A reexamination of pitch angle diffusion of electrons at the boundary of the lunar wake

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Velocity distribution of the solar wind electrons injected into the lunar wake boundary is re-examined by using a simple model structure of inward electric field. The electrons that were flowing along the magnetic field lines undergo pitch angle scattering due to the electric field component perpendicular to the magnetic field. The electrons obtain perpendicular speeds twice as much as the drift speed. On the basis of the GEOTAIL observations of the whistler mode waves and strahl electrons, the intensity of the electric field and the thickness of the wake structure are estimated to be 28–40 $\mathrm{mVm^{-1}}$ and less than 20 km, respectively.

Key words: Lunar wake, pitch angle diffusion, electric field, wake potential structure, electron distribution function.

1. Introduction

In the paper entitled "Pitch angle diffusion of electrons at the boundary of the lunar wake" by Nakagawa and Iizima (2005), the authors assumed double- or triple-layered structures of electric field with equal amount of potential drop and potential rise, which result in equal electric potential between the solar wind and the central part of the wake. At the real wake boundary, it is likely that the magnitude of the electric field as well as the thickness of the electric field layer is different for each layers. On the basis of Lunar Prospector measurements of the electron distribution function, Halekas *et al.* (2005) reported that the electrostatic potential was by several hundred volts negative in the central wake. No double-layered electric field was found at the wake boundary. It seems consistent with previous observations (Ogilvie *et al.*, 1996; Farrell *et al.*, 1996; Futaana *et al.* 2001), and numerical experiments (Farrell *et al.*, 1998; Birch and Chapman, 2001, 2002). It would be better to use such a realistic model of inward electric field to study the pitch angle diffusion of the electrons at the lunar wake boundary.

In this paper, we trace the motion of the electrons injected into the unidirectional electric field of the modeled wake structure. It will be shown that the electrons flowing along the magnetic field gain sufficient velocity component perpendicular to the magnetic field, as expected in excitation of the whistler mode wave detected by GEOTAIL upstream of the wake (Nakagawa *et al.*, 2003). The intensity of the electric field and the width of the wake structure will be discussed.

2. Model configuration

Figure 1 shows the model electric field at the boundary of the lunar wake. To reproduce the observation by Lunar Prospector (Halekas *et al.*, 2005) and several numerical experiments (Farrell *et al.*, 1998; Birch and Chapman, 2001), a simplified model of the electric field

$$\mathbf{E} = \begin{cases} 0.6\ E_0 \mathbf{e}_E & \left(0 < \mathbf{r}\cdot\mathbf{e}_E < \frac{D}{4} \right) \\ 1.4\ E_0 \mathbf{e}_E & \left(\frac{D}{4} < \mathbf{r}\cdot\mathbf{e}_E < \frac{3D}{4} \right) \\ 0.6\ E_0 \mathbf{e}_E & \left(\frac{3D}{4} < \mathbf{r}\cdot\mathbf{e}_E < D \right) \end{cases} \tag{1}$$

as illustrated in Fig. 1 is employed in this study, where $E_0 = \phi_0/D$ is the average electric field through the thickness D of the potential drop ϕ_0, $\mathbf{e}_E$ is the unit vector of electric field direction, and $\mathbf{r}$ is the position. Differently from Nakagawa and Iizima (2005), the electric fields of each layers have the same sign. The magnetic field is set to intersect the layers at an angle of 20°, which reproduces the configuration of interplanetary magnetic field at the time of GEOTAIL observation of the whistler mode waves associated with lunar wake (Nakagawa *et al.*, 2003).

3. Test particle experiment

We follow the motion of an electron as a test particle injected into the model structure of the electric field by solving the equation of motion. In order to simulate the "strahl" component of the solar wind electrons, the test particle was injected into the electric field along the background magnetic field. Figure 2(a) shows an example orbit. Here we take Cartesian coordinates with its x-axis in the direction of the magnetic field $\mathbf{B} = (B_0, 0, 0)$, and select the z-axis so that the y-component of electric field vanishes as $\mathbf{E} = (E_x, 0, E_z)$. The thickness of the potential structure D is $4L$ for this example, where the scale length $L = u\Omega_e^{-1}$ is determined from the speed $u = E_0/B_0$ and the inverse electron cyclotron frequency $\Omega_e^{-1} \equiv m/qB_0$.

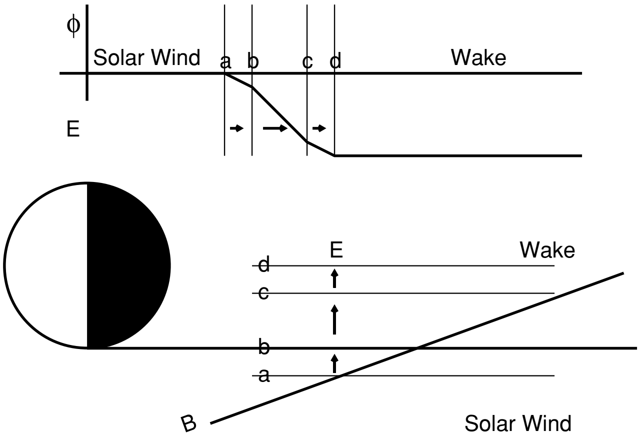

Fig. 1. A simplified model of electric structure at the boundary of the lunar wake. Top: Electric potential ϕ and direction of electric field (arrows). Bottom: Layers of electric field at the boundary of the wake.

Left two panels show the trajectory in velocity space, and the right two panels are for configuration space. As the initial velocity $\mathbf{v}_0 = (4u, 0, 0)$ is parallel to the magnetic field, the test particle is at the origin in $v_y - v_z$ space in the left panel at the beginning. As the test particle enters into the electric field, it starts drift motion with drift speed $u_D = E_\perp/B_0$, getting perpendicular velocity $\mathbf{v}_\perp$, and begins to gyrate around the magnetic field. In the first layer of weak electric field, it traces a small circular orbit (orange) centered at $(u_D,\ 0)$ in $v_y - v_z$ space. As it enters the second layer of intense field, the drift speed u_D becomes large, the center shifts toward higher speed, and the electron begins to go round on a larger orbit (red). At the same time, the electron decelerates in x direction because of the parallel component $E_\parallel$ of the electric field, as recognized in the second left panel. Next the electron enters into the weak field, and finally it gets out of the electric field with significant perpendicular velocity and less parallel speed.

Figure 2(b) shows another model with thickness $D = 2L$. The final perpendicular speed $v_\perp$ was $2.6u$, which is nearly twice as much as the drift speed u_D in the intense field. Figure 2(c) is an example of thin structure, $D = L$. The pitch angle diffusion is not effective, since the electron passes through the thin structure not being affected by the electric field.

4. Velocity distribution of penetrating electrons

Figure 3 shows the results of similar calculations made for a number of particles injected into the model field of thickness $D = 4L, 2L$, or L, with initial speed ranging from $0.02u$ to $5u$, and the pitch angle from 0° to 10° as measured from the direction of the magnetic field. Left panel shows the initial velocity distribution. Each of the test particles represents electrons in each solid angle $0.04u \times 1^\circ \times 1^\circ$. After calculation of their orbits, the number of test particles multiplied by each solid angle is counted for each bins of $0.2u \times 1^\circ \times 1^\circ$.

The final velocity distribution is presented separately in two panels, one of which shows the electrons reflected back by the potential barrier, and the other is for the electrons that passed through the potential difference. After the passage through the model electric field, the velocity distribution was modified to have significant perpendicular component

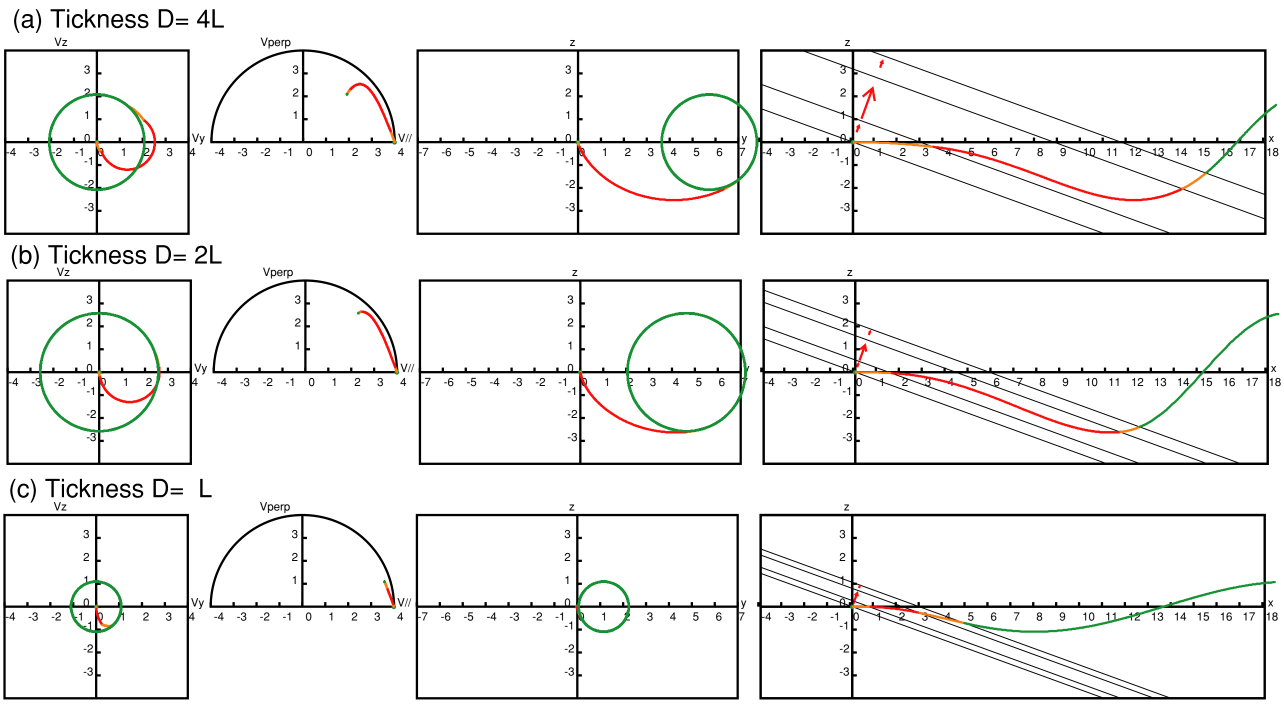

Fig. 2. Examples of the trajectories of an electron with initial velocity $\mathbf{v}_0 = (4u, 0, 0)$, injected into model structures of the electric field along the magnetic field line. (a) The case of thickness of the model structure $D = 4L$, (b) $D = 2L$, and (c) $D = L$. From left to right: the trajectory in velocity space $v_y - v_z$ perpendicular to the magnetic field, the trajectory in $v_\parallel - v_\perp$ space, the orbit in configuration space $y - z$ perpendicular to the magnetic field, and that in configuration space $x - z$ where x axis is parallel to the magnetic field line. Orange and red colors indicate that the test particle is in the weak or intense electric field, respectively, while green color indicates that the particle is out of the electric fields.

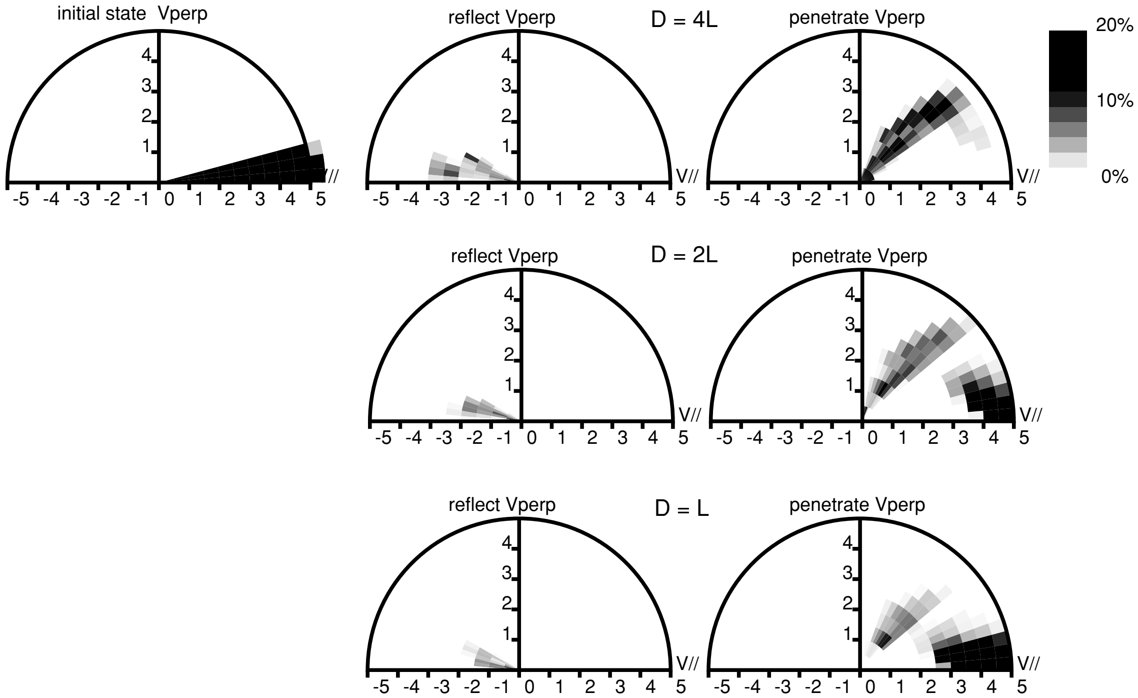

Fig. 3. Velocity distributions of (left) the incident electrons, (center) the electrons reflected by, and (right) the electrons which passed through the model electric field with thickness $D = 4L$, $2L$, and L. The initial speed is in the range $0.02u$ – $5u$ and the pitch angle is between 0° and 10° from the direction of the magnetic field.

$v_\perp$ and reduced parallel component $v_\parallel$, which is suitable for excitation of waves through cyclotron resonance. Top panel ($D = 4L$) shows that the perpendicular component $v_\perp$ became nearly equal to the parallel component $v_\parallel$ in the range $0 < v_\parallel < 3.5u$. Differently from Nakagawa and Iizima (2005), no lower cutoff is observed in $v_\parallel$. In the middle right panel, pitch angle diffusion occurs in the range $0 < v_\parallel < 3u$. The faster electrons with $v_\parallel > 3u$ pass through the wake without being affected by the electric field. In the bottom right panel which shows the result from a thin layer of small potential drop, we see that the pitch angle diffusion occurs only in limited range of speed.

5. Discussion

It is confirmed that the significant pitch angle diffusion occurs in the simple model wake structure of the unidirectional electric field. However, because of the potential drop, the electrons are decelerated in the wake, without regaining their parallel speed as seen in the double layered model field proposed by Nakagawa and Iizima (2005). It might seem inconsistent with the GEOTAIL observations of equal energy (1 keV) of incident strahl electrons (Nakagawa and Iizima, 2005) and resonant electrons estimated from the dispersion relation of the whistler mode wave (Nakagawa *et al.*, 2003). It may suggest that the energy of resonant electrons was over-estimated. In the estimation, Nakagawa *et al.*, (2003) used a dispersion curve for the solar wind parameters in place of that for the wake plasma, and neglected possible difference of bulk speed of the electrons between the wake and the solar wind.

We have seen in Fig. 3 that the perpendicular component of the electron velocity $v_\perp$ becomes nearly equal to the parallel component $v_\parallel$, as much as $3u$, where $u = E_0/B_0$. Thus we can estimate the intensity of the electric field from the relation

$$v_\perp \sim v_\parallel \sim \frac{3E_0}{B_0} \tag{2}$$

only if we know the parallel speed of the resonant electrons in the wake. As the background magnetic field was 6 nT at the detection of the whistler mode wave associated with lunar wake (Nakagawa *et al.*, 2003), we obtain $E_0 \sim 40$ mVm^{-1} for 1 keV ($v_\parallel \sim 2 \times 10^4$ kms^{-1}) and $E_0 \sim 28$ mVm^{-1} for 0.5 keV ($v_\parallel \sim 1.4 \times 10^4$ kms^{-1}) . It is much larger than the average electric field estimated from the potential drop of 300 V observed by Lunar Prospector across the radius (1738 km) of the global structure of the lunar wake (Halekas *et al.*, 2005). Rather, the electric field of 40 mVm^{-1} is close to the near-surface potential drop of 50 V over a distance of 1 km above the lunar surface inside the wake region (Halekas *et al.*, 2002, 2003). If we substitute the global electric field, 0.17 mVm^{-1}, we obtain u as small as 30 kms^{-1} and thus $v_\perp \sim 90$ kms^{-1}, which is too small with respect to $v_\parallel$, to excite the whistler wave as observed by GEOTAIL.

This shows that the global electric field as observed by Lunar Prospector was not responsible for the pitch angle diffusion of the electrons that would excite the whistler mode wave. Rather, more intense electric field in a confined region should be considered. One of the candidates is the wake structure in the close vicinity of the moon. Since the gradient of electric potential is thought to be steep near the lunar limb while it is much more gradual farther downstream, because the solar wind flows down as they rush into the void region of the wake. It is expected that the potential drop of 300 V over a lunar radius at altitudes of 20–115 km (Halekas *et al.*, 2005) is confined in a thin layer near the lunar limb, where the magnitude of the electric field might be of the order of 40 mVm^{-1}. If we simply assume the self-similarity of the potential structure whose thickness is of the order of 1000 km at an altitude of 100 km from the lunar surface (that is, 1738 + 100 km from the flank side of the moon), and that the potential drop 300 V is unchanged, the layer of electric field whose thickness is estimated to be 7.5–10.7 km from the electric field 40–28 mVm^{-1} should be found at around 18 km from the limb of the moon. It is so close to the moon that it might be difficult to distinguish the wake electric field from the near-surface electric field.

Another possibility is that there might be local fluctuations or small scale structures in the electric field, but there is no theoretical reason to expect them. Rather, near-surface potential drops as reported by Halekas *et al.* (2002, 2003) is likely to be associated with the GEOTAIL detection of the whistler mode waves.

The electric field must not be too large. Too intense electric field would reflect back the incident electrons before they gyrate significant angles to have their pitch angle scattered. Figure 2 shows that the most effective pitch angle diffusion occurs when the incident electron gyrates as much phase angle as π by the time of exit from the electric field. Thus, after a half gyration, the parallel speed $v_{\parallel 0}$ must be positive,

$$v_{\parallel 0} - \frac{q}{m}\int_0^{\frac{T}{2}} E_{\parallel} dt > 0, \quad (3)$$

where T is the gyration period $2\pi\Omega_e^{-1}$. Although $E_{\parallel}$ is not constant, we approximate $E_{\parallel}$ with an average value $E_0 \sin 20^\circ$ as an estimate, and obtain

$$E_0 \sin 20^\circ < \frac{v_{\parallel 0} B_0}{\pi}. \quad (4)$$

According to the GEOTAIL observation, the energy of the electrons injected into the wake is less than 1 keV, that is, $v_{\parallel 0} < 2 \times 10^4\ \text{kms}^{-1}$ (Nakagawa and Iizima, 2005), we have $E_0 < 110\ \text{mVm}^{-1}$. It is consistent with the previous estimation of 28–40 mVm^{-1}.

It is expected that the layer of the electric field has a significant thickness, otherwise the electrons get out of the layer before they gyrate by the angle π. It is no more than the depth penetrated by the fastest incident particle (1keV) within a half gyration period without deceleration, $\frac{1}{2} v_{\parallel 0} T \sin 20^\circ$. Using $\Omega_e = 2\pi \times 174$ Hz at the GEOTAIL detection of the whistler wave, we obtain the depth 20 km. Actually, there must be deceleration by the electric field, less thick layer of electric field would be enough.

6. Conclusion

It is confirmed that the simple model structure of the inward electric field at the lunar wake causes significant pitch angle diffusion as expected in the generation of the whistler mode wave. The intensity of the electric field, 28–40 mVm^{-1}, is much stronger than that observed in the central wake by Lunar Prospector, but is rather close to that observed in close vicinity of the lunar surface. The thickness of the electric field layer is estimated to be less than 20 km.

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