Earth Planets Space, **60**, 743–751, 2008

Effects of interactions between stations on the calculation of geomagnetically induced currents in an electric power transmission system

Risto Pirjola

Finnish Meteorological Institute, P. O. Box 503, FIN-00101 Helsinki, Finland

(Received December 17, 2007; Revised February 27, 2008; Accepted March 4, 2008; Online published August 4, 2008)

"Geomagnetically induced currents" (GIC) in ground-based technological networks are a manifestation of space weather. GIC are a potential source of problems to the systems and therefore important in practice. GIC in a power system (or in principle in any other discretely-earthed system) can be calculated conveniently by using matrix equations presented earlier. Since temporal variations associated with GIC are slow compared to the 50/60 Hz frequency used in power transmission, a dc treatment is acceptable. An essential quantity in calculations of GIC in a power grid is the earthing impedance matrix, which is the transfer function coupling GIC flowing to (from) the Earth with the voltages between the earthing points, called nodes or (sub)stations, and a remote earth. The diagonal elements of the matrix equal the earthing resistances of the nodes whereas an off-diagonal element expresses how much GIC at one earthing point affects the voltage at another node. In GIC calculations, except for some special treatments of individual sites, the off-diagonal elements are usually neglected by saying simply that the earthing points (are assumed to) lie distantly enough. In this paper, we examine the effects of off-diagonal elements of the earthing impedance matrix, i.e. the effects of interactions between different stations, on GIC calculations in greater detail and more quantitatively than before. We consider a fictitious system that represents a high-voltage power grid and a simple "network" consisting of two stations with a line connecting them. For both systems, the conclusion can be drawn that the off-diagonal elements do not play a major role in practice. Modelling them only approximately, or even ignoring them, is not of great significance compared to other shortcomings involved in GIC calculations. This is particularly true when looking at a power grid as a whole although at some individual stations the neglect may lead to larger errors in GIC values.

Key words: Geomagnetically induced current, GIC, space weather, power grid, earthing impedance matrix.

1. Introduction

"Geomagnetically induced currents" (GIC) flowing in networks, such as electric power transmission grids, oil and gas pipelines, telecommunication cables and railway systems, are a manifestation of "Space Weather" at the Earth's surface. The origin of space weather is in the activity of the Sun. The plasmaphysical and electromagnetic phenomena associated with space weather constitute a complicated chain of processes, and technological systems in space and on the ground can experience problems due to space weather (e.g. Lanzerotti *et al.*, 1999).

The physical principle of GIC, which have already been known for about 150 years (e.g. Boteler *et al.*, 1998; Lanzerotti *et al.*, 1999; and references therein), can easily be explained based on Faraday's and Ohm's laws: Rapidly-changing currents in the Earth's space environment during a space weather storm create temporal variations of the geomagnetic field. They induce a (geo)electric field, which drives currents in all conductors. Besides technological networks, the Earth is a conductor as well. Currents in the ground also contribute to geomagnetic variations and to induced geoelectric fields occurring at the Earth's surface (e.g. Watermann, 2007).

Nowadays electric power transmission systems constitute the most critical infrastructures regarding GIC. Transformers may be saturated due to the flow of a dc-like GIC in the windings possibly leading to problems that may even extend to a collapse of the whole system and/or to permanent transformer damage (e.g. Kappenman and Albertson, 1990; Kappenman, 1996; Bolduc, 2002; Molinski, 2002; and references therein). Two well-known events are the blackouts in Québec, Canada, in March 1989 and in southern Sweden in October 2003, which have also been documented and reported in detail (Bolduc, 2002; Pulkkinen *et al.*, 2005). GIC are a problem in particular in high-latitude auroral regions. However, during geomagnetic storms, the auroral oval may move to mid and even low latitudes. Furthermore, GIC magnitudes are much affected by the network configuration and resistances. The GIC sensitivities of systems also depend on many technical matters, so that a small GIC that is not harmful to one grid may cause problems to another. All this means that lower-latitude networks may experience serious GIC impacts as well. The large sizes of power grids, their complex interconnections, and the extensive transport of energy make GIC issues more and more important at all latitudes today (Kappenman, 2004).

Theoretical calculations of GIC in technological systems are usually carried out by a two-step procedure (e.g. Pirjola, 2002): 1. ("*geophysical step*") the determination of the horizontal geoelectric field at the Earth's surface, and 2. ("*engineering step*") the computation of GIC produced by the geoelectric field. The input of the geophysical step consists of information about the ground conductivity and of data or assumptions of magnetospheric-ionospheric currents or of geomagnetic variations at the Earth's surface. The engineering step uses the geoelectric field and the network configuration and resistances as the input. This paper is focused on the engineering step for a power grid, which is a discretely-earthed network having earthings at transformer neutrals at (sub)stations, called the nodes or earthing points.

Due to the low frequencies involved in geoelectromagnetic phenomena and GIC (compared to the 50/60 Hz frequency used in electric power transmission), a dc treatment is acceptable in the engineering step, for which Lehtinen and Pirjola (1985) present a technique involving matrix formulas and summarised in Section 2 of this paper. Pirjola (2007) derives an alternative, but equivalent, matrix formulation for the engineering step. In principle, these calculations can be performed exactly provided that the configuration, connections and resistances of the system are known. In practice, however, an accurate description of all components that affect the GIC flow in a complex high-voltage power grid is difficult.

In the engineering step for a power grid, the key quantities are the (geo)voltages obtained by integrating the geoelectric field along the transmission lines and two matrices, the network admittance matrix and the earthing impedance matrix, which include the system resistances. The former matrix is simple to be determined as it is based on known resistances of the transmission lines whereas the latter matrix is more tricky. It is the transfer function that couples GIC flowing to (from) the Earth from (to) the network with the voltages between the earthing points and a remote earth. The diagonal elements of the matrix equal the earthing resistances, and the off-diagonal elements are associated with the effects of GIC at one earthing point on the voltages at other nodes, i.e. with interactions between stations. Except for treatments of two different voltage levels discussed by Mäkinen (1993) and Pirjola (2005a) or other explicit inclusions of more than one earthing point at single sites, as for example in the study described by Wik *et al.* (2008), the off-diagonal elements are usually neglected in GIC calculations by assuming simply that the distances between the earthing points are large enough. Considering the Finnish 400 kV power grid, Pirjola (2008) indicates that the effect of the off-diagonal elements generally seems to be of minor importance in practice. In this paper, we examine the issue in detail and quantitatively by considering a fictitious system that represents a high-voltage power grid (Section 3) and a simple "grid" consisting of two stations with a line connecting them (Section 4). To avoid unnecessary complications and to make the interpretations of the results easier, the horizontal geoelectric field is assumed to be uniform in this paper. Its magnitude is set equal to 1 V/km, thus representing a possible, though not the highest, value to be expected during a geomagnetic storm (e.g. Pirjola, 1983; Kappenman, 2006).

2. Engineering Step of the Calculation of GIC in a Power System

As mentioned in Section 1, the low frequencies accompanying GIC enable a dc modelling in the engineering step (at least as the first approximation). It makes the quantities involved real. Let us consider a system having N discrete nodes, called earthing points or (sub)stations, earthed by the resistances R_i^e ($i = 1, ..., N$). The nodes are connected to each other by conductor lines with resistances R_{ij}^n ($i, j = 1, ..., N$). These conductors are located between any station pairs in the network, and if two nodes k and l do not have a real conductor between them, it is formally expressed by setting R_{kl}^n equal to infinity. The system discussed in this paper is a power grid but the formulation presented in this section is in principle valid for any discretely-earthed network. Lehtinen and Pirjola (1985) derive the following formula for the $N \times 1$ column matrix $\mathbf{I}_e$ consisting of GIC ($= I_{e,j}$, $j = 1, ..., N$) to (from) the Earth with the positive direction into the Earth, called the earthing GIC, at the nodes:

$$\mathbf{I}_e = (\mathbf{1} + \mathbf{Y}_n\mathbf{Z}_e)^{-1}\mathbf{J}_e \quad (1)$$

This equation is presented in many other publications as well (e.g. Pirjola, 2002, 2005a, 2007, 2008). The symbols $\mathbf{1}$, $\mathbf{Y}_n$ and $\mathbf{Z}_e$ denote the $N \times N$ (unit) identity matrix, the $N \times N$ network admittance matrix and the $N \times N$ earthing impedance matrix, respectively. The $N \times 1$ column matrix $\mathbf{J}_e$ includes the information of the geoelectric field via the geovoltages. Its elements are called "perfect-earthing" currents because, assuming perfect earthings (i.e. $\mathbf{Z}_e = 0$), the GIC included in $\mathbf{I}_e$ equal the elements of $\mathbf{J}_e$. An exact definition of $\mathbf{J}_e$ is given by Eqs. (4) and (5) below. Equation (1) provides a solution for the engineering step. It refers to GIC between the Earth and the network, i.e. to the earthing currents. A formula for GIC in the conductor between the nodes i and j could also be given (e.g. Lehtinen and Pirjola, 1985; Pirjola, 2007, 2008) but it is omitted here because conductor GIC are less important than earthing GIC in practice, as the latter are responsible for possible transformer saturation.

The definition of the earthing impedance matrix states that multiplying $\mathbf{I}_e$ by $\mathbf{Z}_e$ gives the voltages between the earthing points and a remote earth associated with the flow of the currents $I_{e,j}$ ($j = 1, ..., N$). Expressing the voltages in an $N \times 1$ column matrix $\mathbf{U}_{cur}$, we thus obtain

$$\mathbf{U}_{cur} = \mathbf{Z}_e\mathbf{I}_e \quad (2)$$

The subscript 'cur' indicates that voltages associated with earthing currents are considered. Consequently, an off-diagonal element $Z_e(i, j)$ ($i \neq j$) of the earthing impedance matrix expresses the contribution of a current at the node j to the voltage at the node i. By the reciprocity theorem for electric networks, $Z_e(i, j) = Z_e(j, i)$, i.e. $\mathbf{Z}_e$ is symmetric. As mentioned in Section 1, the diagonal elements $Z_e(i, i)$ simply equal the earthing resistances R_i^e, which actually directly corresponds to the definition of an earthing resistance (cf. Ohm's law). The off-diagonal elements of $\mathbf{Z}_e$ obviously have to be smaller than (or equal to) the diagonal ones on the same rows and columns. If the stations are distant enough, the influence of one earthing current on the voltage at another station is negligible, and $\mathbf{Z}_e$ is diagonal.

The symmetric $N \times N$ network admittance matrix $\mathbf{Y}_n$ is defined by

$$(i \neq j) : Y_{n,ij} = -\frac{1}{R^n_{ij}}, \quad (i = j) : Y_{n,ij} = \sum_{k=1,k\neq i}^{N} \frac{1}{R^n_{ik}} \tag{3}$$

The elements $J_{e,i}$ of the $N \times 1$ column matrix $\mathbf{J}_e$ are given by

$$J_{e,i} = \sum_{j=1,j\neq i}^{N} \frac{V_{ji}}{R^n_{ji}} \tag{4}$$

where V_{ji} is the geovoltage between the nodes j and i ($j, i = 1, ..., N$) associated with the horizontal geoelectric field $\mathbf{E}$, i.e.

$$V_{ji} = \int_j^i \mathbf{E} \cdot d\mathbf{s} \tag{5}$$

Generally, the integral in (5) is path-dependent, and the integration route has to follow the conductor from j to i (Boteler and Pirjola, 1998; Pirjola, 2000). An equivalent way to express the path-dependence is to say that the geoelectric field is rotational, i.e. its curl is non-zero. By Faraday's law, the curl of the electric field equals the negative time derivative of the magnetic field, which is generally not zero. We consider the horizontal geoelectric field at the Earth's surface, and so the vertical component of the curl of the geoelectric field, which equals the time derivative of the vertical component of the magnetic field, plays a role. For most ionospheric-magnetospheric current sources, usually located in the auroral region, this time derivative differs from zero. The geoelectric field is largest in the vicinity of the source and decreases with distance from the source. Thus the risk for high GIC is largest in high-latitude areas. In this paper, however, the focus is not on the investigation of the spatial variation of the geoelectric field, so the geoelectric field is simply assumed uniform (see Section 1) and thus irrotational. This well corresponds to the situation at mid and low latitudes.

A power grid operates as a three-phase system, but in the application of the above formulas to GIC calculations, as well as in this paper, the three phases are usually treated as one circuit element, whose resistance is one third of that of a single phase and which carries a GIC three times the current in a single conductor. It is also convenient to define the "earthing resistances" of the nodes to include the actual earthing resistances of the stations, the transformer resistances and the resistances of possible neutral point reactors (or any other resistors) in the earthing leads of transformer neutrals (e.g. Pirjola, 2005b, 2007). For additional items on power grid modelling for GIC studies, Mäkinen (1993) and Pirjola (2005a) are also referred to. They present details of handling full-wound and autotransformers when GIC in a power grid with two different voltage levels are considered. The technique is based on defining "virtual nodes" and "virtual lines" and on using off-diagonal elements of the earthing impedance matrix. At this point, we may mention that the technique described by Mäkinen (1993) and Pirjola (2005a) for full-wound transformers is, though correct, obviously unnecessarily complicated and a more straightforward modelling would also be possible. This is, however, an item outside the scope of this paper, and it is not discussed more here.

3. Fictitious High-voltage Power System

Except for special treatments of individual sites referred to in Sections 1 and 2, the earthing impedance matrix $\mathbf{Z}_e$ is usually assumed diagonal in calculations of GIC in a power system by stating vaguely that the stations are distant enough, so interactions between them may be ignored. By discussing the Finnish 400 kV system, Pirjola (2008) shows that the neglect of the off-diagonal elements of $\mathbf{Z}_e$ is acceptable in practice if the power grid is considered as a whole, but at some individual stations GIC values may experience clear influences from the off-diagonal elements. In the network investigated by Pirjola (2008), there is no station in the close vicinity of another station, as the shortest distance is as large as about 20 km.

In this paper, we calculate GIC produced by a uniform horizontal geoelectric field of 1 V/km pointing to the east ('E') or to the north ('N') in a fictitious power grid, in which there are two pairs of closely-located stations (Fig. 1). The grid has similarities with the Finnish 400 kV system but many details make it quite different. It consists of 14 earthing points with the stations 2 and 10 and the stations 3 and 5 located so near each other (distances = 250...300 m) that they belong to the same square mark in Fig. 1. The number of lines is 17 including those 16 seen in the figure and a short line between the stations 3 and 5. The lines ending at the points (3&5) and (2&10) are [1–5, 5–9, 2–3] and [2–3, 10–11, 2–6, 10–12], respectively. (Note that station 6 is separate from the line 10–12 although it is

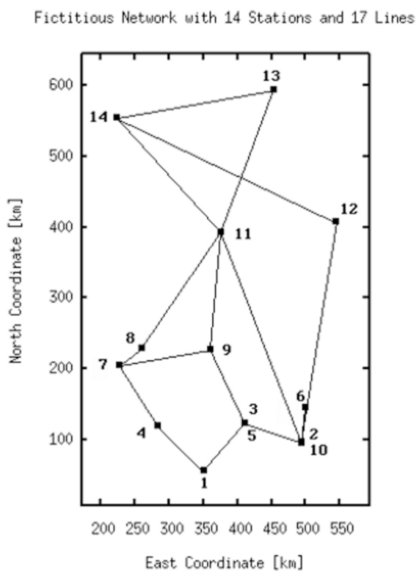

Fig. 1. Fictitious network consisting of 14 earthing points, called nodes or stations, and 17 lines between them presented in a Cartesian east-north coordinate system. Note that the stations 3 and 5, as well as the stations 2 and 10, lie very close to each other (distances = 250...300 m). There is a short line between the stations 3 and 5, which cannot be distinguished in the scale of the figure, but there is no line between stations 2 and 10. The lines ending at the point (3&5) are 1–5, 5–9 and 2–3. The lines ending at the point (2&10) are 2–3, 10–11, 2–6 and 10–12. Note that station 6 is separate from the line 10–12 although it is located close to it.

Table 1. Averages and maxima of the absolute values of the earthing GIC in the fictitious grid shown in Fig. 1. The geoelectric field is uniform and equal to 1 V/km and points to the east ('E') or to the north ('N'). Additional information about the grid and details of resistance values are given in the text. The three different cases regarding the off-diagonal elements of the earthing impedance matrix and denoted by '*nn*', '*00*' and '*hs*' are explained in the text. In each case, the eastward and northward electric fields produce the maximum absolute GIC at the stations 14 and 10, respectively.

	Average			Maximum		
|GIC| [A]	*nn*	*00*	*hs*	*nn*	*00*	*hs*
E	59.5	59.4	59.2	195.9	197.3	196.7
N	57.5	62.2	60.4	211.3	212.0	210.4

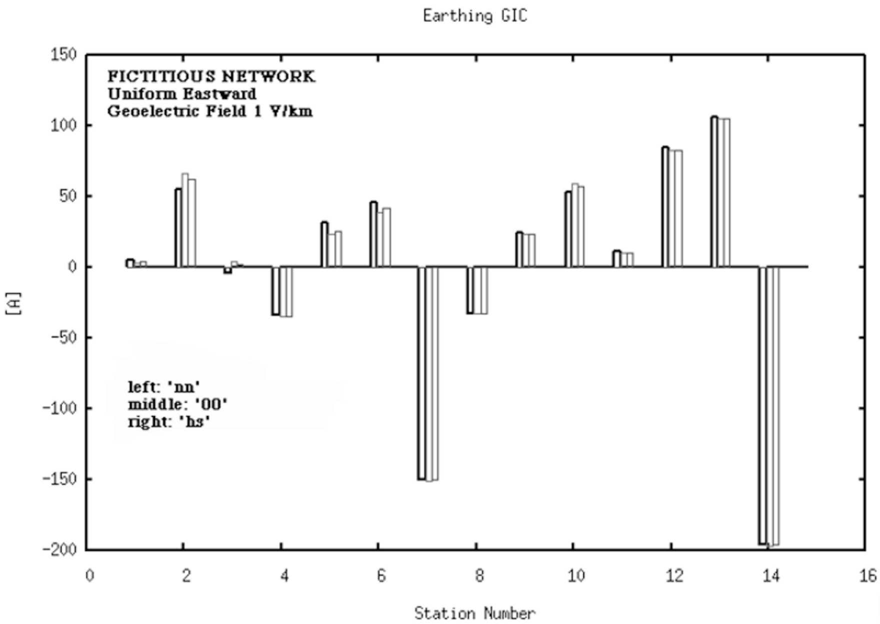

Fig. 2. Earthing GIC at the 14 stations of the fictitious grid shown in Fig. 1. The geoelectric field is uniform and equal to 1 V/km and points to the east. Additional information about the grid and details of the resistance values are given in the text. The three different cases regarding the off-diagonal elements of the earthing impedance matrix and denoted by '*nn*', '*00*' and '*hs*' are explained in the text and correspond to the left-hand, middle and right-hand bars, respectively.

located close to it.)

The earthing resistances of the 14 stations range from 0.33 Ω to 1.14 Ω with a mean of 0.62 Ω. The corresponding values for the 17 line resistances are 0.076 Ω, 3.20 Ω and 1.03 Ω. All these data can be considered reasonable for a high-voltage power system, and comparing the values with those presented by Pirjola and Lehtinen (1985) supports the above-mentioned similarity with the Finnish power grid.

The short distances between the stations 2 and 10 and between the stations 3 and 5 imply that the off-diagonal elements $Z_e(2, 10)$ and $Z_e(3, 5)$ (as well as $Z_e(10, 2)$ and $Z_e(5, 3)$ due to symmetry) are non-zero. The values of these elements are evidently related to the actual earthing resistances of the stations. Therefore we assume that

$$Z_e(2,10) = Z_e(10,2) = \frac{(R_2^e - R_t) + (R_{10}^e - R_t)}{2} = \frac{R_2^e + R_{10}^e}{2} - R_t \tag{6}$$

and

$$Z_e(3,5) = Z_e(5,3) = \frac{(R_3^e - R_t) + (R_5^e - R_t)}{2} = \frac{R_3^e + R_5^e}{2} - R_t \tag{7}$$

The symbol R_t denotes the transformer resistance assumed to be equal at each station. It has to be subtracted from the earthing resistances R_i^e ($i = 2, 3, 5, 10$) to obtain the actual earthing resistances. (The assumption of no neutral point reactors nor other resistors is made, see the end of Section 2.) The value of R_t is set to be 0.28 Ω including all three phases in parallel (Pirjola and Lehtinen, 1985). The other off-diagonal elements of the earthing impedance matrix are kept equal to zero. These grid data refer to the "normal" situation denoted by '*nn*' and considered in Table 1 and in Figs. 2 and 3.

In the second situation discussed for testing purposes in this paper, we set all off-diagonal elements of the earthing impedance matrix equal to zero, including $Z_e(2, 10)$, $Z_e(10, 2)$, $Z_e(3, 5)$ and $Z_e(5, 3)$ even though the distances between the stations are very small. The third situation considered in this paper refers to the "half-sphere-electrode" approximation explained below. In Table 1 and in Figs. 2 and 3, these two cases are denoted by '*00*' and '*hs*', respectively.

Let us assume that a station is earthed with an electrode having the shape of a half-sphere with a radius a. We assume that the Earth is uniform with a conductivity σ and that the plane surface of the half-sphere lies at the Earth's surface with the centre at the earthing point. If a current I is

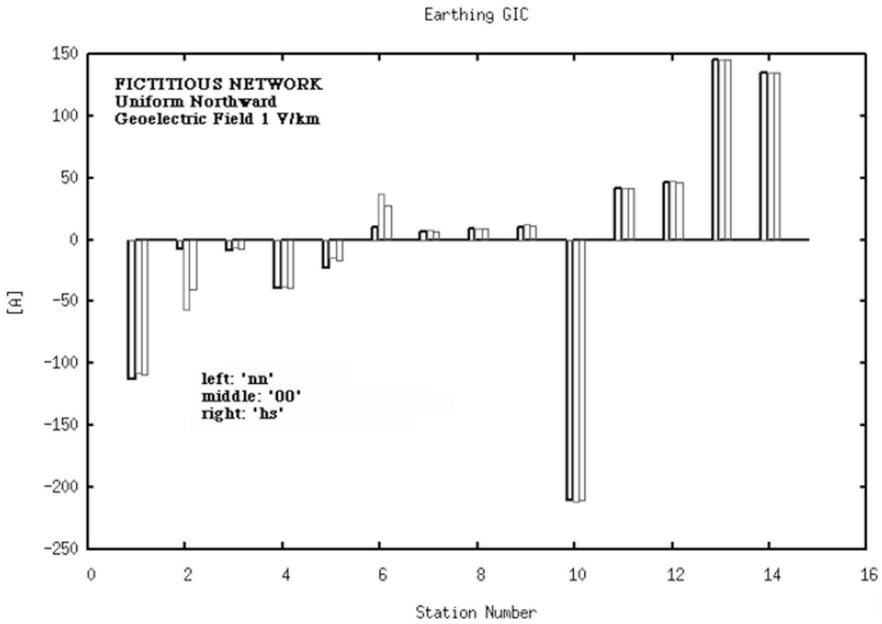

Fig. 3. Similar to Fig. 2 except that the geoelectric field points to the north.

injected at the station into the Earth it flows symmetrically from the spherical surface of the electrode towards a remote earth. Thus, in the Earth, the current density $\mathbf{j}$ is

$$\mathbf{j} = \mathbf{j}(r) = \frac{I}{2\pi r^2}\hat{\mathbf{e}}_r \tag{8}$$

and the electric field $\mathbf{E}_\mathrm{e}$ is by Ohm's law ($\mathbf{j} = \sigma \mathbf{E}_\mathrm{e}$)

$$\mathbf{E}_\mathrm{e} = \frac{I}{2\pi r^2 \sigma}\hat{\mathbf{e}}_r \tag{9}$$

where r is the distance between the point of observation and the station (= the centre of the half-sphere) and $\hat{\mathbf{e}}_r$ is a unit vector in the direction from the station to the point of observation. The voltage U from the station to a remote earth is obtained by taking the line integral of $\mathbf{E}_\mathrm{e}$ given by Eq. (9) from the spherical surface of the electrode to infinity, i.e. by integrating the magnitude of $\mathbf{E}_\mathrm{e}$ from $r = a$ to $r = \infty$. (Here we implicitly assume that the electrode is a perfect conductor, so the voltage from $r = 0$ to $r = a$ is zero.) Consequently

$$U = \frac{I}{2\pi a \sigma} \tag{10}$$

and, by definition, the actual earthing resistance of the station is

$$R^\mathrm{e}_\mathrm{actual} = \frac{1}{2\pi a \sigma} \tag{11}$$

We thus conclude that the actual earthing resistances are inversely proportional to the Earth's conductivity when the Earth is assumed uniform (cf. Pirjola and Viljanen, 1991).

Let another station lie at a distance L from the station at which the current I is injected into the Earth. The voltage U' from this second station to a remote Earth associated with the current I is obtained by a similar integration of the electric field $\mathbf{E}_\mathrm{e}$ given by Eq. (9) but now the lower limit of the integration is $r = L$. Thus

$$U' = \frac{I}{2\pi L \sigma} \tag{12}$$

and, by definition, the off-diagonal element of the earthing impedance matrix between the two stations considered is equal to

$$Z_\mathrm{e}(\mathrm{off-diag}) = \frac{1}{2\pi L \sigma} \tag{13}$$

We see from Eqs. (11) and (13) that the ratio of an off-diagonal element of the matrix $\mathbf{Z}_\mathrm{e}$ to the actual earthing resistance part of the corresponding diagonal element is a/L. This gives us a way to estimate all off-diagonal elements of $\mathbf{Z}_\mathrm{e}$ for the 14-station network considered. Consequently, a "half-sphere-electrode" approximation ('*hs*') means, by definition, that the off-diagonal elements are calculated as follows

$$\begin{aligned} Z_\mathrm{e}(j,k) = Z_\mathrm{e}(k,j) &= \frac{a}{L_{jk}} \frac{(R^\mathrm{e}_j - R_\mathrm{t}) + (R^\mathrm{e}_k - R_\mathrm{t})}{2} \\ &= \frac{a}{L_{jk}}\left(\frac{R^\mathrm{e}_j + R^\mathrm{e}_k}{2} - R_\mathrm{t}\right) \\ &(j,k = 1, ..., 14,\ j \neq k) \end{aligned} \tag{14}$$

where L_{jk} is the distance between the stations j and k. We now assume that $a = 100$ m since it is obviously the correct order of magnitude of sizes of high-voltage substation earthings. If L_{jk} would be smaller than a (which is not the case with the numerical values used now) it would be natural to set a/L_{jk} equal to one. This is also the motivation of the use of Eqs. (6) and (7) but, as the distances between the stations 2 and 10 and between the stations 3 and 5 are 250...300 m, Eq. (14) leads to smaller values of $Z_\mathrm{e}(2, 10)$ and $Z_\mathrm{e}(3, 5)$ than Eqs. (6) and (7).

With the numerical values used, neither Eqs. (6) and (7) nor Eq. (14) lead to the violation of the requirement mentioned in Section 2 that the off-diagonal elements of $\mathbf{Z}_\mathrm{e}$ should be smaller than (or equal to) the diagonal ones on the same rows and columns. This is not self-evident, and in the case of a violation the value of the diagonal element would be assigned to the off-diagonal one.

We now consider GIC in the power grid of Fig. 1 due to an eastward ('E') or a northward ('N') geoelectric field of 1 V/km. The discussion is limited to the earthing GIC, which, as pointed out in Section 2, are more important than GIC in the lines in practice. Table 1 shows the averages and maxima of the absolute values of the earthing GIC corresponding to the three different treatments ('*nn*', '*00*', '*hs*') of the off-diagonal elements of the earthing impedance matrix. In each case, the maximum for 'E' is obtained at the station 14 and for 'N' at the station '10', both of which are corner stations (see e.g. Pirjola, 2008). Table 1 shows that the differences between the three cases are very small. Consequently, when looking at the power grid as a whole, details of the modelling of the off-diagonal elements of the earthing impedance matrix are not important. The results for the case '*00*' even indicate that neglecting the off-diagonal elements between stations very close to each other may be acceptable.

Figure 2 depicts the earthing GIC, due to an eastward ('E') geoelectric field of 1 V/km, at the 14 stations in the three cases. The left-hand, middle and right-hand bars refer to '*nn*', '*00*' and '*hs*', respectively. Figure 3 is similar to Fig. 2 but the geoelectric field is northward ('N'). It is seen that the treatment of the off-diagonal elements of $\mathbf{Z}_e$ may sometimes have a major influence on GIC at individual stations. A good example is the station 3 for 'E', at which even the sign of GIC changes between '*nn*' and the other cases. It should, however, be noted that all these GIC values at the station 3 are very small, thus having no practical importance. Looking at Figs. 2 and 3, we see that only at the stations 2 and 6 for 'N' the difference between '*nn*' and the other cases may play a major role but this does not invalidate the above conclusion of a minor overall impact on the grid.

4. Two-station Power System

We now discuss the effect of off-diagonal elements of the earthing impedance matrix on calculated GIC by considering a power "network" consisting of two stations and a line between them. This simple model enables the use of easy formulas and analytic expressions. Let us denote the (geo)voltage from the station 1 to the station 2 by U, obtained by integrating the geoelectric field according to Eq. (5). The earthing resistances of the stations and the resistance of the line are R_j^e ($j = 1, 2$) and R^n, respectively. The off-diagonal element of the 2×2 earthing impedance matrix $\mathbf{Z}_e$ is G (≥ 0). (Note that $\mathbf{Z}_e$ is symmetric and that, in order not to violate the requirement that on each row and column of $\mathbf{Z}_e$ the largest elements are on the diagonal, G should not exceed R_j^e ($j = 1, 2$).) A simple calculation based on the general formulas presented in Section 2 shows that the GIC (denoted by I) that flows from the Earth at the station 1, along the line and into the Earth at the station 2 is

$$I = \frac{U}{R_1^e + R_2^e + R^n - 2G} \tag{15}$$

This equation shows that including the off-diagonal elements of $\mathbf{Z}_e$ enhances the magnitude of GIC.

We now investigate the effect of the distance x between the stations 1 and 2 on GIC. The quantities U, R^n and G are functions of x whereas R_1^e and R_2^e are independent of x. Assuming a uniform geoelectric field component E parallel to the line and denoting the resistance per unit length of the line by r_u, we obtain

$$U = Ex \tag{16}$$

$$R^n = r_u x \tag{17}$$

Based on Eq. (14), G is assumed to have the expression

$$G = \frac{a}{x} \frac{R_{m1} + R_{m2}}{2} \tag{18}$$

where, as before, a is the radius of a half-sphere electrode. The quantities R_{mj} ($j = 1, 2$) are the actual earthing resistances of the stations, i.e.

$$R_j^e = R_{mj} + R_{tj} \quad (j = 1, 2) \tag{19}$$

where the resistances of the transformers are denoted by R_{t1} and R_{t2}, in principle not necessarily equal. As indicated in connection with Eq. (14), x should be replaced by a in formula (18) if $x < a$. When using formula (18), it is necessary to check that the requirement mentioned in Section 2 that the off-diagonal elements of $\mathbf{Z}_e$ are smaller than (or equal to) the diagonal ones on the same rows and columns is satisfied, i.e. that $G \leq R_j^e$ ($j = 1, 2$) (and if necessary, G has to be decreased to $\min(R_1^e, R_2^e)$).

Substituting Eqs. (16)–(18) into Eq. (15) gives

$$I = I(x) = \frac{Ex}{\alpha + r_u x - \dfrac{\beta}{x}} = \frac{Ex^2}{r_u x^2 + \alpha x - \beta} \tag{20}$$

where $\alpha = R_1^e + R_2^e$ and $\beta = a(R_{m1} + R_{m2})$. The function $I(x)$ has a minimum (with the implicit assumption that $E > 0$) equal to

$$I_{\min} = \frac{E}{r_u + \dfrac{\alpha^2}{4\beta}} = \frac{E}{r_u + \dfrac{(R_1^e + R_2^e)^2}{4a(R_{m1} + R_{m2})}} = \frac{E}{r_u + \dfrac{R_1^e + R_2^e}{4a}\left(1 + \dfrac{R_{t1} + R_{t2}}{R_{m1} + R_{m2}}\right)} \tag{21}$$

at

$$x = x_{\min} = \frac{2\beta}{\alpha} = a\frac{2(R_{m1} + R_{m2})}{R_1^e + R_2^e} = a\frac{2}{1 + \dfrac{R_{t1} + R_{t2}}{R_{m1} + R_{m2}}} \tag{22}$$

If the sum of the resistances of the transformers ($= R_{t1} + R_{t2}$) is smaller than the sum of the actual earthing resistances ($= R_{m1} + R_{m2}$) the minimum $x_{\min}$ thus lies in the region $x \geq a$, in which Eqs. (18) and (20) are valid. For values $x > x_{\min}$, $I(x)$ is an increasing function of x approaching the limit E/r_u. If the off-diagonal elements of $\mathbf{Z}_e$ are ignored, G is zero in Eq. (15), or equivalently β is replaced by zero in Eq. (20), which makes GIC a monotonically growing function of x. The increase of $I(x)$ with x

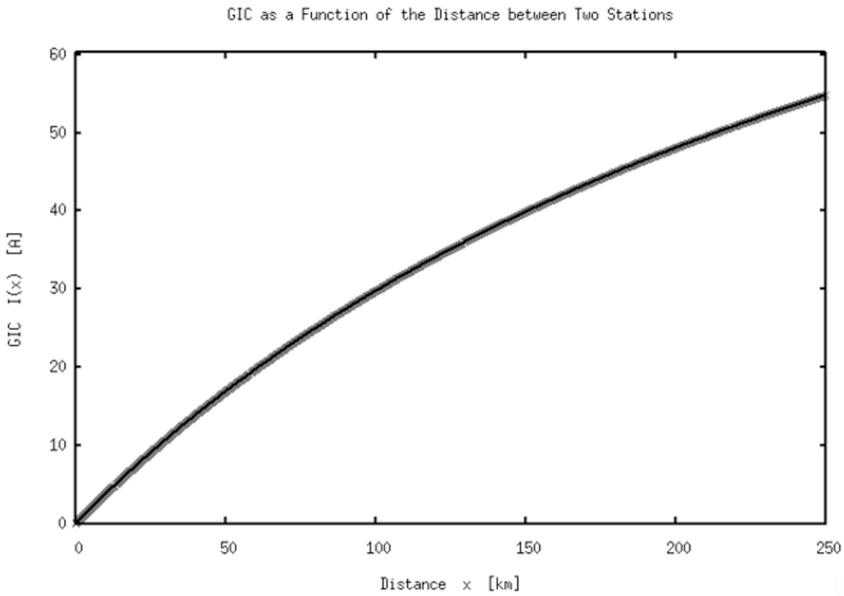

Fig. 4. GIC in a two-station power system due to a parallel geoelectric field of 1 V/km as a function of the length of the line between the stations, which are assumed identical. The stations are earthed by "half-sphere electrodes" (radius = 100 m). The actual earthing resistances of the stations are 1 Ω, and the (total) earthing resistances (= actual + transformer) are 1.28 Ω. The resistance of the line is 8 mΩ/km.

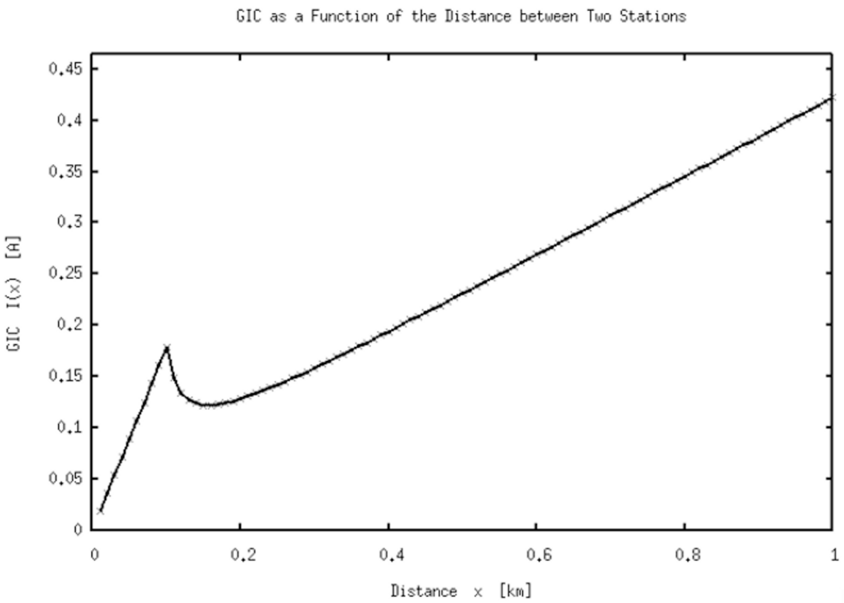

Fig. 5. Similar to Fig. 4 but only the smallest values of the length of the line are included.

includes the well-known fact that longer transmission lines carry larger GIC (Pirjola, 2000).

Let us now use the following values of the parameters:

$$
\begin{aligned}
E &= 1 \ \mathrm{V/km} \\
a &= 100 \ \mathrm{m} \\
R_{t1} = R_{t2} &= 0.28 \ \Omega \\
R_{m1} = R_{m2} &= 1 \ \Omega \\
r_u &= 8 \ \mathrm{m}\Omega/\mathrm{km}
\end{aligned}
$$

These data do not lead to a violation of the requirement that the off-diagonal elements of $\mathbf{Z}_e$ must not exceed the diagonal ones on the same rows and columns. By applying Eq. (20), Fig. 4 shows the GIC ($= I = I(x)$) in the range of x values from 10 m to 250 km. In this figure, $I(x)$ looks increasing with x, but it has a minimum $I_{\min} = 0.12$ A at about $x = 156$ m (Eqs. (21) and (22)). This can be seen in Fig. 5, in which only the smallest values of x are included. For $x \le a$, G is independent of x, which explains the peculiar, and possibly somewhat unrealistic, behaviour of $I(x)$ for very small x values. With $E = 1$ V/km and $r_u = 8$ mΩ W/km, the limit value E/r_u is 125 A. Thus, with the above numerical values of the parameters, I is still less than a half of the limit when $x = 250$ km (= 55 A; see Fig. 4), and even for $x = 2500$ km I is "only" about 111 A.

In the scale of Fig. 4, there would be no distinction between $I(x)$ shown in the figure and GIC obtained by ignoring the off-diagonal elements of $\mathbf{Z}_e$, which justifies the im-

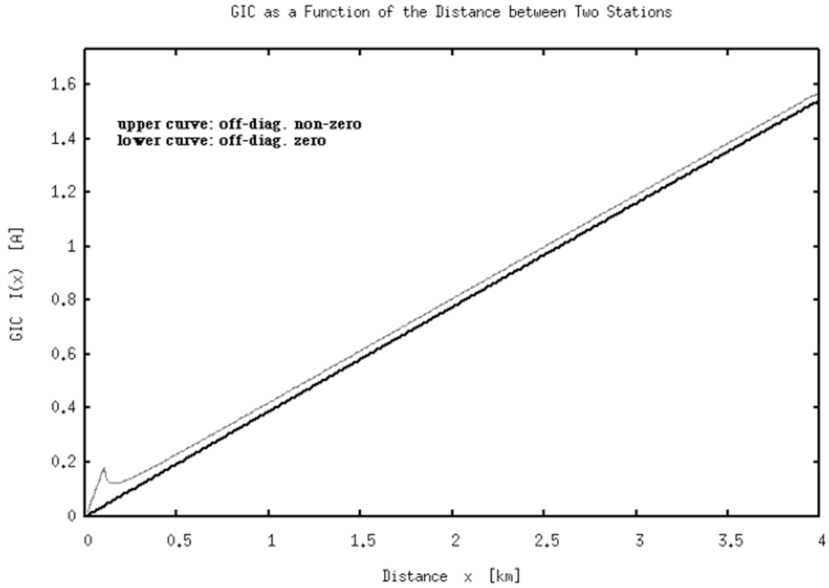

Fig. 6. The upper curve corresponds to Fig. 4 for small values of the length of the line. The lower curve (practically a straight line) depicts GIC obtained when the off-diagonal elements of the earthing impedance matrix are set equal to zero in the calculation.

portant conclusion that the inclusion of the off-diagonal elements is not of great significance. Figure 6 shows those two cases in another scale, i.e. for small x values, with the upper and lower curves representing I from Eq. (20) and GIC calculated by neglecting the off-diagonal elements of $\mathbf{Z}_e$, respectively. The above-mentioned observation that including the off-diagonal elements enhances GIC can be seen but the difference is completely insignificant in practice. In fact, it should also be noted that all GIC values included in Figs. 5 and 6 are uninterestingly small in practice, and so the figures should rather be regarded as curiosities here.

The numerical value 1 Ω applied to the actual earthing resistances R_{m1} and R_{m2} may be considered large in the light of resistance data used in Section 3. Therefore test computations have also been made with $R_{m1} = R_{m2} = 0.62$ Ω and $R_{m1} = R_{m2} = 0.34$ Ω, where the former choice refers to the average of the earthing resistances of the 14 stations discussed in Section 3 and, more properly, in the latter the assumed transformer resistance 0.28 Ω is subtracted. As seen from Eqs. (15) and (19), a decrease of the actual earthing resistances increases the magnitude of GIC, so the $I(x)$ values for 0.62 Ω and 0.34 Ω are larger than those shown in Figs. 4–6. The curves for 0.62 Ω and 0.34 Ω are not shown in this paper but the principal conclusion about the unimportance of the consideration of the off-diagonal elements of $\mathbf{Z}_e$ remains valid. For $x = 250$ km and $x = 2500$ km, $I = 66$ A and $I = 115$ A, respectively, when $R_{m1} = R_{m2} = 0.62$ Ω. The corresponding values for $R_{m1} = R_{m2} = 0.34$ Ω are $I = 77$ A and $I = 118$ A. These data should be compared with the above-mentioned values $I = 55$ A and $I = 111$ A when $R_{m1} = R_{m2} = 1$ Ω.

5. Discussion and Concluding Remarks

At the Earth's surface, "Space Weather" manifests itself as geomagnetically induced currents (GIC) in ground-based technological systems. The history of GIC dates back to early telegraph equipment in the 1800's being thus much longer than the concept of space weather. Today the most important networks regarding GIC are electric power transmission grids, in which GIC may lead to saturation of transformers possibly resulting in problems in the operation of the system or in permanent damage.

Modelling the physical processes in the whole chain from solar activity to GIC in a network is a complicated task and impossible in practice. But if the horizontal (geo)electric field induced by a space weather storm at the Earth's surface and impacting on a ground-based system can be estimated, it is in principle straightforward to apply Ohm's and Kirchhoff's laws for electric circuits to the computation of GIC flowing in each element and at each site of the system considered. For power grids, Lehtinen and Pirjola (1985) may be referred to, and a technique suitable for calculating GIC and pipe-to-soil voltages in buried pipelines is presented by Boteler (1997) and by Pulkkinen *et al.* (2001).

In practice, however, the calculation of GIC in a power system based on knowledge of the geoelectric field encounters problems, one of which is the difficulty in getting grid configuration and resistance data from the companies in a proper form. This partly results from the fact that in a GIC calculation only dc descriptions and values are needed, whereas the power transmission is performed at the 50/60 Hz frequency, and it is partly due to the confidential nature of power grid data. The other problem in calculating GIC is that an exact modelling of all details of a complicated power grid is practically impossible. An item is the influence of overhead shield wires usually neglected by "hand-waving" arguments in GIC studies. They may affect the geoelectric field but a recent investigation by Pirjola (2007) justifies this effect to be minor.

Another issue not fully clear in GIC computations is the interaction between different earthing points accounted for by off-diagonal elements of the earthing impedance matrix, which is usually assumed to be diagonal unless more earthing points than one explicitly lie at practically one site. Otherwise it is often just vaguely stated that the stations are distant enough to enable the neglect of the off-diagonal elements (e.g. Pirjola and Lehtinen, 1985).

In a recent study, Pirjola (2008) concludes that the off-diagonal elements may be ignored in practice but in the network investigated the distances between the stations are about 20 km or more. Therefore, in this paper we consider a fictitious, but realistic, power grid in which there are two closely-located-station pairs. Three different assumptions are used when computing GIC due to uniform geoelectric fields in the grid. In the normal case, the only off-diagonal elements of the earthing impedance matrix refer to the closely-located stations. In the second case, we force even these elements to be zero, and in the third case all off-diagonal elements are included by adjusting them according to the distances between the stations. The computations show that, though GIC magnitudes at some individual sites may vary with the assumption about the off-diagonal elements, the overall influence of the off-diagonal elements on GIC flowing through transformers is minor. This conclusion also has practical significance because it is important to know the sum of GIC flowing through all transformers simultaneously. Pulkkinen *et al*. (2000) present statistical predictions of this sum for the Finnish high-voltage power grid.

In this paper, we also consider a power "network" consisting of two stations with a line connecting them. Such a system is, of course, an idealization but its simplicity enables the use of analytic formulas, which are handy for basic physical examinations. We consider the behaviour of GIC flowing in the system as a function of the length of the line. The study supports the conclusion obtained from the discussion of the larger grid that the inclusion of the off-diagonal elements is of minor practical significance.

References

Bolduc, L., GIC observations and studies in the Hydro-Québec power system, *J. Atmos. Sol.-Terr. Phys*., **64**(16), 1793–1802, 2002.

Boteler, D. H., Distributed-source transmission line theory for electromagnetic induction studies, *Proceedings of the 1997 Zurich EMC Symposium, URSI Supplement*, 401–408, 1997.

Boteler, D. H. and R. J. Pirjola, Modelling Geomagnetically Induced Currents produced by Realistic and Uniform Electric Fields, *IEEE Trans. Power Delivery*, **13**(4), 1303–1308, 1998.

Boteler, D. H., R. J. Pirjola, and H. Nevanlinna, The effects of geomagnetic disturbances on electrical systems at the Earth's surface, *Adv. Space Res*., **22**(1), 17–27, 1998.

Kappenman, J. G., Geomagnetic storms and their impact on power systems, *IEEE Power Eng. Rev*., May 1996, 5–8, 1996.

Kappenman, J. G., An Overview of the Increasing Vulnerability Trends of Modern Electric Power Grid Infrastructures and the potential consequences of Extreme Space Weather Environments, in *Effects of Space Weather on Technology Infrastructure*, edited by I. A. Daglis, NATO Science Series, Kluwer Academic Publishers, II. Mathematics, Physics and Chemistry, 176, Chapter 14: Space Weather and the Vulnerability of Electric Power Grids, 257–286, 2004.

Kappenman, J. G., Great geomagnetic storms and extreme impulsive geomagnetic field disturbance events—An analysis of observational evidence including the great storm of May 1921, *Adv. Space Res*., **38**(2), doi:10.1016/j.asr.2005.08.055, 188–199, 2006.

Kappenman, J. G. and V. D. Albertson, Bracing for the geomagnetic storms, *IEEE Spectrum*, March 1990, 27–33, 1990.

Lanzerotti, L. J., D. J. Thomson, and C. G. Maclennan, Engineering issues in space weather, in *Modern Radio Science 1999*, edited by M. A. Stuchly, 25–50, International Union of Radio Science (URSI), Oxford University Press, 1999.

Lehtinen, M. and R. Pirjola, Currents produced in earthed conductor networks by geomagnetically-induced electric fields, *Ann. Geophys*., **3**(4), 479–484, 1985.

Mäkinen, T., Geomagnetically induced currents in the Finnish power transmission system, Finnish Meteorological Institute, *Geophys. Publ*., 101 pp., No. 32, Helsinki, Finland, 1993.

Molinski, T. S., Why utilities respect geomagnetically induced currents, *J. Atmos. Sol.-Terr. Phys*., **64**(16), 1765–1778, 2002.

Pirjola, R., Induction in power transmission lines during geomagnetic disturbances, *Space Sci. Rev*., **35**(2), 185–193, 1983.

Pirjola, R., Geomagnetically Induced Currents During Magnetic Storms, *IEEE Trans. Plasma Sci*., **28**(6), 1867–1873, 2000.

Pirjola, R., Review on the calculation of surface electric and magnetic fields and of geomagnetically induced currents in ground-based technological systems, *Surv. Geophys*., **23**(1), 71–90, 2002.

Pirjola, R., Effects of space weather on high-latitude ground systems, *Adv. Space Res*., **36**(12), doi:10.1016/j.asr.2003.04.074, 2231–2240, 2005a.

Pirjola, R., Averages of geomagnetically induced currents (GIC) in the Finnish 400 kV electric power transmission system and the effect of neutral point reactors on GIC, *J. Atmos. Sol.-Terr. Phys*., **67**(7), 701–708, 2005b.

Pirjola, R., Calculation of geomagnetically induced currents (GIC) in a high-voltage electric power transmission system and estimation of effects of overhead shield wires on GIC modelling, *J. Atmos. Sol.-Terr. Phys*., **69**(12), 1305–1311, 2007.

Pirjola, R., Study of effects of changes of earthing resistances on geomagnetically induced currents in an electric power transmission system, *Radio Sci*., **43**, RS1004, doi:10.1029/2007RS003704, 13 pp., 2008.

Pirjola, R. and M. Lehtinen, Currents produced in the Finnish 400 kV power transmission grid and in the Finnish natural gas pipeline by geomagnetically-induced electric fields, *Ann. Geophys*., **3**(4), 485–491, 1985.

Pirjola, R. J. and A. T. Viljanen, Geomagnetic Induction in the Finnish 400 kV Power System, in *Environmental and Space Electromagnetics*, edited by H. Kikuchi, 276–287, Chapter 6.4, Springer-Verlag, Tokyo, 1991.

Pulkkinen, A., A. Viljanen, R. Pirjola, and BEAR Working Group, Large geomagnetically induced currents in the Finnish high-voltage power system, *Reports, No. 2000:2, Finnish Meteorological Institute*, Helsinki, Finland, 99 pp., 2000.

Pulkkinen, A., R. Pirjola, D. Boteler, A. Viljanen, and I. Yegorov, Modelling of space weather effects on pipelines, *J. Appl. Geophys*., **48**(4), 233–256, 2001.

Pulkkinen, A., S. Lindahl, A. Viljanen, and R. Pirjola, Geomagnetic storm of 29–31 October 2003: Geomagnetically induced currents and their relation to problems in the Swedish high-voltage power transmission system, *Space Weather*, **3**, S08C03, doi:10.1029/2004SW000123, 19 pp., 2005.

Watermann, J., The magnetic environment—GIC and other ground effects, in Space Weather, in *Research Towards Applications in Europe*, edited by J. Lilensten, Springer, 269–275, Chapter 5.0, 2007.

Wik, M., A. Viljanen, R. Pirjola, A. Pulkkinen, P. Wintoft, and H. Lundstedt, Calculation of Geomagnetically Induced Currents in the 400 kV Power System in Southern Sweden, *Space Weather*, 2008 (in press).

R. Pirjola (e-mail: risto.pirjola@fmi.fi)