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Spherical cap harmonic analysis of magnetic variations data from mainland Australia

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Spherical cap harmonic analysis (SCHA) has been applied to geomagnetic data from an array of magnetometers deployed across the Australian mainland during 1989–90 in order to examine features of the ionospheric S_q current system. The external contours, corresponding to ionospheric currents, are generally in good agreement with the data in that the direction of current flow is perpendicular to the measured horizontal magnetic field. The derived internal (induced) current systems are less reliable but exhibit fairly consistent behaviour on some occasions. As the S_q current system whorl passes over Australia, the reversed internal current whorl lags by a bit less than an hour. When the external system moves off the continent into the Indian Ocean, the derived internal system sometimes appears to remain over the continent for about two hours but this is found to be caused by the large coastal effect in the west. The variations obtained from the analysis at particular sites are compared with the original data and give generally good agreement. Internal and external variations at several sites are derived and behave as expected to some extent. In particular we find that the internal component of the vertical element adds to the external component on the west coast but subtracts from it on the east coast. Further analyses are performed using data from the CM4 model. The SCHA gives a separation of internal and external components in good agreement with the original for the horizontal CM4 magnetic fields while the agreement is not so good for the vertical component.

Key words: Spherical cap harmonic analysis, S_q currents, induced currents, Australia.

1. Introduction

The method of spherical cap harmonic analysis (SCHA) was devised by Haines (1985) for use where data were available from observatories in a restricted geographical area. SCHA differs from global spherical harmonic analysis (GSHA) in that Legendre functions of non-integral degree are required in a SCHA while the Legendre functions in a GSHA are of integral degree. This difference arises from the boundary conditions imposed at the edge of the spherical cap. De Santis *et al*. (1999) discuss the mathematical relationship of spherical cap harmonics to global spherical harmonics.

A prime motivation for using the analysis method was that it should enable the separation of the externally driven variations from those generated internally. In our case it should separate the magnetic variations caused by ionospheric currents from those induced below the earth's surface. There have been several reports that the separation of the internal and external components using SCHA is unreliable but not all have been published in the open literature. One of the earliest of these is Lowes (1999). In this paper we will demonstrate some apparent successes and some failures of the method using a unique and reasonably well-spaced array of magnetometers spread over the Australian mainland.

The original paper of Haines (1985) treats the earth's main magnetic field. Düzgit and Malin (2000) found that SCHA was useful for modelling anomalies in the main field. Torta *et al*. (2006) give a comprehensive list of references to various uses of SCHA, including analyses of the Earth's main field, anomaly models, secular variation models and applications in geodesy. Haines and Torta (1994) applied SCHA to daily magnetic variations from an array of 40 observatories in Europe. The full Northern Hemisphere S_q current whorl is only just visible in this work as the southern-most observatories are near the S_q focus. The derived internal currents flow in the opposite direction to the external currents, as expected, but their form is rather different, especially near noon when current amplitudes are largest. The reliability of the method was further investigated by Torta and De Santis (1996). They state that problems with the method occur when an "inherently long-wavelength field is forced to be fitted by inherently short-wavelength functions". They generated an "artificial" set of values for the three magnetic elements X (northward), Y (eastward) and Z (vertical) for 50 regularly spaced sites over Europe and performed a SCHA on these, using a cap with 18° half-angle. The external equivalent current function obtained showed a good resemblance to the original. Errors introduced by the SCHA were about 24% in the externally produced X and Y elements near the centre of the cap and up to 43% near the edge of the cap. Errors in Z

were larger and errors in the internal elements were more than twice larger than those for the external. So this study is already intimating that the method works better for the external than for the internal elements. Torta *et al.* (1997) used SCHA on the European observatory array on magnetically "quiet" days to derive the seasonal variation of the ionospheric S_q system. In this work they did not display the internal system.

2. Data and Analysis

The present work uses data gathered from an array of 54 magnetometers specially set up to cover the Australian mainland and run from August 1989 to June 1990, though not all of these were running all the time. This project is known as the Australia-Wide Array of Geomagnetic Stations (AWAGS) and further information on this may be found in Chamalaun and Barton (1993a, b). The magnetometers recorded the northward, eastward and vertical components of the magnetic field, X, Y, and Z, and all three components were used in the analysis. Data were recorded every minute. We first calculated hourly means and then removed a linear trend. Also, at each station, the average of the values at the local midnight at the beginning and the midnight at the end of each day is subtracted so that only the daily variation component remains.

The spherical cap used is centred at 132.5E, 27.5S and generally has a half width of 20°. Haines and Torta (1994) showed, from the European data set, that similar results were obtained from their "hourly model" and their "spatio-temporal model". In the latter case the data are expanded in terms of a temporal series, such as a Fourier series and the results for each hour are then extracted. Here we decided to use the hourly model in which only the data for each UT hour are used in each separate calculation. The maximum spatial index for both internal and external sources was generally set at 5 (the value used by Haines and Torta, 1994) and the temporal degree for both internal and external sources is set to 0 for the hourly model. De Santis and Torta (1997) provide the following formula relating the maximum spatial index K to the half-width of the cap θ_0 and the minimum wavelength resolvable $w_{\min}$:

$$K \approx \frac{2\theta_0}{\pi}\left(\frac{2\pi}{w_{\min}} + 0.5\right) - 0.5 \qquad (1)$$

In our case we have $K = 5$ and $\theta_0 = 20°$, giving $w_{\min} = 14.8°$.

3. Results

Figure 1 shows some typical outputs from the analysis. The data are from 13 February 1990, selected because it has low K_p values of 0^+, 1^0 and 0^+. The arrows, drawn at each magnetometer location, give an approximate indication of ionospheric current direction and magnitude and are obtained by rotating the horizontal magnetic field components clockwise through 90° in accordance with the Biot-Savart law (Winch, 1966). The contours represent the equivalent external current function derived from the spherical cap harmonic analysis with 5 kA flowing between one contour and the next. The asterisks represent the limits of the spherical cap used in the analysis. Good agreement between the arrow directions and the clockwise direction of flow of the dominant external currents round the contours shows that the SCHA is performing satisfactorily here and the focus of the external system can be pinpointed on each map at around 10–11 h local time. The westward movement of the system across northern Australia can be clearly seen in these figures.

If we look at the internal current system contours derived from the SCHA for the same day, we find that the currents are reversed in direction, as expected, but they seem to lag a little behind the external system. The internal system can be clearly seen over Western Australia at 3 h UT while the external system has already passed into the Indian Ocean. This internal system remains in much the same position and with similar intensity at 4 and 5 h UT and disappears at 6 h UT. We have many examples where this behaviour is similar—first a small lag of the internal system relative to the external system when it is over the continent and then the internal system remains after the external system has moved away westwards. Matsushita and Maeda (1965) showed in a GSHA that the position of the focus of the internal current system generally lagged that of the external system by about one hour.

Since Torta and de Santis (1996) suggest that a smaller cap leads to greater errors in the separation of the external and internal components, we looked at the effect of increasing the cap half width to 30°. The second pair of diagrams in Fig. 1 for 2 h UT shows that, with the 30° cap, the current systems are somewhat simpler, as expected, but not significantly different.

Figure 2 shows the derived internal current whorl at 5 UT on 27 January 1990. The external system (not shown) follows the arrows marked in the figure but the westward progress of the internal system appears to be delayed, as was also seen above for 13 February 1990. In seeking the cause of this we reduced the Z component at three stations in south-west Australia: we reduced Z at ABY from 61.0 to 30.0, at GER from 54.2 to 30.0 and at GNA from 53.2 to 30.0 nT. When the data were re-processed the internal current whorl at 5 h UT completely disappeared. Similarly, if we increased Z at the same three stations to 80.0 nT, the internal current whorl greatly increased in intensity. All this is happening at a time when the external current focus is way out in the Indian Ocean and so no internal current whorl is expected to be seen. Thus the apparent lag of the internal current system arises from the large Z variations in south-west Australia associated with the coastal effect. The presence of a larger coastal effect in this region is supported by the results of Stening and Reztsova (2007) where it is shown to extend from Albany (ABY) to Learmonth (LRM).

An example where SCHA fails to give an acceptable internal current system is shown in Fig. 3. At the same time the external current system generated is quite good.

We are able to separate out the internal and external contributions at an individual location. In Fig. 4 we show this for Tennant Creek (TCK), a station far from the coast. We only show data from the middle of the day (09–15 h LT). The internal and external contributions are summed and compared to the original data in the top set of diagrams. The agreement is good comparing the totals to the origi-

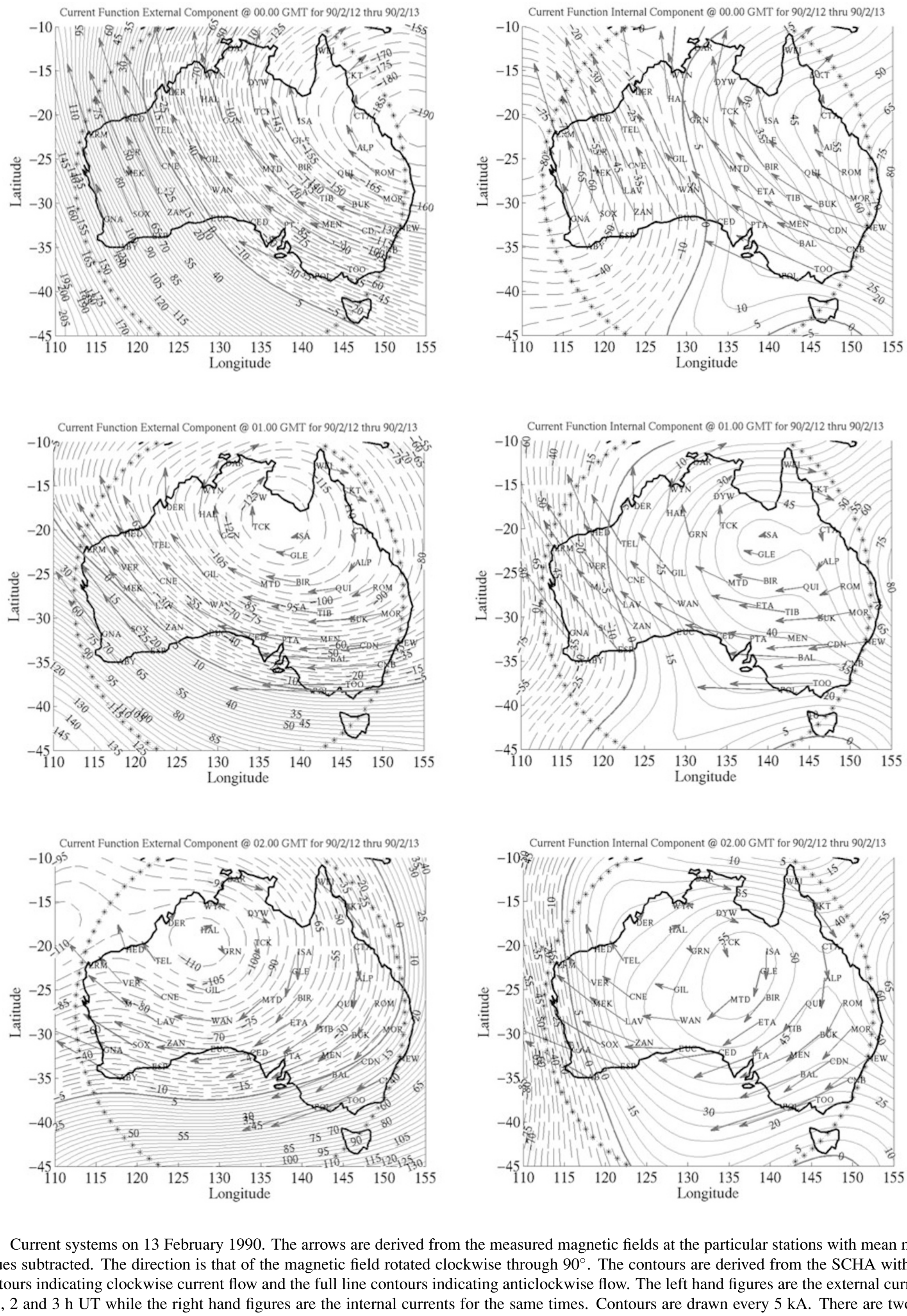

Fig. 1. Current systems on 13 February 1990. The arrows are derived from the measured magnetic fields at the particular stations with mean midnight values subtracted. The direction is that of the magnetic field rotated clockwise through 90°. The contours are derived from the SCHA with dashed contours indicating clockwise current flow and the full line contours indicating anticlockwise flow. The left hand figures are the external currents for 0, 1, 2 and 3 h UT while the right hand figures are the internal currents for the same times. Contours are drawn every 5 kA. There are two sets of diagrams shown for 2 h UT. The first pair uses a cap half-width of 20°, while the second pair, which has no asterisks indication the edge of the cap, uses a half-width of 30°.

nal data. The internal and external variations of X are both positive but the internal X variation is not similar to the external variation. The internal and external Y variations are quite similar and the internal Z variation is opposite to the external as theoretically expected. It is opposite both in its sense of variation and in sign.

By contrast we show in Fig. 5 the internal/external separation at Albany (ABY) on 27 January 1990, the day also

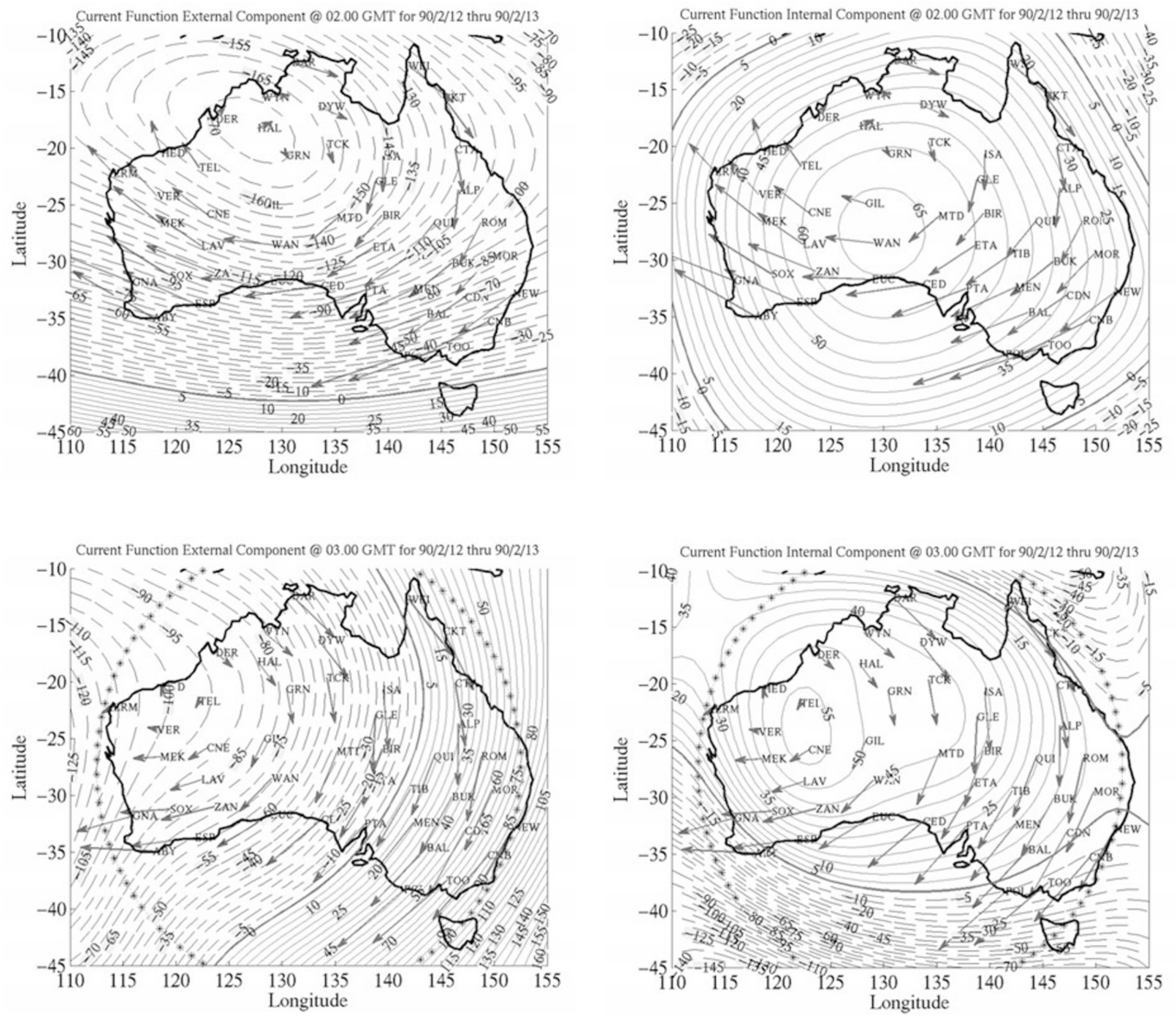

Fig. 1. (continued).

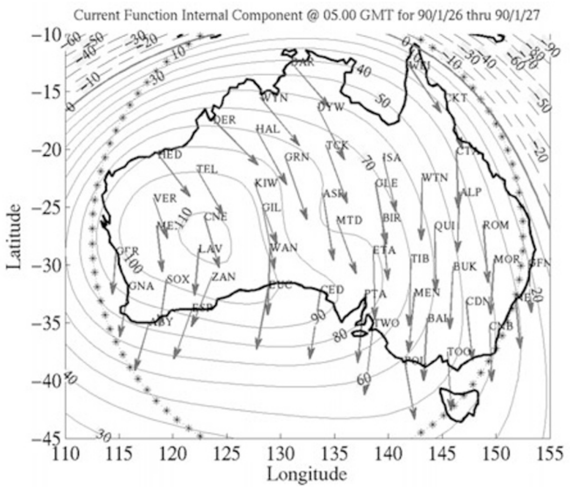

Fig. 2. Internal current system derived from SCHA for 5 h UT on 27 January 1990.

shown in Fig. 2. Whereas the internal X and Y variations are only very vaguely similar to the corresponding external variations, the internal Z variations are remarkably similar to the external. This differs from the behaviour of Z in Fig. 4 where the internal variation is opposite to the external. The behaviour of Z at Geraldton (GER) on this day is similar to that at Albany while, again on the same day, Z at the inland station at Tennant Creek (TCK) behaves "normally" as it did on 13 December 1989 (Fig. 4) with the internal component of Z ranging from -30 to $+30$ nT. On the east coast, Newcastle (NEW) has internal and external Z varying in an opposite fashion but the internal Z is larger than at TCK, being of similar amplitude to the external variation (Fig. 6). Here we have extended the plot to cover all of daytime. The coastal effect on the east coast is in the opposite sense to that on the west coast and yields a quite different observed variation in Z (Fig. 7). The internal component of Z here is so large compared to the external component that it completely changes the observed daily variation of Z. This again supports the findings of Stening and Reztsova (2007) where the coastal effects at ABY and NEW are found to be of opposite phase at both 12 h and 24 h periods.

We also found that at the time the snapshot of the currents was taken for Fig. 3, the Z variations at GER, CVN and LRM were close to zero. So the transition from negative to positive, which for GER is at about 9 h LT in Fig. 7, occurs about two hours later on 15 May 1990 (Fig. 3). So again an

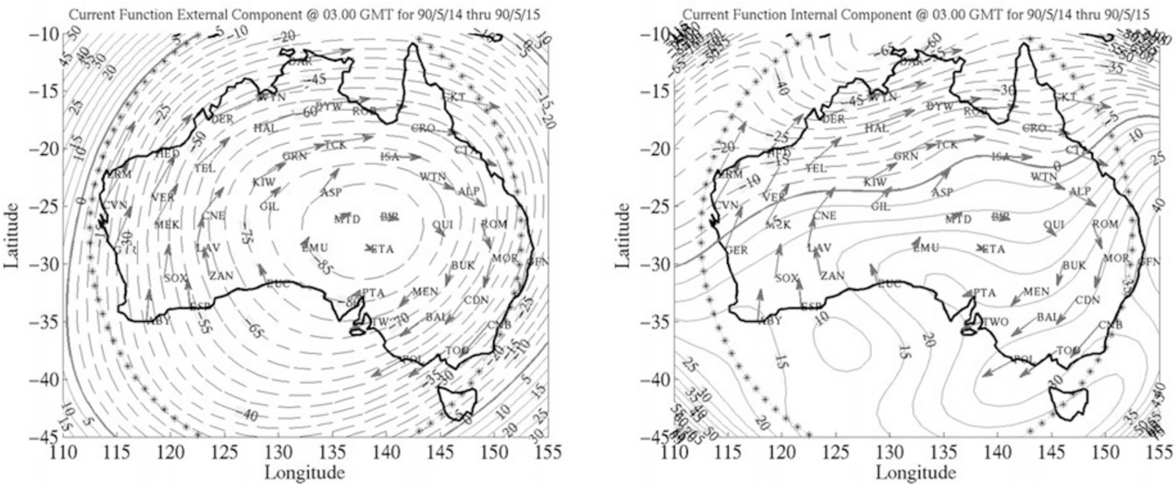

Fig. 3. External (left) and internal (right) current systems derived by SCHA for 3 h UT on 15 May 1990.

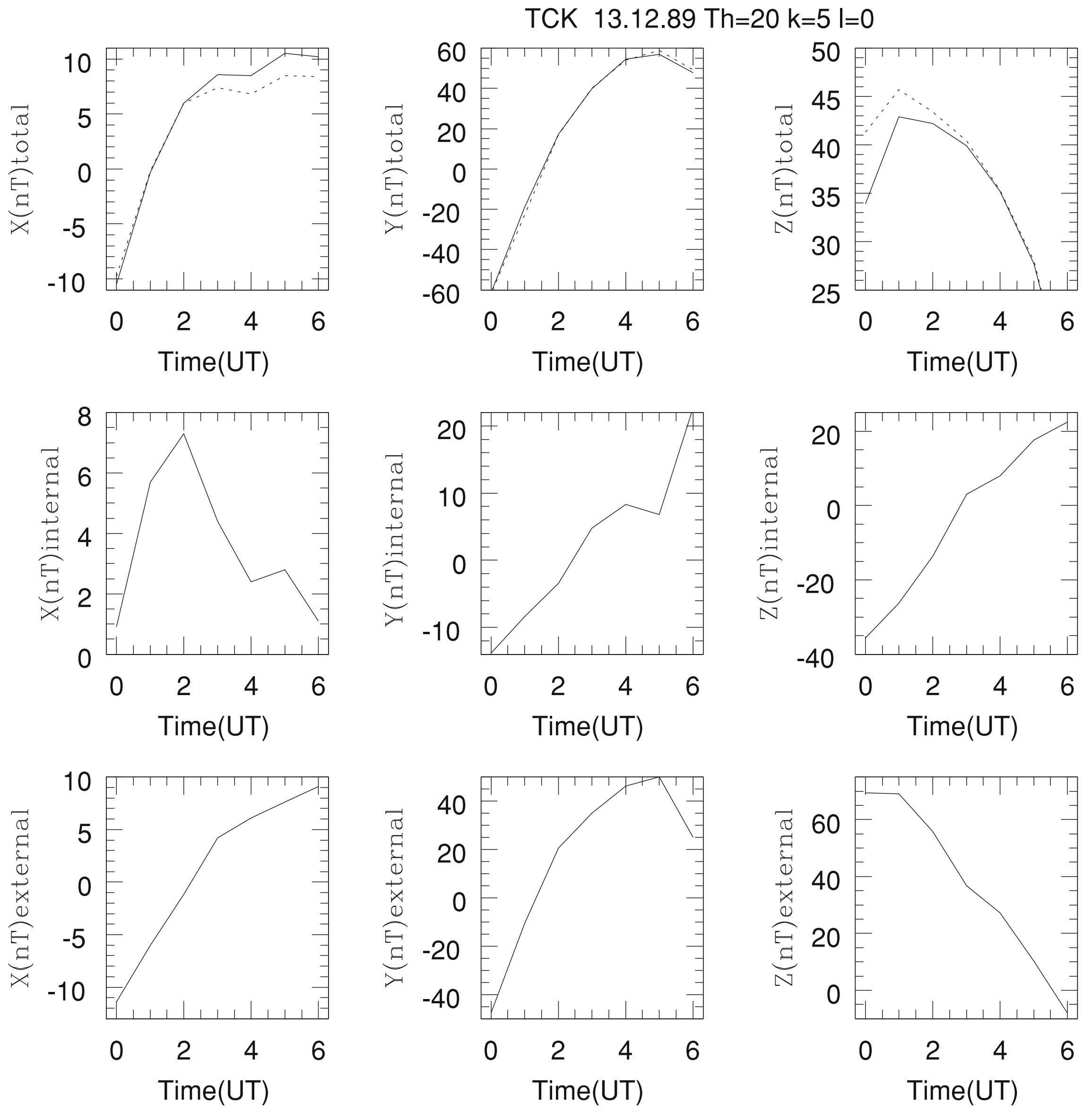

Fig. 4. Decomposition of magnetic variations into external and internal contributions at Tennant Creek during daylight hours on 13 December 1989. The top figures show the total variation (external plus internal) from the analysis with the full line and the original data with the dashed line. The middle figures show the internal components and the bottom figures show the external components.

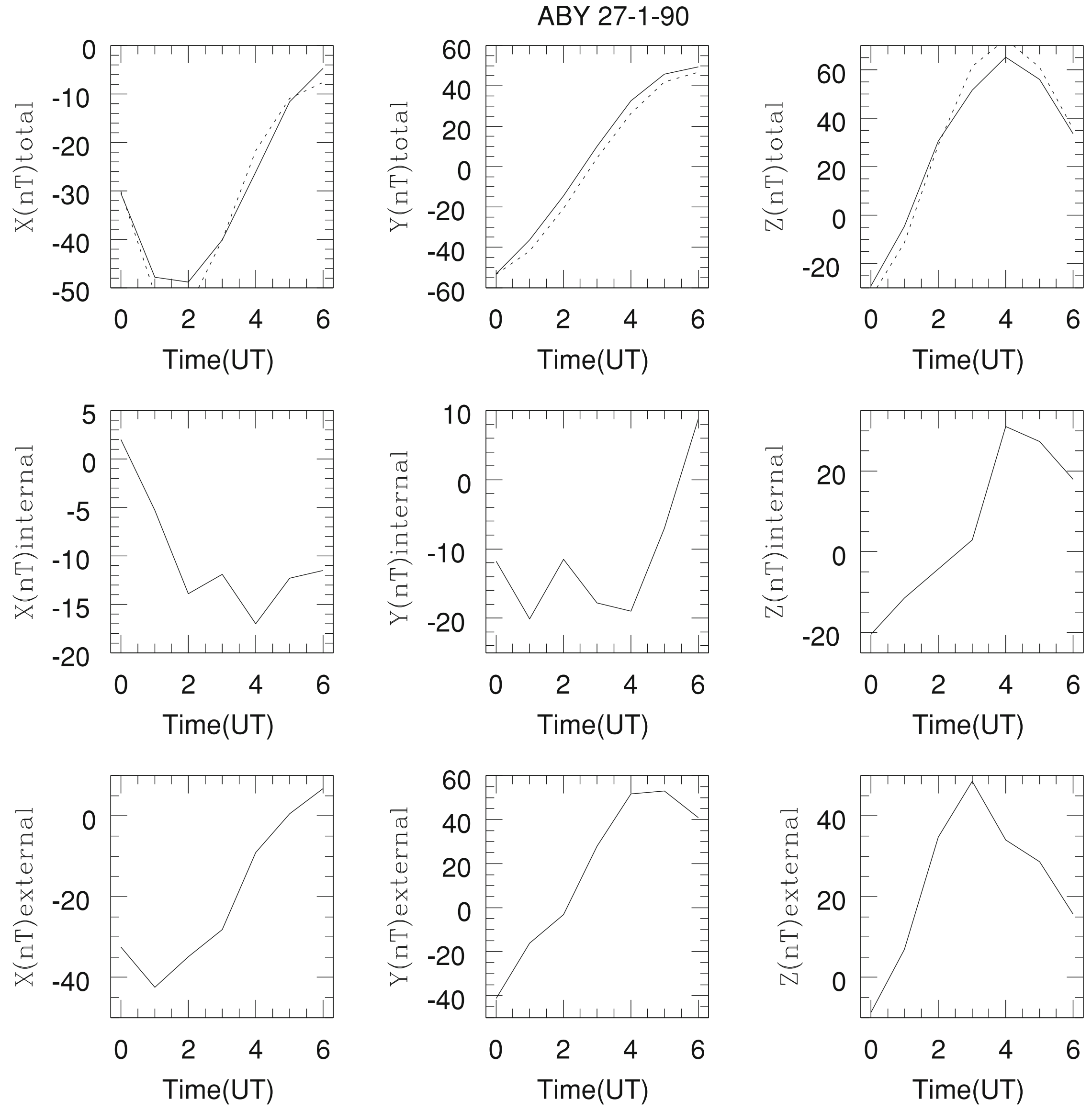

Fig. 5. As for Fig. 4 but for Albany (ABY) on 27 January 1990.

anomalous Z variation may be the cause of the disruption of the performance of the SCHA in giving correct internal currents.

4. Comparisons with the CM4 Model

The development of the CM4 model of the Earth's magnetic field by Sabaka *et al.* (2002, 2004) provides an opportunity for testing the SCHA effectiveness. The model provides, among other things, values of both the external and, separately, the internal contributions to the daily magnetic variations at any specified position on the Earth. This gives the opportunity to input the total field variations derived from the model into the SCHA and to test whether the analysis can correctly recover the external and internal components. Our trust in the model is somewhat affected by the fact that data from only three stations in Australia are included, two closely spaced in the south east corner and one in the south-west corner (figure 1 of Sabaka *et al.*, 2004). The model does not exhibit the coast effect in the same way as does the AWAGS data, so that model variations of Z have a similar form from east to the west coast in contrast to the differences displayed in Fig. 7.

We used the same stations as in the AWAGS analysis but substituted the magnetic fields derived from the CM4 model. Only fields from ionospheric currents and their associated induced currents were included. In Fig. 8 we see that clear current whorls are obtained both for the external and internal components. However the whorl is two hours later than that derived from AWAGS data on that day. We then proceeded to derive the internal and external components at the Tennant Creek station (TCK) and compared them with the known internal and external contributions from CM4. These are shown in Fig. 9. The agreement is good for the Y component, fair for the X component and not very good for Z. Another analysis was performed in a different season using data for 13 February 1990. The CM4 data yielded a focus much further south than the AWAGS data, being at 30–35° latitude compared to 20° from AWAGS, as seen in Fig. 1. As the focus was so far south, the internal analysis of the CM4 data did not

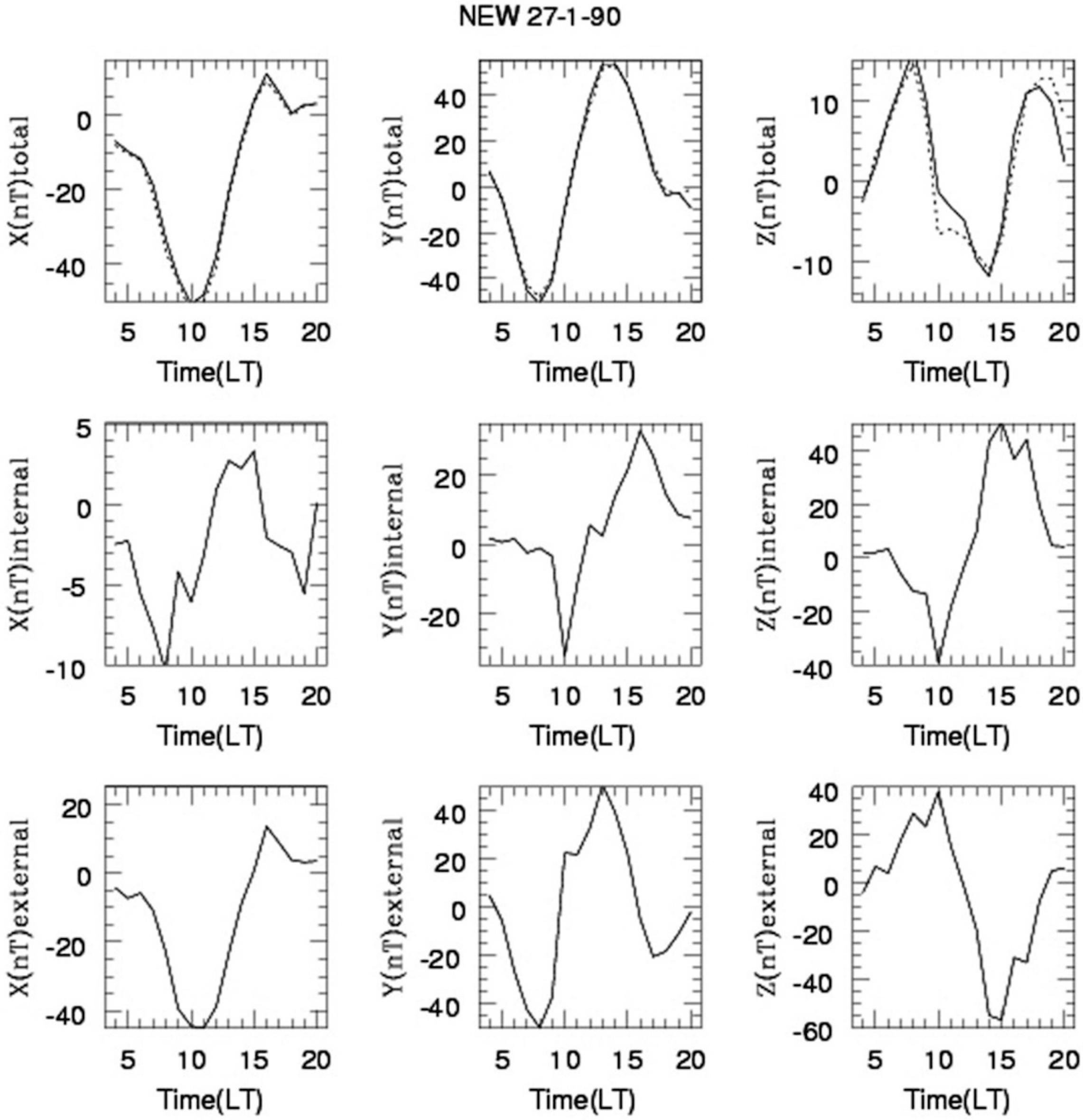

Fig. 6. As for Fig. 4 but for Newcastle (NEW) on 27 January 1990. Note that here we have an extended plot against local time.

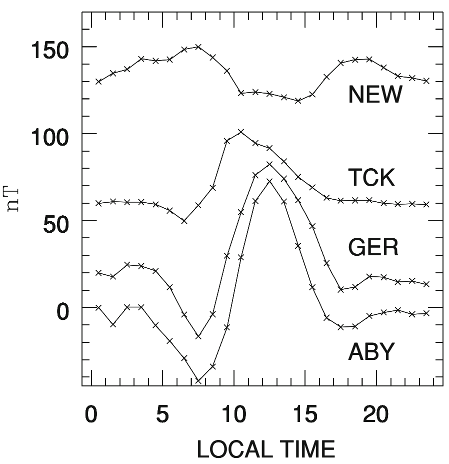

Fig. 7. Daily variations (hourly mean values) for the vertical Z component at selected stations on 27 January 1990. Curves have been vertically shifted to separate them. Midnight values have been subtracted so that each curve starts at 0 nT at local midnight.

form a current whorl. We tested the separation at the station QUI with results shown in Fig. 10. Again the Y variations fit very well and, if a baseline shift is made, the X variations also agree. The Z variations do not fit so well but the fit is slightly better than for the June data. The fact that no internal current whorl was derived did not seem to affect the separation at an individual station.

5. Discussion

From the above examples, which are typical, we conclude that the SCHA method successfully derives the form of the external current system at a particular time on a particular day. One of the aims of this work has been to locate the position of the focus of the current system and to reveal the general shape of the current whorl. In this respect we see that, generally, the agreement between the arrows, derived directly from the magnetic field measurements, and the current function contours is good. Further detail on the shape of the S_q system has been published elsewhere (Sten-

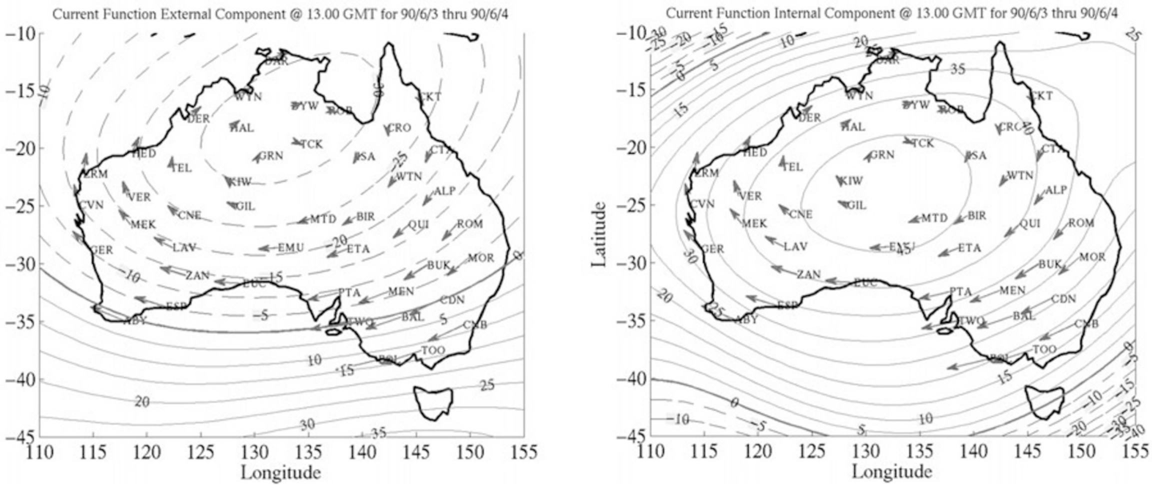

Fig. 8. External (left) and internal (right) current systems derived by SCHA from CM4 data for 4 June 1990 at 13 h Central Local Time with θ_0 size 30°.

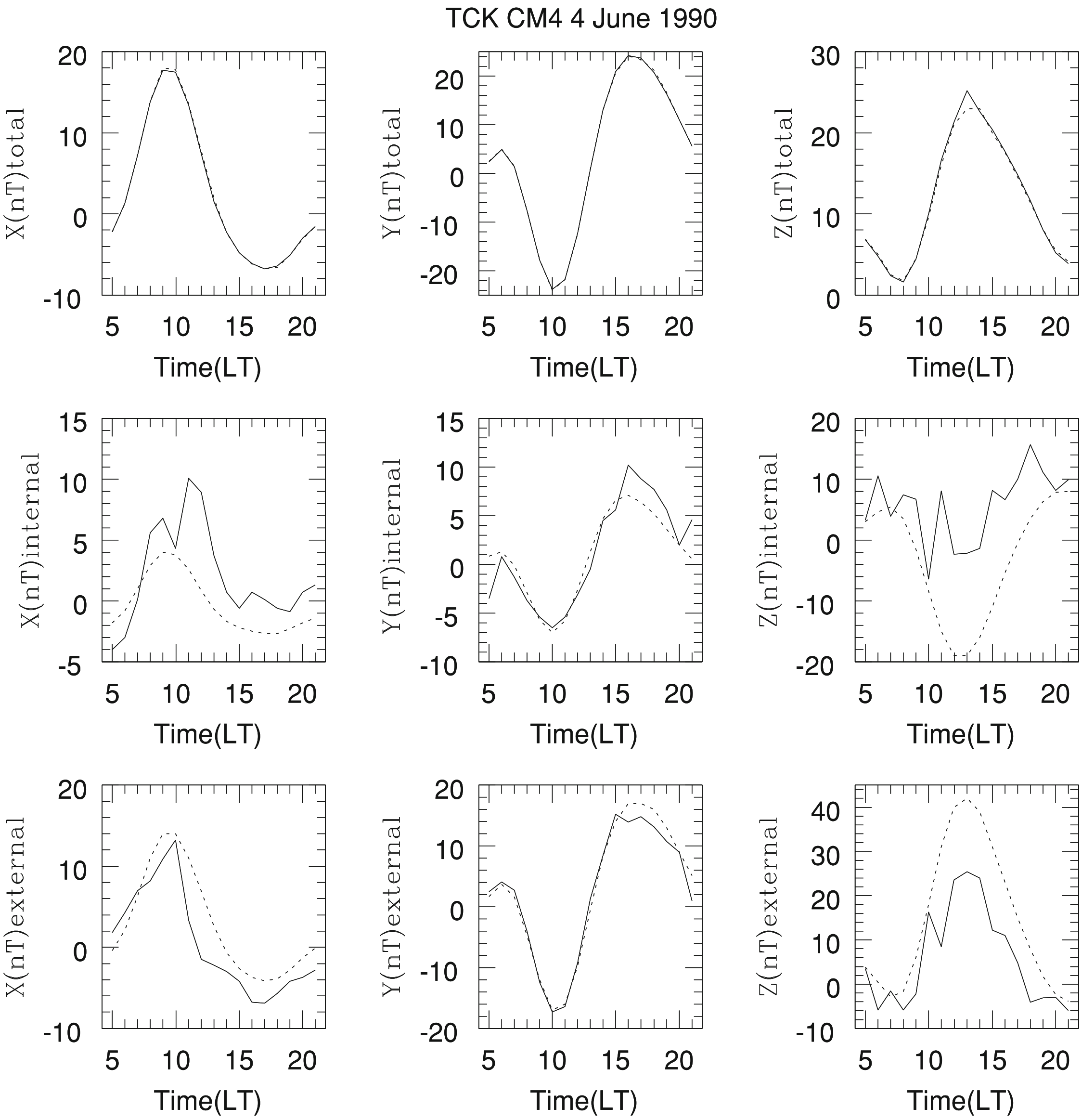

Fig. 9. Total (top), internal (middle) and external (bottom rows) components at Tennant Creek (TCK) from CM4 data on 4 June 1990. Full line, SCHA derived values; dotted line, original CM4 data.

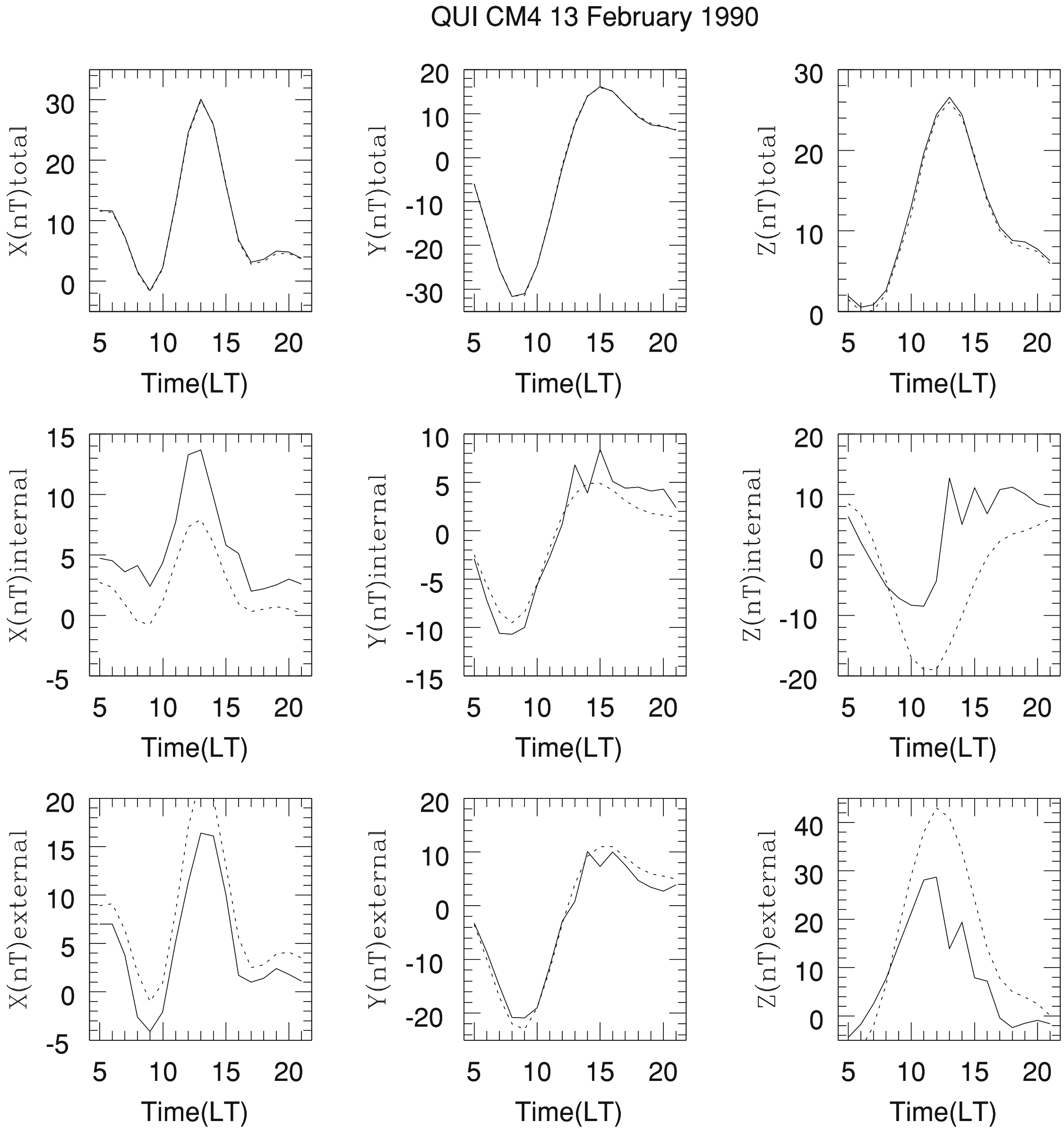

Fig. 10. Total (top), internal (middle) and external (bottom rows) components at Quilpie (QUI) (26.6°S, 144.2°E) from CM4 data on 4 June 1990. Full line, SCHA derived values; dotted line, original CM4 data.

ing, 2008).

The analysis is less reliable in respect of the internal system and it is hard to judge when it is performing badly. Some workers (e.g. Lowes, 1999) believe that SCHA will never be able to separate internal from external fields with any accuracy. Thébault *et al*. (2004) also emphasise that the method fails to correctly model the radial dependence of the field and so is unlikely to give a good internal/external separation. Others (G. V. Haines, private communication, 2006) still believe the method can yield useful information if used carefully. Thébault *et al*. (2006) have devised a revised SCHA which they suggest will remove some of the problems associated with the Haines formalism but which is beyond the scope of this paper.

The analysis of CM4 model data shows that the SCHA can at least separate the internal and external components of the horizontal magnetic field. We have found that changing the cap size from 20° to 30° also has little effect on the currents obtained from the CM4 data and this is expected since the model field has less geographic variation than the AWAGS fields. We have further shown how the coast effect sometimes appears to disrupt the SCHA in the sense that no internal current whorl can be identified, or it lags behind the external whorl by an unrealistic amount. But the separations at Newcastle (NEW) and Albany (ABY) are consistent with other analyses and one might therefore expect that the overall picture of internal currents may be quite complicated as it must incorporate the regular induced S_q current effect as well as the additional currents associated with the coast effect.

The large lags of the internal current system with respect to the external system are a persistent feature which can now be seen as an artefact produced by the intense coastal effect on the southwest coast.

6. Conclusions

1. On some days SCHA presents a realistic picture of the westward movement of the S_q current system across Australia, with the internal current system lagging slightly behind the external currents.
2. On other days the SCHA fails to provide a typical internal current system. On many occasions the vertical magnetic field, Z, variations on the west coast were found to disrupt the SCHA, either because they were exceptionally large or anomalous in some other way. Otherwise we are unable to predict when SCHA will be able to separate out the internal current system.
3. The coastal effect on the vertical Z component is elucidated in that we see that the internal component adds to the external component on the west coast while it subtracts on the east coast, giving rise to the quite different observed daily variations in Z and confirming earlier results of Stening and Reztsova (2007).
4. The total field variations from SCHA at particular stations generally agree well with the original data.

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