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The IZMIRAN main magnetic field candidate model for IGRF-10, produced by a spherical harmonic-natural orthogonal component method

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A simple method is proposed for constructing a space-time model of the main magnetic field based on the high-accuracy satellite survey data. At the first stage, we expand the CHAMP daily mean data into spherical harmonics with constant coefficients. It provides us with a series of the daily mean spherical-harmonic models (DMM) over a survey interval of several years, which are, then, expanded into the natural orthogonal components (NOC). It is shown that the NOC series converges rapidly, and that the accuracy of the space-time model over the time interval under consideration is no worse than the accuracy of the traditional models.

Key words: Secular variation, satellite data, natural orthogonal components, spherical harmonic models.

1. Introduction

Traditionally, the main magnetic field and its secular variations were modeled by expanding into spherical harmonics (SH) of the annual means from the global magnetic observatory network. Because of averaging within a year, the high-frequency fields generated by currents in the outer magnetosphere did not virtually affect the annual mean field values. The magnetic surveys from low-altitude satellites have changed the original data both qualitatively and quantitatively. The instant field measured at a certain point is the sum of the fields caused by intra- and extra-terrestrial sources and having different spectral composition. Consequently, modeling becomes a matter of separating the fields of different origin, most of which may be qualified as the main field measuring error. A simple method for eliminating the errors is the selection of data based on certain criteria, such as geomagnetic activity indices, solar zenith angle, and parameters of the interplanetary magnetic field. As a result of such selection, most data are rejected, and only the field values, for which the contribution of external sources is negligible, are used for further analysis (Olsen *et al.*, 2002).

In this paper, we propose an alternative approach based on the assumption that the fields of different origin caused by different processes in different media must have different temporal and spatial characteristics; i.e., the spatial structures and time variations of such fields do not correlate with each other. If it is true, then the expansion into natural orthogonal components can be applied to all data without any selection (Langel, 1987). The objective of our present work is to validate the above statement.

2. Modeling

We processed the data in two steps. The first one consisted in producing DMMs of the field as SH expansion up to $n = m = 13$, using all vector data available for each individual day. (See Eq. (1) and (2)).

$$U(r,\theta,\lambda) = a \cdot \sum_{n=1}^{N} \sum_{m=0}^{n} \left(\frac{a}{r}\right)^{n+1} (g_n^m \cos m\lambda + h_n^m \sin m\lambda) \times P_n^m(\cos\theta) \quad (1)$$

$$X = -\frac{1}{r}\frac{dU}{d\theta}; \quad Y = \frac{-1}{r\sin\theta}\frac{dU}{d\lambda}; \quad Z = -\frac{dU}{dr}, \quad (2)$$

where U is the geomagnetic potential in the point with geographic coordinates r, θ, λ; X, Y and Z are the northern, eastern and vertical components of the field, **a** is the mean radius of the Earth; $P_n^m(\cos\theta)$ are the associated Legendre functions of degree n and order m, normalized according to the convention of Schmidt; g_n^m and h_n^m are the constant coefficients.

The second step consisted in expansion of the obtained time set of DMMs in a series of natural orthogonal components. If data set can be described by a rectangular matrix $I \times J$, containing elements H_{ij} ,where i is the column number ($i \in [1, I]$) and j is the line number ($j \in [1, J]$), then they can be represented as (Faynberg, 1975; Langel, 1987):

$$H_{ij} = \sum_{k}^{K} C_{kj} \cdot T_{ki} + \delta_{ij}, \quad (3)$$

where C_{kj} are the numerical functions independent of time and T_{ki} are the numerical functions independent of the point position. K is the min$[I, J]$. The main quality of obtained T_{ki} and C_{kj} is their full orthogonality on the data set. Hence, as it follows from (3), a time variation of the field in some point j is represented by a linear combination of k temporal functions T_{ki} with coefficients C_{kj}. These temporal functions are common for all observation points and do not correlate with each other.

We applied this method to g_n^m, h_n^m time series coefficients of the DMMs, where j corresponds to particular combination of n, m, g and h, i corresponds to time (in days). We developed DMMs for each fourth day, as when using one year data (2002) the similar results with every day or every fourth day data were obtained (Golovkov and Zvereva, 1998, 2000). Taking into account that the condition of the orthogonality is satisfied we can say that the NOC method separates DMMs into number of models, which have different spatial structure and vary in time independently.

Then the varying in time the potential (1) can be rewritten as:

$$U(r,\theta,\lambda,t) = a\cdot\sum_{n=1}^{N}\sum_{m=0}^{n}\left(\frac{a}{r}\right)^{n+1}(g_n^m(t)\cos m\lambda + h_n^m(t)\sin m\lambda) \times P_n^m(\cos\theta) \quad (4)$$

The $g_n^m(t)$ and $h_n^m(t)$ in this equation are functions depending on time. They can be expressed in time as that follows from (3):

$$g_n^m(t_i) = \sum_{k=1}^{K} g_{nk}^m T_{ki}, \quad (5)$$

where g_{nk}^m do not depend on time and $K \le \min[J, I]$. It is similar for h_{nk}^m.

Substituting (5) in (4) we obtain

$$U(r,\theta,\lambda,t_i) = a\cdot\sum_{k=1}^{K}T_{ki}\sum_{n=1}^{N}\sum_{m=0}^{n}\left(\frac{a}{r}\right)^{n+1} \times (g_{nk}^m\cos m\lambda + h_{nk}^m\sin m\lambda)\times P_n^m(\cos\theta). \quad (6)$$

Therefore we can use some numerical functions describing T_{ki} in the SH analysis in the same way as analytical ones, traditionally used, to describe the temporal changes of g_n^m, h_n^m. Requirements to these numerical functions are the same as to any analytical ones. They must be orthogonal to each other on the whole time interval, approximate the observed series with the required accuracy and create the rapidly converging series of SH expansion.

3. Data

The choice of the initial data for constructing the geomagnetic field space-time model depends on the method applied. As seen from the previous section, it is of fundamental importance that the obtained DMMs are uniform. This condition restricts the use of the OERSTED data as having gaps associated with the altitude problems. As a result, the distribution of measurements over the Earth surface is often non-uniform, which may cause additional errors in calculating the model coefficients for the corresponding DMMs. Therefore, we have used the CHAMP vector data with one second resolution during the interval from May 2001 to August 2004.

The short time series of data from the OERSTED satellite were only used to corroborate the results derived from the CHAMP measurements. In particular, it was necessary to estimate the role of the attitude uncertainty of the CHAMP data retrieved from the ISDC database (level 2) (http://isdc.gfz-potsdam.de/champ/).

Unlike the other models submitted for IGRF 2000.0–2005,0, our model is constructed without data selection. We have used the vector survey data obtained along all satellite orbits, including high latitudes. It was done to eliminate the latitudinal selection of data, which could introduce errors in the model associated with the latitude of transition from vector to scalar data, and to avoid the problem of scalar linearization.

In addition to the satellite data, the annual means from the global geomagnetic observatory network have been used to estimate the model accuracy. The number of the observatories involved for different epochs is given in Table 2.

4. Construction of DMMs

The process was divided in two steps. The first step consisted in developing the geomagnetic field DMMs for every four days during the period from May 2001 till August 2004 (the total of 305 models). The models were based on the CHAMP vector measurements alone. The OERSTED satellite data were only used for a few days to check the DMMs obtained. The data for both very quiet and disturbed days were involved in the analysis. Figure 1(a) illustrates the deviation ΔZ between the models based on the CHAMP and OERSTED data for two quiet days (August 16 and 19, 2001; $\Sigma k_p = 10$). Figures 1(b) and 1(c) represent, respectively, the data coverage maps for the CHAMP and OERSTED satellites for the same days.

A comparison of the DMMs based on the CHAMP and OERSTED satellite data with equally good coverage shows that the difference between the corresponding coefficients is insignificant in spite of the attitude uncertainty of the CHAMP data. The DMMs for the days with high geomagnetic activity may differ significantly. It is, mainly, due to inhomogeneous distribution of the OERSTED data over the Earth surface.

Below, we justify the neglect of the external field in our models. The main objective of the present work is modeling the main field and its variations, while the variations of external origin can be regarded as observation errors. Their effect on the main field modeling can be demonstrated by the following test. Let us take four days with different level of geomagnetic activity. For each day, calculate the models using the internal terms of the expansion (i) alone ($n = m = 10$, the number of the spherical coefficients $N = 120$) and both the internal and external terms $(i + e)$ ($N = 120+3$). Table 1 provides the internal field coefficient g_1^0 for both models (lines 3 and 4) and the external field coefficient q_1^0 for model $(i + e)$ (line 5).

One can see from Table 1 that the first external term in expansion q_1^0 is positive and large enough (about 40 nT) even for very quiet days. For disturbed days, it reaches 175 nT. The term g_1^0 for (i) also depends on the disturbance level. The deviation from the quiet-day value is positive both for (i) (~25 nT) and for $(i + e)$ (~50 nT). Thus, the effect of the geomagnetic disturbance on the accuracy of the model of internal fields decreases if the model equation (1) does not contain the external terms of the expansion. Line 6 in Table 1 provides the r.m.s. deviations of the DMMs (i)

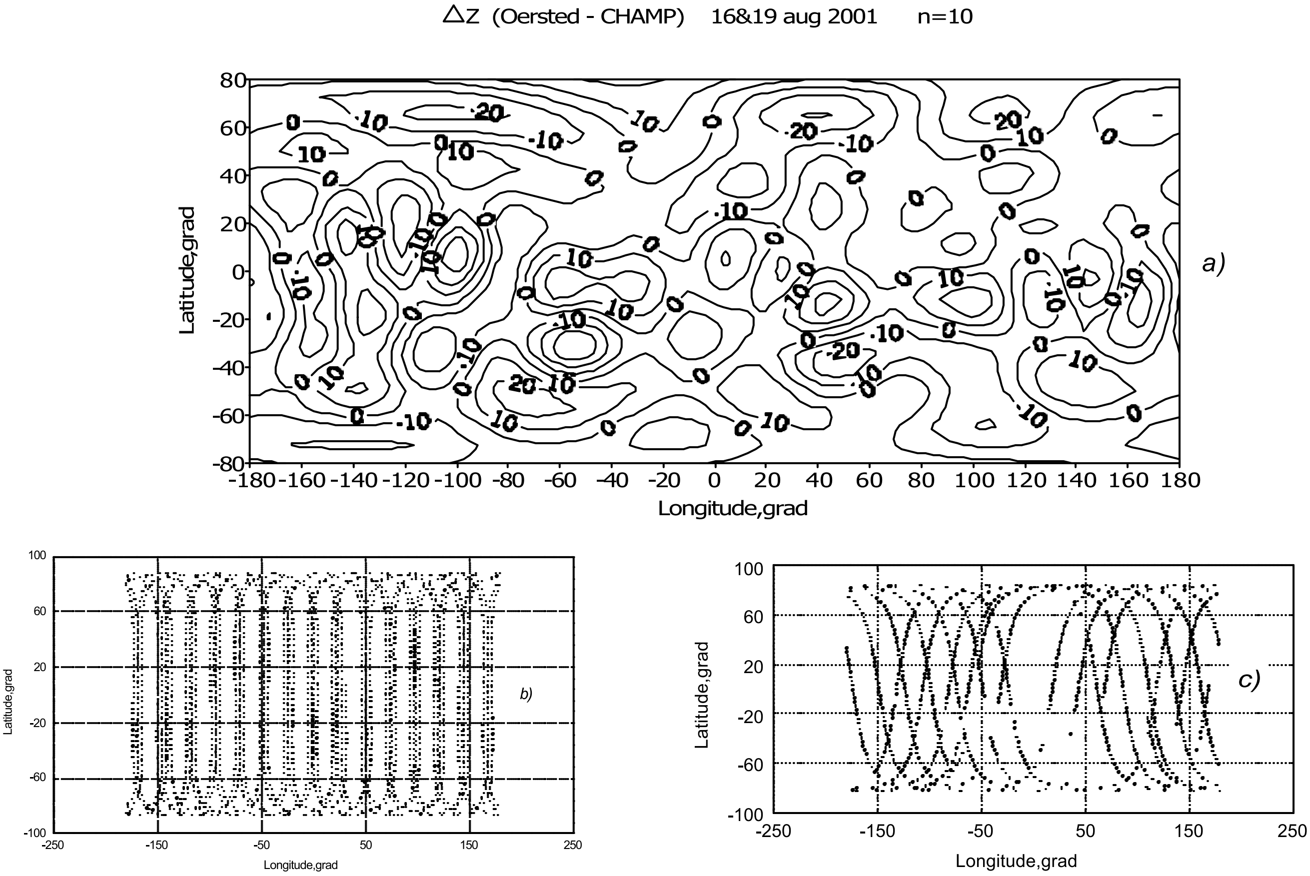

Fig. 1. The ΔZ deviation between CHAMP and OERSTED models for the two quite days (16 and 19 Aug 2001) is presented (a). Covering the globe with data from CHAMP (b) and OERSTED (c) for these days.

Table 1. Comparison of purely inner (i) and mixed ($i+e$) DMMs. As objects for comparing there were taken g_1^0 of (i) and g_1^0 of $i(i+e)$, $q_1^0 e(i+e)$; r.m.s. deviations between (i) and $i(i+e)$ for quite and disturbed days.

Date		12.11.2001	8.11.2001	7.11.2001	6.11.2001
Daily Σk_p		4+	12+	23+	54
g_1^0	i	−29598.5	−29597.8	−29594.8	−29575.3
	$i(i+e)$	−29590.9	−29585.3	−29574.4	−29540.6
q_1^0	$e(i+e)$	36.4	62.1	101.3	175.6
Deviation	$i-i(i+e)$	11.8	19.9	32.3	56.8

from the internal parts of $i(i+e)$ models for the days with different disturbance level. One can see that they do not exceed 60 nT.

It should be noted that similar calculations were performed for many other days, and they corroborated our conclusions.

Thus, we have calculated DMMs taking into account only the internal field coefficients up to $n=m=13$, as required for the IGRF candidate models.

Such modeling plays the role of a space-time frequency filtering of data spaced uniformly over the Earth surface and over one-day interval. The second step deals with DMM's coefficients, or to be more exact, with DMM's deviations relative to the mean model for the entire interval under consideration.

5. NOC Analyses

Before making the NOC analysis, we have averaged the time series of the coefficients g_n^m, h_n^m over the time interval from May 2001 to August 2004. Thus, we obtained the mean model coefficients corresponding to the middle of the interval, i.e., January 1, 2003. This model is the main field model for the epoch of 2003. Subtracting it from the DMM's time series, we obtained data for the NOC analysis.

This procedure yields the numerical functions T_{ki} and C_{kj}, where k varies from $k=1$ to $k=\min[I,J]$. The functions T_{ki} are the time variations of the kth NOC (NOC_k), and C_{kj} is the set of spherical coefficient for the latter.

In our case, the models with NOC_1, NOC_2, and NOC_3 describe 92.3, 1.1, and 0.9% of the field analyzed. Time variations of the first three NOCs (T_1, T_2, and T_3) are represented in Fig. 2.

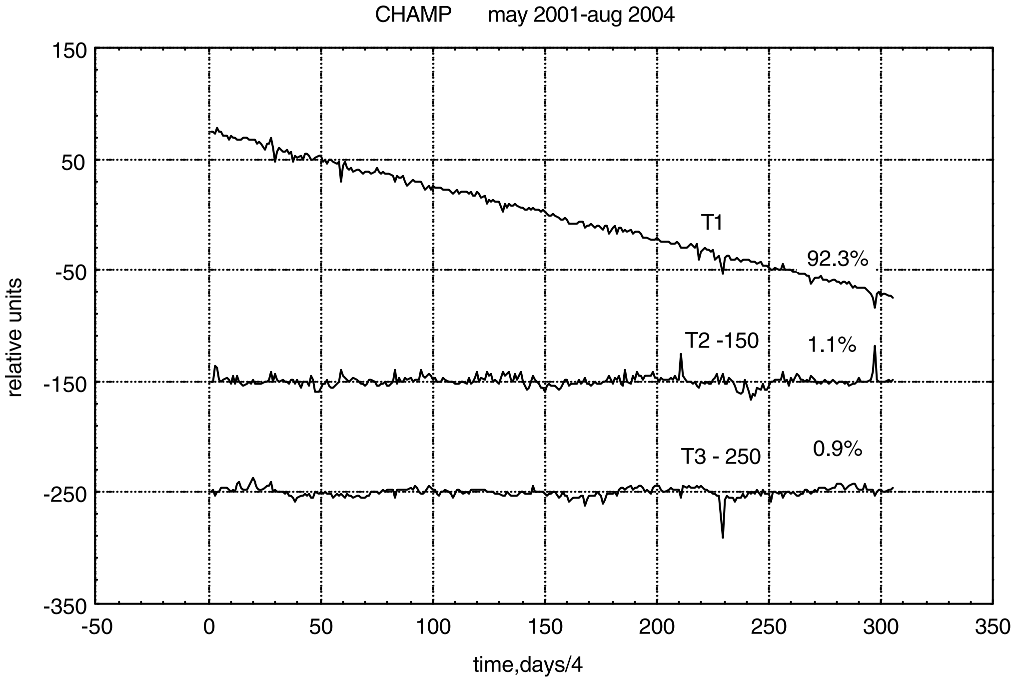

Fig. 2. Three temporal orthogonal components T_1, T_2, T_3 received from the NOC expansion of series of coefficients of DMMs for interval from May 2001 till August 2004. The contribution of the NOC1 in total modeling field is 92.3%, NOC2 - 1.1% and NOC3 - 0.9%. The T_2 and T_3 curves are omitted by 150 and 250 units respectively. On the horizontal axis—time in days/4, and on the vertical axis—relative units.

As seen from Fig. 2, T_1 is a slightly disturbed straight line all over the time interval under examination. The other two are rather high-frequency variations, which do not contain any trend that could be interpreted as secular variation. The maps of the field Z-component described by the first three spatial NOCs C_1–C_3 are represented, respectively, in Figs. 3(a)–3(c). The NOC's contribute for $k > 3$ insignificantly to the total field energy all over the modeling interval.

Thus, the model with the mean coefficients was used as the main field model for the epoch 2003.0. The SV model was obtained as a result of the NOC analysis of C_1 using, however, a linear approximation instead T_1.

The full SH-NOC model consists of $k + 1$ temporally and spatially orthogonal models, where $k = \min[I, J]$. The accuracy of the main field model 2003.0 is determined as the accuracy of the mean of 305 values. The temporal part of the first NOC (T_1) is mainly linear with the zero mean. The deviation of this curve, which is the white noise, has a stochastic nature and does not exceed 5 nT. The percentage contribution of each NOC to the total field is represented in Fig. 4. Variation of the r.m.s. error between the total field and the sum of k NOC models is shown in Fig. 5 up to $k = 50$.

To obtain model MF2005.0, we extended model MF2003.0 to this epoch under the assumption that the secular variation for 2001–2004 is linear. The comparison of our model MF 2005.0 with the mean of four candidate models for IGRF 2005 yields 12 nT in accordance with the formula proposed by F. Lowes:

$$\sigma = \sqrt{\sum_{n=1}^{13} \sum_{m=0}^{n} (n+1)[(\Delta g_n^m)^2 + (\Delta h_n^m)^2]},$$

where Δg_n^m and Δh_n^m are the differences between corresponding coefficients of the comparing models.

Then, we compared the annual means from all magnetic observatories available in 2000–2003 with the values obtained from our SH-NOC models and followed year-to-year variation of the biases by the formula:

$$D_t = \sqrt{\sum_i^I (Z_{\mathrm{IGRFt}} - Z_{\mathrm{obst}})_i^2 - (Z_{\mathrm{SH-NOCt}} - Z_{\mathrm{obst}})_i^2},$$

where D_t is r.m.s deviation particular biases for epoch t, and $t \in [2000.0 - 2003.0]$. Number of observatories I is for each year different. The same for X and Y components. Results are presented in Table 2 for components X, Y, Z independently. One can see that D_t is stable all over the interval under examination, including 2000.

6. Discussion

As follows from the above consideration, the basic problem in developing a potential space-time model of the main geomagnetic field is separation of the measured values into the parts produced by different sources both inside and outside the Earth. The traditional method is based on the data selection according to certain criteria of the magnetic activity. These are mainly the indices of geomagnetic activity obtained by routine procedures from the groundbased observatory network data, as well as some space-time criteria, such as the solar zenith angle, geomagnetic latitude, etc., which control the level of external variations. The huge bulk of the initial data is reduced many times by such selection, but it still remains too large for the space and time expansion by method of least squares.

The method proposed in this paper differs significantly

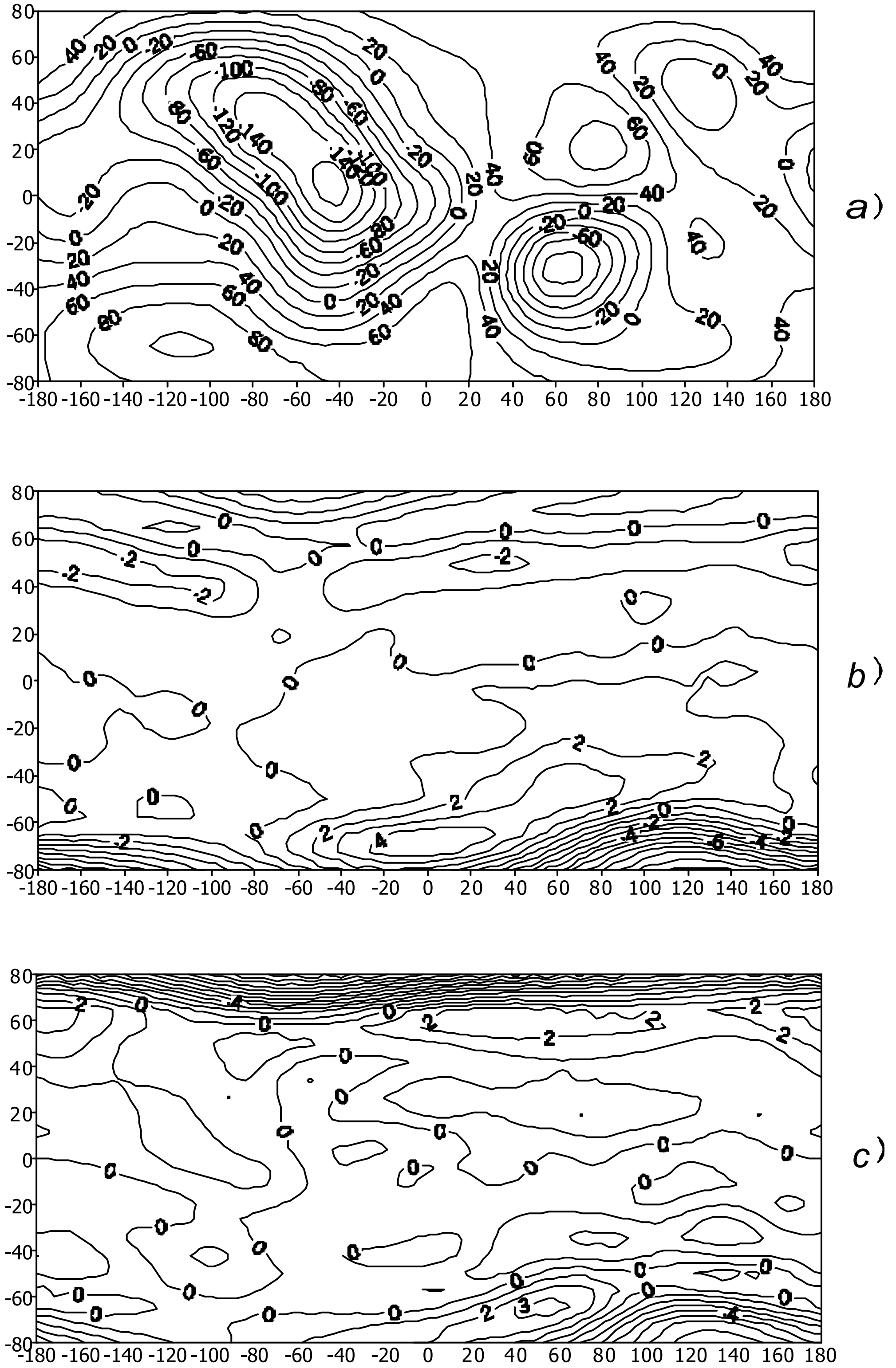

Fig. 3. Maps of Z components of the field described by first three NOCs.

from the traditional one, since it does not use data selection at all. The space-time model is developed using the method of natural orthogonal components for interconnected expansion in time and space. Our approach is based on a single assumption that the sources producing different field variations differ both in their spatial structure and in time scales. So, we suggest that variations from each source are adequately described by natural orthogonal components.

The algorithm for determining NOC requires spatially uniform data. The uniformity of the survey data can be ensured by averaging them over a time it takes the satellite to complete the orbits evenly spaced all over the Earth surface. For obtaining a final model of degree and order $n = m = 13$, the minimum time interval is one day. The averaging over a day is most naturally performed applying the spherical harmonic analysis with constant coefficients.

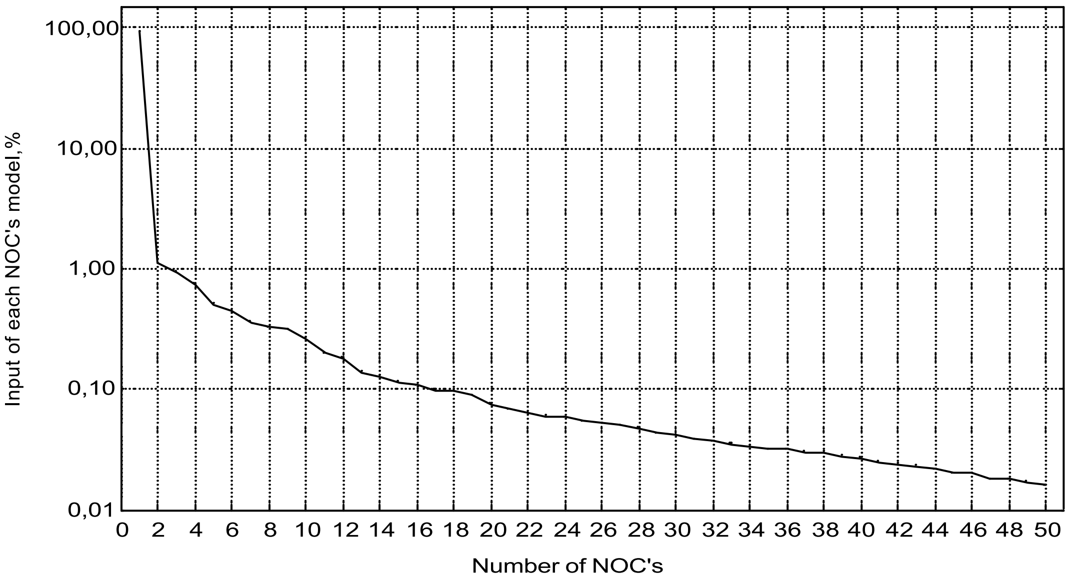

Fig. 4. The contribution of each NOC to the total modeling field.

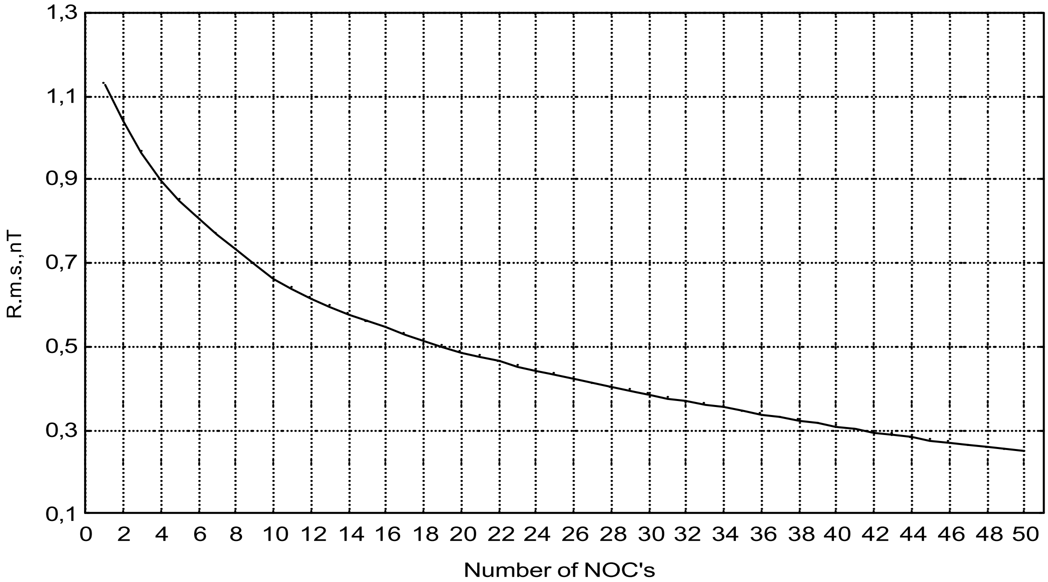

Fig. 5. Root mean square misfit of the total modeling field with the sum of k NOC models.

Table 2. Root mean square deviation particular biases for different epoch.

Year	Number of observatories	X, nT	Y, nT	Z, nT
2000	133	19	17	30
2001	130	18	18	26
2002	114	20	23	25
2003	77	21	22	24

The DMM's series is a set of 195 coefficients for a long time interval (at least a few years). We are using here the interval of 3.3 years. Thus, we have obtained a rectangular matrix of DMM's coefficients multiplied by the number of the days, i.e., just what is necessary to apply the NOC algorithm.

The solution of the problem is somewhat difficult, because the DMM's coefficients span over five orders of magnitude, which complicates the analysis. In order to make the matrix more uniform, we have averaged the identical coefficients over the entire time interval under examination and subtracted the mean values from the lines of the primary matrix. As a result, we, firstly, obtained a 3.3-year mean model of the main field referred to the middle epoch and, secondly, reduced the dispersion of coefficients in the secondary matrix by nearly three orders.

Applying the NOC method to the secondary matrix yields 195 NOCs, which converge rapidly enough to make the r.m.s. values of the first three NOCs decrease by two orders of magnitude. As a result, NOC1 involves 92% of the field variations. Thus, NOC1 provides a good approximation to the field secular variation over the time interval analyzed. A comparison of the MF models referred to 2005 to obtain candidate IGRF has shown that the difference between the models based on the data selection method and our SH-NOC model is about 10 nT.

Thus, the methods of data selection and SH-NOC are shown to yield equal accuracy. The advantage of the SH-NOC method is its simplicity and self-sufficiency as concerns the use of ground-based observation data.

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