Abbas *Mathematical Sciences* 2012, **6**:29
http://www.iaumath.com/content/6/1/29

Mathematical Sciences
a SpringerOpen Journal

ORIGINAL RESEARCH **Open Access**

A note on Weyl pseudo almost automorphic functions and their properties

Syed Abbas

Abstract

Purpose: In this paper we discuss the notion of Weyl pseudo almost automorphic functions. These kind of functions are more general than *almost automorphic functions and Stepanov almost automorphic functions*. We establish the composition theorem for this class of functions.

Methods: The Weyl norm has been used to prove the results. In order to prove the composition theorem, the function has been broken into different parts.

Results: The concept of Weyl pseudo almost automorphic functions are defined. The composition theorem has been proved for this kind of functions.

Conclusion: The concept is new and may be useful in the qualitative theory of differential equations.

Keywords: Almost automorphic functions, Weyl almost automorphic functions

AMS-Classification: 42A75, 34K14

Introduction

In $1924-1926$ Bohr [1] introduced the concept of almost periodic functions in his two papers published in *Acta Mathematica*. Since then there have been many important generalizations of this functions. The generalization includes pseudo almost periodic functions [2], where the function can be decomposed into two parts. These functions are further generalized to weighted pseudo almost periodic function by Diagana, where the weighted mean of the second component is 0 [3]. Another direction of generalization is the almost automorphic functions introduced by Bochner [4]. The pseudo almost automorphic functions are the natural generalization of almost automorphic functions [4] suggested by N'Guerekata in his book [5] and developed by Liang et al. [6]. These functions are further generalized by Blot et al. [7] and named them as weighted pseudo almost automorphic. The authors in [7] have proved very important properties of these functions including the composition theorem and the completeness property. Weighted pseudo almost automorphic sequences have been studied by Abbas [8]. The concept of Weyl almost periodic functions has been introduced by H. Weyl [9]. This class of functions is an extension of Stepanov almost periodic functions.

In this work, we consider the Weyl pseudo almost automorphic functions and prove their composition properties. The concept of Stepanov-like pseudo almost periodicity is introduced by Diagana [10,11], which is a generalization of pseudo almost periodicity. Furthermore, Stepanov-like almost automorphy has been introduced by N'Guerekata and Pankov [12]. The concept of Weyl almost automorphy discussed in this work is a generalization of Stepanov-like pseudo almost automorphy. We further generalize this concept to Weyl pseudo almost automorphic. At the best of the authors, these results are new and complement the existing ones.

Methods

We use the Weyl norm as defined by H. Weyl and the concept of Stepanov almost automorphic functions to establish our results. In order to prove the composition theorem, we break the function into different parts and prove their corresponding properties. This concept is new and may be useful in the area of differential equations.

Correspondence: sabbas.iitk@gmail.com
School of Basic Sciences, Indian Institute of Technology Mandi, Mandi, Himachal Pradesh-175001, India

Results and discussion

Let us denote $B(X)$ be the Banach space of all linear and bounded operators on X endowed with the norm $\|\cdot\|_{B(X)}$ and $\mathcal{C} = \mathcal{C}(\mathbb{R}, X)$ the set of all continuous functions from $\mathbb{R}$ to X.

Definition 0.1. A continuous function $f : \mathbb{R} \to X$ is called almost automorphic if for every real sequence $\{s_n\}$, there exists a subsequence $\{s_{n_k}\}$ such that

$$g(t) = \lim_{n\to\infty} f(t + s_{n_k})$$

is well defined for each $t \in \mathbb{R}$ and

$$\lim_{n\to\infty} g(t - s_{n_k}) = f(t)$$

for each $t \in \mathbb{R}$. The set of all almost automorphic functions from $\mathbb{R}$ to X are denoted by $\mathrm{AA}(X)$.

The set of all almost automorphic functions from $\mathbb{R}$ to X are denoted by $\mathrm{AA}(X)$ and it is a Banach space equipped with the supremum norm

$$\|f\|_\infty = \sup_{t\in\mathbb{R}} \|f(t)\|.$$

Definition 0.2. A continuous function $f : \mathbb{R} \times X \to \mathbb{R}$ is called almost automorphic in t uniform for x the compact subsets of X if for every compact subset K of X and every real sequence $\{s_n\}$, there exists a subsequence $\{s_{n_k}\}$ such that

$$g(t, x) = \lim_{n\to\infty} f(t + s_{n_k}, x)$$

is well defined for each $t \in \mathbb{R}$, $x \in K$,and

$$\lim_{n\to\infty} g(t - s_{n_k}, x) = f(t, x)$$

for each $t \in \mathbb{R}$ $x \in K$. The set of all such functions is denoted by $\mathrm{AA}(\mathbb{R} \times X)$.

We denote

$$\mathrm{AA}_0(X) = \left\{f \in BC(\mathbb{R}, X) : \lim_{r\to\infty} \frac{1}{2r}\int_{-r}^{r} \|f(\xi)\|d\xi = 0\right\},$$

and by $\mathrm{AA}_0(\mathbb{R} \times X \times X, X)$ the set of all continuous functions $f : \mathbb{R} \times X \times X \to X$ such that $f(., u, \phi) \in \mathrm{AA}_0(X)$ and

$$\lim_{r\to\infty} \frac{1}{2r}\int_{-r}^{r} \|f(\xi, u, \phi)\|d\xi = 0,$$

uniformly in $(u, \phi) \in X \times X$.

Now we discuss about Weyl almost and Weyl pseudo almost automorphic functions. The Weyl [9] metric for any two functions f and g is defined by

$$d_{W^p}(f, g) = \left\{\lim_{l\to\infty} \sup_{-\infty<x<\infty} \frac{1}{2l}\int_{x-l}^{x+l} |f(t)-g(t)|^p dt\right\}^{\frac{1}{p}}.$$

The Weyl norm of any function is given by

$$\|f\|^p_{W^p} = \lim_{l\to\infty} \sup_{-\infty<x<\infty} \frac{1}{2l}\int_{x-l}^{x+l} |f(t)|^p dt.$$

Defining

$$\|f\|^p_{W^p_l} = \sup_{-\infty<x<\infty} \frac{1}{2l}\int_{x-l}^{x+l} |f(t)|^p dt,$$

we can see that $\lim_{l\to\infty} \|f\|_{W^p_l} = \|f\|_{W^p}$.

The function f are called Weyl almost periodic if it is almost periodic with respect the metric d_{W^p}. The Weyl almost periodic function is an extension of the class of Stepanov almost periodic function. The Stepanov metric is defined by

$$d_{S^p_l}(f, g) = \sup_{-\infty<x<\infty} \left\{\frac{1}{l}\int_{x}^{x+l} |f(t) - g(t)|^p dt\right\}^{\frac{1}{p}}.$$

Hence, a function which is almost periodic with respect to the metric $d_{S^p_l}$ is called Stepanov almost periodic. Thus, $S_{\mathrm{p}}\mathrm{AP}(X) \subset W_{\mathrm{p}}\mathrm{AP}(X)$, where $S_{\mathrm{p}}\mathrm{AP}$ denotes the set of Stepanov almost periodic functions from $\mathbb{R}$ to X and $W_{\mathrm{p}}\mathrm{AP}$ denotes the set of Weyl almost periodic functions from $\mathbb{R}$ to X.

In reference to the function $f \in L^1_{\mathrm{loc}}(\mathbb{R}, E)$, where E is any compact set of $\mathbb{R}$, we associate the translation $f^h \in L^1_{\mathrm{loc}}(\mathbb{R}, E)$ such that $f^h(t) = f(t + h)$, for each $t \in \mathbb{R}$.

We can see in the following proposition that Stepanov and Weyl metrics have some nice properties.

Proposition 0.3. (Andrea, Proposition 2, [13]). *Assume that $l, l_1, l_2 > 0$, $l_1 < l_2$, $h \in \mathbb{R}$ and $f, g \in L^1_{\mathrm{loc}}(\mathbb{R}, E)$. Then*

(i) (Equivalence)

$$D_{S^p_{l_1}}(f, g) \le \frac{l_2}{l_1} D_{S^p_{l_2}}(f, g)$$

and

$$D_{S^p_{l_2}}(f, g) \le (1 + \frac{l_1}{l_2}) D_{S^p_{l_1}}(f, g).$$

(ii) *(Shift invariance)*

$$D_{S_l^p}(f^h, g^h) = D_{S_l^p}(f, g), \quad D_{W_p}(f^h, g^h) = D_{W_p}(f, g).$$

We define the transformation $f^b(t,s), t \in \mathbb{R}, s \in [-l, l]$ of a function $f : \mathbb{R} \to X$ by

$$f^b(t,s) = \frac{f(t+s)}{(2l)^{\frac{1}{p}}},\ p \geq 1,\ l > 0,$$

which is similar to Bochner transform.

Definition 0.4. A function $h : \mathbb{R} \to \mathbb{R}$ is called Weyl almost automorphic in t if its Bochner transform $f^b : \mathbb{R} \to L^p(-l, l; X)$ is almost automorphic in the sense that in every real sequence $\{s_n\}$, there exists a subsequence $\{s_{n_k}\}$ and a function $h^* \in L^p_{\text{loc}}(\mathbb{R}, X)$ such that

$$\lim_{l\to\infty} \frac{1}{2l} \int_{-l}^{l} \|h(t + s_{n_k} + x) - h^*(t + x))\|^p \to 0$$

and

$$\lim_{l\to\infty} \frac{1}{2l} \int_{-l}^{l} \|h^*(t - s_{n_k} + x) - h(t + x))\|^p \to 0$$

for each $t \in \mathbb{R}$. The set of all such functions are denoted by $W_{\text{p}}\text{AA}(\mathbb{R}; X)$.

Definition 0.5. A function $h : \mathbb{R} \times X \to X$ with $f(\cdot, u) \in L^p_{\text{loc}}(\mathbb{R}, X)$ for each $u \in X$ is called Weyl almost automorphic in t uniform for u in compact subsets of X; if the function $t \to f(t, u)$ is Weyl almost automorphic, that is, for every compact subset K of X and for every real sequence $\{s_n\}$, there exists a subsequence $\{s_{n_k}\}$, a function of $h^*(\cdot, u) \in L^p_{\text{loc}}(\mathbb{R}, X)$ such that

$$\lim_{l\to\infty} \frac{1}{2l} \int_{-l}^{l} \|h(t + s_{n_k} + x, u) - h^*(t + x, u))\|^p \to 0$$

and

$$\lim_{l\to\infty} \frac{1}{2l} \int_{-l}^{l} \|h^*(t - s_{n_k} + x, u) - h(t + x, u))\|^p \to 0$$

for each $t \in \mathbb{R}$ and $u \in K$. The set of all such functions was denoted by $W_{\text{p}}\text{AA}(\mathbb{R} \times X; X)$.

We call a function Weyl pseudo almost automorphic if it can be written as $h + \phi$, where h is Weyl almost automorphic, i.e., almost automorphic with respect to the norm given by the metric d_{W^p} and ϕ satisfies

$$\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \left\{ \lim_{l\to\infty} \frac{1}{2l} \int_{x-l}^{x+l} |\phi(t)|^p dt \right\}^{\frac{1}{p}} dx = 0.$$

The set of all such functions is denoted by $W_{\text{p}}\text{PAA}(\mathbb{R}, X)$.

Definition 0.6. *A function $f : \mathbb{R} \to X$ is called Weyl pseudo almost automorphic if it can be written as $h + \phi$, where $h^b \in AA(L^p(-l, l); X)$ and $\phi^b \in AA_0(L^p(-l, l); X)$.*

Definition 0.7. *A function $F : \mathbb{R} \times X \to X$ is called Weyl pseudo almost automorphic if it can be written as $F(t, u) = H(t, u) + \Phi(t, u)$, where $H^b \in AA(\mathbb{R} \times X, L^p(-l, l; X)$ and $\Phi^b \in AA_0(\mathbb{R} \times X, L^p(-l, l; X)$.*

Now we prove the composition theorem for Weyl pseudo almost automorphic functions. By assumption, we can write $f = h + \phi$. Consider $u = v + w$, where v is Weyl almost automorphic and w is a null component. Let $f(t, u(t)) = H(t) + I(t) + J(t)$ and define

$$\begin{aligned} H(t) =& h(t, v(t)), \quad I(t) = f(t, u(t)) - f(t, v(t)), \\ J(t) =& \phi(t, v(t)). \end{aligned}$$

For any sequence t_{n_k}, we have

$$\begin{aligned}
& d_{W^p}(h(\cdot, v(\cdot + t_{n_k}), h^*(\cdot, v^*(\cdot))^p \\
&= \lim_{l\to\infty} \sup_{-\infty<x<\infty} \frac{1}{2l} \int_{x-l}^{x+l} \Big| h(t + t_{n_k}, v(t + t_{n_k})) \\
&\quad - h^*(t, v^*(t))) \Big|^p dt \\
&\leq \lim_{l\to\infty} \sup_{-\infty<x<\infty} \frac{1}{2l} \int_{x-l}^{x+l} \Big(L_h |v(t + t_{n_k}) - v^*(t)| \\
&\quad + |h(t + t_{n_k}, v^*(t) - h^*(t, v^*(t))| \Big)^p dt \\
&\leq \lim_{l\to\infty} \sup_{-\infty<x<\infty} \frac{L_h^p}{2l} \int_{x-l}^{x+l} |v(t + t_{n_k}) - v^*(t)|^p dt \\
&\quad + \lim_{l\to\infty} \sup_{-\infty<x<\infty} \frac{1}{2l} \int_{x-l}^{x+l} |h(t + t_{n_k}, v^*(t)) \\
&\quad - h^*(t, v^*(t))|^p dt \\
&\leq L_h^p \|v(\cdot + t_{n_k}) - v^*(\cdot)\|^p_{W_p} + \|h(\cdot + t_{n_k}, v^*(\cdot)) \\
&\quad - h^*(\cdot, v^*(\cdot))\|^p_{W_p},
\end{aligned} \tag{1}$$

which goes to zero as $k \to \infty$. Thus, $d_{W^p}(h(\cdot, v(\cdot + t_{n_k}), h^*(\cdot, v(\cdot)) \to 0$. Furthermore, consider the following equations

$$\begin{aligned}
&d_{W^p}(h^*(\cdot, v^*(\cdot - t_{n_k}), h(\cdot, v(\cdot)))^p \\
&= \lim_{l\to\infty} \sup_{-\infty<x<\infty} \frac{1}{2l}\int_{x-l}^{x+l} \Big|h^*(s-t_{n_k}, v^*(s-t_{n_k})) \\
&\quad - h(s, v(s))\Big|^p dt \\
&\le \lim_{l\to\infty} \sup_{-\infty<x<\infty} \frac{1}{2l}\int_{x-l}^{x+l} \Big(L_h|v^*(t-t_{n_k}) - v(t)| \\
&\quad + |h^*(t-t_{n_k}, v(t) - h(t, v(t))|\Big)^p dt \\
&\le \lim_{l\to\infty} \sup_{-\infty<x<\infty} \frac{L_h^p}{2l}\int_{x-l}^{x+l} |v^*(t-t_{n_k}) - v(t)|^p dt \\
&\quad + \lim_{l\to\infty} \sup_{-\infty<x<\infty} \frac{1}{2l}\int_{x-l}^{x+l} |h^*(t-t_{n_k}, v(t)) \\
&\quad - h(t, v(t))|^p dt \\
&\le L_h^p \|v^*(\cdot + t_{n_k}) - v(\cdot)\|_{W_p}^p + \|h^*(\cdot + t_{n_k}, v(\cdot)) \\
&\quad - h(\cdot, v(\cdot))\|_{W_p}^p,
\end{aligned} \tag{2}$$

which tends to zero as $k \to \infty$. Thus, $d_{W^p}(h^*(\cdot, v^*(\cdot - t_{n_k}), h(\cdot, v(\cdot)) \to 0$. From the above analysis, we get that the function $H(t) = h(t, v(t))$ is Weyl almost automorphic.

In order to prove that the Weyl norm of the component I is 0, let us calculate the following:

$$\begin{aligned}
\frac{1}{2T}\int_{-T}^{T} \|I\|_{W_p} ds &= \frac{1}{2T}\int_{-T}^{T} \left\{\lim_{l\to\infty}\frac{1}{2l}\int_{x-l}^{x+l} |I(t)|^p dt\right\}^{\frac{1}{p}} dx \\
&= \frac{1}{2T}\int_{-T}^{T} \left\{\lim_{l\to\infty}\frac{1}{2l}\int_{x-l}^{x+l} |I(t)|^p dt\right\}^{\frac{1}{p}} dx \\
&= \frac{1}{2T}\int_{-T}^{T} \left\{\lim_{l\to\infty}\frac{1}{2l}\int_{0}^{2l} |I(x-l+t)|^p dt\right\}^{\frac{1}{p}} dx \\
&\le \frac{1}{2T}\int_{-T}^{T} \Big\{\lim_{l\to\infty}\frac{1}{2l}\int_{0}^{2l} L_f^p |u(x-l+t) \\
&\quad - v(x-l+t)|^p dt\Big\}^{\frac{1}{p}} dx \\
&\le \frac{L_f}{2T}\int_{-T}^{T} \left\{\lim_{l\to\infty}\frac{1}{2l}\int_{0}^{2l} |w(x-l+t)|^p dt\right\}^{\frac{1}{p}} dx \\
&\le \frac{L_f}{2T}\int_{-T}^{T} \|w\|_{W_p} dx.
\end{aligned} \tag{3}$$

Thus, by assumption we get

$$\lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \|I\|_{W_p} ds = 0.$$

Furthermore, as

$$\begin{aligned}
\frac{1}{2T}\int_{-T}^{T} \|J\|_{W_p} ds &= \frac{1}{2T}\int_{-T}^{T} \left\{\lim_{l\to\infty}\frac{1}{2l}\int_{x-l}^{x+l} |J(t)|^p dt\right\}^{\frac{1}{p}} dx \\
&= \frac{1}{2T}\int_{-T}^{T} \Big\{\lim_{l\to\infty}\frac{1}{2l}\int_{0}^{2l} |\phi(x-l \\
&\quad + t, v(x-l+t))|^p dt\Big\}^{\frac{1}{p}} dx \\
&\le \frac{1}{2T}\int_{-T}^{T} \Big\{\lim_{l\to\infty}\frac{1}{2l}\int_{0}^{2l} \sup_{v\in K} |\phi(x-l \\
&\quad + t, v))|^p dt\Big\}^{\frac{1}{p}} dx,
\end{aligned} \tag{4}$$

which gives

$$\lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \|J\|_{W_p} ds = 0.$$

Hence, we have proven the composition theorem for Weyl pseudo almost automorphic functions, i.e., $f(\cdot, u(\cdot))$ is Weyl pseudo almost automorphic, which is an important result. This result tells us that we can decompose the function in two parts, and this could be useful in proving certain results related to the properties of the solution of differential equations. Now we summarize our result in the following theorem.

Theorem 0.8. *Assume that $p > 1$ and the following conditions:*
(i) $f = h + \phi$ is Weyl pseudo almost automorphic, h is Weyl almost automorphic, and ϕ is the ergodic component in Weyl norm.
(ii) $u = v + w$ is Weyl pseudo almost automorphic, v is Weyl almost automorphic, and w is the ergodic component in Weyl norm.
Then $f(\cdot, u(\cdot))$ is Weyl pseudo almost automorphic, i.e., $f \in W_pAA(\mathbb{R} \times X; X)$.

The above result can be easily generalized for the function of three variables, i.e., for the function $f(t, u(t), v(t))$.

Example. Consider the following function,

$$f(t) = 0,\ t \le 0, \quad f(t) = 1,\ t > 0.$$

It is easy to see that $f \in L^1_{\text{loc}}(\mathbb{R}; \mathbb{R})$. Moreover, for $\tau > 0$, we can observe that $d_{S_l^p}(f^\tau, f) = \frac{1}{l} \times l = 1$. Thus, the relative density of the set $\{\tau : d_{S_l^p}(f^\tau, f) < \epsilon\}$ for some l requires arbitrary large values of τ^s in this set. If we demand that l is a large constant for all τ, then for most of them, we get $\tau > l$. Now $d_{S_l^p}(f^\tau, f) < \epsilon$ is impossible and thus, f is not in S_pAP. On the other hand $d_{W^p}(f^\tau, f) = \lim_{l\to\infty} \frac{1}{l} \times \tau = 0$ and hence, it is Weyl almost periodic and Weyl almost automorphic. □

Now consider the following function

$$\phi(t) = \begin{cases} e^{-kt} & \text{if } t \geq 0, \\ e^{kt} & \text{if } t < 0, \end{cases}$$

where $k > 0$. Then

$$\begin{aligned} \int_{x-l}^{x+l} |\phi(t)|^p dt &= \int_{-l}^{l} |\phi(t+x)|^p dt \\ &= \int_{-l}^{0} e^{p(t+x)} dt + \int_{0}^{l} e^{-p(t+x)} dt \\ &= e^{px} \frac{e^{-lp} - 1}{p} + e^{-px} \frac{1 - e^{-lp}}{p}. \end{aligned} \quad (5)$$

By taking the limit $l \to \infty$, we get

$$\frac{e^{-lp} - 1}{2lp} = 0, \quad \frac{1 - e^{-lp}}{2lp} = 0.$$

Thus,

$$\left\{ \lim_{l\to\infty} \frac{1}{2l} \int_{x-l}^{x+l} |\phi(t)|^p dt \right\}^{\frac{1}{p}} = 0$$

and hence,

$$\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \left\{ \lim_{l\to\infty} \frac{1}{2l} \int_{x-l}^{x+l} |\phi(t)|^p dt \right\}^{\frac{1}{p}} dx = 0,$$

which implies that $\phi \in W_p AA(\mathbb{R}; \mathbb{R})$ and hence, the function $f + \phi$ is Weyl pseudo almost automorphic.

Conclusions

In this work, we define a new class of functions called 'Weyl pseudo almost automorphic.' The Weyl norm and Weyl almost periodic functions were defined by Weyl [9]. Moreover, we establish the composition theorem for this class of functions.

Competing interest

The author declares that he has no competing interest.

Author's information

SA received his M.Sc. and Ph.D. in Mathematics from the Indian Institute of Technology Kanpur, in Kanpur, U.P., India. He is currently working as an assistant professor at the School of Basic Sciences, Indian Institute of Technology Mandi in Mandi, H.P., India. His main areas of research are functional/delay/abstract differential equations, ecological modeling, and fractional differential equations.

Acknowledgements

The author would like to thank the anonymous referees for their suggestions and comments, which helped in improving the manuscript.

Received: 11 May 2012 Accepted: 12 August 2012
Published: 30 August 2012

References

1. Bohr, H: Zur Theorie der Fastperiodischen Funktionen I , Acta. Math. **45**, 29–127 (1925)
2. Zhang, CY: Pseudo almost periodic solutions of some differential equations. J. Math. Anal. Appl. **181**(1), 62–76 (1994)
3. Diagana, T: Weighted pseudo almost periodic functions and applications. Compt. Rendus. Math. **343**(103), 643–646 (2006)
4. Bochner, S, Neumann, von, J: On compact solutions of operational-differential equations. I. Annals of Mathematics. **36**(1), 255–291 (1935)
5. N'Guérékata, GM: Topics in Almost Automorphy. Springer, New York (2005)
6. Liang, J, Zhang, J, Xiao, TJ: Composition of pseudo almost automorphic and asymptotically almost automorphic functions. J. Math. Anal. Appl. **340**(2), 1493–1499 (2008)
7. Blot, J, Mophu, GM, N'Guérékata, GM, Pennequin, D: Weighted pseudo almost automorphic functions and applications to abstract differential equations. Nonlinear Anal . TMA. **71**, 903–909 (2009)
8. Abbas, S: Weighted pseudo almost automorphic sequences and their applications. Electron J. Diff. Equ. **2010**(121), 1–14 (2010)
9. Weyl, H: Integralgleichungen und fastperiodische Funktionen. Math Ann. **97**, 338–356 (1927)
10. Diagana, T: Stepanov-like pseudo almost periodic functions and their applications to differential equations. Comm. Math. Anal. **3**(1), 9–18 (2007)
11. Diagana, T: Stepanov-like pseudo-almost periodicity and its applications to some nonautonomous differential equations. Nonlinear Anal. TMA. **69**(12), 4227–4285 (2008)
12. N'Guérékata, GM, Pankov, A: Stepanov-like almost automorphic functions and monotone evolution equations. Nonlinear Analysis TMA. **68**(9), 2658–2667 (2008)
13. Andres, J, Bersani, AM, Lesniak, K: On some almost periodicity problems in various metrics. Acta Applicandae Math. **65**(1-3), 35–57 (2001)

doi:10.1186/2251-7456-6-29
Cite this article as: Abbas: **A note on Weyl pseudo almost automorphic functions and their properties.** *Mathematical Sciences* 2012 **6**:29.