\Delta v. \tag{53}$$

By choosing $Q = I$, $R = I$,

$$\frac{1}{2} \lambda_m (ER^{-1} + R^{-1}E) = E. \tag{54}$$

Since, $J_a > 0$ and $\bar{J}_a > 0$, as for E ,

$$E = -\frac{\Delta J_a}{\bar{J}_a + \Delta J_a} = \frac{-\Delta J_a - \bar{J}_a + \bar{J}_a}{\bar{J}_a + \Delta J_a} = -1 + \frac{\bar{J}_a}{\bar{J}_a + \Delta J_a} > -1. \tag{55}$$

Since $\rho_{E2} \geq 0$, as for $\|e\|$,

$$\|e\| \leq \|D\| \|x\| + (I + \rho_{E2}) \|\Delta v\|. \tag{56}$$

Then, choose $a = \|D\|$, $b = (1 + \rho_{E2}) \|\Delta v\|$. By Eqs. (54)–(56) and choosing $\rho_{E1} = 0$, $\rho_{E2} = \left| \frac{\Delta J_a}{\bar{J}_a + \Delta J_a} \right|$, the Assumption 3(ii) is satisfied.

Remark 8 It is proper that the uncertainties J_a and μ in the EV model are described by fuzzy set theory. This is because the physical meaning of these two parameters

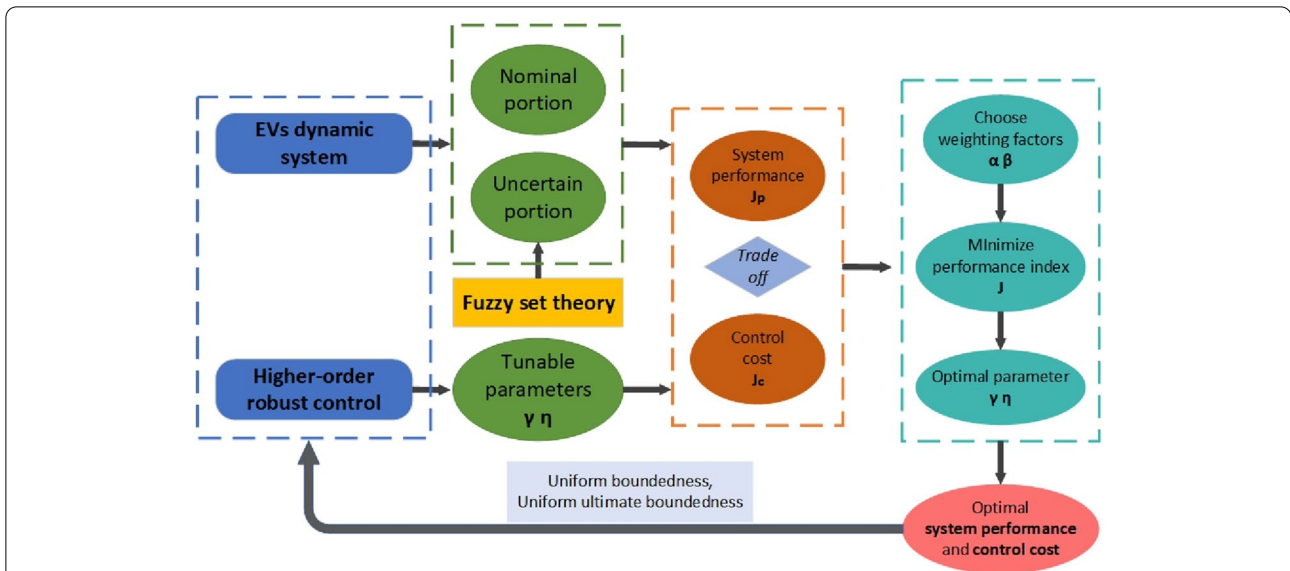


Figure 4 The frame of EVs system and optimal control design

Table 1 Weighting factors & Optimal parameters & J_{min}

α, β	α/β	γ^*, η^*	J_{min}
100, 1	100	0.923, 1.275	2.021
10, 1	10	0.786, 1.224	1.509
1, 1	1	0.579, 1.274	1.239
1, 10	0.1	0.472, 1.258	10.709
1, 100	0.01	0.404, 1.234	96.313

cannot be easily interpreted through the probability theory. Further, the uniform boundedness and uniform ultimate boundedness are ensured for the uncertain EV system based on fuzzy set theory and the Lyapunov approach.

6 Design Procedure

A flowchart is used to summarize the frame this paper as shown in Figure 4.

On the one hand, the dynamical system is established. Furthermore, the uncertain portion of this system is described by fuzzy set theory. On the other hand, a class of novel controls are proposed with two tunable parameters. Then, the cost function which is related to both system performance and control cost is formulated. By minimizing this function, we can obtain the optimal parameters γ and η . Based on that, a better system performance and smaller control cost can be obtained.

7 Simulation Results

The values of the nominal portions in EV system are given as follows: $k_t = 2, k_e = 0.4, R = 1, B_m = 0.1, i = 10, F_n = 2000, r = 0.3, \omega_0 = 5, \dot{\omega}_0 = 0, \bar{J}_a = 5$ and $\bar{\mu} = 0.6$.

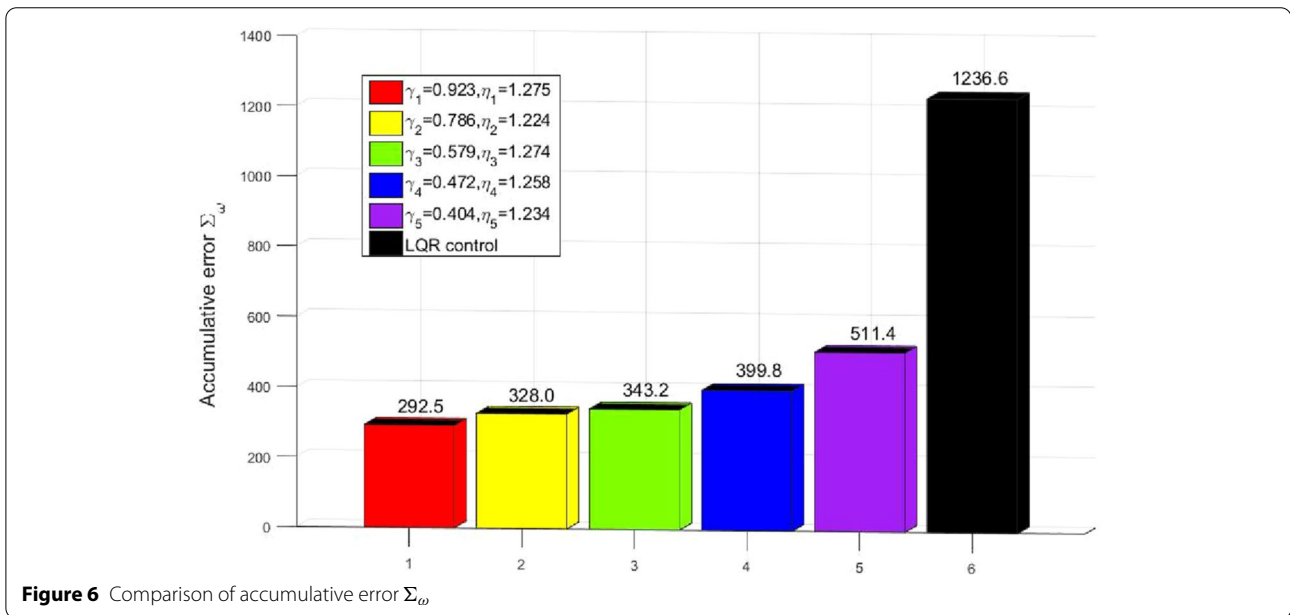
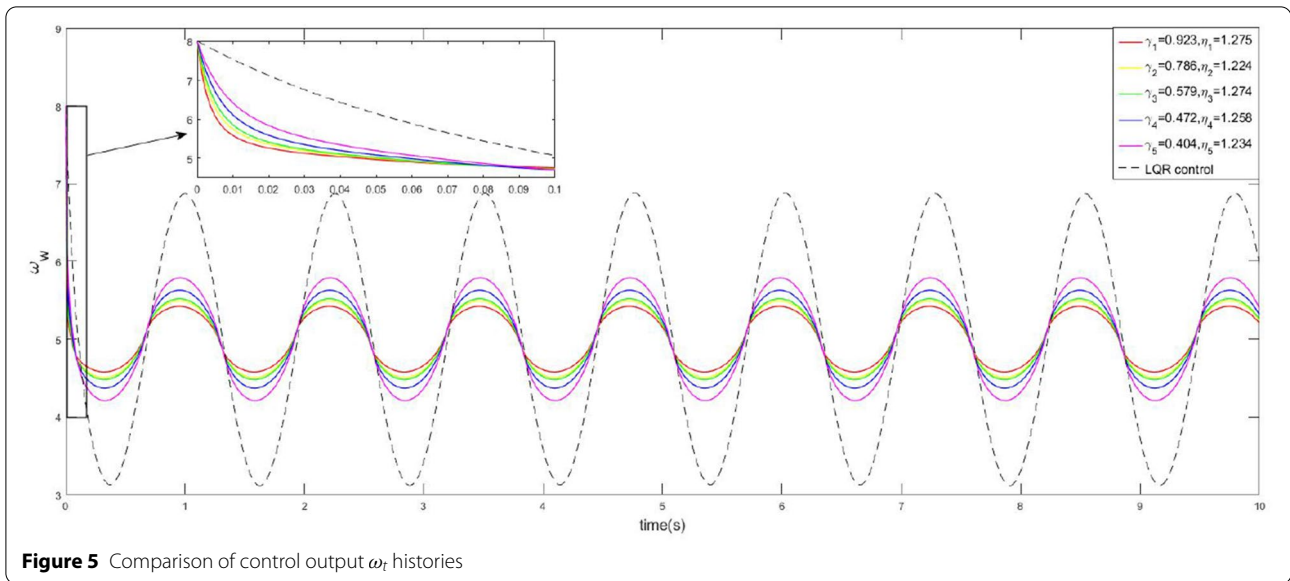
For the uncertainties, the amplitude of ΔJ_a is chosen to be “close to 0.5”, the amplitude of $\Delta \mu$ is chosen to be “close to 0.3”. The associated membership functions are given as follows:

$$\phi_{\Delta J_a}(v) = \begin{cases} 2v, & 0 \leq v \leq 0.5, \\ -2v + 2, & 0.5 \leq v \leq 1, \end{cases} \quad (57)$$

$$\phi_{\Delta \mu}(v) = \begin{cases} \frac{10}{3}v + 1, & -0.3 \leq v \leq 0, \\ -\frac{10}{3}v + 1, & 0 \leq v \leq 0.3. \end{cases} \quad (58)$$

Then, we choose the crisp initial condition $x(t_s) = x(0) = 3$. Thus, $V_s = 0.249$. Based on the fuzzy arithmetic, $\kappa_{1 \sim 7}$ in Eq. (41) can be obtained.

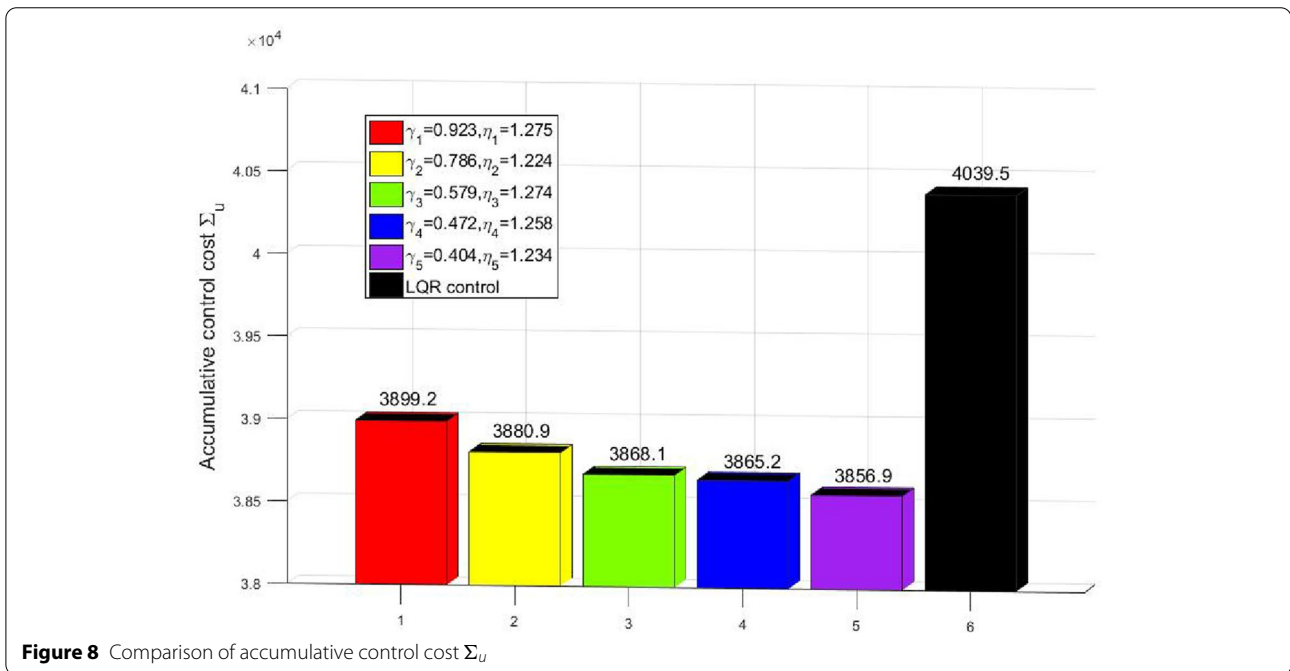
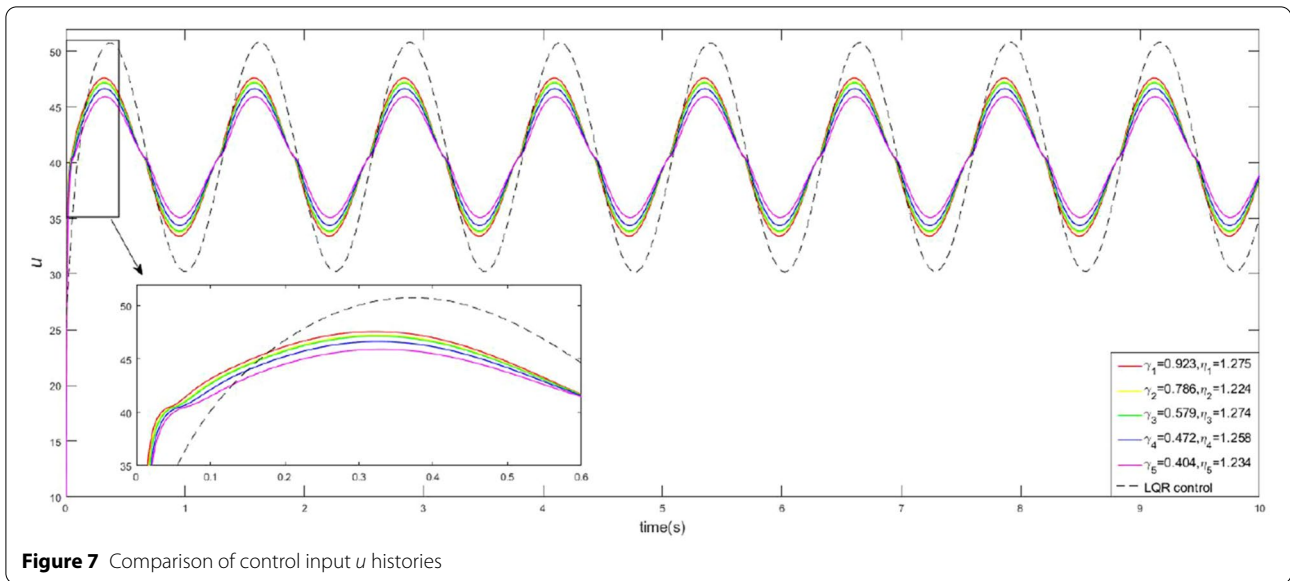
By giving different weighting factors α and β , which means how much we concern about the system performance or control cost, different values of γ, η and J_{min} are obtained by Eqs. (42) and (43). Their values are shown in Table 1. Specifically, the parameters α and β are the weighting factors of system performance and control cost, respectively. In the optimization problem, In this optimization problem, a larger β value represents that we expect to optimize the control first. At this point, the system will reduce the control cost at the expense of performance. Vice versa. Therefore, in the simulation, by comparing different beta values and control costs, the smaller control at larger beta values is expected to



obtain. Finally, the simulations were performed by using $\Delta J_a = 0.5|\cos(3t)|$ and $\Delta \mu = 0.3\sin(5t)$. LQR is selected for comparison. The reasons for choosing LQR are two-fold. First, the LQR is a proven optimal control, which minimizes a performance index including the state and control costs. Second, the LQR is a proven robust control. Many new control methods have been compared with LQR, such as the fuzzy control [41] and the adaptive control [42]. The most common robustness measures attributed to the LQR are a one-half gain reduction in any input channel, an infinite gain amplification in any input channel, or a phase error of plus or minus sixty degrees

in any input channel. In addition, there is robustness to uncertainty in the real coefficients of the model and certain nonlinearities, including control switching and saturation [43].

Figure 5 shows the speed of wheel ω_w histories with different optimal parameters γ and η . LQR control is also plotted as comparison. It shows that the proposed controls are able to render ω_w in a smaller region than LQR control. From the partial figure, we can find that the proposed controls can stabilize the speed of wheel faster than LQR control as well.



In order to show the system performance more clearly, we define the accumulative error Σ_ω as

$$\Sigma_\omega = \int_0^{t_s} (\omega_w(t) - \omega_0) dt. \tag{59}$$

Figure 6 shows the accumulative errors Σ_ω under different controls. The accumulative errors of the system under the proposed controls are much more smaller than the accumulative error under LQR control. Furthermore, Figure 5 and Figure 6 show that the performance of the

system gets better as the ratio of weighting factors α/β becomes larger. Since a larger ratio α/β means that we concern more about the system performance than the control cost.

Figure 7 shows the control input u histories with different optimal parameters γ and η . LQR control is also plotted as comparison. It can be seen that the fluctuations of the proposed controls are smaller than the fluctuations of LQR control.

Similarly, we define the accumulative control cost Σ_u as

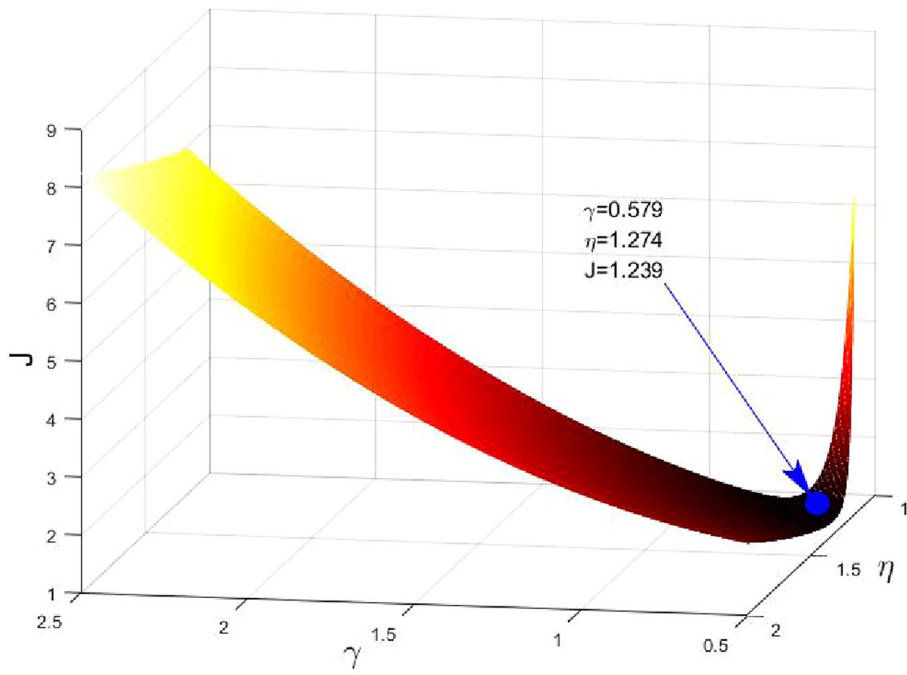


Figure 9 Relation between J , γ and η under $\alpha = \beta = 1$

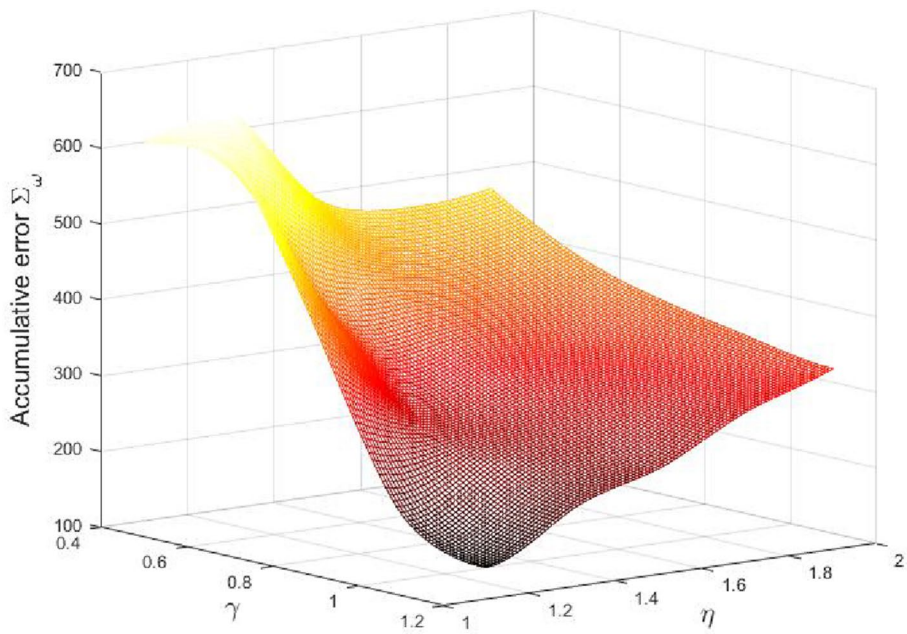


Figure 10 Relation between accumulative error Σ_{ω} , γ and η

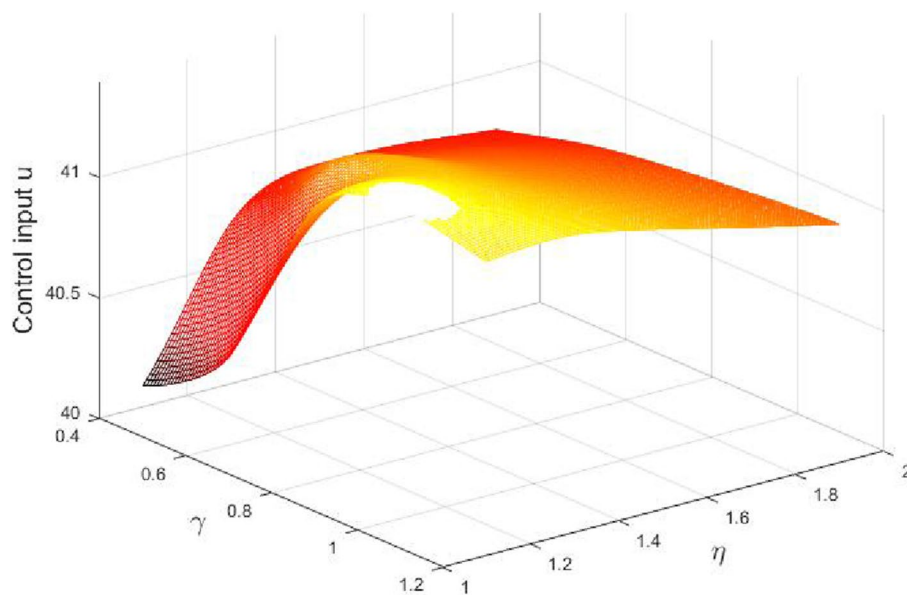


Figure 11 Relation between control input u , γ and η

$$\Sigma_u = \int_0^{t_s} u(t) dt. \quad (60)$$

Figure 8 shows the comparison of the control costs with LQR. As shown in this figure, the control efforts with optimal parameters are significantly less than LQR, while demonstrating superior performance. It can be seen that the accumulative control cost decreases as the ratio of weighting factors α/β becomes smaller. Since a smaller ratio means that we concern more about the control costs than the system performance.

Figure 9 shows the relation between the performance index J , optimal parameters γ and η when $\alpha = \beta = 1$. It verifies the correctness of J_{min} which is obtained by Eqs. (42) and (43).

Figure 10 and Figure 11 show the relation between Σ_ω , u and γ , η , respectively. It can be seen the larger γ renders better performance and larger control input, and vice versa. However, η does not have such monotonicity with Σ_ω and u . Thus, γ decreases when we concern more about control cost, namely α/β decreases while η always remains around a certain value as shown in Table 1.

8 Conclusions

A class of higher-order robust controls are proposed for EVs. The speed of EVs can be kept stable by these controls when there exists uncertainty. It should be noted that the controls are deterministic based on tunable parameters instead of IF-THEN fuzzy rules. They are able to guarantee uniform boundedness and uniform ultimate

boundedness of the system via the Lyapunov minimax approach. Furthermore, the system uncertainty and performance are described by fuzzy set theory. Then, the optimal gain design problem with multi parameters is taken into consideration. A performance index including the tunable parameters γ and η is presented. By giving different weighting factors of system performance and control cost, the optimal parameters γ and η can be solved by minimizing the performance index J . The results of this paper demonstrate that the newly proposed high-order robust controller can be applied to the anti-skid control of electric vehicles. In addition, the optimization method proposed in this paper can find the optimal parameters of the controller to achieve better system performance.

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Authors' Contributions

CL: Software, writing the original draft, formal analysis and simulations. YC: Conceptualization, methodology, writing and review. KH: Data curation and supervision. HZ: Resources, project administration. All authors read and approved the final manuscript.

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Competing Interests

The authors declare that they have no competing interests.

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