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Hybrid generalized contractions

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Abstract

Sintunavarat and Kumam introduced the notion of hybrid generalized multi-valued contraction mapping and established a common fixed point theorem. We extend their result for four mappings and prove common coincidence and common fixed point theorem.

Keywords: Common fixed points, Coincidence points, R-weakly commuting

MSC 2000: 47H10,54H25

Introduction and preliminaries

Alber and Guerre-Delabriere [1] introduced the concept of weak contraction in Hilbert spaces. Rhoades [2] has shown that the result concluded by Alber and Guerre-Delabriere in [1] is also valid in complete metric spaces. Berinde and Berinde [3] extended weak contraction for multi-valued mappings and introduced the notion of multi-valued (θ, L)-weak contraction and multi-valued (α, L)-weak contraction. Kamran [4] extended these contractions for hybrid pair of mappings and introduced multi-valued (f, θ, L)-weak contraction and multi-valued (f, α, L)-weak contraction. Sintunavarat and Kumam [5] introduced the notion of generalized (f, α, β)-weak contraction to extend the notion of multi-valued (f, α, L)-weak contraction. They further extended the notion of generalized (f, α, β)-weak contraction by introducing hybrid generalized multi-valued contraction mappings and established a common fixed point theorem [6]. The purpose of this paper is to extend the notion of hybrid generalized multi-valued contraction and to prove common coincidence and common fixed point theorems.

Let (X, d) be a metric space. For $x \in X$ and $A \subseteq X$, $d(x, A) = \inf\{d(x, y) : y \in A\}$. We denote by CB$(X)$ the class of all nonempty closed and bounded subsets of X. For every $A, B \in$ CB(X)

$$H(A, B) = \max\{\sup_{x\in A} d(x, B), \sup_{y\in B} d(y, A)\}.$$

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Such a map H is called Hausdorff metric induced by d. A point $x \in X$ is said to be a common fixed point of $f : X \to X$ and $T : X \to$ CB(X) if $x = fx \in Tx$. The point $x \in X$ is said to be a coincidence point of $f : X \to X$ and $T : X \to$ CB(X) if $fx \in Tx$. Some works for hybrid pair are available in [7-10].

The mappings $f : X \to X$ and $T : X \to$ CB(X) are called R-weakly commuting [11,12] if for all $x \in X$, $fTx \in$ CB(X), and there exists a positive real number R such that $H(fTx, Tfx) \leq Rd(fx, Tx)$.

Lemma 1. *[4] Let (X, d) be a metric space, $\{A_k\}$ be a sequence in $CB(X)$, and $\{x_k\}$ be a sequence in X such that $x_k \in A_{k-1}$. Let $\phi : [0, \infty) \to [0, 1)$ be a function satisfying $\limsup_{r\to t^+} \phi(r) < 1$ for every $t \in [0, \infty)$. Suppose $d(x_{k-1}, x_k)$ to be a nonincreasing sequence such that*

$$H(A_{k-1}, A_k) \leq \phi(d(x_{k-1}, x_k))d(x_{k-1}, x_k),$$

$$d(x_{k+1}, x_k) \leq H(A_{k-1}, A_k) + \phi^{n_k}(d(x_{k-1}, x_k)),$$

where $n_1 < n_2 < \ldots, k, n_k \in \mathbb{N}$. Then, $\{x_k\}$ is a Cauchy sequence in X.

Lemma 2. *[13] If $A, B \in CB(X)$ and $a \in A$, then for each $\epsilon > 0$, there exists $b \in B$ such that*

$$d(a, b) \leq H(A, B) + \epsilon.$$

Definition 1. [6] Let (X, d) be a metric space, $f : X \to X$ be a single-valued mapping, and $T : X \to$ CB(X) be a multi-valued mapping. T is said to be a hybrid generalized multi-valued contraction mapping if and only if there exist two functions $\phi : [0, \infty) \to [0, 1)$ satisfying

$\limsup_{r\to t^+}\phi(r) < 1$ for every $t \in [0,\infty)$ and $\varphi : [0,\infty) \to [0,\infty)$ such that

$$H(Tx, Ty) \le \phi(M(x,y))M(x,y) + \varphi(N(x,y))N(x,y), \tag{1}$$

for each $x, y \in X$,

where

$$M(x,y) = \max\{d(fx,fy), d(fy, Tx)\},$$

and

$$N(x,y) = \min\{d(fx,fy), d(fx, Tx), d(fy, Ty), d(fx, Ty), d(fy, Tx)\}.$$

Main results

Definition 2. Let (X,d) be a metric space, $f,g : X \to X$ and $T : X \to \mathrm{CB}(X)$. A mapping $S : X \to \mathrm{CB}(X)$ is said to be an extended hybrid generalized multi-valued f contraction if and only if there exist two functions $\phi : [0,\infty) \to [0,1)$ satisfying $\limsup_{r\to t^+}\phi(r) < 1$ for every $t \in [0,\infty)$ and $\varphi : [0,\infty) \to [0,\infty)$ such that

$$H(Sx, Ty) \le \phi(M(x,y))M(x,y) + \varphi(N(x,y))N(x,y), \tag{2}$$

for each $x, y \in X$,

where

$$M(x,y) = \max\{d(fx,gy), \min\{d(fx, Ty), d(gy, Sx)\}\},$$

and

$$N(x,y) = \min\{d(fx,gy), d(fx, Tx), d(gy, Sy), d(fx, Ty), d(gy, Sx)\}.$$

Remark 1. If $f = g$ and $T = S$, then Definition 2 reduces to Definition 1.

Lemma 3. *Let (X,d) be a metric space, $f,g : X \to X$ and $T : X \to CB(X)$. Let $S : X \to CB(X)$ be the extended hybrid generalized multi-valued f contraction. Let $\{gx_{2k+1}\}$ be a g-orbit of S at x_0 and $\{fx_{2k+2}\}$ be an f-orbit of T at x_1 such that*

$$d(y_{k+1}, y_{k+2}) \le H(A_k, A_{k+1}) + \phi^{n_{k+1}}(M(x_k, x_{k+1})), \tag{3}$$

for each $k \in \{0,2,4,6,\ldots\}$, and

$$d(y_{k+2}, y_{k+1}) \le H(A_{k+1}, A_k) + \phi^{n_{k+1}}(M(x_{k+1}, x_k)), \tag{4}$$

for each $k \in \{1,3,5,\ldots\}$, where $y_{2k+1} = gx_{2k+1} \in Sx_{2k} = A_{2k}$, $y_{2k+2} = fx_{2k+2} \in Tx_{2k+1} = A_{2k+1}$, for each $k \ge 0$. Further, $n_1 < n_2 < \ldots$ and $\{d(y_k, y_{k+1})\}$ is a nonincreasing sequence. Then, $\{y_k\}$ is a Cauchy sequence in X.

Proof. Let $y_0 = x_0$. Then, we construct a sequence $\{y_k\}$ in X, A_k in $\mathrm{CB}(X)$ such that $y_{2k+1} = gx_{2k+1} \in Sx_{2k} = A_{2k}$ and $y_{2k+2} = fx_{2k+1} \in Tx_{2k+1} = A_{2k+1}$.

For $k \in \{0,2,4,6,\ldots\}$, it follows from the extended hybrid generalized multi-valued f contraction that

$$\begin{aligned}
&H(A_k, A_{k+1})\\
&\quad = H(Sx_k, Tx_{k+1});\\
&\quad \le \phi(M(x_k, x_{k+1}))M(x_k, x_{k+1})\\
&\qquad + \varphi(N(x_k, x_{k+1}))N(x_k, x_{k+1});\\
&\quad = \phi(\max\{d(fx_k, gx_{k+1}), \min\{d(fx_k, Tx_{k+1}),\\
&\qquad d(gx_{k+1}, Sx_k)\}\})\\
&\qquad \times \max\{d(fx_k, gx_{k+1}), \min\{d(fx_k, Tx_{k+1}),\\
&\qquad d(gx_{k+1}, Sx_k)\}\}\\
&\qquad + \varphi(\min\{d(fx_k, gx_{k+1}), d(fx_k, Tx_k), d(gx_{k+1}, Sx_{k+1}),\\
&\qquad d(fx_k, Tx_{k+1}), d(gx_{k+1}, Sx_k)\}) \min\{d(fx_k, gx_{k+1}),\\
&\qquad d(fx_k, Tx_k), d(gx_{k+1}, Sx_{k+1}), d(fx_k, Tx_{k+1}),\\
&\qquad d(gx_{k+1}, Sx_k)\};\\
&\quad = \phi(d(fx_k, gx_{k+1}))d(fx_k, gx_{k+1});\\
&\quad = \phi(d(y_k, y_{k+1}))d(y_k, y_{k+1}).
\end{aligned}$$

Similarly, we show that for $k = \{1,3,5,\ldots\}$, we have

$$H(A_{k+1}, A_k) \le \phi(d(y_{k+1}, y_k))d(y_{k+1}, y_k).$$

By (3), for $k \in \{0,2,4,6,\ldots\}$, we have

$$\begin{aligned}
d(y_{k+1}, y_{k+2}) &= d(gx_{k+1}, fx_{k+2});\\
&\le H(Sx_k, Tx_{k+1}) + \phi^{n_{k+1}}(M(x_k, x_{k+1}));\\
&= H(A_k, A_{k+1}) + \phi^{n_{k+1}}(\max\{d(fx_k, gx_{k+1}),\\
&\quad \min\{d(fx_k, Tx_{k+1}), d(gx_{k+1}, Sx_k)\}\});\\
&= H(A_k, A_{k+1}) + \phi^{n_{k+1}}(d(fx_k, gx_{k+1}));\\
&= H(A_k, A_{k+1}) + \phi^{n_{k+1}}(d(y_k, y_{k+1})).
\end{aligned}$$

Similarly, for $k = \{1,3,5,\ldots\}$, we have

$$d(y_{k+2}, y_{k+1}) \le H(A_{k+1}, A_k) + \phi^{n_{k+1}}(d(y_{k+1}, y_k)).$$

Given that $\{d(y_k, y_{k+1})\}$ is a nonincreasing sequence, thus, all the conditions of Lemma 1 are satisfied. Hence, $\{y_k\}$ is a Cauchy sequence in X. □

Theorem 1. *Let (X,d) be a complete metric space, $f,g : X \to X$, $T : X \to CB(X)$ are continuous mappings, and $S : X \to CB(X)$ is a continuous extended hybrid generalized multi-valued f contraction such that $SX \subseteq gX$ and $TX \subseteq fX$. Then,*

(i) if g, T and f, S are R-weakly commuting, then g, T and f, S have a common coincidence point say z.

(ii) Moreover, if $ggz = gz$, $fgz = gz$, then f, g, T, and S have a common fixed point.

Proof. Let x_0 be an arbitrary point in X and $y_0 = fx_0$. Then, we construct a sequence $\{y_k\}$ in X, A_k in $\mathrm{CB}(X)$ respectively as follows. Since $SX \subseteq gX$, there exists a point

$x_1 \in X$ such that $y_1 = gx_1 \in Sx_0 = A_0$. We can choose a positive integer n_1 such that

$$\phi^{n_1}(d(y_0, y_1)) \leq [1 - \phi(M(x_0, x_1))] M(x_0, x_1). \tag{5}$$

Since $TX \subseteq fX$, there exists $y_2 = fx_2 \in Tx_1 = A_1$ such that

$$d(y_1, y_2) \leq H(Sx_0, Tx_1) + \phi^{n_1}(d(y_0, y_1)). \tag{6}$$

Using (5) and the notion of extended hybrid generalized multi-valued f contraction in the above inequality, we have

$$\begin{aligned} d(y_1, y_2) &\leq H(Sx_0, Tx_1) + \phi^{n_1}(d(y_0, y_1)); \\ &\leq \phi(M(x_0, x_1))M(x_0, x_1) \\ &\quad + \varphi(N(x_0, x_1))N(x_0, x_1) \\ &\quad +[1 - \phi(M(x_0, x_1))] M(x_0, x_1); \\ &= M(x_0, x_1); \\ &= \min\{d(fx_k, Tx_{k+1}), d(gx_{k+1}, Sx_k)\}\}); \\ &= d(fx_0, gx_1); \\ &= d(y_0, y_1). \end{aligned}$$

Now, we can choose a positive integer $n_2 > n_1$ such that

$$\phi^{n_2}(d(y_2, y_1)) \leq [1 - \phi(M(x_2, x_1))] M(x_2, x_1). \tag{7}$$

There exists $y_3 = gx_3 \in Sx_2 = A_2$ such that

$$d(y_3, y_2) \leq H(Sx_2, Tx_1) + \phi^{n_2}(d(y_2, y_1)). \tag{8}$$

Using (7) and the notion of extended hybrid generalized multi-valued f contraction in the above inequality, we have

$$\begin{aligned} d(y_3, y_2) &\leq H(Sx_2, Tx_1) + \phi^{n_2}(d(y_2, y_1)); \\ &\leq \phi(M(x_2, x_1))M(x_2, x_1) \\ &\quad + \varphi(N(x_2, x_1))N(x_2, x_1) \\ &\quad +[1 - \phi(M(x_2, x_1))] M(x_2, x_1); \\ &= M(x_2, x_1); \\ &= \max\{d(fx_2, gx_1), \min\{d(fx_2, Tx_1), \\ &\quad d(gx_1, Sx_2)\}\}; \\ &= d(fx_2, gx_1); \\ &= d(y_2, y_1). \end{aligned}$$

Now, we can choose a positive integer $n_3 > n_2$ such that

$$\phi^{n_3}(d(y_2, y_3)) \leq [1 - \phi(M(x_2, x_3))] M(x_2, x_3). \tag{9}$$

There exists $y_4 = fx_4 \in Tx_3 = A_3$ such that

$$d(y_3, y_4) \leq H(Sx_2, Tx_3) + \phi^{n_3}(d(y_2, y_3)). \tag{10}$$

Using (9) and the notion of extended hybrid generalized multi-valued f contraction in the above inequality, we have

$$\begin{aligned} d(y_3, y_4) &\leq H(Sx_2, Tx_3) + \phi^{n_3}(d(y_2, y_3)); \\ &\leq \phi(M(x_2, x_3))M(x_2, x_3) \\ &\quad + \varphi(N(x_2, x_3))N(x_2, x_3) \\ &\quad +[1 - \phi(M(x_2, x_3))] M(x_2, x_3); \\ &= M(x_2, x_3); \\ &= \max\{d(fx_2, gx_3), \min\{d(fx_2, Tx_3), \\ &\quad d(gx_3, Sx_2)\}\}; \\ &= d(fx_2, gx_3); \\ &= d(y_2, y_3). \end{aligned}$$

Now, we can choose a positive integer $n_4 > n_3$ such that

$$\phi^{n_4}(d(y_4, y_3)) \leq [1 - \phi(M(x_4, x_3))] M(x_4, x_3). \tag{11}$$

There exists $y_5 = gx_5 \in Sx_4 = A_4$ such that

$$d(y_5, y_4) \leq H(Sx_4, Tx_3) + \phi^{n_4}(d(y_4, y_3)). \tag{12}$$

Using (11) and the notion of extended hybrid generalized multi-valued f contraction in the above inequality, we have

$$\begin{aligned} d(y_5, y_4) &\leq H(Sx_4, Tx_3) + \phi^{n_4}(d(y_4, y_3)); \\ &\leq \phi(M(x_4, x_3))M(x_4, x_3) \\ &\quad + \varphi(N(x_4, x_3))N(x_4, x_3) \\ &\quad +[1 - \phi(M(x_4, x_3))] M(x_4, x_3); \\ &= M(x_4, x_3); \\ &= \max\{d(fx_4, gx_3), \min\{d(fx_4, Tx_3), \\ &\quad d(gx_3, Sx_4)\}\}; \\ &= d(fx_4, gx_3); \\ &= d(y_4, y_3). \end{aligned}$$

By repeating this process for all $k \in \mathbb{W}$, we have the following:

Case (i). For $k \in \{0, 2, 4, 6, \ldots\}$, we can choose a positive integer n_{k+1} such that

$$\phi^{n_{k+1}}(d(y_k, y_{k+1})) \leq [1 - \phi(M(x_k, x_{k+1}))] M(x_k, x_{k+1}). \tag{13}$$

There exists $y_{k+2} = fx_{k+2} \in Tx_{k+1} = A_{k+1}$ such that

$$d(y_{k+1}, y_{k+2}) \leq H(Sx_k, Tx_{k+1}) + \phi^{n_{k+1}}(d(y_k, y_{k+1})). \tag{14}$$

Using (13) and the notion of extended generalized multi-valued f contraction in the above inequality, we have

$$\begin{aligned} d(y_{k+1}, y_{k+2}) &\leq H(Sx_k, Tx_{k+1}) + \phi^{n_{k+1}}(d(y_k, y_{k+1})); \\ &\leq \phi(M(x_k, x_{k+1}))M(x_k, x_{k+1}) \\ &\quad + \varphi(N(x_k, x_{k+1}))N(x_k, x_{k+1}) \\ &\quad + [1 - \phi(M(x_k, x_{k+1}))]M(x_k, x_{k+1}); \\ &= M(x_k, x_{k+1}); \\ &= \max\{d(fx_k, gx_{k+1}), \min\{d(fx_k, Tx_{k+1}), \\ &\quad d(gx_{k+1}, Sx_k)\}\}; \\ &= d(fx_k, gx_{k+1}); \\ &= d(y_k, y_{k+1}). \end{aligned}$$

Case (ii). For $k \in \{1, 3, 5, 7, \ldots\}$, we can choose a positive integer n_k such that

$$\phi^{n_{k+1}}(d(y_{k+1}, y_k)) \leq [1 - \phi(M(x_{k+1}, x_k))]M(x_{k+1}, x_k). \tag{15}$$

There exists $y_{k+2} = gx_{k+2} \in Sx_{k+1} = A_{k+1}$ such that

$$d(y_{k+2}, y_{k+1}) \leq H(Sx_{k+1}, Tx_k) + \phi^{n_{k+1}}(d(y_{k+1}, y_k)). \tag{16}$$

Using (15) and the notion of extended generalized multi-valued f contraction in the above inequality, we have

$$\begin{aligned} d(y_{k+2}, y_{k+1}) &\leq H(Sx_{k+1}, Tx_k) + \phi^{n_{k+1}}(d(y_{k+1}, y_k)); \\ &\leq \phi(M(x_{k+1}, x_k))M(x_{k+1}, x_k) \\ &\quad + \varphi(N(x_{k+1}, x_k))N(x_{k+1}, x_k) \\ &\quad + [1 - \phi(M(x_{k+1}, x_k))]M(x_{k+1}, x_k); \\ &= M(x_{k+1}, x_k); \\ &= \max\{d(fx_{k+1}, gx_k), \min\{d(fx_{k+1}, Tx_k), \\ &\quad d(gx_k, Sx_{k+1})\}\}; \\ &= d(fx_{k+1}, gx_k); \\ &= d(y_{k+1}, y_k). \end{aligned}$$

Hence, $\{d(y_k, y_{k+1})\}$ is a nonincreasing sequence for each $k \geq 0$. Thus, by Lemma 3, $\{y_k\}$ is a Cauchy sequence in X. Then, (2) ensures that $\{A_k\}$ is a Cauchy sequence in CB(X). As we know that if X is complete, then CB(X) is also complete. Therefore, there exist $z \in X$ and $A \in$ CB(X) such that $y_k \to z$ and $A_k \to A$. Moreover, $gx_{2k+1} \to z$ and $f_{2k+2} \to z$, since

$$d(z, A) = \lim_{k\to\infty} d(y_k, A_k) \leq \lim_{k\to\infty} H(A_{k-1}, A_k) = 0. \tag{17}$$

It follows that $z \in A$, since A is closed. Thus, we have

$$\lim_{k\to\infty} gx_{2k+1} = z \in A = \lim_{k\to\infty} Sx_{2k} \text{ and}$$

$$\lim_{k\to\infty} fx_{2k+2} = z \in A = \lim_{k\to\infty} Tx_{2k+1}$$

As g, T and f, S are R-weakly commuting, we have

$$\begin{aligned} d(gfx_{2k+2}, Tgx_{2k+1}) &\leq H(gTx_{2k+1}, Tgx_{2k+1}) \\ &\leq Rd(gx_{2k+1}, Tx_{2k+1}). \end{aligned} \tag{18}$$

$$d(fgx_{2k+1}, Sfx_{2k}) \leq H(fSx_{2k}, Sfx_{2k}) \leq Rd(fx_{2k}, Sx_{2k}). \tag{19}$$

Letting $k \to \infty$ in (18) and (19) and using (17) and the continuity of f, g, T, and S, we get

$$gz \in Tz \text{ and } fz \in Sz.$$

By condition (ii) of Theorem 1, we have $ggz = gz, fgz = gz$. Let $v = gz$ and then we have $gv = v = fv$. From (2), we have

$$\begin{aligned} H(Sv, Tz) &\leq \phi(\max\{d(fv, gz), \\ &\quad \min\{d(fv, Tz), d(gz, Sv)\}\}) \\ &\quad \times \max\{d(fv, gz), \\ &\quad \min\{d(fv, Tz), d(gz, Sv)\}\} \\ &\quad + \varphi(\min\{d(fv, gz), d(fv, Tv), d(gz, Sz), \\ &\quad d(fv, Tz), d(gz, Sv)\}) \\ &\quad \times \min\{d(fv, gz), d(fv, Tv), d(gz, Sz), \\ &\quad d(fv, Tz), d(gz, Sv)\}. \end{aligned}$$

Note that $fv = gz$ and $fv \in Tz$. Hence, we have $H(Sv, Tz) = 0$, i.e., $Sv = Tz$. Again from (2), we have

$$\begin{aligned} H(Sv, Tv) &\leq \phi(\max\{d(fv, gv), \\ &\quad \min\{d(fv, Tv), d(gv, Sv)\}\}) \\ &\quad \times \max\{d(fv, gv), \\ &\quad \min\{d(fv, Tv), d(gv, Sv)\}\} \\ &\quad + \varphi(\min\{d(fv, gv), d(fv, Tv), d(gv, Sv), \\ &\quad d(fv, Tv), d(gv, Sv)\}) \\ &\quad \times \min\{d(fv, gv), d(fv, Tv), d(gv, Sv), \\ &\quad d(fv, Tv), d(gv, Sv)\}. \end{aligned}$$

Note that $fv = gv$ and $gv \in Sv$. Hence, we have $H(Sv, Tv) = 0$, i.e., $Sv = Tv$. Therefore, we have $v = fv = gv \in Sv = Tv$. □

Remark 2. Theorem 1 improves and extends some known results of Kamran [12], Nadler [13], Hu [14], Kaneko [15], and Mizoguchi and Takahashi [16].

Example 1. *Let $X = [0, \infty)$ be endowed with the metric*

$$d(x, y) = \begin{cases} x + y & \text{if } x \neq y, \\ 0 & \text{if } x = y. \end{cases} \tag{20}$$

Define $T, S : X \to CB(X)$ and $f, g : X \to X$ by $Tx = [0, \frac{x}{3}]$, $Sx = \{0\}$, $fx = \frac{3x}{2}$, and $gx = \frac{x}{2}$ for all $x \in X$. Let $\phi(t) = \frac{2t}{3}$ and $\varphi(t) = t$ for all $t \geq 0$. It is easy to show that S is the *extended hybrid generalized multi-valued f contraction. It is easy to check that all the conditions of Theorem 1 hold, and 0 is a common fixed point of f, g, T, and S.*

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
MUA and TK contributed equally in this article. Both authors read and approved the final manuscript.

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