Nonlinear variability of SYM-H over two solar cycles

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Fractal fluctuation analysis is applied to ground-based SYM-H data during quiet times and during magnetic storm times spanning two solar cycles between 1981–2002. On the basis of Kp, intervals were selected that corresponded to quiet and active magnetospheric dynamics. A nonlinear detrended fluctuation analysis (DFA) was applied to monitor nonlinear variability over the solar cycles. We find significant variations in nonlinear statistics between quiet and active intervals, which indicates a difference in statistical variability for quiet times, and storm times.

Key words: SYM-H, fractals, nonlinear, space weather, indices, geomagnetism, Dst, prediction.

1. Introduction

The most widely used statistical descriptor of magnetic storm activity is the D_{st} index. This index is considered to reflect variations in the intensity of the symmetric part of the ring current that circles Earth at altitudes ranging from about 3–8 earth radii (R_E), and is proportional to the total energy in the drifting particles that form the ring current. It is calculated as an hourly index from the horizontal magnetic field component at four observatories, namely, Hermanus (33.3° south, 80.3° in magnetic dipole latitude and longitude), Kakioka (26.0° north, 206.0°), Honolulu (21.0° north, 266.4°), and San Juan (29.9° north, 3.2°). These four observatories were chosen because they are close to the magnetic equator and thus are not strongly influenced by auroral current systems. Convolution of their magnetic variations forms the D_{st} index, measured in nanoTesla, which is considered to provide a reasonable global estimate of the variation of the horizontal field near the equator. It is calculated once every hour. In this work we examine the statistical nature of the scaling properties in the ground-based SYM-H magnetic index. This index was developed as part of an effort to describe geomagnetic disturbance fields in mid-latitudes with high-time resolution (Iyemori *et al.*, 1999). It is essentially the same as the D_{st} index, although it uses one-minute values from a different set of stations and a slightly different coordinate system. As such, this index also provides an excellent measure of the large-scale behavior of the ring current and space storm dynamics.

In this communication we examine SYM-H, over two solar cycles, to determine the characteristic nonlinear statistical differences, if any, between quiet and active magnetospheric dynamics. The SYM-H series is obtained from the WDC-Kyoto (Iyemori *et al.*, 1999). As well, the length of the data series—one minute sampling from 1981 through 2002—makes possible a determination of the variation, if any, of the statistics over the solar cycles. We will determine whether statistically stable, but dissimilar, nonlinear processes are involved in quiet and active periods. Our discrimination between quiet and active intervals relies on the Kp index, which has been shown to be a good tool for such studies (Rangarajan and Iyemori, 1997).

2. Detrended Fluctuation Analysis Technique

The scaling properties of space physics data during dynamic magnetospheric activity were investigated by Ohtani *et al.* (1995, 1998). They found that magnetic fluctuations in the magnetotail were well described as self-affine data with a power law spectrum. Studies of geomagnetic indices have served as particularly fruitful examples of fractional Brownian motion in space physics (Sharma, 1995; Takalo *et al.*, 1999; Price and Newman, 2001; Wanliss and Reynolds, 2003). In this communication we examine the scaling properties of SYM-H over two solar cycles using a relatively new analysis technique.

Rather than more traditional techniques used to estimate fractal properties of time-series (Higuchi, 1988), we have elected to use a more recent method that eliminated some difficulties in calculating such statistics. Novel ideas from statistical physics led to the development of the detrended fluctuation analysis (DFA) (Peng *et al.*, 1995). The method is a modified root mean squared analysis of a random walk designed specifically to be able to deal with nonstationarities in nonlinear data. In comparison with other techniques it has been found to be among the most robust of statistical techniques designed to detect long-range correlations in time series (Taqqu *et al.*, 1996; Cannon *et al.*, 1997; Blok, 2000). DFA has been shown to be robust to the presence of trends (Hu *et al.*, 2001) and nonstationary time series (Kantelhardt *et al.*, 2002; Chen *et al.*, 2002).

The technique begins by the division of the time series into boxes of varying length n (Fig. 1). In this example, $n = 1000$. After this, a least-squares linear fit to the data signal is performed for each box; this linear fit represents the local trend in each box. Next, for each box the root mean squared deviations $F(n)$ of the signal from the local trend is determined. This procedure is repeated for different box

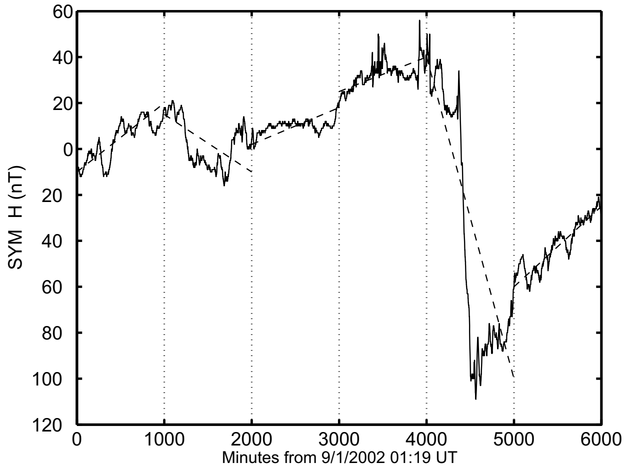

Fig. 1. 6000 minutes of SYM-H data are shown in this figure. The vertical dotted lines indicate box of size $n = 1000$, and the dashed straight-line segments represent the trend estimated in each box by a least-squares fit.

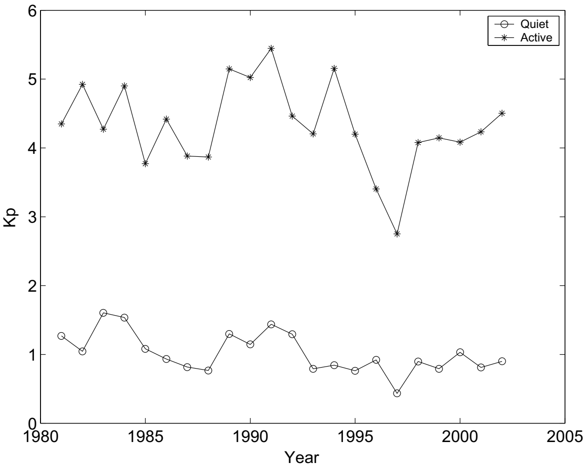

Fig. 2. Smallest (circles) and largest (stars) mean values of Kp for 10,000 consecutive minutes in each year from 1981 through 2002.

sizes. Finally, the log-log plot of the deviation, $F(n)$, versus box size is used to calculate the slope which gives the scaling exponent, α. For this analysis technique, an uncorrelated time series gives $\alpha = 1/2$, as for standard Brownian motion. If $\alpha > 1/2$ ($\alpha < 1/2$) the data demonstrate correlation (anticorrelation).

3. Data Selection

The analysis is employed on the SYM-H time-series covering 22 years of data, from 1981 through 2002. The detrended fluctuation analysis was applied to quiet and active intervals selected for each year; a single quietest, and a single most active event, was selected for each year. We found that the scaling exponent for the quiet intervals was generally close to 1/2, but for active intervals it was usually greater than 1/2, irrespective of solar cycle.

To ensure the data were selected that were representative of a magnetospheric quiet or active state, we relied not only on SYM-H, but also on the Kp index (Rangarajan and Iyemori, 1997). Generally, when SYM-H indicates significant activity, there is usually significant Kp activity also. Since SYM-H is calculated exclusively from low- to mid-latitude magnetometer stations, and Kp includes higher-latitude stations, quiet interval data selection based on Kp ensures that data are selected for which the entire magnetosphere is as close as possible to a ground state.

On the other hand, during active times such as space storms, Kp is large and SYM-H reaches large negative values. However, sometimes SYM-H shows only small activity even when Kp is large, demonstrating dynamic activity (for example, magnetospheric substorms) at higher-latitude regions of the magnetosphere. Thus, for the most part, use of Kp to select events ensures that data are selected for which the magnetosphere is typically quiet or active over a wide range of latitudes.

For each year, 10,000 consecutive minutes (i.e. 10,000 data points) that have the largest/smallest mean Kp were selected as representative of active and quiet states. We note that for each of the 22 years between 1981 and 2002, the active times corresponded to magnetic storms. The average mean values of Kp for each year are shown in Fig. 2. The upper curve (stars) shows Kp for the most active intervals, and the lower curve (circles) shows the corresponding average values for quiet intervals.

The solar cycle influence on the data selection criterion is weakly evident as both quiet and active events have smallest values near solar minimum (solar minimum around 1986 and 1996). As well, during the solar minimum near 1996 the most active periods have Kp<4, which places these events outside the range of truly ‘disturbed’ intervals (Rangarajan and Iyemori, 1997). Overall, 5 years have their most disturbed intervals with Kp<4. Furthermore, as shown by Rangarajan and Iyemori (1997), the entire data interval of 22 years spans a time when quiet events were relatively rare; this is clear in that 10 years have minimum Kp>1. Intervals of low activity, representing the quiet periods, generally have SYM-H > -40 nT (100% of events) and active periods have SYM-H < -80 nT (82% of events). For the active events, most of the data correspond to intense space storm events (Gonzalez *et al.*, 1994).

4. Statistical Analysis of Data

Figure 3 shows the fluctuation versus box size calculated for each active interval for years from 1981–2002. The dotted line with slope 1/2, corresponding to Brownian motion, is shown for reference. The same is shown for the quiet intervals in Fig. 4. The actual fluctuations for the quiet and active intervals follow almost perfect power-law scaling over nearly two decades (between $n = 16$ to 1000), whereafter the data appear more variable. This highlights one difficulty in this analysis, namely that the low and high box-number edges should be treated with caution. The first few points at the low end should be disregarded because in this region the detrending removes too much of the fluctuation. For larger values of box size, there are too few boxes for a proper averaging to be made, and we also disregard those values.

Figures 3 and 4 show that the curves for active intervals tend to have a slope that is larger than the dotted reference curve, which has $\alpha = 0.50$, corresponding to a random walk. This means that the active interval data are correlated, or

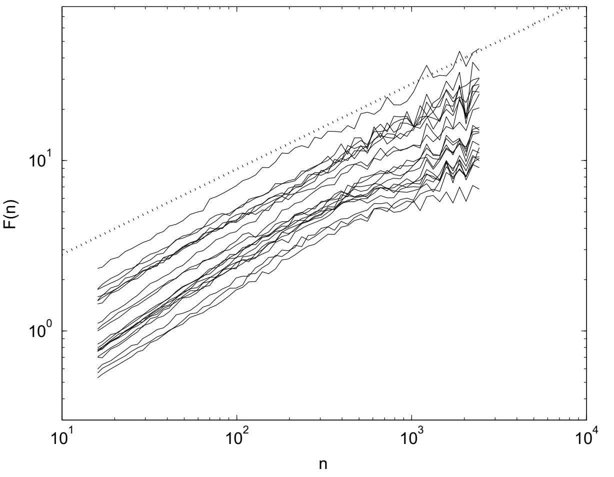

Fig. 3. Fluctuation versus box size for active intervals from 1981–2002. The dotted reference line has slope 0.5 and is shown for reference.

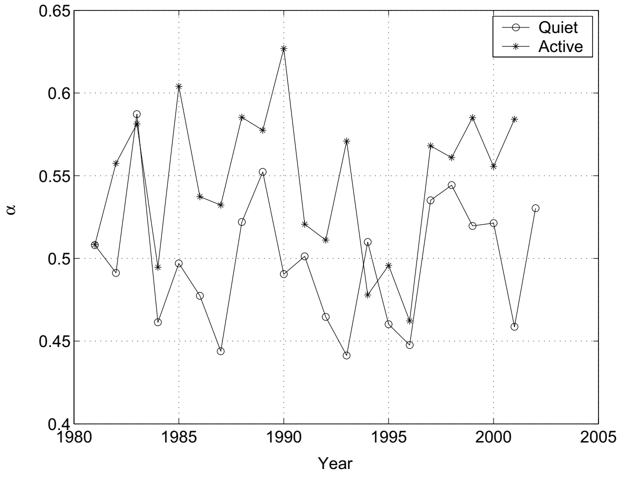

Fig. 5. Scaling exponents for both quiet and active periods for 1981–2002.

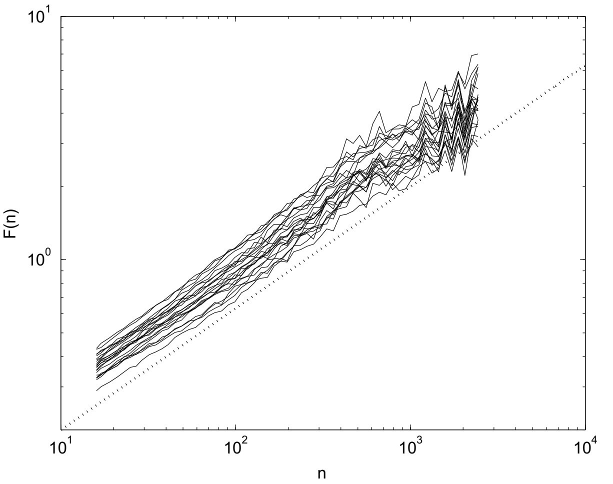

Fig. 4. Fluctuation versus box size for quiet intervals from 1981–2002. The dotted reference line has slope 1/2 and is shown for reference.

persistent. The nonlinear statistical behavior of the quiet intervals are quite different. They appear to fit $\alpha = 0.50$, consistent with a random walk.

Figure 5 shows the summary of calculated scaling exponents for each of the 22 years in the dataset. Around solar minimum (near 1996) the scaling exponents for quiet/active intervals are quite similar, and there is considerable overlap. This is probably an indication that during solar minimum the Kp selection criterion fails to properly separate dynamical behaviors. For example, the most active interval during 1996 (Fig. 2) had mean Kp<3! Most other active intervals typically had Kp∼4, usually larger. We find for all but two years $\alpha_A > \alpha_Q$.

For all events, the averages are $\alpha_Q = 0.498 \pm 0.039$ and $\alpha_A = 0.548 \pm 0.044$. To determine whether these averages are significantly different from the null hypothesis—that the difference is due purely to randomness—we applied the students-t test, and found $t = 3.880$, $p = 3.726 \times 10^{-4}$. These results imply that the difference between the statistics computed for quiet and active intervals are significant; the likelihood that the means are different due to random processes is only a small fraction of 1% probability.

5. Conclusion

In this brief communication we have attempted to characterize the differences in the scaling behavior between magnetospheric quiet and active dynamics. We applied the DFA method to quantify long-range correlations embedded in the nonstationary SYM-H time series. The DFA method was selected because it copes very well with nonlinearity and nonstationarity. The method is particularly well suited to analysis of the SYM-H data because it avoids spurious detection of correlations that are artifacts of nonstationarity. In particular, we analyzed the most quiet and active periods for each of the 22 years between 1981 and 2002. For statistical robustness, each quiet and active data subset selected for analysis comprised 10,000 consecutive SYM-H values, at one-minute sampling interval. Through selection of only one representative quiet and active interval for each year, it is possible to characterize the variation of the scaling exponents over two solar cycles.

The nonlinear statistical properties between quiet and active events were noticeably different. In general, however, for each year, quiet and active subsets generally clustered around their own unique values. A statistically consistent difference was found between the nonlinear scaling exponents for quiet and active events; $\alpha_Q = 0.498 \pm 0.039$ and $\alpha_A = 0.548 \pm 0.044$. No clear dependence was found between nonlinear statistics and solar cycle. This implies that the nonlinear behavior is more strongly a function of internal magnetospheric dynamics, rather than simply a product of the external forcing due to the solar cycle. It is interesting also that for quiet intervals during 1983 (solar min) and 1988–1989 and 1997–2000 (solar max) the statistical exponent tends to be higher than 0.5, which is different from other years, although for all years the equivalent active interval exponents are consistently larger. If not for the 1983 observation, it might be suggested that there is some solar cycle modulation of the statistical variability. This would be an interesting result. It could be that the 1983 event has high α-exponent simply due to the choice of the interval analyzed. A future, more detailed, study of the continuous SYM-H series may provide answers to this question.

In conclusion, these results demonstrate that the nonlinear statistical behavior of the magnetosphere, as derived from SYM-H, is markedly different during quiet and active intervals. The transition from quiet and active phases represents a period of potentially increased statistical variability. In future studies we will consider statistics during a space storm, and hypothesise that a clear transition, in terms of nonlinear statistical variability, will occur between quiet and active phases. In this study quiet and active events behaved in statistically dissimilar manners, possibly because quiet and active events represent a transition from weakly anticorrelated ($\alpha \sim 0.5$) to weakly correlated ($\alpha \sim 0.6$) regulation of the magnetospheric dynamics. The active intervals selected in this study included space storms in 100% of the cases. Since space storms frequently are preceded by quiet intervals, and are themselves characterized by global dynamic activity, the results presented here raise interesting questions that will be investigated further. For example, does the transition to storm times feature a repeatable change in nonlinear statistical behavior? Preliminary results indicate that the onset of storm times (quiet to active intervals) are presaged by alterations in the scale-invariant properties of SYM-H. By monitoring the variations of these scale-invariant properties one may be able to develop a method to predict the coming storm onset.

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