Ahmad and Hussain *Mathematical Sciences* 2012, **6**:64
http://www.iaumath.com/content/6/1/64

Mathematical Sciences
a SpringerOpen Journal

ORIGINAL RESEARCH Open Access

On some structures of soft topology

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Abstract

In this paper, we define soft exterior and study its basic properties. We establish several important results relating soft interior, soft exterior, soft closure, and soft boundary in soft topological spaces. Moreover, we characterize soft open sets, soft closed sets, and soft clopen sets via soft boundary. All these findings will provide a base to researchers who want to work in the field of soft topology and will help to establish a general framework for applications in practical fields.

Keywords: Soft set, Soft topology, Soft open(closed) set, Soft nbd, Soft interior(exterior), Soft closure(boundary)

Introduction

In 1999, Molodtsov [1] initiated the theory of soft sets as a new mathematical tool to deal with uncertainties while modelling the problems in engineering, physics, computer science, economics, social sciences, and medical sciences. In [2], Molodtsov et al. successfully applied soft sets in directions such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability, and theory of measurement. Maji et al. [3,4] gave the first practical application of soft sets in decision-making problems. In 2003, Maji et al. [4] defined and studied several basic notions of the soft set theory. In 2005, Pei and Miao [5] and Chen [6] improved the work of Maji et al. [3,4].

Many researchers have contributed towards the algebraic structure of the soft set theory [7-19]. The application of the soft set theory in algebraic structures was introduced by Aktas and Cagman [8]. They established the basic notions of soft groups as a generalization of the idea of fuzzy groups. Jun [9] investigated BCK/BCI algebras and studied their applications in ideal theory. Feng et al. [20] worked on soft semirings, soft ideals, and idealistic soft semirings. Ali et al. [21] and Shabir and Irfan Ali [22] studied semigroups and soft ideals over a semigroup which characterized generalized fuzzy ideals and fuzzy ideals with thresholds of a semigroup.

Recently, in 2011, Shabir and Naz [23] and Cagman et al. [24] initiated the study of soft topology and soft topological spaces independently. Shabir and Naz [23] defined soft topology on the collection τ of soft sets over X. Consequently, they defined basic notions of soft topological spaces such as soft open and closed sets, soft subspace, soft closure, soft nbd of a point, soft T_i spaces, $i = 1, 2, 3, 4$, soft regular spaces, and soft normal spaces and established their several properties. In [25], we presented further several important properties of notions defined and studied in [23]. On the other hand, Cagman et al. [24] introduced a soft topology on a soft set and defined a soft topological space. They defined basic notions and concepts of soft topological spaces such as soft open and closed sets, soft interior, soft closure, soft basis, soft nbd of a point, soft limit point of a soft set, soft complement, soft difference, and soft boundary and established several properties of these soft notions. The work in both papers is appreciable.

The notion of soft topology by Cagman et al. [24] is more general than that by Shabir and Naz [23]. Therefore, we continue investigating the work of Cagman et al. [24] and follow their notations and mathematical formalism. In this paper, first, we define and study soft exterior. We characterize soft open sets, soft closed sets, and soft clopen sets in terms of soft boundary. We establish several interesting properties of soft interior, soft exterior, soft closure, and soft boundary and their relationship which are fundamental for further research on soft topology and will strengthen the foundations of the theory of soft topological spaces.

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Preliminaries

Now, we recall some definitions and results defined and discussed in [1,4,24]. Hereafter, U refers to an initial universe, E is a set of parameters, $P(U)$ is the power set of U, and A is a nonempty subset of E.

Definition 1. A soft set F_A on the universe U is defined by the set of ordered pairs $F_A = \{(x, f_A(x)) : x \in E, f_A(x) \in P(U)\}$, where $f_A : E \to P(U)$ such that $f_A(x) = \phi$ if $x \notin A$. Here, f_A is called an approximate function of the soft set F_A. The value of $f_A(x)$ may be arbitrary. Some of them may be empty, and some may have nonempty intersection. The class of all soft sets over U will be denoted by $S(U)$.

Example 1. Suppose that there are six houses in the universe $U = \{h_l, h_2, h_3, h_4, h_5, h_6\}$ under consideration and that $E = \{x_1, x_2, x_3, x_4, x_5\}$ is a set of decision parameters. The $x_i \quad (i = 1, 2, 3, 4, 5)$ stand for the parameters 'expensive,' 'beautiful,' 'wooden,' 'cheap,' and 'in green surroundings,' respectively.

Consider the mapping f_A given by 'houses (.),' where (.) is to be filled in by one of the parameters $x_i \in E$. For instance, $f_A(x_1)$ means 'houses (expensive),' and its functional value is the set $\{h \in U : h \text{ is an expensive house }\}$.

Suppose that $A = \{x_1, x_3, x_4\} \subseteq E$ and $f_A(x_1) = \{h_2, h_4\}$, $f_A(x_3) = U$, and $f_A(x_4) = \{h_1, h_3, h_5\}$. Then, we can view the soft set F_A as consisting of the following collection of approximations:

$$F_A = \{(x_1, \{h_2, h_4\}), (x_3, U), (x_4, \{h_1, h_3, h_5\})\}.$$

Definition 2. Let $F_A \in S(U)$. If $f_A(x) = \phi$, for all $x \in E$, then F_A is called an empty soft set, denoted by F_Φ. $f_A(x) = \phi$ means that there is no element in U related to the parameter $x \in E$. Therefore, we do not display such elements in the soft sets as it is meaningless to consider such parameters.

Definition 3. Let $F_A \in S(U)$. If $f_A(x) = U$, for all $x \in A$, then F_A is called an A-universal soft set, denoted by $F_{\tilde{A}}$. If $A = E$, then the A-universal soft set is called a universal soft set, denoted by $F_{\tilde{E}}$.

Definition 4. Let $F_A, F_B \in S(U)$. Then, F_A is a soft subset of F_B, denoted by $F_A \tilde{\subseteq} F_B$, if $f_A(x) \subseteq f_B(x)$, for all $x \in E$.

Definition 5. Let $F_A, F_B \in S(U)$. Then, F_A and F_B are soft equal, denoted by $F_A = F_B$, if $f_A(x) = f_B(x)$, for all $x \in E$.

Definition 6. Let $F_A, F_B \in S(U)$. Then, the soft union $F_A \tilde{\cup} F_B$, the soft intersection $F_A \tilde{\cap} F_B$, and the soft difference $F_A \tilde{\setminus} F_B$ of F_A and F_B are defined by the approximate functions $f_{A\tilde{\cup}B}(x) = f_A(x) \cup f_B(x)$, $f_{A\tilde{\cap}B}(x) = f_A(x) \cap f_B(x)$, and $f_{A\tilde{\setminus}B}(x) = f_A(x) \setminus f_B(x)$, respectively, and the soft complement $F_A^{\tilde{c}}$ of F_A is defined by the approximate function $f_A^{\tilde{c}}(x) = f_A^c(x)$, where $f_A^{\tilde{c}}(x)$ is the complement of the set $f_A(x)$, that is, $f_A^{\tilde{c}}(x) = U \tilde{\setminus} f_A(x)$, for all $x \in E$. It is easy to see that $(F_A^{\tilde{c}})^{\tilde{c}} = F_A$ and $F_\Phi^{\tilde{c}} = F_{\tilde{E}}$.

Proposition 1. *Let $F_A \in S(U)$. Then,*

(1) $F_A \tilde{\cup} F_A = F_A$, $F_A \tilde{\cap} F_A = F_A$.
(2) $F_A \tilde{\cup} F_\Phi = F_A$, $F_A \tilde{\cap} F_\Phi = F_\Phi$.
(3) $F_A \tilde{\cup} F_{\tilde{E}} = F_{\tilde{E}}$, $F_A \tilde{\cap} F_{\tilde{E}} = F_A$.
(4) $F_A \tilde{\cup} F_A^{\tilde{c}} = F_{\tilde{E}}$, $F_A \tilde{\cap} F_A^c = F_\Phi$.

Proposition 2. *Let $F_A, F_B, F_C \in S(U)$. Then,*

(1) $F_A \tilde{\cup} F_B = F_B \tilde{\cup} F_A$, $F_A \tilde{\cap} F_B = F_B \tilde{\cap} F_A$.
(2) $(F_A \tilde{\cup} F_B)^{\tilde{c}} = F_B^{\tilde{c}} \tilde{\cap} F_A^{\tilde{c}}$, $(F_A \tilde{\cap} F_B)^{\tilde{\tilde{c}}} = F_B^{\tilde{c}} \tilde{\cup} F_A^{\tilde{c}}$.
(3) $F_A \tilde{\cup} (F_B \tilde{\cap} F_C) = (F_A \tilde{\cup} F_B) \tilde{\cap} (F_A \tilde{\cup} F_C)$.
(4) $F_A \tilde{\cap} (F_B \tilde{\cup} F_C) = (F_A \tilde{\cap} F_B) \tilde{\cup} (F_A \tilde{\cap} F_C)$.

Definition 7. Let $F_A \in S(U)$. The soft power set of F_A is defined by $\tilde{P}(F_A) = \{F_{A_i} : F_{A_i} \tilde{\subseteq} F_A, i \in I \subseteq N\}$, and its cardinality is defined by $|\tilde{P}(F_A)| = 2^{\sum_{x \in E} |f_A(x)|}$, where $|f_A(x)|$ is the cardinality of $f_A(x)$.

Example 2. Let $U = \{u_1, u_2, u_3\}$, $E = \{x_1, x_2, x_3\}$, $A = \{x_1, x_2\} \subseteq E$, and

$F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2, u_3\})\}$. Then,
$F_{A_1} = \{(x_1, \{u_1\})\}$,
$F_{A_2} = \{(x_1, \{u_2\})\}$,
$F_{A_3} = \{(x_1, \{u_1, u_2\})\}$,
$F_{A_4} = \{(x_2, \{u_2\})\}$,
$F_{A_5} = \{(x_2, \{u_3\})\}$,
$F_{A_6} = \{(x_2, \{u_2, u_3\})\}$,
$F_{A_7} = \{(x_1, \{u_1\}), (x_2, \{u_2\})\}$,
$F_{A_8} = \{(x_1, \{u_1\}), (x_2, \{u_3\})\}$,
$F_{A_9} = \{(x_1, \{u_1\}), (x_2, \{u_2, u_3\})\}$,
$F_{A_{10}} = \{(x_1, \{u_2\}), (x_2, \{u_2\})\}$,
$F_{A_{11}} = \{(x_1, \{u_2\}), (x_2, \{u_3\})\}$,
$F_{A_{12}} = \{(x_1, \{u_2\}), (x_2, \{u_2, u_3\})\}$,
$F_{A_{13}} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2\})\}$,
$F_{A_{14}} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_3\})\}$,
$F_{A_{15}} = F_A$,
$F_{A_{16}} = F_\Phi$

are all soft subsets of F_A. So, $|\tilde{P}(F_A)| = 2^4 = 16$.

Definition 8. Let $F_A \in S(U)$. A soft topology on F_A, denoted by $\tilde{\tau}$, is a collection of soft subsets of F_A having the following properties:

(1) $F_\Phi, F_A \in \tilde{\tau}$.
(2) $\{F_{A_i} \tilde{\subseteq} F_A : i \in I \subseteq N\} \subseteq \tilde{\tau} \Rightarrow \tilde{\bigcup}_{i\in I} F_{A_i} \in \tilde{\tau}$.
(3) $\{F_{A_i} \tilde{\subseteq} F_A : 1 \leq i \leq n, n \in N\} \subseteq \tilde{\tau} \Rightarrow \tilde{\bigcap}_{i=1}^{n} F_{A_i} \in \tilde{\tau}$.

The pair $(F_A, \tilde{\tau})$ is called a soft topological space.

Example 3. Let us consider the soft subsets of F_A that are given in Example 2. Then, $\tilde{\tau}_1 = \{F_\Phi, F_A\}$, $\tilde{\tau}_2 = P(\tilde{F}_A)$, and $\tilde{\tau}_3 = \{F_\Phi, F_A, F_{A_2}, F_{A_{11}}, F_{A_{13}}\}$ are soft topologies on F_A.

Definition 9. Let $(F_A, \tilde{\tau})$ be a soft topological space. Then, every element of $\tilde{\tau}$ is called a soft open set. Clearly, F_Φ and F_A are soft open sets.

Definition 10. Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B \tilde{\subseteq} F_A$. Then, F_B is said to be soft closed if the soft set $F_B^{\tilde{c}}$ is soft open.

Theorem 1. *Let $(F_A, \tilde{\tau})$ be a soft topological space. Then, the following hold:*

(1) The universal soft set $F_{\tilde{E}}$ and $F_A^{\tilde{c}}$ are soft closed sets.
(2) Arbitrary soft intersections of the soft closed sets are soft closed sets.
(3) Finite soft unions of the soft closed sets are soft closed sets.

Definition 11. Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B \tilde{\subseteq} F_A$. Then, the soft interior of a soft set F_B is denoted by F_B° and is defined as the soft union of all soft open subsets of F_B. Thus, F_B° is the largest soft open set contained in F_B.

Theorem 2. *Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B, F_C \tilde{\subseteq} F_A$. Then,*

(1) $F_\Phi^\circ = F_\Phi$.
(2) $F_B^\circ \tilde{\subset} F_B$.
(3) $(F_B^\circ)^\circ = F_B^\circ$.
(4) F_B is a soft open set if and only if $F_B^\circ = F_B$.
(5) $F_B \tilde{\subseteq} F_C$ implies $F_B^\circ \tilde{\subseteq} F_C^\circ$.
(6) $F_B^\circ \tilde{\cap} F_C^\circ = (F_B \tilde{\cap} F_C)^\circ$.
(7) $F_B^\circ \tilde{\cup} F_C^\circ \tilde{\subseteq} (F_B \tilde{\cup} F_C)^\circ$.

Definition 12. Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B \tilde{\subseteq} F_A$. Then, the soft closure of F_B, denoted $\overline{F_B}$, is defined as the soft intersection of all soft closed supersets of F_B. Note that $\overline{F_B}$ is the smallest soft closed set containing F_B.

Theorem 3. *Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B \tilde{\subseteq} F_A$. F_B is a closed soft set if and only if $\overline{F_B} = F_B$.*

Theorem 4. *Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B, F_C \tilde{\subseteq} F_A$. Then,*

(1) $\overline{\overline{F_B}} = \overline{F_B}$.
(2) $F_C \tilde{\subseteq} F_B$ implies $\overline{F_C} \tilde{\subseteq} \overline{F_B}$.
(3) $\overline{F_B \tilde{\cup} F_C} = \overline{F_B} \tilde{\cup} \overline{F_C}$.
(4) $\overline{F_B \tilde{\cap} F_C} \tilde{\supseteq} \overline{F_B} \cap \overline{F_C}$.

Definition 13. Let $(F_A, \tilde{\tau})$ be a soft topological and $F_B \tilde{\subseteq} F_A$. Then, the soft boundary of soft set F_B is denoted by $F_B^{\tilde{b}}$ and is defined as $F_B^{\tilde{b}} = \overline{F_B} \cap \overline{(F_B^{\tilde{c}})}$.

Definition 14. Let $(F_A, \tilde{\tau})$ be a soft topological space, $F_B \tilde{\subseteq} F_A$, and $\alpha \in F_A$. If every soft nbd of α soft intersects F_B in some points other than α itself, then α is called a soft limit point of F_B. The set of all soft limit points of F_B is denoted by $F_B^{'}$. In other words, if $(F_A, \tilde{\tau})$ is a soft topological space, $F_B, F_C \tilde{\subseteq} F_A$, and $\alpha \in F_A$, then $\alpha \in F_B^{'}$ if and only if $F_C \tilde{\cap} (F_B \tilde{\setminus} \{\alpha\}) \neq F_\Phi$, for all $F_C \in \tilde{\nu}(\alpha)$.

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Definition 15. Let $(F_A, \tilde{\tau})$ be a soft topological space and $\alpha \in F_A$. If there is a soft open set F_B such that $\alpha \in F_B$, then F_B is called a soft open neighborhood (soft nbd) of α. The set of all soft nbds of α, denoted $\tilde{\nu}(\alpha)$, is called the family of soft nbds of α, that is, $\tilde{\nu}(\alpha) = \{F_B : F_B \in \tilde{\tau} : \alpha \in F_B\}$.

Example 4. Let us consider the topological space $(F_A, \tilde{\tau}_3)$ in Example 3 and $\alpha = (x_1, \{u_1, u_2\}) \in F_A$. Then, $\tilde{\nu}(\alpha) = \{F_A, F_{A_{13}}\}$.

The following theorem gives important properties of the soft nbd system:

Proposition 3. *Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B, F_C \tilde{\subseteq} F_A$. Then, the collection of soft nbd $\tilde{\nu}(\alpha)$ at α in $(F_A, \tilde{\tau})$ has the following properties:*

(1) If $F_B \in \tilde{\nu}(\alpha)$, then $\alpha \in F_B$.
(2) If $F_B, F_C \in \tilde{\nu}(\alpha)$, then $F_B \tilde{\cap} F_C \in \tilde{\nu}(\alpha)$.
(3) If $F_B \in \tilde{\nu}(\alpha)$ and $F_B \tilde{\subseteq} F_C$, then $F_C \in \tilde{\nu}(\alpha)$.
(4) If $F_B \in \tilde{\nu}(\alpha)$, then there is an $F_C \in \tilde{\nu}(\alpha)$ such that $F_B \in \tilde{\nu}(\beta)$, for each $\beta \in F_C$.
(5) $F_B \tilde{\subseteq} F_A$ is soft open if and only if F_B contains a soft nbd of each of its points.

Proof. (1) is obvious since F_B is a soft open nbd of $\alpha \in F_A$. So, F_B is a soft open set such that $\alpha \in F_B$.
(2) If $F_B, F_C \in \tilde{\nu}(\alpha)$, then there exist soft open sets F_D and F_G such that $\alpha \in F_D \tilde{\subseteq} F_B$ and $\alpha \in F_G \tilde{\subseteq} F_C$. Therefore, $\alpha \in F_D \tilde{\cap} F_G \tilde{\subseteq} F_B \tilde{\cap} F_C$, and hence, $F_B \tilde{\cap} F_C \in \tilde{\nu}(\alpha)$.

(3) Since $F_B \in \tilde{\nu}(\alpha)$, therefore there exists a soft open set F_D such that $\alpha \in F_D \tilde{\subseteq} F_B$. Then, $\alpha \in F_D \tilde{\subseteq} F_B \tilde{\subseteq} F_C$ or $\alpha \in F_D \tilde{\subseteq} F_C$. Hence, $F_C \in \tilde{\nu}(\alpha)$.

(4) Since $F_B \in \tilde{\nu}(\alpha)$, then $\alpha \in F_C \tilde{\subseteq} F_B$, for F_C soft open in F_A. Since $\alpha \in F_C \tilde{\subseteq} F_C$, then $F_C \in \tilde{\nu}(\alpha)$. If $\beta \in F_C$, then by (3) $F_C \tilde{\subseteq} F_B$ implies that $F_B \in \tilde{\nu}(\beta)$, for each $\beta \in F_C$.

(5) (i) Suppose F_B is a soft open in F_A, then $\alpha \in F_B \tilde{\subseteq} F_B$ implies that F_B is a soft nbd of each $\alpha \in F_B$.

(ii) If each $\alpha \in F_B$ has a soft nbd $F_{C_\alpha} \tilde{\subseteq} F_B$, then $F_B = \{\alpha : \alpha \in F_B\} \tilde{\subseteq} \tilde{\bigcup}_{\alpha \in F_B} F_{C_\alpha} \tilde{\subseteq} F_B$ or $F_B = \tilde{\bigcup}_{\alpha \in F_B} F_{C_\alpha}$. This implies that F_B is soft open in F_A.

□

Definition 16. Let $(F_A, \tilde{\tau})$ be a soft topological space. A soft nbd base at $\alpha \in F_A$ is a subcollection $\tilde{\delta}(\alpha)$ of soft nbd $\tilde{\nu}(\alpha)$ having the property that each $F_B \in \tilde{\nu}(\alpha)$ contains some $F_C \in \tilde{\delta}(\alpha)$, that is, $\tilde{\nu}(\alpha)$ must be determined by $\tilde{\delta}(\alpha)$ as follows:

$$\tilde{\nu}(\alpha) = \{F_B \tilde{\subseteq} F_A : F_C \tilde{\subseteq} F_B, \text{ for some } F_C \in \tilde{\delta}(\alpha)\}.$$

Each $F_C \in \tilde{\delta}(\alpha)$ is called a basic soft open neighborhood of α.

For the soft basic nbd system, we have the following properties:

Proposition 4. *Let $(F_A, \tilde{\tau})$ be a soft topological space and for each $\alpha \in F_A$, let $\tilde{\delta}(\alpha)$ be a soft nbd base at α. Then,*

(1) If $F_C \in \tilde{\delta}(\alpha)$, then $\alpha \in F_C$.
(2) If $F_{C_1}, F_{C_2} \in \tilde{\delta}(\alpha)$, then there is some $F_{C_3} \in \tilde{\delta}(\alpha)$ such that $F_{C_3} \tilde{\subseteq} F_{C_1} \tilde{\cap} F_{C_2}$.
(3) If $F_C \in \tilde{\delta}(\alpha)$, then there is some $F_{C_0} \in \tilde{\delta}(\alpha)$ such that if $\gamma \in F_{C_0}$, then there is some $F_D \in \tilde{\delta}(\gamma)$ with $F_D \tilde{\subseteq} F_C$.
(4) $F_B \tilde{\subseteq} F_A$ is soft open if and only if F_B contains a soft basic nbd of each of its points.

Proof. These properties are easily verified for soft basic nbds by referring to the corresponding properties of soft nbds in Proposition 3. □

Next, we prove the following theorem which relates the concepts of soft interior and soft closure:

Theorem 5. *Let F_B be a soft set of soft topological space $(F_A, \tilde{\tau})$. Then,*

(1) $\overline{(F_B^{\tilde{c}})} = (F_B^{\circ})^{\tilde{c}}$.
(2) $(F_B^{\tilde{c}})^{\circ} = (\overline{F_B})^{\tilde{c}}$.
(3) $F_B^{\circ} = (\overline{(F_B^{\tilde{c}})})^{\tilde{c}}$.
(4) $\overline{F_B} = (((F_B^{\tilde{c}})^{\circ})^{\tilde{c}}$.
(5) $(F_B \tilde{\setminus} F_C)^{\circ} \tilde{\subseteq} F_B^{\circ} \tilde{\setminus} F_C^{\circ}$.

Proof. (1) Let $\alpha \in F_B$ such that $\alpha \notin F_B^{\circ}$. Then, for each soft open nbd F_C of α, F_C soft intersects $F_B^{\tilde{c}}$. Otherwise, for some soft open nbd F_C of α, $F_C \tilde{\cap} F_B^{\tilde{c}} = F_\Phi$ or $F_C \tilde{\subseteq} F_B$. Since F_B° is the largest soft open set in F_B, therefore $\alpha \in F_C \tilde{\subseteq} F_B^{\circ}$, which is a contradiction. Therefore, by Theorem 12(a) [24], $\alpha \in \overline{(F_B^{\tilde{c}})}$. Hence, $(F_B^{\circ})^{\tilde{c}} \tilde{\subseteq} \overline{F_B^{\tilde{c}})}$.

Conversely, suppose $\alpha \in \overline{F_B^{\tilde{c}})}$, then by Definition 17 [24], $\alpha \in F_B^{\tilde{c}}$ or α is a soft limit point of $F_B^{\tilde{c}}$. If $\alpha \in F_B^{\tilde{c}}$, then $\alpha \in (F_B^{\circ})^{\tilde{c}}$. In the second case, $\alpha \notin F_B^{\circ}$. Otherwise, by the definition of soft limit point [24], $F_B^{\circ} \tilde{\cap} F_B^{\tilde{c}} \neq F_\Phi$, which is false. Therefore, $\alpha \in \overline{F_B^{\tilde{c}})}$. This shows that $\overline{F_B^{\tilde{c}})} \tilde{\subseteq} (F_B^{\circ})^{\tilde{c}}$. Combining, we get (1).

(2) is proved in [24].

(3) and (4) are directly obtained by taking the complements of (1) and (2), respectively.

$$\begin{aligned}
(5)\quad (F_B \tilde{\setminus} F_C)^{\circ} &= (F_B \tilde{\cap} F_C^{\tilde{c}})^{\circ} \\
&= F_B^{\circ} \tilde{\cap} (F_C^{\tilde{c}})^{\circ} && \text{(by Theorem 8(3)[24])} \\
&= F_B^{\circ} \tilde{\cap} (\overline{F_C})^{\tilde{c}} && \text{(by Theorem 5(2))} \\
&\tilde{\subseteq} F_B^{\circ} \tilde{\cap} (F_C^{\circ})^{\tilde{c}} \\
&= F_B^{\circ} \tilde{\setminus} F_C^{\circ}.
\end{aligned}$$

□

Now, we define the following:

Definition 17. Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B \tilde{\subseteq} F_A$. Then, the soft exterior of a soft set F_B is denoted by $(F_B)^{\tilde{e}}$ and is defined as $(F_B)^{\tilde{e}} = (F_B^{\tilde{c}})^{\circ}$.

Thus, α is called a soft exterior point of F_B if there exists a soft open set F_C such that $\alpha \in F_C \tilde{\subseteq} F_B^{\tilde{c}}$. We observe that $(F_B)^{\tilde{e}}$ is the largest soft open set contained in $F_B^{\tilde{c}}$.

Example 5. In Example 2, we take $F_B = \{(x_1, U), (x_2, \{u_1, u_2\})\}$. Then, the soft exterior of F_B is F_Φ since $(F_B)^{\tilde{c}} = \{(x_2, \{u_3\})\}$, and thus, $(F_B)^{\tilde{e}} = ((F_B)^{\tilde{c}})^{\circ} = F_\Phi$.

Theorem 6. *Let F_B and F_C be soft sets of a soft topological space $(F_A, \tilde{\tau})$. Then,*

(1) $(F_B)^{\tilde{e}} = (F_B^{\tilde{c}})^{\tilde{e}}$.
(2) $(F_B \tilde{\cup} F_C)^{\tilde{e}} = (F_B)^{\tilde{e}} \tilde{\cap} (F_B)^{\tilde{e}}$.
(3) $(F_B)^{\tilde{e}} \tilde{\cup} (F_C)^{\tilde{e}} \tilde{\subseteq} (F_B \tilde{\cap} F_C)^{\tilde{e}}$.

Proof. (1) The proof follows from the definition.

$$\begin{aligned}
(2)\quad (F_B \tilde{\cup} F_C)^{\tilde{e}} &= ((F_B \tilde{\cup} F_C)^{\tilde{c}})^{\circ} \\
&= (F_B^{\tilde{c}} \tilde{\cap} F_C^{\tilde{c}})^{\circ}
\end{aligned}$$

$$= (F_B^{\tilde{c}})^\circ \tilde{\cap} (F_C^{\tilde{c}})^\circ \qquad \text{(by Theorem 8(3)[24])}$$
$$= (F_B)^{\tilde{e}} \tilde{\cap} (F_C)^{\tilde{e}}.$$

$$\begin{aligned}(3)\ (F_B)^{\tilde{e}} \tilde{\cup} (F_C)^{\tilde{e}} &= (F_B^{\tilde{c}})^\circ \tilde{\cup} (F_C^{\tilde{c}})^\circ \\ &\tilde{\subseteq} \quad (F_B^{\tilde{c}} \tilde{\cup} F_C^{\tilde{c}})^\circ \qquad \text{(by Theorem 8(4)[24])} \\ &= ((F_B \tilde{\cap} F_C)^{\tilde{c}})^\circ \\ &= (F_B \tilde{\cap} F_C)^{\tilde{e}}.\end{aligned}$$

□

Theorem 7. *Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B \tilde{\subseteq} F_A$. Then, the following hold:*

(1) $(F_B^{\tilde{b}})^{\tilde{c}} = F_B^\circ \tilde{\cup} (F_B^{\tilde{c}})^\circ = F_B^\circ \tilde{\cup} (F_B)^{\tilde{e}}$.
(2) $\overline{F_B} = F_B \tilde{\cup} F_B^{\tilde{b}}$.
(3) $F_B^\circ = (F_B) \tilde{\setminus} F_B^{\tilde{b}}$.

Proof. We use Theorem 5(1) and (2) to prove (1):

$$\begin{aligned}(1)\quad F_B^\circ \tilde{\cup} (F_B^{\tilde{c}})^\circ &= ((F_B^\circ)^{\tilde{c}})^{\tilde{c}} \tilde{\cup} (((F_B^{\tilde{c}})^\circ)^{\tilde{c}})^{\tilde{c}} \\ &= [\,(F_B^\circ)^{\tilde{c}} \tilde{\cap} (((F_B)^{\tilde{c}})^\circ)^{\tilde{c}}]^{\tilde{c}} \\ &= [\,\overline{F_B^{\tilde{c}}} \tilde{\cap} \overline{(F_B)}]^{\tilde{c}} \\ &= (F_B^{\tilde{b}})^{\tilde{c}}.\end{aligned}$$

$$\begin{aligned}(2)\quad F_B \tilde{\cup} F_B^{\tilde{b}} &= F_B \tilde{\cup} (\overline{F_B} \tilde{\cap} \overline{F_B^{\tilde{c}}}) \\ &= [\,F_B \tilde{\cup} \overline{F_B}]\,\tilde{\cap} [\,F_B \tilde{\cup} \overline{F_B^{\tilde{c}}}] \\ &= \overline{F_B} \tilde{\cap} [\,F_B \tilde{\cup} (\overline{F_B^{\tilde{c}}}] \\ &= \overline{F_B} \tilde{\cap} F_A \\ &= \overline{F_B}.\end{aligned}$$

$$\begin{aligned}(3)\quad F_B \tilde{\setminus} F_B^{\tilde{b}} &= F_B \tilde{\cap} (F_B^{\tilde{b}})^{\tilde{c}} \\ &= F_B \tilde{\cap} (F_B^\circ \tilde{\cup} (F_B^{\tilde{c}})^\circ) \qquad (by\ \ (1)) \\ &= [\,F_B \tilde{\cap} F_B^\circ]\,\tilde{\cup} [\,(F_B \tilde{\cap} F_B^{\tilde{c}})^\circ] \\ &= F_B^\circ \tilde{\cup} F_\Phi \\ &= F_B^\circ\end{aligned}$$

□

Remark 1. (a) In [24], it is known that $F_B^{\tilde{b}} = (F_B^{\tilde{c}})^{\tilde{b}}$.
(b) From Theorem 7(1), it follows that $F_A = F_B^\circ \tilde{\cup} (F_B)^{\tilde{e}} \tilde{\cup} F_B^{\tilde{b}}$.

Theorem 8. *Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B \tilde{\subseteq} F_A$. Then, the following hold:*

(1) $F_B^{\tilde{b}} \tilde{\cap} F_B^\circ = F_\Phi$.
(2) $F_B^{\tilde{b}} \tilde{\cap} (F_B)^{\tilde{e}} = F_\Phi$.

Proof.

$$\begin{aligned}(1)\quad F_B^\circ \tilde{\cap} F_B^{\tilde{b}} &= F_B^\circ \tilde{\cap} (\overline{F_B} \tilde{\cap} \overline{F_B^{\tilde{c}}}) \\ &= F_B^\circ \tilde{\cap} \overline{F_B} \tilde{\cap} (F_B^\circ)^{\tilde{c}} \qquad \text{(by Theorem 5(1))} \\ &= F_\Phi.\end{aligned}$$

$$\begin{aligned}(2)\quad F_B^{\tilde{b}} \tilde{\cap} (F_B)^{\tilde{e}} &= (F_B^{\tilde{c}})^\circ \tilde{\cap} (\overline{F_B} \tilde{\cap} \overline{F_B^{\tilde{c}}}) \\ &= (F_B^{\tilde{c}})^\circ \tilde{\cap} \overline{F_B} \tilde{\cap} \overline{F_B^{\tilde{c}}} \\ &= (\overline{F_B})^{\tilde{c}} \tilde{\cap} \overline{F_B} \tilde{\cap} \overline{F_B^{\tilde{c}}} \qquad \text{(by Theorem 5(1))} \\ &= F_\Phi.\end{aligned}$$

□

Theorem 9. *Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B \tilde{\subseteq} F_A$. Then,*

(1) *F_B is soft open if and only if $F_B \tilde{\cap} F_B^{\tilde{b}} = F_\Phi$.*
(2) *F_B is soft closed if and only if $F_B^{\tilde{b}} \tilde{\subseteq} F_B$.*

Proof. (1) Let F_B be a soft open set. Then, $F_B^\circ = F_B$. Thus, $F_B \tilde{\cap} F_B^{\tilde{b}} = F_B^\circ \tilde{\cap} F_B^{\tilde{b}} = F_\Phi$ (by Theorem 8(1)). Conversely, let $F_B \tilde{\cap} \overline{F_B} = F_\Phi$. Then, $F_B \tilde{\cap} [\,\overline{F_B} \tilde{\cap} \overline{F_B^{\tilde{c}}}] = F_\Phi$, $F_B \tilde{\cap} \overline{F_B^{\tilde{c}}} = F_\Phi$, or $\overline{F_B^{\tilde{c}}} \tilde{\subseteq} F_B^{\tilde{c}}$, which implies that $F_B^{\tilde{c}}$ is soft closed, and hence, F_B is soft open.
(2) Let F_B be a soft closed set. Then, $\overline{F_B} = F_B$. Now, $F_B^{\tilde{b}} = \overline{F_B} \tilde{\cap} \overline{F_B^{\tilde{c}}} \tilde{\subseteq} \overline{F_B} = F_B$, or $F_B^{\tilde{b}} \tilde{\subseteq} F_B$ and conversely.

□

Theorem 10. *Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B, F_C \tilde{\subseteq} F_A$. Then, the following hold:*

(1) $(F_B \tilde{\cup} F_C)^{\tilde{b}} \tilde{\subseteq} [\,F_B \tilde{\cap} F_C^{\tilde{c}}]^{\tilde{b}}\,\tilde{\cup} [\,F_C^{\tilde{b}} \tilde{\cap} \overline{F_B^{\tilde{c}}}]$.
(2) $[\,F_B \tilde{\cap} F_C]^{\tilde{b}} \tilde{\subseteq} [\,F_B{}^{\tilde{b}} \tilde{\cap} \overline{F_C}]\,\tilde{\cup} [\,F_C{}^{\tilde{b}} \tilde{\cap} \overline{F_B}]$.

Proof.

$$\begin{aligned}(1)\ (F_B \tilde{\cup} F_C)^{\tilde{b}} &= \overline{(F_B \tilde{\cup} F_C)} \tilde{\cap} \overline{(F_B \tilde{\cup} F_C)^{\tilde{c}}} \\ &= (\overline{F_B} \tilde{\cup} \overline{F_C}) \tilde{\cap} \overline{(F_B^{\tilde{c}} \tilde{\cap} F_C^{\tilde{c}})} \qquad \text{(by Theorem 11(5)[24])} \\ &\tilde{\subseteq} (\overline{F_B} \tilde{\cup} \overline{F_C}) \tilde{\cap} (\overline{F_B^{\tilde{c}}} \tilde{\cap} \overline{F_C^{\tilde{c}}}) \qquad \text{(by Theorem 11(4)[24])} \\ &= [\,(\overline{F_B} \tilde{\cap} \overline{(F_B^{\tilde{c}})} \tilde{\cap} \overline{F_C^{\tilde{c}}})]\,\tilde{\cup} [\,(\overline{F_C} \tilde{\cap} \overline{(F_B^{\tilde{c}})} \tilde{\cap} \overline{F_C^{\tilde{c}}})] \\ &= [\,(\overline{F_B} \tilde{\cap} (\overline{F_B^{\tilde{c}}})) \tilde{\cap} (\overline{F_C^{\tilde{c}}})]\,\tilde{\cup} [\,((\overline{F_C}) \tilde{\cap} (\overline{F_C^{\tilde{c}}})) \tilde{\cap} (\overline{F_B^{\tilde{c}}})] \\ &= [\,F_B^{\tilde{b}} \tilde{\cap} (\overline{F_B})]\,\tilde{\cup} [\,F_C^{\tilde{b}} \tilde{\cap} (\overline{F_B^{\tilde{c}}})] \\ &\tilde{\subseteq} F_B^{\tilde{b}} \tilde{\cup} F_C^{\tilde{b}}.\end{aligned}$$

(2)
$$
\begin{aligned}
[F_B\tilde{\cap}F_C]^{\tilde{b}} &= \overline{(F_B\tilde{\cap}F_C)}\tilde{\cap}\overline{(F_B\tilde{\cap}F_C)^{\tilde{c}}}\\
&\tilde{\subseteq} [\,\overline{F_B}\tilde{\cap}\overline{F_C}]\,\tilde{\cap}[\,\overline{F_B^{\tilde{c}}\tilde{\cup}F_C^{\tilde{c}})}] \quad \text{(by Theorem 11(4)[24])}\\
&= [\,\overline{F_B}\tilde{\cap}\overline{F_C}]\,\tilde{\cap}[\,\overline{F_B^{\tilde{c}}}\tilde{\cup}\overline{F_C^{\tilde{c}}}] \quad \text{(by Theorem 11(5)[24])}\\
&= [\,(\overline{F_B}\tilde{\cap}\overline{F_C})\tilde{\cap}\overline{F_B^{\tilde{c}}}]\,\tilde{\cup}[\,(\overline{F_B}\tilde{\cap}\overline{F_C})\tilde{\cap}\overline{F_C^{\tilde{c}}}]\\
&= (F_B^{\tilde{b}}\tilde{\cap}\overline{F_C})\tilde{\cup}(\overline{F_B}\tilde{\cap}F_C^{\tilde{b}}).
\end{aligned}
$$

□

Theorem 11. *Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B\tilde{\subseteq}F_A$. Then, the following holds: $((F_B^{\tilde{b}})^{\tilde{b}})^{\tilde{b}} = (F_B^{\tilde{b}})^{\tilde{b}}$.*

Proof.

$$
\begin{aligned}
((F_B^{\tilde{b}})^{\tilde{b}})^{\tilde{b}} &= \overline{(F_B^{\tilde{b}})^{\tilde{b}}}\tilde{\cap}\overline{((F_B^{\tilde{b}})^{\tilde{b}})^{\tilde{c}})}\\
&= (F_B^{\tilde{b}})^{\tilde{b}}\tilde{\cap}\overline{((F_B^{\tilde{b}})^{\tilde{b}})^{\tilde{c}}} \qquad \dots \quad (1)
\end{aligned}
$$

Now, consider

$$
\begin{aligned}
((F_B^{\tilde{b}})^{\tilde{b}})^{\tilde{c}} &= [\,\overline{(F_B^{\tilde{b}})}\tilde{\cap}\overline{(F_B^{\tilde{b}})^{\tilde{c}}}]^{\tilde{c}}\\
&= [\,F_B^{\tilde{b}}\tilde{\cap}\overline{(F_B^{\tilde{b}})^{\tilde{c}}}]^{\tilde{c}}\\
&= (F_B^{\tilde{b}})^{\tilde{c}}\tilde{\cup}(\overline{(F_B^{\tilde{b}})^{\tilde{c}}})^{\tilde{c}}.
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
\overline{((F_B^{\tilde{b}})^{\tilde{b}})^{\tilde{c}})} &= \overline{[\,(F_B^{\tilde{b}})^{\tilde{c}}\tilde{\cup}((\overline{(F_B^{\tilde{b}})^{\tilde{c}}}))^{\tilde{c}}]}\\
&= \overline{((F_B^{\tilde{b}})^{\tilde{c}})}\tilde{\cup}\overline{(((\overline{(F_B^{\tilde{b}})^{\tilde{c}}}))^{\tilde{c}})} \quad \text{(by Theorem 11(5)[24])}\\
&= (F_C\tilde{\cup}\overline{((F_C)^{\tilde{c}})} = F_A \qquad \dots \quad (2)
\end{aligned}
$$

where $F_C = \overline{(((F_B^{\tilde{b}}))^{\tilde{c}})}$. From (1) and (2), we have $((F_B^{\tilde{b}})^{\tilde{b}})^{\tilde{b}} = (F_B^{\tilde{b}})^{\tilde{b}}\tilde{\cap}F_A = (F_B^{\tilde{b}})^{\tilde{b}}$. □

Theorem 12. *Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B\tilde{\subseteq}F_A$. $F_B^{\tilde{b}} = F_\Phi$ if and only if F_B is a soft clopen set.*

Proof. Suppose that $F_B^{\tilde{b}} = F_\Phi$.

(1) First, we prove that F_B is a soft closed set. Consider

$$
\begin{aligned}
&F_B^{\tilde{b}} = F_\Phi \Rightarrow \overline{F_B}\tilde{\cap}\overline{(F_B^{\tilde{c}})} = F_\Phi\\
&\Rightarrow \overline{F_B}\tilde{\subseteq}(\overline{(F_B^{\tilde{c}})})^{\tilde{c}} = F_B^{\circ}\tilde{\subseteq}F_B \qquad \text{(by Theorem 5(3))}\\
&\Rightarrow \overline{F_B}\tilde{\subseteq}F_B \Rightarrow \overline{F_B} = F_B.
\end{aligned}
$$

This implies that F_B is a soft closed set.

(2) Using Theorem 5(3), we now prove that F_B is a soft open set:

$$
\begin{aligned}
F_B^{\tilde{b}} &= F_\Phi \Rightarrow \overline{F_B}\tilde{\cap}\overline{F_B^{\tilde{c}}} \text{ or } F_B\tilde{\cap}(F_B^{\circ})^{\tilde{c}}\\
&= F_\Phi \Rightarrow F_B\tilde{\subseteq}F_B^{\circ} \Rightarrow F_B^{\circ} = F_B.
\end{aligned}
$$

This implies that F_B is a soft open set.

Conversely, suppose that F_B is a soft clopen set. Then,

$$
\begin{aligned}
F_B^{\tilde{b}} &= \overline{F_B}\tilde{\cap}\overline{(F_B^{\tilde{c}})}\\
&= \overline{F_B}\tilde{\cap}(F_B^{\circ})^{\tilde{c}} \qquad \text{(by Theorem 5(1))}\\
&= F_B\tilde{\cap}F_B^{\tilde{c}} = F_\Phi.
\end{aligned}
$$

□

Conclusions

In the present work, we have defined and studied the important properties of soft exterior. We have established several results relating soft interior, soft exterior, soft closure, and soft boundary. Moreover, we have characterized soft open sets, soft closed sets, and soft clopen sets in terms of soft boundary. We have presented the base of the theory of soft topological spaces. These findings will strengthen the foundation of soft topological spaces and will help to establish a general framework for practical applications.We hope that researchers working on soft topological structures will be benefited.

Competing interests
The authors declare that they have no competing interests.

Author's contributions
Both authors contributed equally and significantly in writing this article. Both authors read and approved the final manuscript.

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Received: 3 September 2012 Accepted: 7 October 2012
Published: 2 November 2012

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doi:10.1186/2251-7456-6-64
Cite this article as: Ahmad and Hussain: On some structures of soft topology. *Mathematical Sciences* 2012 **6**:64.

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