ORIGINAL RESEARCH

Asymptotically λ-invariant statistical equivalent sequences of fuzzy numbers

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Abstract

This paper presents the following definition which is a natural combination of the definitions for asymptotically equivalent λ-statistical convergence and σ-convergence of fuzzy numbers. Two sequences X and Y of fuzzy numbers are said to be asymptotically λ-invariant statistical equivalent of multiple L provided that for every $\varepsilon > 0$,

$$\lim_n \frac{1}{\lambda_n} \left| \left\{ k \in I_n : \overline{d} \left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L \right) \geq \varepsilon \right\} \right| = 0, \text{ uniformly in } m$$

$\left(\text{denoted by } X \overset{S_{\sigma,\lambda}^L(F)}{\sim} Y \right)$ and simply asymptotically λ-invariant statistical equivalent if $L = \overline{1}$.

Keywords: λ-statistical convergence, σ-convergence, Fuzzy numbers

Introduction

Let σ be a one-to-one mapping of the set of positive integers into itself such that $\sigma^k(m) = \sigma\left(\sigma^{k-1}(m)\right)$, $k = 1, 2, 3, \ldots$. The generalized de la Vallee-Pousin mean is defined by

$$t_n(x) = \frac{1}{\lambda_n} \sum_{k \in I_n} x_k,$$

where (λ_n) is a non-decreasing sequence of positive numbers such that $\lambda_{n+1} \leq \lambda_n + 1$, $\lambda_1 = 1$, and $\lambda_n \to \infty$ as $n \to \infty$ and $I_n = [n - \lambda_n + 1, n]$. A sequence $x = (x_k)$ is said to be (V, λ)-summable to a number L if $t_n(x) \to L$ as $n \to \infty$ [1]. (V, λ)-summability reduces to $(C, 1)$-summability when $\lambda_n = n$ for all $n \in N$.

Let D denote the set of all closed and bounded intervals on $\mathbb{R}$, the real line. For $X, Y \in D$, we define

$$d(X, Y) = \max(|a_1 - b_1|, |a_2 - b_2|),$$

where $X = [a_1, a_2]$ and $Y = [b_1, b_2]$. It is known that (D, d) is a complete metric space. A fuzzy real number X is a fuzzy set on $\mathbb{R}$, i.e., a mapping $X : \mathbb{R} \to I(= [0, 1])$ associating each real number t with its grade of membership $X(t)$.

The set of all upper semicontinuous, normal, and convex fuzzy real numbers is denoted by $\mathbb{R}(I)$. Throughout the paper, by a fuzzy real number X, we mean that $X \in \mathbb{R}(I)$.

The α-cut or α-level set $[X]^\alpha$ of the fuzzy real number X, for $0 < \alpha \leq 1$, is defined by $[X]^\alpha = \{t \in \mathbb{R} : X(t) \geq \alpha\}$; for $\alpha = 0$, it is the closure of the strong 0-cut, i.e., closure of the set $\{t \in R : X(t) > 0\}$. The linear structure of $\mathbb{R}(I)$ induces the addition $X + Y$ and the scalar multiplication $\mu X, \mu \in \mathbb{R}$, in terms of α-level sets, defined by

$$[X + Y]^\alpha = [X]^\alpha + [Y]^\alpha, \quad [\mu X]^\alpha = \mu [X]^\alpha$$

for each $\alpha \in (0, 1]$.

Let $\overline{d} : \mathbb{R}(I) \times \mathbb{R}(I) \to \mathbb{R}$ be defined by

$$\overline{d}(X, Y) = \sup_{0 \leq \alpha \leq 1} d\left([X]^\alpha, [Y]^\alpha\right).$$

Then, $\overline{d}$ defines a metric on $\mathbb{R}(I)$. It is well known that $\mathbb{R}(I)$ is complete with respect to $\overline{d}$.

A sequence (X_k) of fuzzy real numbers is said to be convergent to the fuzzy real number X_0 if, for every $\varepsilon > 0$, there exists $n_0 \in N$ such that $\overline{d}(X_k, X_0) < \varepsilon$, for all $k \geq n_0$. Let $c(F)$ denote the set of all convergent sequences of fuzzy numbers.

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A sequence (X_k) of fuzzy real numbers is said to be bounded if the set $\{X_k : k \in N\}$ of fuzzy numbers is bounded. We denote by $\ell_\infty (F)$ the set of all bounded sequences of fuzzy numbers. In [2], it was shown that $c (F)$ and $\ell_\infty (F)$ are complete metric spaces.

A subset E of N is said to have density (asymptotic or natural) $\delta (E)$ if

$$\delta (E) = \lim_{n \to \infty} \frac{1}{n} \sum_{k=1}^{n} \varkappa_E (k) \text{ exists,}$$

where $\varkappa_E$ is the characteristic function of E. The definition of statistical convergence was introduced by Fast [2] and studied by several authors [3-9]. The sequence x is statistically convergent to s if for each $\epsilon > 0$,

$$\lim_{n \to \infty} \frac{1}{n} |\{k \le n : |x_k - s| \ge \epsilon\}| = 0,$$

where $|A|$ denotes the number of elements in A. Schoenberg [10] studied statistical convergence as a summability method and listed some of the elementary properties of statistical convergence.

Nuray and Savaş [11] defined the notion of statistical convergence for sequences of fuzzy real numbers and studied some properties. A fuzzy real number (X_k) is said to be statistically convergent to the fuzzy real number X_0 if for every $\varepsilon > 0$,

$$\delta \left(\left\{ k \in N : \bar{d} (X_k, X_0) \ge \varepsilon \right\} \right) = 0.$$

Fuzzy sequence are spaces studied by several authors such as [12-19].

In 1993, Marouf [20] presented definitions for asymptotically equivalent sequences of real numbers and asymptotic regular matrices. In 2003, Patterson [21] extended these concepts by presenting an asymptotically statistical equivalent analog of these definitions and natural regularity conditions for nonnegative summability matrices. In 2006, Savaş and Başarir [22] introduced and studied the concept of (σ, λ)-asymptotically statistical equivalent sequences. In 2008, Esi and Esi [23] introduced and studied the concept of asymptotically equivalent difference sequences of fuzzy numbers. In 2009, Esi [24] introduced and studied asymptotically equivalent sequences for double sequences. For sequences of fuzzy numbers, Savaş [25,26] introduced and studied the concepts of strongly λ-summable λ-statistical convergence and asymptotically λ-statistical equivalent sequences, respectively. Recently, Braha [27] defined asymptotically generalized difference lacunary sequences. The goal of this paper is to extend the idea on asymptotically equivalent and λ_{σ^F}-statistical convergence of fuzzy numbers.

Methods
Definitions and notations

Definition 1. Two sequences $X = (X_k)$ and $Y = (Y_k)$ of fuzzy numbers are said to be σ^F-asymptotically equivalent if

$$\lim_k \bar{d} \left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, \bar{1} \right)$$
$$= 0 \text{, uniformly in } m \left(\text{denoted by } X \overset{\sigma^F}{\sim} Y \right).$$

Definition 2. A sequence of fuzzy numbers, $X = (X_k)$, is said to be $S_{\sigma,\lambda}^L (F)$-statistically convergent or $S_{\sigma^F}^\lambda$-convergent to the fuzzy number L if for every $\varepsilon > 0$,

$$\lim_n \frac{1}{\lambda_n} \left| \left\{ k \in I_n : \bar{d} \left(X_{\sigma^k(m)}, L \right) \ge \varepsilon \right\} \right| = 0, \text{ uniformly in } m.$$

In this case, we write $S_{\sigma^F}^\lambda - \lim X = L$ or $X_k \to L \left(S_{\sigma^F}^\lambda \right)$.

Following this result, we introduce two new notions asymptotically $S_{\sigma,\lambda}^L (F)$-statistical equivalent of multiple L and strong $V_{\sigma,\lambda}^L (F)$-asymptotically equivalent of multiple L.

The next definition is a natural combination of Definitions 1 and 2.

Definition 3. Two sequences $X = (X_k)$ and $Y = (Y_k)$ of fuzzy numbers are said to be asymptotically λ-invariant statistical equivalent of multiple L provided that for every $\varepsilon > 0$,

$$\lim_n \frac{1}{\lambda_n} \left| \left\{ k \in I_n : \bar{d} \left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L \right) \ge \varepsilon \right\} \right|$$
$$= 0, \text{ uniformly in } m \left(\text{denoted by } X \overset{S_{\sigma,\lambda}^L (F)}{\sim} Y \right)$$

and simply asymptotically $S_{\sigma,\lambda} (F)$-statistical equivalent if $L = \bar{1}$.

Example 1. Let $\lambda_n = n$ and $\sigma (m) = m + 1$ for all $n, m \in \mathbb{N}$. Consider the sequences of fuzzy numbers $X = (X_k)$ and $Y = (Y_k)$ defined by $X_n = \overline{n^{-2}}$ and $Y_n = \overline{n^{-1}}$ for all $n \in \mathbb{N}$. Then,

$$\lim_n \frac{1}{\lambda_n} \left| \left\{ k \in I_n : \bar{d} \left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L \right) \ge \varepsilon \right\} \right|$$
$$= \lim_n \frac{1}{n} \left| \left\{ k \in [1, n] : \bar{d} \left(\overline{n^{-1}}, \bar{0} \right) \ge \varepsilon \right\} \right| = 0.$$

If we take $\lambda_n = n$ for all $n \in \mathbb{N}$, the above definition reduces to following definition:

Definition 4. Two sequences $X = (X_k)$ and $Y = (Y_k)$ of fuzzy numbers are said to be asymptotically invariant statistical equivalent of multiple L provided that for every $\varepsilon > 0$,

$$\lim_n \frac{1}{n} \left| \left\{ k \le n : \bar{d} \left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L \right) \ge \varepsilon \right\} \right|$$

$$= 0, \text{ uniformly in } m \left(\text{denoted by } X \overset{S_\sigma^L(F)}{\sim} Y \right)$$

and simply asymptotically $S_\sigma(F)$-statistical equivalent if $L = \bar{1}$.

Definition 5. Two sequences $X = (X_k)$ and $Y = (Y_k)$ of fuzzy numbers are said to be strong $V_{\sigma,\lambda}^L(F)$-asymptotically equivalent of multiple L provided that

$$\lim_n \frac{1}{\lambda_n} \sum_{k \in I_n} \bar{d} \left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L \right)$$

$$= 0, \text{ uniformly in } m \left(\text{denoted by } X \overset{V_{\sigma,\lambda}^L(F)}{\sim} Y \right)$$

and simply strong $V_{\sigma,\lambda}(F)$-asymptotically statistical equivalent if $L = \bar{1}$.

Example 2. Let $\lambda_n = n$ and $\sigma(m) = m+1$ for all $n, m \in \mathbb{N}$. Consider the sequences of fuzzy numbers $X = (X_k)$ and $Y = (Y_k)$ defined by $X_n = \overline{n^{-2}}$ and $Y_n = \overline{n^{-1}}$ for all $n \in \mathbb{N}$. Then,

$$\lim_n \frac{1}{\lambda_n} \sum_{k \in I_n} \bar{d} \left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, \bar{0} \right) = \lim_n \frac{1}{n} \sum_{k=1}^n \bar{d} \left(\frac{\overline{n^{-2k}}}{\overline{n^{-k}}}, \bar{0} \right)$$

$$= \lim_n \frac{1}{n} \sum_{k=1}^n \frac{1}{n^k} < \infty.$$

If we take $\lambda_n = n$ for all $n \in \mathbb{N}$, the above definition reduces to the following definition:

Definition 6. Two sequences X and Y of fuzzy numbers are said to be strong Cesaro $C_\sigma^L(F)$-asymptotically equivalent of multiple L provided that

$$\lim_n \frac{1}{n} \sum_{k=1}^n \bar{d} \left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L \right)$$

$$= 0, \text{ uniformly in } m \left(\text{denoted by } X \overset{C_{\sigma,\lambda}^L(F)}{\sim} Y \right)$$

and simply strong Cesaro $C_\sigma(F)$-asymptotically equivalent if $L = \bar{1}$.

If we take $\sigma(m) = m + 1$, the above definitions reduce the following definitions:

Definition 7. Two sequences $X = (X_k)$ and $Y = (Y_k)$ of fuzzy numbers are said to be asymptotically almost equivalent if

$$\lim_k \bar{d} \left(\frac{X_{k+m}}{Y_{k+m}}, \bar{1} \right)$$

$$= 0, \text{ uniformly in } m \left(\text{denoted by } X \overset{\widehat{F}}{\sim} Y \right).$$

Definition 8. A sequence of fuzzy numbers $X = (X_k)$ is said to be $\lambda_{\widehat{F}}$-statistically almost convergent or $S_{\widehat{F}}^\lambda$-convergent to the fuzzy number L if for every $\varepsilon > 0$,

$$\lim_n \frac{1}{\lambda_n} \left| \left\{ k \in I_n : \bar{d}(X_{k+m}, L) \ge \varepsilon \right\} \right| = 0 \text{ uniformly in } m.$$

In this case, we write $S_{\widehat{F}}^\lambda - \lim X = L$ or $X_k \to L \left(S_{\widehat{F}}^\lambda \right)$.

Definition 9. Two sequences $X = (X_k)$ and $Y = (Y_k)$ of fuzzy numbers are said to be asymptotically almost λ-statistical equivalent of multiple L provided that for every $\varepsilon > 0$,

$$\lim_n \frac{1}{\lambda_n} \left| \left\{ k \in I_n : \bar{d} \left(\frac{X_{k+m}}{Y_{k+m}}, L \right) \ge \varepsilon \right\} \right|$$

$$= 0, \text{ uniformly in } m \left(\text{denoted by } X \overset{S_\lambda^L(\widehat{F})}{\sim} Y \right)$$

and simply asymptotically *almost* λ-statistical equivalent if $L = \bar{1}$.

If we take $\lambda_n = n$ for all $n \in \mathbb{N}$, the above definition reduces to the following definition:

Definition 10. Two sequences $X = (X_k)$ and $Y = (Y_k)$ of fuzzy numbers are said to be asymptotically *almost* statistical equivalent of multiple L provided that for every $\varepsilon > 0$,

$$\lim_n \frac{1}{n} \left| \left\{ k \le n : \bar{d} \left(\frac{X_{k+m}}{Y_{k+m}}, L \right) \ge \varepsilon \right\} \right|$$

$$= 0, \text{ uniformly in } m \left(\text{denoted by } X \overset{S^L(\widehat{F})}{\sim} Y \right)$$

and simply asymptotically *almost* statistical equivalent if $L = \bar{1}$.

Definition 11. Two sequences $X = (X_k)$ and $Y = (Y_k)$ of fuzzy numbers are said to be strong asymptotically almost λ-equivalent of multiple L provided that

$$\lim_n \frac{1}{\lambda_n} \sum_{k \in I_n} \bar{d} \left(\frac{X_{k+m}}{Y_{k+m}}, L \right)$$

$$= 0, \text{ uniformly in } m \left(\text{denoted by } X \overset{V_\lambda^L(\widehat{F})}{\sim} Y \right)$$

and simply strong asymptotically almost λ-equivalent if $L = \bar{1}$.

If we take $\lambda_n = n$ for all $n \in \mathbb{N}$, the above definition reduces to following definition.

Definition 12. Two sequences $X = (X_k)$ and $Y = (Y_k)$ of fuzzy numbers are said to be strong asymptotically almost equivalent of multiple L provided that

$$\lim_n \frac{1}{n} \sum_{k=1}^{n} \bar{d} \left(\frac{X_{k+m}}{Y_{k+m}}, L \right)$$

$$= 0, \text{ uniformly in } m \left(\text{denoted by } X \overset{C^L(\widehat{F})}{\sim} Y \right)$$

and simply strong asymptotically almost equivalent if $L = \bar{1}$.

Results and discussion

Theorem 1. *Let $X = (X_k)$ and $Y = (Y_k)$ be two fuzzy real valued sequences. Then, the following conditions are satisfied:*

(i) If $X \overset{V^L_{\sigma,\lambda}(F)}{\sim} Y$, then $X \overset{S^L_{\sigma,\lambda}(F)}{\sim} Y$.

(ii) If $X \in \ell_\infty(F)$ and $X \overset{S^L_{\sigma,\lambda}(F)}{\sim} Y$, then $X \overset{V^L_{\sigma,\lambda}(F)}{\sim} Y$; hence, $X \overset{C^L_{\sigma,\lambda}(F)}{\sim} Y$.

(iii) $X \overset{S^L_{\sigma,\lambda}(F)}{\sim} Y \cap \ell_\infty(F) = X \overset{V^L_{\sigma,\lambda}(F)}{\sim} Y \cap \ell_\infty(F)$.

Proof. (i) If $\varepsilon > 0$ and $X \overset{V^L_{\sigma,\lambda}(F)}{\sim} Y$, then

$$\sum_{k \in I_n} \bar{d} \left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L \right) \geq \sum_{\substack{k \in I_n \\ \bar{d}\left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L\right) \geq \varepsilon}} \bar{d} \left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L \right)$$

$$\geq \varepsilon \left| \left\{ k \in I_n : \bar{d} \left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L \right) \geq \varepsilon \right\} \right|.$$

Therefore, $X \overset{S^L_{\sigma,\lambda}(F)}{\sim} Y$.

(ii) Suppose that $X = (X_k)$ and $Y = (Y_k)$ are in $\ell_\infty(F)$ and $X \overset{S^L_{\sigma,\lambda}(F)}{\sim} Y$. Then, we can assume that

$$\bar{d} \left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L \right) \leq T, \text{ for all } k \text{ and } m.$$

Given $\varepsilon > 0$,

$$\frac{1}{\lambda_n} \sum_{k \in I_n} \bar{d} \left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L \right) = \frac{1}{\lambda_n} \sum_{\substack{k \in I_n \\ \bar{d}\left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L\right) \geq \varepsilon}} \bar{d} \left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L \right)$$

$$+ \frac{1}{\lambda_n} \sum_{\substack{k \in I_n \\ \bar{d}\left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L\right) < \varepsilon}} \bar{d} \left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L \right)$$

$$\leq \frac{T}{\lambda_n} \left| \left\{ k \in I_n : \bar{d} \left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L \right) \geq \varepsilon \right\} \right| + \varepsilon.$$

Therefore, $X \overset{V^L_{\sigma,\lambda}(F)}{\sim} Y$. Further, we have

$$\frac{1}{n} \sum_{k=1}^{n} \bar{d} \left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L \right)$$

$$= \frac{1}{n} \sum_{k=1}^{n-\lambda_n} \bar{d} \left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L \right) + \frac{1}{n} \sum_{k \in I_n} \bar{d} \left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L \right)$$

$$\leq \frac{1}{\lambda_n} \sum_{k=1}^{n-\lambda_n} \bar{d} \left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L \right) + \frac{1}{\lambda_n} \sum_{k \in I_n} \bar{d} \left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L \right)$$

$$\leq \frac{2}{\lambda_n} \sum_{k \in I_n} \bar{d} \left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L \right).$$

Hence, $X \overset{C^L_{\sigma,\lambda}(F)}{\sim} Y$ since $X \overset{V^L_{\sigma,\lambda}(F)}{\sim} Y$.

(iii) Follows from (i) and (ii).

$\square$

In the next theorem, we prove the following relation:

Theorem 2. $X \overset{S^L_\sigma(F)}{\sim} Y$ implies $X \overset{S^L_{\sigma,\lambda}(F)}{\sim} Y$ if

$$\lim\inf \left(\frac{\lambda_n}{n} \right) > 0. \tag{1}$$

Proof. For a given $\varepsilon > 0$, we have

$$\left\{ k \leq n : \bar{d} \left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L \right) \geq \varepsilon \right\}$$

$$\supset \left\{ k \in I_n : \bar{d} \left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L \right) \geq \varepsilon \right\}.$$

Therefore,

$$\frac{1}{n} \left| \left\{ k \leq n : \bar{d} \left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L \right) \geq \varepsilon \right\} \right|$$

$$\geq \frac{1}{n} \left| \left\{ k \in I_n : \bar{d} \left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L \right) \geq \varepsilon \right\} \right|$$

$$\geq \frac{\lambda_n}{n} \cdot \frac{1}{\lambda_n} \left| \left\{ k \in I_n : \bar{d} \left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L \right) \geq \varepsilon \right\} \right|.$$

Taking the limit as $n \to \infty$ and using Equation 1, we get the desired result. This completes the proof. □

Conclusions

The concept of asymptotic equivalence was first suggested by Marouf [20] in 1993. After that, several authors introduced and studied some asymptotically equivalent sequences. The results obtained in this study are much more general than those obtained earlier.

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