**International Journal of
Advanced Structural Engineering**
a SpringerOpen Journal

ORIGINAL RESEARCH

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Shear response and design of RC beams strengthened using CFRP laminates

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Abstract

The present investigation addresses the shear strengthening of deficient reinforced concrete (RC) beams using carbon fiber-reinforced polymer (CFRP) sheets. The effect of the pattern and orientation of the strengthening fabric on the shear capacity of the strengthened beams were examined. Three beams with various lay-ups of strengthening fabric, 45°, 0°/90°, and 0°/90°/45° were examined, in addition to an unstrengthened control beam. Principal and shear strains were measured at different locations at the critical sections of the strengthened beams corresponding to each applied shear force. Experimental results showing the advantage of beam strengthened using the various lay-ups of CFRP sheets are discussed. It is concluded that Beam-45°, Beam-0°/90°, and Beam-0°/90°/45° show about 25%, 19%, and 40% increases in shear-load carrying capacity in comparison to the control beam, respectively. Also, there exists a critical value of shear force up to which there is no appreciable shear strain in the CFRP sheets/beam. This shear force marks the ultimate shear resistance of the control beam. However, the strengthened beams exhibited significant strength and stiffness even beyond the critical value of the shear force. A design example for shear strengthening shows that the design equations available in the literature underestimate the actual shear strength of the beams.

Keywords: CFRP; Reinforced concrete beams; Shear strengthening

Introduction

There is an increasing demand for damaged reinforced concrete structures to be rehabilitated without increasing their self-weight. These structures may need to be strengthened because their life expectancy has been met, but they also may not be compatible with upgraded design standards, they may have mistakes in their design or construction, or being exposed to unpredicted loads such as truck hits or powerful earthquakes. Any change in the usage or exposure to a load that exceeds the initial design specifications for a structure may cause damage. There are many references in the literature for the strengthening of beams using fiber-reinforced polymer (FRP); however, there are few which deal with the shear strengthening of reinforced concrete beams using fibrous composite materials.

Although steel plates/strips are being used for strengthening structural elements including columns and piers for highway bridges, steel plates cannot be considered a future solution for shear strengthening because of corrosion

problems, especially in outdoor applications. However, carbon fiber-reinforced polymer (CFRP) materials are being used to rehabilitate deficient reinforced concrete structures (ACI Committee 440 1996) because of their higher resistance to electro-chemical corrosion, higher strength-to-weight ratio, ease of handling, high fatigue resistance, availability in various lengths and shapes, and their direction-dependent strength.

Berset (1992) first tested reinforced concrete beams with and without shear strengthening reinforcements. These reinforcements were in the form of glass fiber-reinforced polymer (GFRP) laminates bonded to the vertical sides of the beam in shear critical zones. He developed a simple analytical model to determine the contribution of the external reinforcement to the shear capacity of the beam. Uji (1992) tested reinforced concrete beams strengthened in shear with CFRP fabrics/laminates and developed a model for the FRP contribution to the shear capacity based on arbitrarily defined FRP-concrete bonding interfaces. Dolan et al. (1992) tested prestressed concrete beams strengthened with aramid fabric reinforcement and

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concluded that aramid fiber-reinforced polymer fabrics performed quite well as shear retrofit reinforcements.

Al-Sulaimani et al. (1994) applied shear strengthening using GFRP plates/strips and developed a model for the contribution of the GFRP composites to the beam's shear capacity. Their model is based on the assumption that FRP-concrete interfaces at plates and strips are capable of carrying shear stresses averaging 0.8 and 1.2 MPa, respectively, before debonding. Ohuchi et al. (1994) developed a model for the contribution of CFRP sheets to the shear capacity of the beam, using an analogy with steel stirrups. They assumed a limiting strain for the external reinforcement equal to either the tensile failure strain of the CFRP or 2/3 of it, depending on the thickness of the fabric. Chajes et al. (1995) used various types of fibers (glass, aramid, and carbon) to develop a model for the FRP contribution to the shear capacity, analogous to that used for steel stirrups, based on a limiting FRP strain equal to 0.005.

Malavar et al. (1995) tested reinforced concrete beams strengthened in shear with CFRP fabric and determined the contribution of the fabric to the overall shear capacity of the beam, analogous to the case for steel stirrups, with the limiting FRP strain equal to the strain at tensile fracture of the material. Vielhaber and Limberger (1995) conducted tests on large-scale reinforced concrete beams strengthened in shear with CFRP fabrics and stated that external shear strengthening provided considerable safety against brittle shear failure. Sato et al. (1996) conducted tests on concrete beams strengthened in shear using CFRP strips and continuous fabrics, and expressed the failure mode (debonding of external reinforcements) through a simple model which accounted for partial shear transfer by the debonded CFRP.

Alfarabi et al. (1994a) recommended the use of I-jackets to prevent diagonal tension failure and to minimize the decrease in ductility of the beam. In another investigation, Alfarabi et al. (1994b) examined the feasibility of utilizing GFRP plates for shear strengthening, using configurations such as plates, wings, and I-jacket patterns, and observed that only I-jacket shape could change the mode of failure to flexural failure. Malek et al. (1998) and (He 1997) examined local shear failure in concrete beams strengthened with CFRP plates. They observed that local shear failure in concrete, which occurs between the FRP plate and the steel reinforcement, is attributed to shear and normal stress concentrations at the plate ends and around the flexural cracks.

Takahashi et al. (1997) examined a U-Jacket pattern of CFRP sheets and observed that this pattern of strengthening is capable of controlling peeling, changing the mode of failure from peeling to rupture of the sheet. Triantafillou (1998) developed an analytical model in association with experimental results for the shear capacity of FRP laminates, analogous to that used for steel stirrups, using an effective FRP strain that decreases with increasing FRP axial rigidity.

Grace et al. (1999) studied the behavior of reinforced concrete beams strengthened with CFRP/GFRP materials. They concluded that the combination of vertical and horizontal sheets, together with a proper epoxy, could lead to a doubling of the ultimate shear load carrying capacity of the beam. They also recommended that a higher factor of safety is needed for the design of FRP strengthened beams, in view of the potential for brittle failure of strengthened beams.

Most recently, Singh et al. (2012) have developed CFRP rebars at Structural Testing Center of BITS Pilani and used them as near-surface mounted FRP rebars for shear and flexural strengthening of reinforced concrete (RC) and engineered cementitious composite beams.

Research significance

The mechanical behavior, including the shear strength, of beams strengthened with externally applied CFRP fabric sheets with different fiber orientations is presented in this investigation. The results are useful for providing design guidelines and for determining the most efficient combination of fabric lay-up. Also, this work provides an additional experimental data source, which will be useful in developing suitable analytical models for the evaluation of the shear strengths of externally strengthened RC beams.

Methods

Four precast RC beams designed to fail in shear (due to an inadequate number of stirrups) were delivered to the Structural Testing Center by Baker Concrete Technologies, Inc. (Columbus, OH, USA). Three beams were strengthened in the shear span with uni-directional CFRP sheets, while the fourth beam was left unstrengthened and served as the control beam. The experimental program focused on finding the optimum lay-ups for the sheets used to strengthen the RC beams in shear. Testing was performed with combinations of sheets with fibers oriented in different directions so that the most efficient pattern of shear strengthening could be determined.

Three u-shaped lay-ups of CFRP sheets were used for shear strengthening of the RC beams. The first beam (Beam-45°) had sheets applied with the fibers orientated at 45° to the longitudinal axis of the beam. The second beam (Beam-0°/90°) had two layers of CFRP sheets; the first layer ran parallel to the length of the beam (at 0°), while the second layer was placed perpendicular to the longitudinal axis of the beam (at 90°) resulting in a combined 0°/90° configuration. The third beam (Beam-0°/

90°/45°) had three layers. These sheets were placed parallel (at 0°), perpendicular (at 90°), and diagonal (at ±45°) to the longitudinal axis of the beam resulting in a combined 0°/90°/45° configuration. Note that all specimens were designed and strengthened to accommodate three tests per beam to maximize the use of the beams. All beams were instrumented and loaded to failure.

Beam design and strengthening

All beams had a rectangular cross section with a height of 381 mm (15 in.), a width of 228 mm (9 in.), and a length of 4,877 mm (16 ft). Internal longitudinal (flexural) reinforcement was provided by two bars (#8), each of 25.4 mm (1 in.) in diameter. Two other bars (#3) placed at the top of the beams, each of 9.5 mm (3/8 in.) in diameter, served as hanging bars for the stirrups. The bars were Grade 60 steel, having yield strength of 413.7 MPa (60 ksi). These reinforcing bars (#8) were placed 330.2 mm (13 in.) from the top of the beams, resulting in an effective depth of 342.9 mm (13.5 in.). Shear reinforcement was provided by stirrups formed from 9.5-mm (3/8 in.)-diameter Grade 60 steel bars. The center-to-center spacing between adjacent stirrups was 305 mm (12 in.). Details of the cross section and reinforcement are given in Figure 1a.

Initial strengthening

In order to maximize the use of each test beam, the CFRP sheets were placed along the length of each beam so that three independent tests could be conducted on each strengthened beam. The lay-ups (Figures 1, 2, and 3) for the initial strengthening of all the beams were determined upon initial testing of the control beam and were set to mimic the control beam's testing span. Figure 1 refers to the initial strengthening pattern for Beam-45°, while Figures 2 and 3 refer to the patterns for Beam-0°/90° and Beam-0°/90°/45°, respectively.

Before the application of the CFRP sheets, the surfaces of the beams were prepared. A hand grinder was used to round off the corners of all the beams, and the bottom and longitudinal sides of the beams were sand blasted to create an ideal bonding surface for the CFRP sheets. Three areas in each beam were strengthened. These consisted of a strengthened portion beginning at 228 mm (9 in.) from the left end of the beam and continuing for 1,067 mm (3.5 ft), followed by a 1,067-mm (3.5 ft) unstrengthened portion, another 1,067-mm (3.5 ft) strengthened portion, followed by a 152-mm (6 in.) unstrengthened portion, and finally a 1,067-mm (3.5 ft) strengthened portion with a 228-mm (9 in.) unstrengthened portion at the right end of the beam.

Strengthening of these areas consisted of first preparing a batch of primer mixed with sand and applying grout mixture to the three areas of strengthening. This was done so that any large voids in the concrete exposed during sand blasting would be filled, allowing for proper bonding between the epoxy adhesive and the concrete.
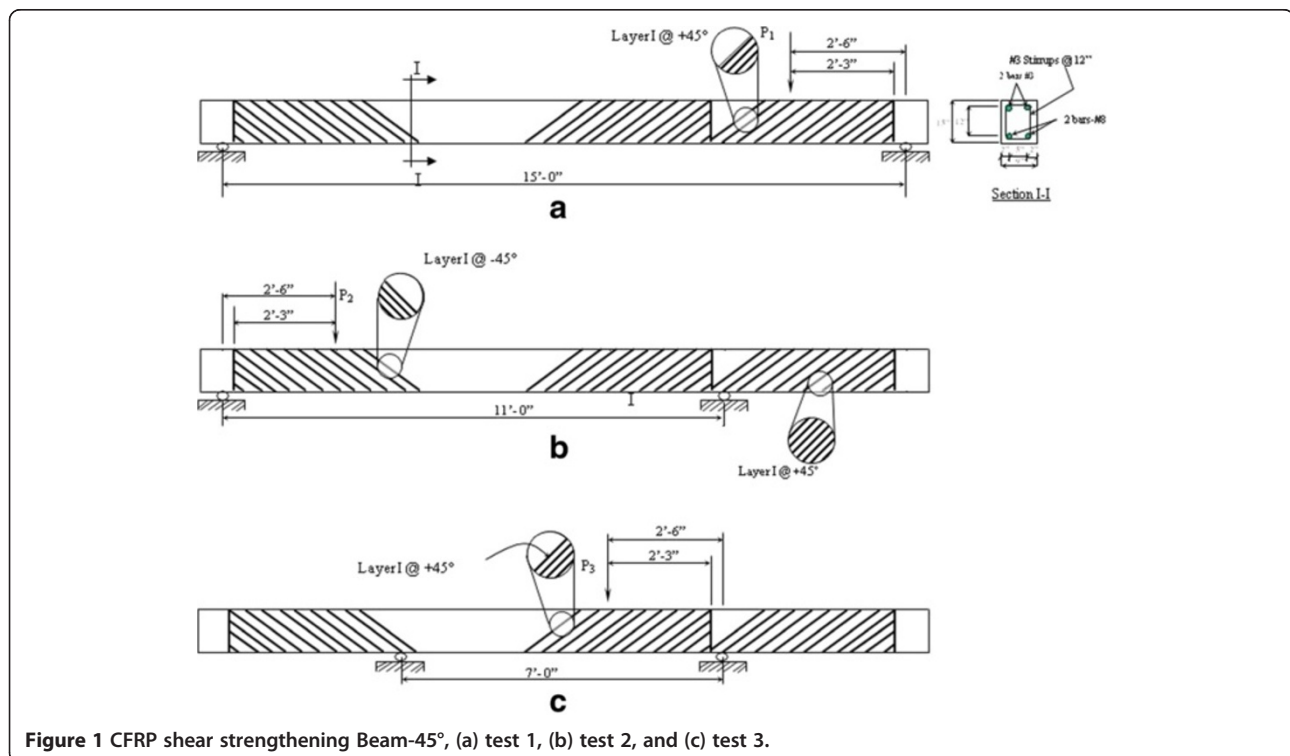

Figure 1 CFRP shear strengthening Beam-45°, (a) test 1, (b) test 2, and (c) test 3.
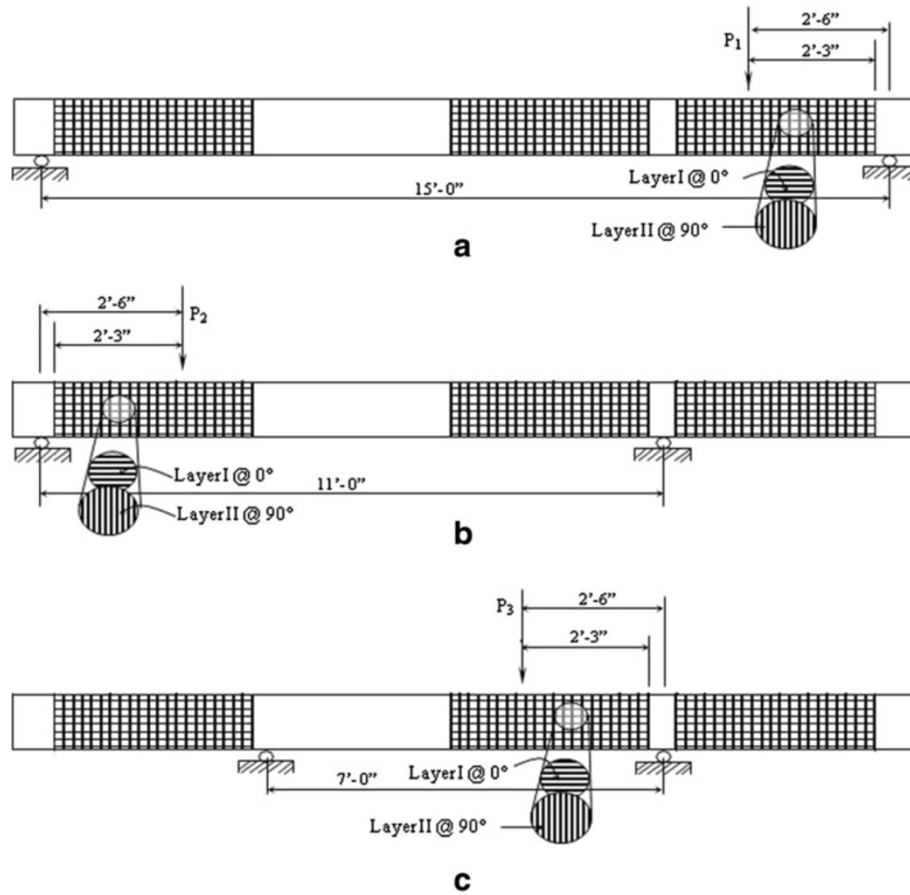
Figure 2 CFRP shear strengthening Beam-0°/90°, (a) test 1, (b) test 2, and (c) test 3.

The primer was allowed to set, and a batch of epoxy adhesive was mixed. A coat of epoxy adhesive was applied to the areas of the beam that were to be strengthened. The CFRP sheets were then impregnated with epoxy adhesive and applied to the prepared surfaces. Impregnation of the sheet was accomplished by placing the CFRP sheet on a clean plastic sheet and pouring a portion of the epoxy onto the CFRP sheet. The epoxy was impregnated into the sheet with a steel grooved roller. This process was repeated to ensure that the CFRP sheet formed a proper bond with the concrete. The strengthened beams were cured for 7 days to allow proper curing of the epoxy adhesive. Typical properties of the CFRP sheets are presented in Table 1. All CFRP sheets had an as received width of 305 mm (12 in.). They were cut (prior to application) into appropriate pieces on site. A typical application of CFRP sheets is shown in Figure 4, (Beam 0°/90°, test 1).

Strengthening of Beam-45°
Figure 1a,b,c shows the portions of Beam-45° strengthened for tests 1, 2, and 3, respectively. For test 1 (Figure 1a), the first piece of CFRP sheet was placed so that the upper left corner was 175.3 mm (6.9 in.) to the left of the load position and at an angle of 45° to the longitudinal axis of the beam. A second piece of CFRP sheet with the same dimensions as the first was placed immediately to the right of the first sheet without overlapping; this was followed by placement of a smaller piece of CFRP sheet to the right of the previously placed sheet. Similarly, fabric pieces of the same dimensions were positioned on the opposite face of the beam to complete the strengthening process. The strengthening patterns for test 2 (Figure 1b) and test 3 (Figure 1c) were similar to that for test 1, with the exception that the fiber orientation of the CFRP sheets for test 2 was opposite to that for test 1 (i.e., −45° for test 2 and 45° for test 1).

Strengthening of Beam-0°/90°
For Beam-0°/90°, each of the three sections to be strengthened had CFRP sheets 1,066.8 mm (42 in.) long positioned parallel (at 0°) to the longitudinal axis of the beam on bottom and sides of the beam. The second layer of sheets was placed perpendicular (at 90°) to the face of the beam on top of the 0° layer. This layer consisted of three and one half sheets, each 990.6 mm
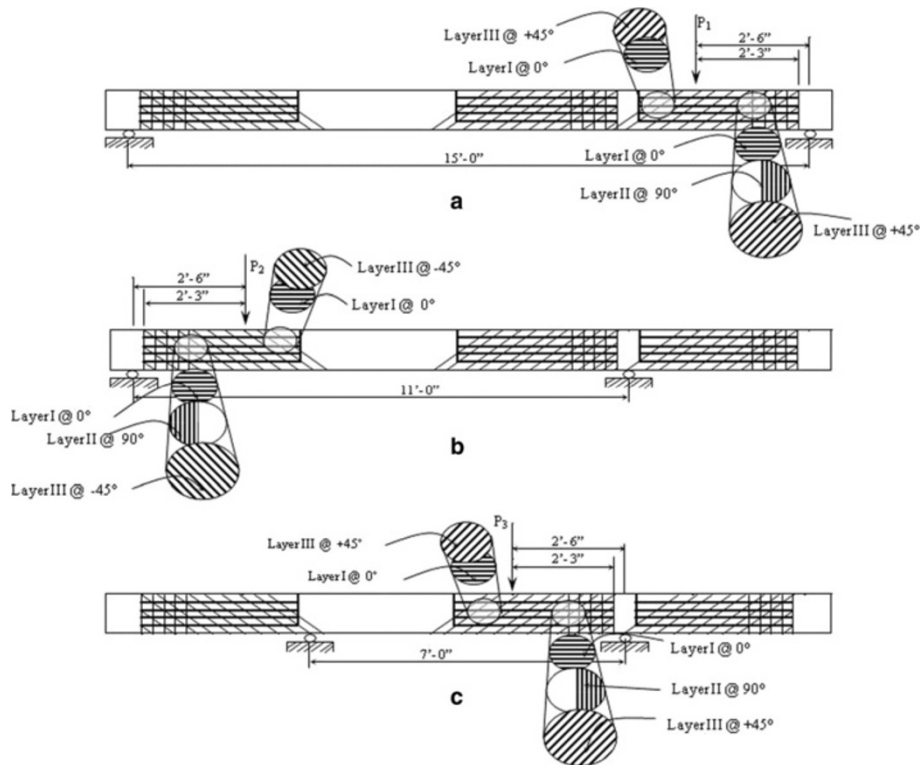
Figure 3 CFRP shear strengthening Beam-0°/90°/45°, (a) test 1, (b) test 2, and (c) test 3.

(39 in.) in length, so that the bottom and both sides of the beam were covered. Note that each sheet was of 305 mm (12 in.) wide, as three and one half sheets covered 1,066.8 mm (42 in.) of the beam's length, as shown in Figure 2a,b,c for tests 1, 2, and 3, respectively.

Strengthening of Beam-0°/90°/45°
In Beam-0°/90°/45°, fibers were placed parallel (at 0°), perpendicular (at 90°), and diagonally (at 45°) to the longitudinal axis of the beam. For test 1 (Figure 3), strengthening consisted of a 1,066.8 mm (42 in.) length of fabric with its edge placed along the top longitudinal corner of the beam. Similarly, a piece of fabric was also positioned on the opposite face of the beam. Next, a single CFRP sheet, 990.6 mm (39 in.) long, was placed perpendicular to the beam face, covering the bottom and sides of the beam. This sheet was positioned so that its upper left corner was on top of the upper left corner of

the sheet parallel to the longitudinal axis of the beam. The position of the diagonal layer was the same as for Beam-45°. The strengthening patterns for all tests of Beam-0°/90°/45° were the same, except that the fiber orientations for tests 1 and 2 were mirror image of each other, as shown in Figure 3a,b.

Supplemental strengthening
After the first test of Beam-45°, inadequacies in the initial strengthening pattern consisting of failure in the unstrengthened zone, as shown in Figure 5a, were found.

Table 1 Typical properties of CFRP fabric (Fiber Reinforced Systems™ 2000)

Width, in. (mm)	12 (304.8)
Thickness, in. (mm)	0.007 (0.178)
Ultimate tensile strength, ksi (MPa)	400 (2758)
Average modulus of elasticity, ksi (GPa)	33,000 (228)
Average ultimate strain (%)	1.8

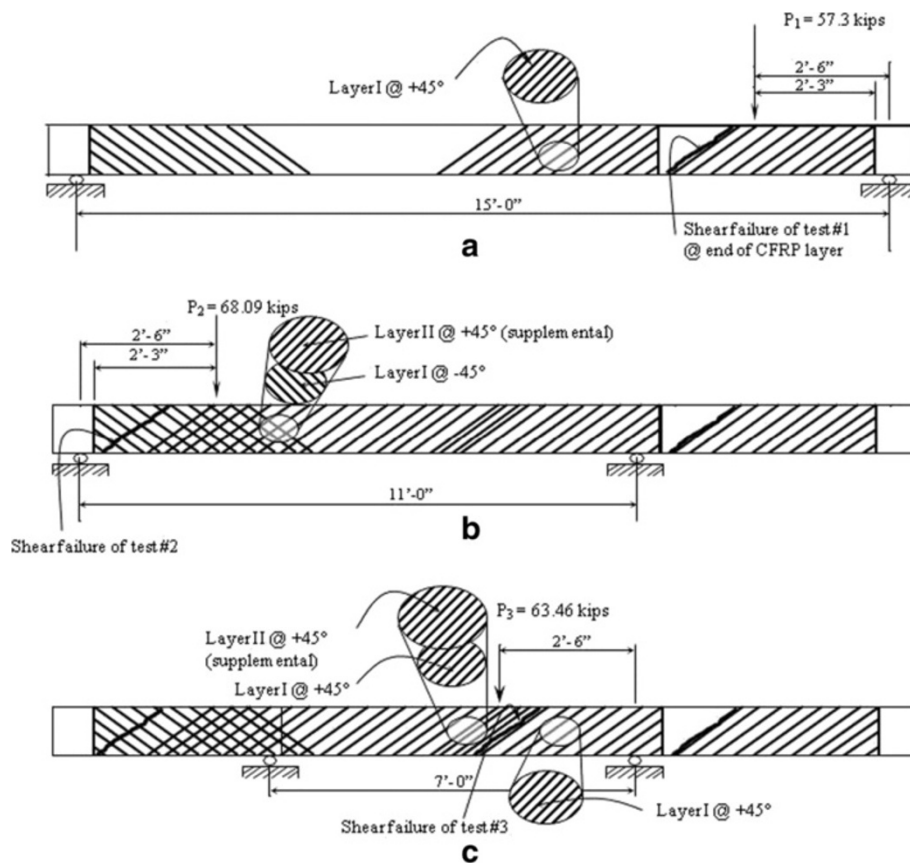**Figure 4** Application of CFRP sheet for Beam-0°/90°, test 1.

Figure 5 Supplemental CFRP shear strengthening for Beam-45°, (a) test 1, (b) test 2, and (c) test 3.

Therefore, supplemental strengthenings for Beam-45°, Beam-0°/90°, and Beam-0°/90°/45° were added and were shown in Figures 5, 6, and 7, respectively. Note that for Beam-45°, test 1 was conducted without supplemental strengthening. However, for Beam-0°/90° and Beam-0°/90°/45°, test 1 was conducted after supplemental strengthening. Tests 2 and 3 were conducted only after supplemental strengthening of all the beams.

Beam-45°

In order to strengthen this beam for test 2, CFRP sheets were placed perpendicular to the sheets initially placed at −45° to the longitudinal axis of the beam, as shown in Figure 5. The first supplemental sheet was placed so that the upper left corner of the fabric was directly under the load, resulting in a 175.3-mm (6.9 in.) overlap with the initial layer. Three more sheets were positioned at the same angle (+45°) to the longitudinal axis of the beam, each butting up against the other until the final applied sheet overlapped the existing CFRP sheet (for test 3) by 71.1 mm (2.8 in.). Similarly, four sheets were applied on the opposite face of the beam.

Beam-0°/90°

Supplemental strengthening for this beam consisted of placing CFRP sheets parallel (at 0°) to the longitudinal axis of the beam in beam's unstrengthened areas, that is, the 152.4-mm (6 in.) section near the right support and the 1,066.8-mm (3.5 ft) section near the left support. Perpendicular (at 90°) layers of fabric were then placed on top of the 0° layers (as shown in Figure 6). There was no overlap of the supplemental CFRP sheets (Figure 6) with the existing CFRP sheets (Figure 2). This resulted in four seams in the final strengthened pattern. These seams should have been avoided, as will be discussed later.

Beam-0°/90°/45°

Supplemental strengthening for this beam consisted of placing CFRP sheets parallel (at 0°) to the longitudinal axis of the beam in areas where there was no initial strengthening (Figure 3). Then, a CFRP sheet was placed perpendicular (at 90°) to the longitudinal axis of the beam, with the left corner positioned 1,447.8 mm (4 ft 9 in.) from the left end of the beam. This sheet covered
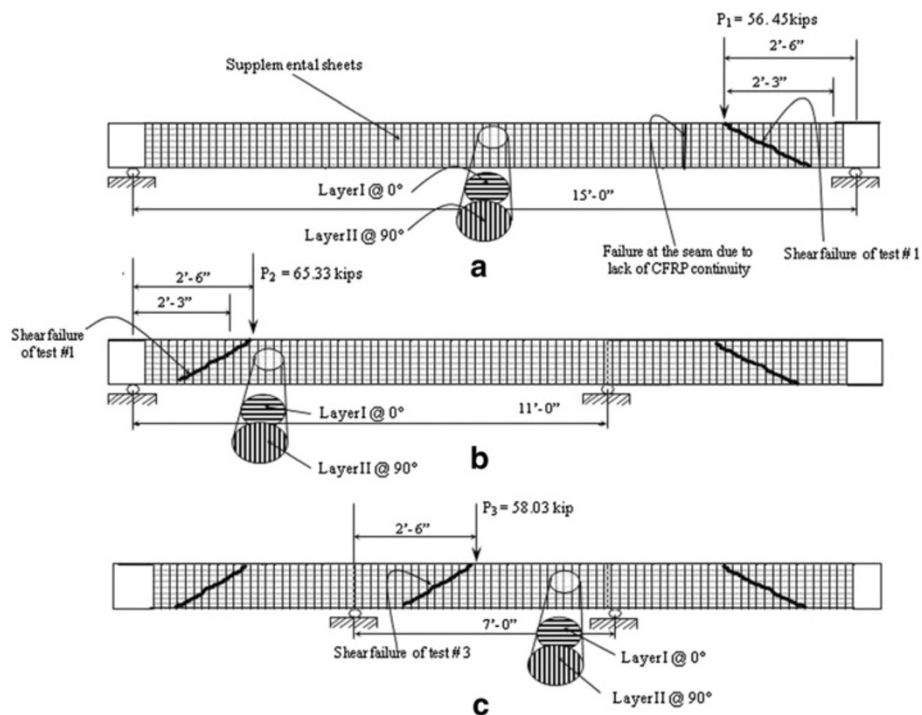
Figure 6 Supplemental CFRP shear strengthening for Beam-0°/90°, (a) test 1, (b) test 2, and (c) test 3.

the bottom and two sides of the beam. Next, seven sheets of CFRP fabric were placed over the existing layers, at 135° (−45°) to the longitudinal axis of the beam. The placement of these sheets started with the upper right corner of the first sheet placed 914.4 mm (3 ft) from the right side of the beam. Subsequent sheets were butted up to the sheets placed earlier. Finally, six additional sheets were applied to the beam at 45° with the longitudinal axis of the beam. Placement of these sheets began with the first sheet being placed at right side of the beam. As shown in Figure 7, the similar strengthening pattern was kept on the opposite face side of the beam also. Diagonal shear failure modes observed in each test are also indicated in this figure.

Instrumentation, loading, and data acquisition

All test beams were instrumented with special sensors such as load cells, linear voltage displacement transducer, and rosettes. Strain measurements were made with electrical resistance strain gage rosettes used in conjunction with demountable mechanical strain gage (DEMEC, Wexham Developments, Wokingham, Berkshire, UK) strain measurement system. A typical rosette arrangement for Beam-0°/90° is shown in Figure 8. A 908-kN (200 kips) compression Whetstone bridge-type load cell was used to monitor the applied load. A linear potentiometer was mounted on the loading actuator to monitor the downward vertical deflection. All beams were loaded in 22.2-kN

(5 kips) increments. At each load increment, readings from the DEMEC strain gages were taken. Visible cracks in the control beam were marked. The data acquisition system recorded data only during the incremental loading.

Results and discussion

Figure 9 shows typical shear force and deflection (under applied load) relationships for the control beam and the strengthened beams (i.e., the 45°, 0°/90°, and 0°/90°/45° beams). For a given deflection between 6.4 mm (0.25 in.) and 25.4 mm (1 in.), the shear-load carrying capacity of Beam-0°/90°/45° is the highest, while for deflection less than 6.4 mm (0.25 in.), the beams all exhibit almost the same shearing resistance. After the ultimate shear load, the strengthened beams exhibit an almost complete loss of load carrying capacity within a very small deflection range (about one half of that of the control beam); this is due to the loss of stiffness caused by side cover delamination and/or rupture of sheet. Note that the ultimate failure of the beam was initiated due to the crushing of concrete leading to the rupture of the CFRP sheets along with pulling off concrete in Beam-45°, rupture of CFRP sheets at the seams in Beam-0°/90°, and rupture of CFRP sheets with side cover delamination in Beam-0°/90°/45°. The failures of strengthened beams occurred all of a sudden after the beams reached their ultimate shear strength. This is indicative of brittle failure in strengthened beams. Although the control beam has a low shear

Figure 7 Supplemental CFRP shear strengthening for Beam-0°/90°/45°, (a) test 1, (b) test 2, and (c) test 3.
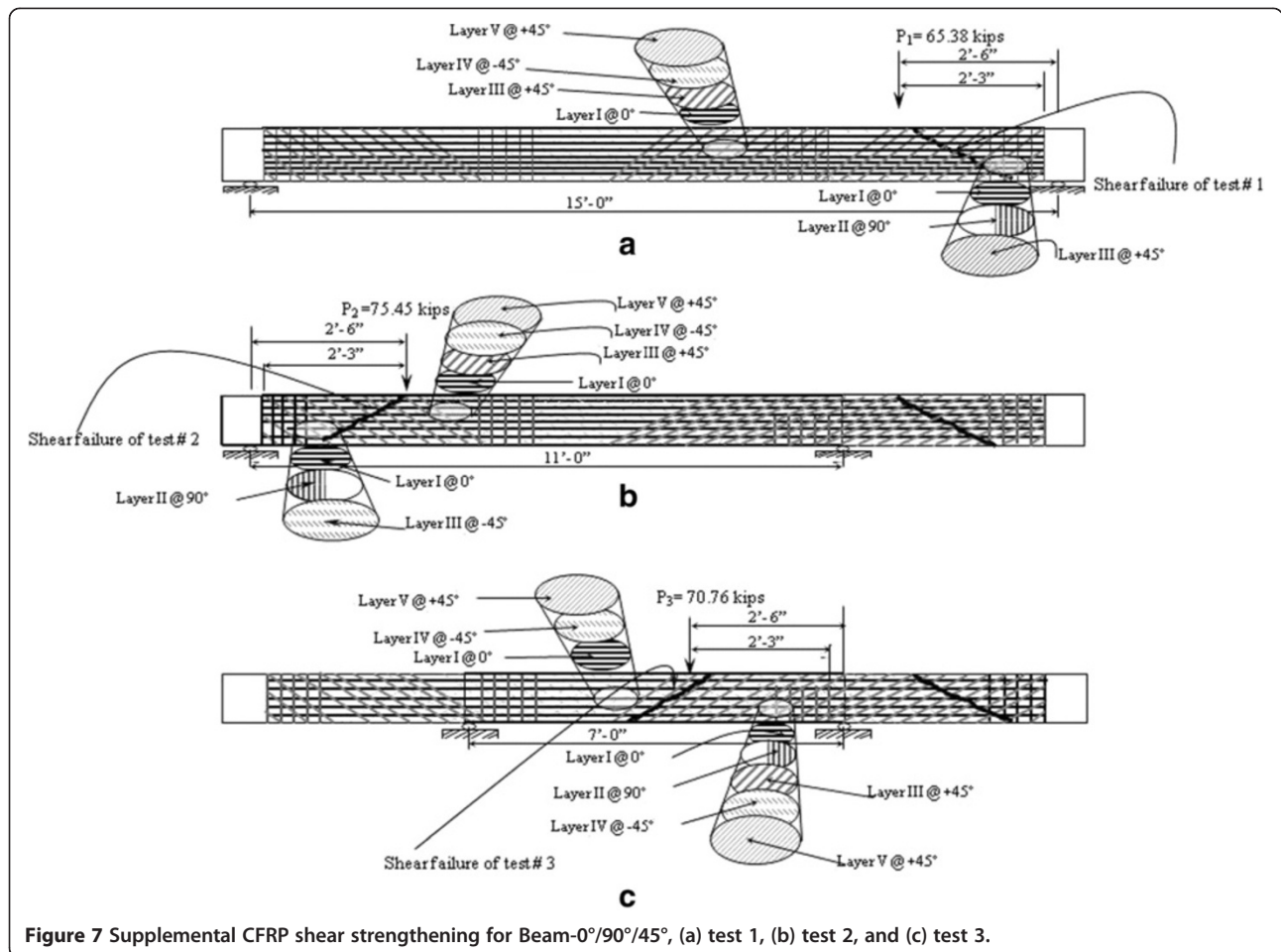
capacity in comparison to the strengthened beams, it has sufficient ductility, as is indicated by the large area under the shear load versus deflection curve. Therefore, strengthening of the beams leads to a loss in their ductility. Typical relationships between shear force and strains (shear and principal strains at the point of rosette #5) for the control beam and Beam-45° are shown in Figure 10. The corresponding relationships for Beam-0°/90° and Beam-0°/90°/45° are shown in Figures 11 and 12, respectively. Note from Figure 10 that there is no significant strain up to a certain value of the shear force in the control beam and Beam-45°. This limit of the shear force
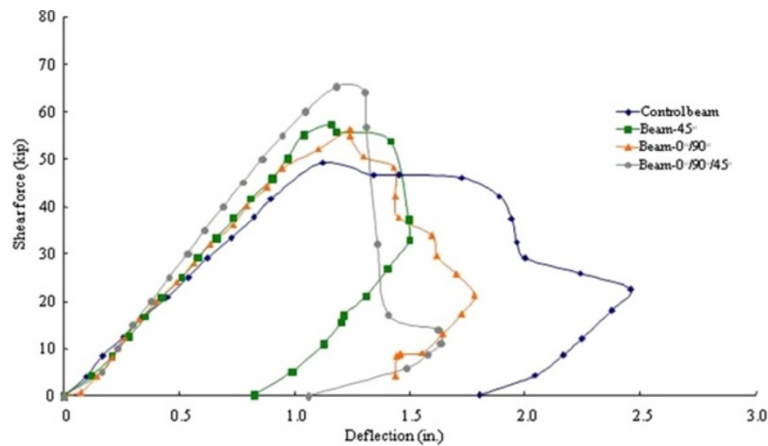

Figure 8 Arrangement of rosette strain gages for Beam-0°/90°.

Figure 9 Shear force versus deflection relationship for test 1 supplemental strengthening.

up to which no significant strain develops is defined as the critical shear force in the discussion that follows. The control beam experiences very large shear and principal strains after very small increases in shear force beyond the critical value so that the critical shear force 80.1 kN (18 kips) marks the ultimate shear load carrying capacity of the control beam (Figure 10). This behavior may be due to development of cracks in the control beam. In contrast, Figure 10 shows that Beam-45° exhibits appreciable shear-load carrying capacity beyond its critical shear force. Similar observations were made for Beam-0°/90° and Beam-0°/90°/45° (see Figures 11 and 12, respectively).

Figure 13a,b,c shows shear force versus shear strain relationships at the location of rosette #5 for the control and strengthened beams in tests 1, 2, and 3, respectively (curves for Beam-45° and Beam-0°/90° are not presented

in Figure 13c because of the non-availability of results). There is no significant shear strain in any beam up to the critical shear force; however, the control beam underwent substantial straining (distortion) beyond the critical shear force. From Figure 13a, test 1, Beam-45° has the highest shear resistance for the largest part of the strain range. A similar observation is made from Figure 13b (test 2) for Beam-0°/90° and Beam-45° for shear strains less than $2,500 \times 10^{-6}$. In this range of shear strain, Beam-45° and Beam-0°/90° show nearly same shear resistance. However, for higher values of shear strain, Beam-0°/90°/45° shows the highest shear resistance. In general, Beam-45° has reasonable shear resistance for both smaller and larger shearing deformations, as indicated by almost the same stiffness after the critical shear force. For shearing deformation both smaller and larger than that at the critical shear force,
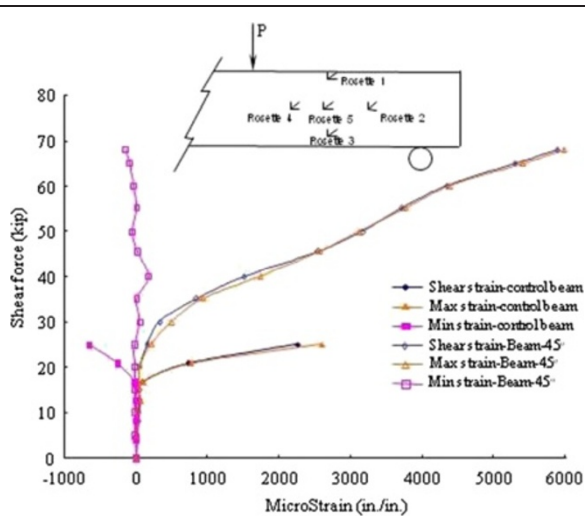

Figure 10 Strains at rosette #5 location for control beam and Beam-45°, test 2.
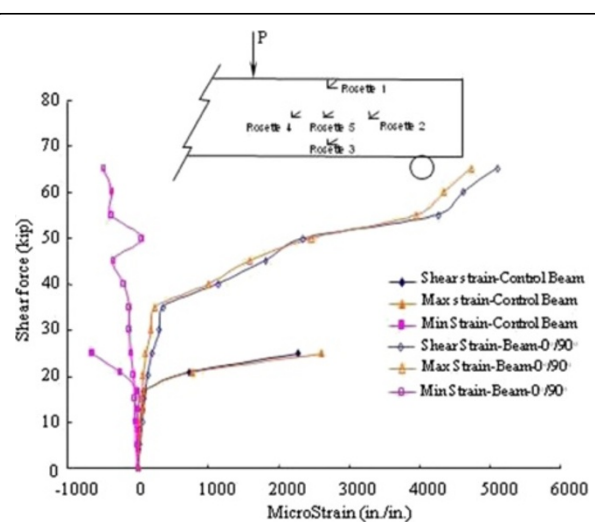

Figure 11 Strains at rosette #5 location for control beam and Beam-0°/90°, test 2.
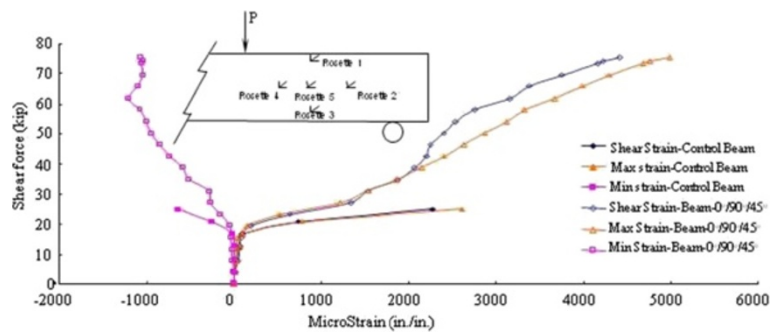
Figure 12 Strains at rosette #5 location for control beam and Beam-0°/90°/45°, test 2.

Beam-45° has a stable shear response over large deformations. Beam-0°/90°/45° has the highest shear resistance, but this occurs at the expense of large shear strains (see Figure 13a,b).

Figure 14a,b,c compares the shear force versus strain relationships for the control and strengthened beams for tests 1, 2, and 3, respectively (results for Beam-45° and

Beam-0°/90° are not shown in Figure 14c because of their non-availability for test 3). In these figures, 'maximum' and 'minimum' strains refer to the principal strains at rosette #5. The strengthened beams have very large stiffnesses in the direction of the minimum principal strain. However, in the maximum principal strain direction, the strengthened beams show a finite stiffness and the
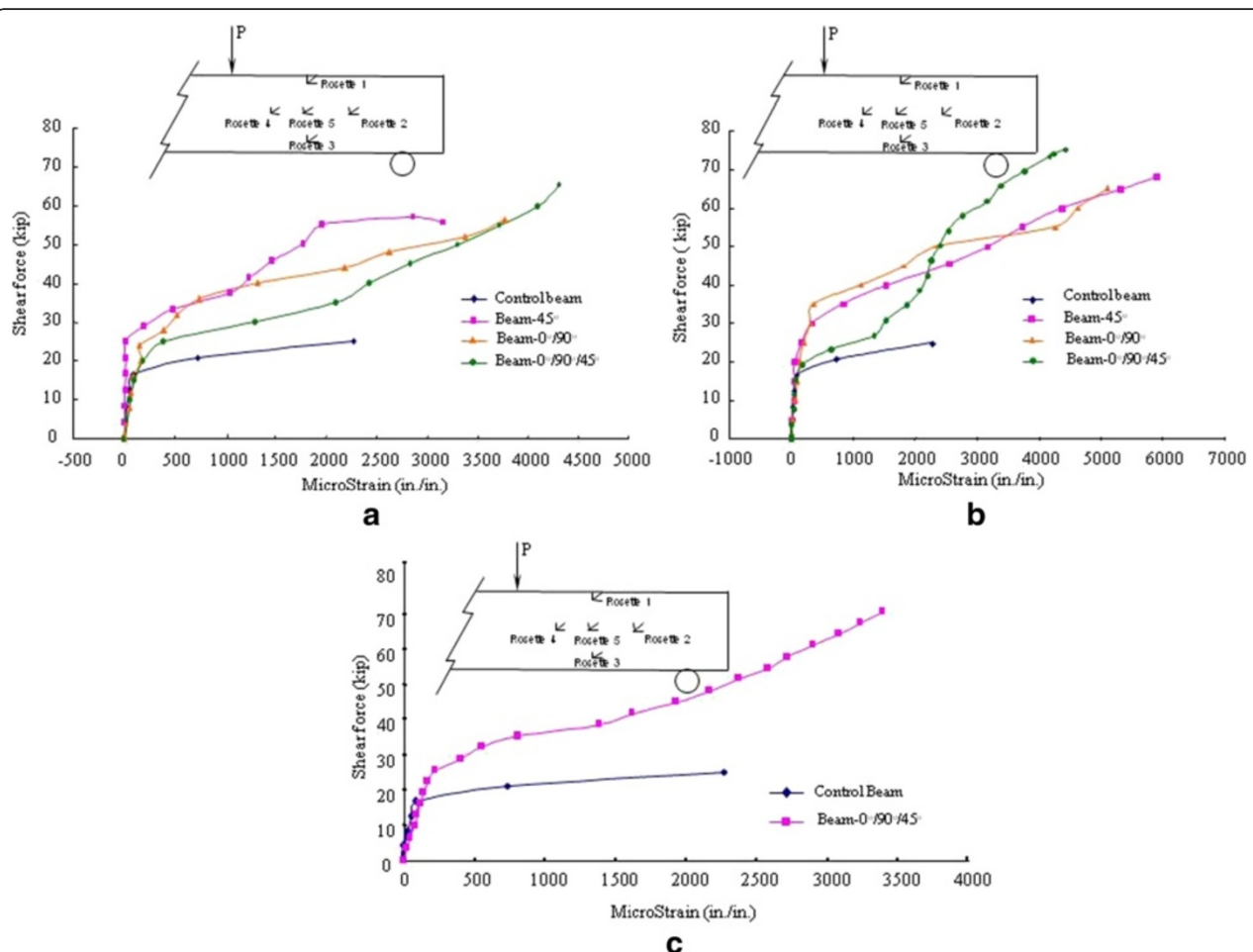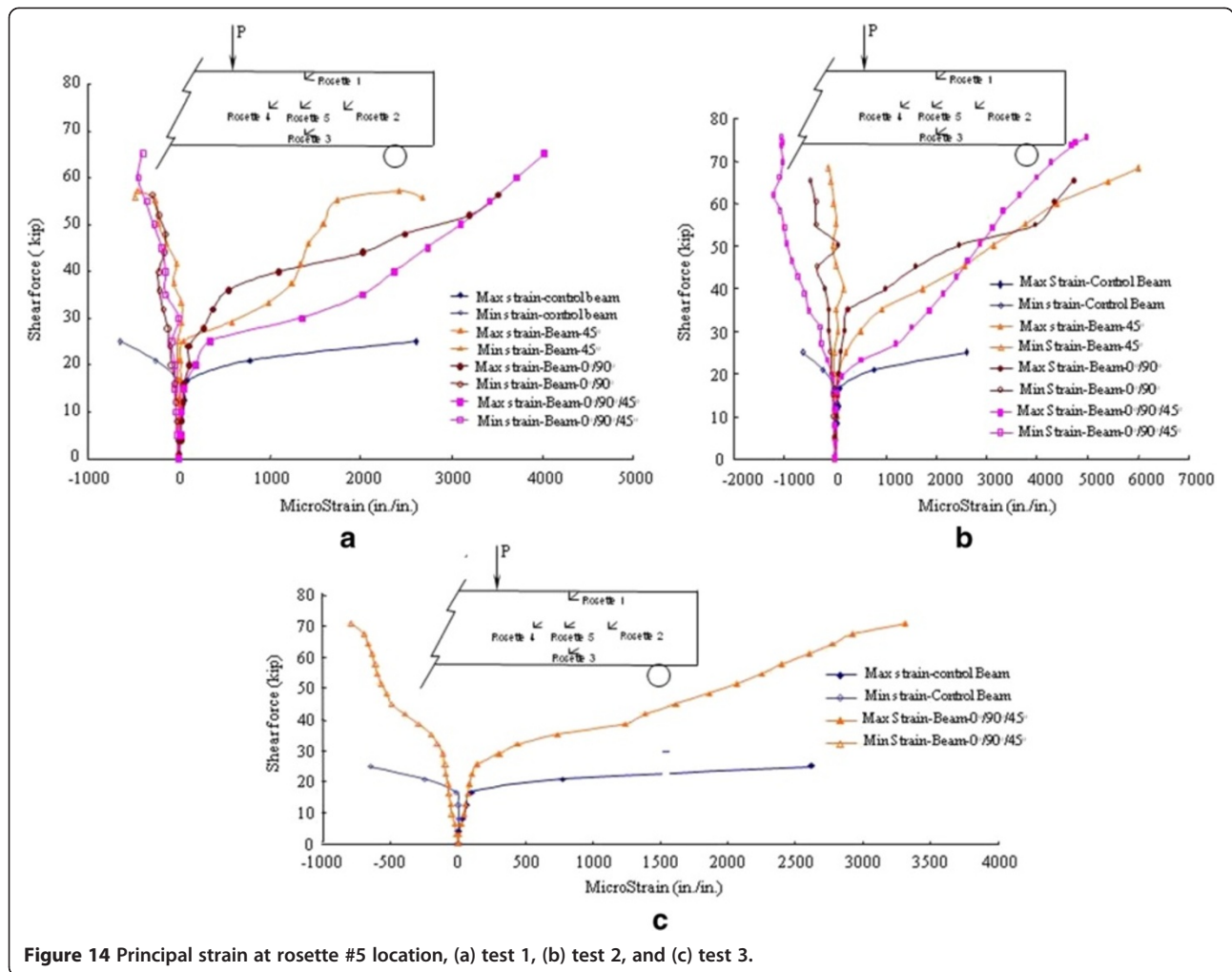

Figure 13 Shear strain at rosette #5 location, (a) test 1, (b) test 2, and (c) test 3.

Figure 14 Principal strain at rosette #5 location, (a) test 1, (b) test 2, and (c) test 3.

control beam shows an almost complete loss of stiffness. In test 1, Beam-0°/90° has the highest shear resistance in the maximum principal strain direction for maximum principal strains less than $1,400 \times 10^{-6}$. In test #2, Beam-0°/90° has the highest shear resistance for maximum principal strains less than $3,000 \times 10^{-6}$. However, in the advanced stage of deformation, Beam-45° shows the largest shear resistance in the maximum principal strain direction.

Figures 15, 16, 17, and 18 show typical failure modes of the control beam, Beam-45°, Beam-0°/90°, and Beam-0°/90°/45°, respectively. The control beam (Figure 15) failed by diagonal cracking near the support. The CFRP

Figure 15 Failure of control beam.

Figure 16 Failure of Beam-45°.

Figure 17 Failure of Beam-0°/90°.

sheets of Beam-45° (Figure 16) ruptured at beam failure in addition to pulling off the concrete cover on both sides. Beam-0°/90° failed by crushing of the concrete, which led to rupture of the fabric sheets at the seam (see Figure 17). This mode of failure could have been avoided if adequate overlapping had been provided. A similar failure is shown for Beam-0°/90°/45° in Figure 18, wherein crushing of concrete led to side cover delamination and rupture of the sheet.

The shear resistance observed from tests 1, 2, and 3, and failure modes of control and strengthened beams are presented in Table 2. On average, Beam-45°, Beam-0°/90°, and Beam-0°/90°/45° have 24.6%, 18.6%, and 39.6% higher shear strengths than the control beam, respectively. It may be noted that as a case study, design examples for three types of laminates have been presented in the 'Appendix' section, citing the difference in experimental and analytical shear strengths of laminates.

Conclusions

Based on the experimental results, the following conclusions may be drawn.

1. External strengthening of the beams using CFRP sheets delays the onset of cracking and provides

Figure 18 Failure of Beam-0°/90°/45°.

additional reserve strength beyond cracking with increase in the deformation.

2. The increase in the shear capacities of strengthened beams was round about 25%, 19%, and 40% in the case of Beam-45°, Beam-0°/90°, and Beam-0°/45°/90°, respectively, in comparison to that of the control beam.

3. There exists a critical value of shear force up to which there is no appreciable strain in the control beam, this force marks the ultimate shear resistance of the control beam. However, the strengthened beams show appreciable strength well beyond their critical shear force.

4. Shear design equations slightly underestimate the actual strength of Beam-45°, Beam-0°/90°, and Beam-0°/90°/45° by about 11.9%, 7.7%, and 10.8%, respectively.

5. The ultimate failure of the control beam (beam without external strengthening) occurs due to diagonal shear cracking. However, the ultimate failure of strengthened beams occurred with delayed cracking of concrete eventually leading to the rupture of CFRP sheets and pulling of concrete on side and/or side cover delamination depending on the strengthening patterns. In the case of Beam-45°, CFRP sheets ruptured at beam failure along with pulling off concrete on both sides of the beam. Beam-0°/90° failed by crushing of concrete, which led to the rupture of the fabric sheet at the seams between CFRP sheets, and in Beam-0°/90°/45°, the crushing of concrete led to the side cover delamination and rupture of the CFRP sheet.

Notations

A_{fv}: cross sectional area of each CFRP sheet mm^2
A_v: cross sectional area of steel stirrups mm^2
d: effective depth of beam mm
d_{frp}: effective depth of CFRP sheet mm
E_f: modulus of elasticity of CFRP sheet GPa
f_c': specified compressive strength of concrete MPa
f_{fe}: effective stress in CFRP sheet MPa
f_{fu}: ultimate tensile strength of CFRP sheet MPa
f_y: yield strength of steel MPa
L_e: effective bond length mm
n: number of CFRP sheets of a specific fiber orientation
R: ratio of effective stress to ultimate tensile strength of CFRP sheet
s: center-to-center distance between adjacent steel stirrups mm
S_{frp}: spacing between adjacent CFRP sheets mm
t_{frp}: thickness of CFRP sheet mm
V_c: shear resistance provided by concrete with steel flexural reinforcement kN
V_{frp}: shear resistance provided by CFRP sheet kN
V_{fo}: shear contribution of CFRP sheet with fibers oriented at 0° kN

Table 2 Ultimate shear capacities of test beams

Beam designation	Shear capacity, kN (kips)				Increase in shear capacity due to initial strengthening (%)	Increase in shear capacity due to supplemental strengthening (%)		
	Initial strengthening	Supplemental strengthening						
	Test 1[a]	Test 1	Test 2	Test 3	Test 1	Test 1	Test 2	Test 3
Unstrengthened	182.5	-	175.8	146.4	-		-	-
Beam	(41.0)		(39.5)	(32.9)[b]				
Beam-45°	212.7	-	234.1	181.6	16.6	-	33.2	24.0
	(47.8)		(52.6)	(40.8)[c]				
Beam-0°/90°	-	209.2	224.7	166.0	-	14.6	27.8	13.4
		(47.0)	(50.5)	(37.3)[d]				
Beam-0°/90°/45°	-	242.5	259.4	202.5	-	32.9	47.6	38.3
		(54.5)	(58.3)	(45.5)[e]				

[a]Test 1 for Beam-45° was conducted after initial strengthening, while for other strengthened beams, tests 1, 2, and 3 were conducted after supplemental strengthening. [b]Control beam failed by diagonal shear cracking. [c]Beam-45° failed by rupture of CFRP sheet along with pulling of concrete cover on sides of beam. [d]Beam-0°/90° failed by rupture of CFRP sheets at seams between CFRP sheets on sides of beam. [e]Beam-0°/90°/45° failed by rupture of CFRP sheets along with side cover delamination.

V_{f45}: shear contribution of CFRP sheet with fibers oriented at 45° kN

V_{f90}: shear contribution of CFRP sheet with fibers oriented at 90° kN

V_n: nominal shear strength of a reinforced concrete section kN

V_s: shear resistance provided by steel stirrups kN

w_{frp}: width of CFRP sheet mm

w_{fe}: effective width of CFRP sheet mm

ε_{fu} design rupture strain of CFRP sheet

β: orientation of CFRP fibers with respect to longitudinal axis of beam degrees

ϕ: shear strength reduction factor.

Appendix
Design example

During the construction of a reinforced concrete beam (shown in Figure 1a), half of the steel reinforcement was accidentally omitted. The beam was originally designed with #3 stirrups [9.5 mm (3/8 in.)] spaced at 152 mm (6 in.) center-to-center. The yield strength of the steel reinforcement is 413.7 MPa (60 ksi), and the concrete strength is 27.58 MPa (4 ksi). Calculate the amount of externally bonded CFRP fabric sheet required to restore the beam to its original shear design capacity. Properties of the CFRP sheet are given in Table 1. The method of installation will be a 'U' wrap using laminates made of CFRP sheet. Possible fiber orientations are 0°, 90°, and 45°. The effective width of the beam is 228 mm (9 in.), depth 381 mm (15 in.). The effective depth (*d*) of the beam is 343 mm (13.5 in.).

Design procedure

The general design methodology presented here is based on a design algorithm (Khalifa et al. 1998), a design manual (Replark System 2000), ACI 318 (2005), and experimental results of the present investigation.

Shear strength of concrete and steel reinforcements (as per ACI 318 2005):

$$V_c = \frac{\sqrt{f'_c}\,b_w d}{6} = \frac{\sqrt{27.58} \times 228 \times 343}{6}$$
$$= 68.45 \text{ kN}$$

$$V_s, \text{Design} = \frac{A_v f_y d}{s}$$
$$= \frac{2 \times 70.88 \times 413.7 \times 343}{152}$$
$$= 132.3 \text{ kN}$$

$$V_s, \text{As–built} = \frac{A_v f_y d}{s}$$
$$= \frac{2 \times 70.88 \times 413.7 \times 343}{304}$$
$$= 66.2 \text{ kN}.$$

$$\phi V_n, Design = 0.85(V_c + V_s, \text{Design})$$
$$= 0.85(6.45 + 132.3)$$
$$= 170.6 \, kN$$
$$\phi V_n As–built = 0.85(V_c + V_s, \text{As–built}) = 114.45 \, kN$$

Case 1: Assume 45° lay-up of strengthening (continuous 'U' wraps)

Contribution of CFRP reinforcement: $V_{frp} = \frac{A_{fv} f_{fe}(\sin\beta + \cos\beta) d_{frp}}{S_{frp}}$ (Khalifa et al. 1998).

Cross-sectional area of CFRP fabric sheets = A_{fv}

$A_{fv} = 2n\, t_{frp}\, w_{frp} = 2 \times 1 \times 0.178 \times 305 = 108.6 \text{ mm}^2$

$\beta = 45°$

$d_{frp} = d = 343$ mm

$S_{frp} = w_{frp} = 305$ mm (since strengthening pattern is continuous)

$f_{fe} = R\, f_{fu}$

Factor R based on effective stress:

$R = 0.5622\,(\rho_f\, E_f)^2 - 1.2188\,(\rho_f\, E_f) + 0.778$ (Khalifa et al. 1998),

where

$$\rho_f = \frac{2t_{frp}}{b_w} \times \frac{w_{frp}}{S_{frp}} = \frac{2 \times 0.178}{228} \times \frac{305}{305} = 1.56 \times 10^{-3}$$

$$\rho_f \ E_f = 1.56 \times 10^{-3} \times 228 = 0.36 \ \text{GPA} \le 1.1 \ \text{GPA};$$

$$R = 0.5622 \ (0.36)^2 - 1.2188 \ (0.36) \ + 0.778 = 0.412$$

(Khalifa et al. 1998).

Factor R based on bond failure mechanism (Maeda et al. 1997):

Effective bond length, L_e

$$L_e = \frac{461.3}{(E_f t_{frp})^{0.58}} = \frac{461.3}{[228 \times 0.178]^{0.58}} = 53.8 \ \text{mm}$$

$$W_{fe} = d_{frp} \text{-} L_e = 343 - 53.8 = 289.2 \ \text{mm}$$

$$R = \frac{0.0042 \left(f_c'\right)^{2/3} W_{fe}}{(E_f t_{frp})^{0.58} \varepsilon_{fu} d_{frp}}$$

$$= \frac{0.0042(27.58)^{2/3} \times 289.2}{(228 \times 0.178)^{0.58} \times 0.018 \times 343} = 0.210.$$

Factor R based on limiting strain (Replark System 2000):

$$R = \frac{0.004}{\varepsilon_{fu}} = \frac{0.004}{0.018} = 0.222.$$

The lowest value of factor R is based on bond failure mechanism.

Hence, $R = 0.210$ control design.

Effective stress, $f_{fe} = R f_{fu} = (0.210 \times 2{,}758) = 579.2$ MPa

$$V_{frp} = \frac{108.6 \times 579.2 \times (\sin 45 + \cos 45) \times 343}{305} = 100.0 \ \text{kN}.$$

Check maximum shear reinforcement:

$$V_s + V_{frp} \le \frac{2\sqrt{f_c'}}{3} b_w d$$

$$V_{frp} = 100.0 \le \frac{2\sqrt{27.58}}{3 \times 1000} \times 228 \times 343 - 66.2 = 207.6 \ \text{kN}.$$

Determine shear capacity of CFRP reinforced beam:

$$\phi V_n = 0.85(V_c + V_s) + 0.70 V_{frp}$$

$$= 0.85 \ (68.45 + 66.2) + 0.70 \times 100.0$$

$$= 184.5 \ \text{kN} \ge \phi V_{n,} \text{Design} = 170.6 \ \text{kN}.$$

Evaluate difference in experimental and theoretical shear strength values:

Average experimental shear strength = 209.5 kN
Theoretical shear strength = 184.5 kN

$$\% \text{difference} = \frac{209.5 - 184.5}{209.5} \times 100 = 11.9\%.$$

Thus, one layer of 45° ['U' continuous wrap] provides adequate shear strengthening. However, the design equations underestimate the actual shear strength of Beam-45° by about 11.9%.

Case 2: Assume 0°/90° lay-up of strengthening [continuous 'U' wraps]

Cross-sectional area (A_{fv}) of each of fabric sheet with fiber orientation of 0° and 90°

$$A_{fv} = 2n t_{frp} w_{frp} = 2 \times 1 \times 0.178 \times 305 = 108.6 \ \text{mm}^2.$$

Factor R based on effective stress:

$$R = 0.5622(\rho_f E_f)^2 + 1.2188(\rho_f E_f) + 0.778,$$

where

$$\rho_f = \frac{2t_{frp}}{b_w} \times \frac{w_{frp}}{S_{frp}} = \frac{2 \times 2 \times 0.178}{228} \times \frac{305}{305} = 3.1 \times 10^{-3}$$

$$\rho_f E_f = 3.1 \times 10^{-3} \times 228 = 0.709 \ GPa \le 1.1 \ GPa$$

$$R = 0.5622(0.707)^2 \ 1.2188(0.707) + 0.778 = 0.197$$

Factor R based on bond failure mechanism:
Effective bond length, L_e

$$L_e = \frac{461.3}{(E_f t_{frp})^{0.58}} = \frac{461.3}{[228 \times 2 \times 0.178]^{0.58}} = 36 \ \text{mm}$$

$$W_{fe} = d_{frp} - L_e = 343 - 36 = 307 \ \text{mm}$$

$$R = \frac{0.0042 \left(f_c'\right)^{2/3} w_{fe}}{(E_f t_{frp})^{0.58} \varepsilon_{fu} d_{frp}}$$

$$= \frac{0.0042 (27.58)^{2/3} \times 307}{(228 \times 2 \times 0.178)^{0.58} \times 0.018 \times 343} = 0.149.$$

Factor R based on limiting strain:

$$R = \frac{0.004}{\varepsilon_{fu}} = \frac{0.004}{0.018} = 0.222.$$

The minimum value of factor R is based on bond failure mechanism. Hence, $R = 0.149$ controls design.

Effective stress, $f_{fe} = R \times f_{fu} = (0.149 \times 2758) = 410.9$ MPa

Total shear contribution of 0°/90° lay-up:

The shear contribution of CFRP sheet with 0° fiber orientation, V_{fo}

$$V_{fo} = \frac{108.6 \times 410.9 \times (\sin 0 + \cos 0) \times 343}{305} = 50.2 \ \text{kN}.$$

The shear contribution of CFRP sheet with 90° fiber orientation, V_{f90}

$$V_{f90} = \frac{108.6 \times 410.9 \times (\sin 90 + \cos 90) \times 343}{305} = 50.2 \ \text{kN}.$$

Thus, total shear contribution of 0°/90° lay-up

$$V_{frp} = V_{fo} + V_{f90} = 50.2 + 50.2 = 100.4 \text{ kN}.$$

Check maximum shear reinforcement:

$$V_s + V_{frp} \leq \frac{2\sqrt{f'_c}}{3} b_w d;$$

$$V_{frp} = 100.4 \leq \frac{2\sqrt{27.58}}{3 \times 1000} \times 228 \times 343 - 66.2 = 207.6 \text{ kN}.$$

Determine shear capacity of CFRP reinforced beam:

$$\phi V_n = 0.85(V_c + V_s) + 0.70 V_{frp}$$
$$= 0.85 \ (68.45 + 66.2) + 0.70 \times 100.0$$
$$= 184.5 \text{ kN} \geq \phi V_{n,} \text{Design} = 170.6 \text{ kN}$$

Evaluate the difference in experimental and theoretical shear strength values:
Average experimental shear strength = 200 kN
Theoretical shear strength =184.7 kN

$$\% \text{difference} = \frac{200.0 - 184.7}{200.0} \times 100 = 7.7\%.$$

Thus, 0°/90° lay-up provides adequate shear strengthening. Note that the shear resistance contributions of a single 45° lay-up and 0°/90° lay-up are almost the same, which is consistent with experimental results (see Table 2). However, the design equations underestimate the actual strength of Beam-0°/90° by about 7.7%.

Case 3: Assume three layers of fabric sheet having 0°/90°/45° lay-up
Cross-sectional area (A_{fv}) of each fabric sheet with fiber orientation of 0°, 90°, and 45°
$A_{fv} = 2 \ n \ t_{frp} \ w_{frp} = 2 \times 1 \times 0.178 \times 305 = 108.6 \text{ mm}^2$
Factor R based on effective stress:

$$R = 0.5622(r_f E_f)^2 - 1.2188(r_f E_f) + 0.778,$$

where

$$\rho_f = \frac{2t_{frp}}{b_w} \times \frac{w_{frp}}{S_{frp}} = \frac{2 \times 3 \times 0.178}{228} \times \frac{305}{305} = 4.68 \times 10^{-3}$$

$$\rho_f \ E_f = 4.68 \times 10^{-3} \times 228 = 1.067 \text{ GPa} \leq 1.1 \text{ GPa}$$

$$R = 0.5622 \ (1.067)^2 - 1.2188 \ (1.067) + 0.778 = 0.118.$$

Factor R based on bond failure mechanism:

Effective bond length, L_e

$$L_e = \frac{461.3}{(E_f t_{frp})^{0.58}} = \frac{461.3}{[228 \times 3 \times 0.178]^{0.58}} = 28.5 \text{ mm}$$

$$w_{fe} = d_{frp} - L_e = 343 - 28.5 = 314.5 \text{ mm}$$

$$R = \frac{0.0042 \left(f'_c\right)^{2/3} w_{fe}}{(E_f t_{frp})^{0.58} \varepsilon_{fu} d_{frp}}$$

$$= \frac{0.0042 \ (27.58)^{2/3} \times 314.5}{(228 \times 3 \times 0.178)^{0.58} \times 0.018 \times 343} = 0.121.$$

Factor R based on limiting strain:

$$R = \frac{0.004}{\varepsilon_{fu}} = \frac{0.004}{0.018} = 0.222.$$

The minimum value of factor R is based on effective stress. Hence, $R = 0.118$ control design.
Effective stress, $f_{fe} = R \times f_{fu} = (0.118 \times 2758) = 325.4$ MPa
Total shear contribution of 0°/90°/45° lay-up:
The shear contribution of CFRP sheet with 0° fiber orientation, V_{fo}

$$V_{fo} = \frac{108.6 \times 325.4 \times (\sin 0 + \cos 0) \times 343}{305} = 39.7 \text{ kN}.$$

The shear contribution of CFRP sheet with 90° fiber orientation, V_{f90}

$$V_{f90} = \frac{108.6 \times 325.4 \times (\sin 90 + \cos 90) \times 343}{305} = 39.7 \text{ kN}.$$

The shear contribution of CFRP sheet with 45° fiber orientation, V_{f45}

$$V_{f45} = \frac{108.6 \times 325.4 \times (\sin 45 + \cos 45) \times 343}{305} = 56.2 \text{ kN}.$$

Thus, total shear contribution of 0°/90°/45° lay-up
$V_{frp} = V_{fo} + V_{f90} + V_{f45} = 39.7 + 39.7 + 56.2 = 135.6 \text{ kN}.$
Check maximum shear reinforcement:

$$V_s + V_{frp} \leq \frac{2\sqrt{f'_c}}{3} b_w d$$

$$V_{frp} = 135.6 \leq \frac{2\sqrt{27.58}}{3 \times 1000} \times 228 \times 343 - 66.2 = 207.6 \text{ kN}.$$

Determine shear capacity of CFRP strengthened beam:

$$\phi V_\mathrm{n} = 0.85\,(V_\mathrm{c} + V_\mathrm{s}) + 0.70\,V_\mathrm{frp}$$

$$= 0.85\,(68.45 + 66.2) + 0.70 \times 135.6$$

$$= 209.4\ \mathrm{kN} \geq \phi V_\mathrm{n},\mathrm{Design} = 170.6\ \mathrm{kN}.$$

Evaluate the difference in experimental and theoretical shear strength values:

Average experimental shear strength = 234.8 kN
Theoretical shear strength = 209.4 kN

$$\%\mathrm{difference} = \frac{234.8 - 209.4}{234.8} \times 100 = 10.8\%.$$

Thus, it is seen that the shear capacity of Beam-0°/90°/45° is the maximum and provides adequate shear strengthening. However, the design equations underestimate the strength of Beam-0°/90°/45° by about 10.8%.

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