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Mathematical modelling of HIV epidemic and stability analysis

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Abstract

A nonlinear mathematical model of differential equations with piecewise constant arguments is proposed. This model is analyzed by using the theory of both differential and difference equations to show the spread of HIV in a homogeneous population. Because of the solution of this differential equations being established in a certain subinterval, solutions will be analyzed as a system of difference equations. After that, results will be considered for differential equations as well. The population of the model is divided into three subclasses, which are the HIV negative class, the HIV positive class that do not know they are infected and the HIV positive class that know they are infected. As an application of the model we took the spread of HIV in India into consideration.

MSC: 39A10; 39A11

Keywords: logistic differential equations; difference equations; piecewise constant arguments; stability

1 Introduction

Sexually transmitted diseases such as HIV are overwhelmingly observed in some populations. Countries are dealing with the growing impact of the epidemics on the youngest and most productive population groups; increasing numbers in children and adolescents; worsening situation among the poor and marginalized populations; a continuous aggravation of the existent health problems (such as tuberculosis); and above all, the diversion of resources from other health, welfare, and educational priorities. It is very important to understand how the transmission process of the infectious diseases works in order to avoid ulterior spread of these epidemics [1]. So mathematical models for transmission dynamics in HIV play an important role in better understanding of epidemiological patterns for disease control as they provide short and long term prediction for HIV and AIDS incidence [2].

The first mathematical model used for the explicit study of a sexually transmitted disease was a one sex model that was constructed by Cooke and Yorke in 1973 [1, 3]. A two sex model developed specifically for gonorrhoea was formulated by Lajmanovich and Yorke in 1976 [4]. One STD (sexually transmitted disease) that many people are worried about getting is HIV. Mathematical modeling of the HIV epidemic has been studied by various authors recently [5–12]. In 1994, Velasco-Hernandez and Hsieh analyzed the HIV epidemic in a male population [13]. In 2004, Bachar and Dorfmatr considered the mathematical model where the population was divided into three sub populations

[14]. Expanding [14], Naresh has studied the effect of contact tracing on reducing the spread of HIV/AIDS in a homogeneous population with constant immigration of susceptibles.

Changes in population involved in a HIV transmission model can take place by discrete steps and continuous processes. In discrete models, difference equations reflect the change over the whole time step, whereas in continuous models, differential equations are developed to explore the changes in one variable with a decreasingly small change in another variable [1]. Because HIV population dynamics involves both continuous and discrete time arguments, for this type of problems, a different point of view came up and Buseenberg and Cooke used piecewise constant arguments to construct the mathematical modeling of biological structures in 1983. By taking into account the work of Naresh [2] we have constructed a new mathematical model. We added new terms and took into consideration both discrete time and continuous time. This model is analyzed by using the theory of both differential and difference equations to show the spread of HIV/AIDS in a homogeneous population. Because of the solution of these differential equations established in a certain subinterval, the solutions will be analyzed as a difference equations system. Furthermore, these results will be considered for differential equations, as well. So our mathematical model that is named a differential equation systems with piecewise constant arguments is as follows:

$$\begin{cases} \frac{dS}{dt} = S(t)r_1(p_1 - \alpha_1 S(t) - \beta_1 I_1([t]) - \beta_2 I_2([t])), \\ \frac{dI_1}{dt} = I_1(t)r_2(1 - \alpha_2 I_1(t) + \beta_1(1 - \varepsilon_1)S([t]) - \gamma I_2([t]) - \theta I_1([t]) \\ \quad + \beta_2(1 - \varepsilon_2)S([t])I_2([t])), \\ \frac{dI_2}{dt} = I_2(t)r_3(1 - \alpha_3 I_2(t) + \beta_2 \varepsilon_2 S([t]) + \gamma I_1([t]) + \theta I_1([t]) \\ \quad + \beta_1 \varepsilon_1 S([t])I_1([t])). \end{cases} \quad (1.1)$$

In the equations above, t is the time and $[t]$ is the exact value of t for $t \geq 0$. The model monitors three populations; susceptibles $S(t)$, HIV positives that do not know they are infected $I_1(t)$, HIV positives that know they are infected $I_2(t)$. The susceptibles are composed of individuals that have not contracted the infection but may get infected through contacts (sexual, blood transfusion *etc.*) with infectives. r_1 is the population growth rate of the susceptible population. The deaths (of natural causes) are given at a rate α_1 . p_1 is the rate of susceptible population per year. The susceptibles are lost from their class following contacts with the infectives I_1 and I_2 at a rate β_1 and β_2 , respectively. I_1 are populations that are HIV positive but do not know it, because disease symptoms have not yet appeared in this class. This population is generated by the HIV infection of susceptibles. r_2 is the population growth rate of the population I_1 . The deaths (of natural causes) are given at a rate α_2 . The population of this class decreases and becomes aware after screening at a rate θ . γ is the rate of individuals in class I_1 that detect the infection by tracing contacts with class I_2 . Following the contacts of class S and I_1 , class S becomes aware of the infection and this class is detected further which has the rate of ε_1 . And also, following the contacts of class S and I_2 , class S becomes aware of the infection and is detected further with the rate of ε_2 . r_3 is the population growth rate of the population I_2 . The deaths (of natural causes) are given at a rate α_3 .

2 Local and global asymptotic stability analysis

In this section, the local and global behavior of the nonlinear system (1.1) under specific conditions is investigated. To show the consistence of the population classes with the model constructed in Section 2, the data of [2] is used. Examples show the spread of the population over the time.

2.1 Equilibrium point of the model (1.1)

For $t \in [n, n + 1]$ every equation in system (1.1) is a Bernoulli differential equation as follows:

$$\begin{cases} \frac{dS}{dt} - r_1(p_1 - \beta_1 I_1(n) - \beta_2 I_2(n))S(t) = -\alpha_1 r_1 S(t)^2, \\ \frac{dI_1}{dt} - r_2(1 + \beta_1(1 - \varepsilon_1)S(n) - \gamma I_2(n) - \theta I_1(n) + \beta_2(1 - \varepsilon_2)S(n)I_2(n))I_1(t) \\ = -\alpha_2 r_2 I_1(t)^2, \\ \frac{dI_2}{dt} - r_3(1 + \beta_2 \varepsilon_2 S(n) + \gamma I_1(n) + \theta I_1(n) + \beta_1 \varepsilon_1 S(n)I_1(n))I_2(t) = -\alpha_3 r_3 I_2(t)^2. \end{cases} \quad (2.1)$$

Solving (2.1) for $t \in [n, n + 1]$ and letting t go to $n + 1$ where $n = 0, 1, 2, 3, \dots$, we obtain a system of difference equations as follows:

$$\begin{cases} S(n + 1) = \frac{S(n)U_1}{(U_1 - \alpha_1 S(n))e^{-r_1 U_1} + \alpha_1 S(n)}, \\ I_1(n + 1) = \frac{I_1(n)U_2}{(U_2 - \alpha_2 I_1(n))e^{-r_2 U_2} + \alpha_2 I_1(n)}, \\ I_2(n + 1) = \frac{I_2(n)U_3}{(U_3 - \alpha_3 I_2(n))e^{-r_3 U_3} + \alpha_3 I_2(n)}, \end{cases} \quad (2.2)$$

where

$$\begin{aligned} U_1 &= p_1 - \beta_1 I_1(n) - \beta_2 I_2(n), \\ U_2 &= 1 + \beta_1(1 - \varepsilon_1)S(n) - \gamma I_2(n) - \theta I_1(n) + \beta_2(1 - \varepsilon_2)S(n)I_2(n), \\ U_3 &= 1 + \beta_2 \varepsilon_2 S(n) + \gamma I_1(n) + \theta I_1(n) + \beta_1 \varepsilon_1 S(n)I_1(n). \end{aligned}$$

It is obvious that (2.3) must hold:

$$\begin{cases} p_1 - \beta_1 I_1(n) - \beta_2 I_2(n) \neq 0, \\ 1 + \beta_1(1 - \varepsilon_1)S(n) - \gamma I_2(n) - \theta I_1(n) + \beta_2(1 - \varepsilon_2)S(n)I_2(n) \neq 0, \\ 1 + \beta_2 \varepsilon_2 S(n) + \gamma I_1(n) + \theta I_1(n) + \beta_1 \varepsilon_1 S(n)I_1(n) \neq 0. \end{cases} \quad (2.3)$$

System (2.2) is a system of difference equations. Hence, to explain the global behavior of system (2.1) we must consider the solution of (2.1), that is, the difference equations system (2.2). Firstly, we need to obtain the equilibrium points of this system (2.2), which are also the critical points of system (2.1). We have

$$\begin{cases} \beta_1 I_1(n) + \beta_2 I_2(n) + \alpha_1 S(n) = p_1, \\ I_1(n)(\alpha_2 + \theta) + \gamma I_2(n) - \beta_1(1 - \varepsilon_1)S(n) - \beta_2(1 - \varepsilon_2)S(n)I_2(n) = 1, \\ \alpha_3 I_2(n) - \beta_2 \varepsilon_2 S(n) - (\gamma + \theta)I_1(n) - \beta_1 \varepsilon_1 S(n)I_1(n) = 1. \end{cases} \quad (2.4)$$

From (2.4) we can write

$$\begin{aligned} I_1(n)(\alpha_2 + \theta) + \gamma I_2(n) - \beta_1(1 - \varepsilon_1)S(n) - \beta_2(1 - \varepsilon_2)S(n)I_2(n) \\ = \alpha_3 I_2(n) - \beta_2 \varepsilon_2 S(n) - (\gamma + \theta)I_1(n) - \beta_1 \varepsilon_1 S(n)I_1(n). \end{aligned} \quad (2.5)$$

Simplifying this, we get

$$\begin{aligned}
 &(\alpha_2 + 2\theta + \gamma)I_1 + (\gamma - \alpha_3)I_2 + (\beta_2\varepsilon_2 - (1 - \varepsilon_1)\beta_1)S \\
 &+ (\beta_1\varepsilon_1I_1 - \beta_2(1 - \varepsilon_2)I_2)S = 0.
 \end{aligned} \tag{2.6}$$

Substituting in (2.6) the expression

$$I_2(n) = \frac{p_1 - \alpha_1 S - \beta_1 I_1}{\beta_2} \tag{2.7}$$

we obtain

$$\begin{aligned}
 &((\beta_2(\alpha_2 + 2\theta + \gamma) - \beta_1(\gamma - \alpha_3))I_1(n) + p_1(\gamma - \alpha_3)) \\
 &+ (\beta_2(\beta_2\varepsilon_2 - \beta_1 + \beta_1\varepsilon_1) - \alpha_1(\gamma - \alpha_3) + \beta_2\beta_1\varepsilon_1I_1)S(n) \\
 &- \beta_2(1 - \varepsilon_2)(p_1 - \alpha_1 S(n) - \beta_1 I_1)S(n) = 0.
 \end{aligned} \tag{2.8}$$

To find the positive equilibrium point of (2.2), we have some assumptions in view of the demographic data in [2].

- (i) $\beta_1 = 3\beta_2$,
- (ii) $\beta_2 < \sqrt{\frac{\alpha_1(\alpha_3 - \gamma)}{3 - 4\varepsilon_1}}$,
- (iii) $\varepsilon_1 = \varepsilon_2$,
- (iv) $\varepsilon_1 \neq (1 - \varepsilon_2)$,
- (v) $(1 - \varepsilon_1)(\alpha_2 + 2\theta - 2\gamma + 3\alpha_3) > 3(\alpha_3 - \gamma)$,
- (vi) $\alpha_3 > \gamma$,
- (vii) $\varepsilon_1 = \varepsilon_2 < 0.75$,
- (viii) $\theta > \gamma$.

Thus, the equilibrium points of system (2.2) are $\bar{X} = (\bar{S}, \bar{I}_1, \bar{I}_2)$. Here

$$\bar{S} = \frac{\beta_2(3 - 4\varepsilon_1)}{\alpha_1(1 - \varepsilon_1)} + \frac{p_1}{\alpha_1} - \left(\frac{3p_1(\alpha_3 - \gamma)}{\alpha_1(1 - \varepsilon_1)(\alpha_2 + 2\theta - 2\gamma + 3\alpha_3)} + \frac{\alpha_3 - \gamma}{\beta_2(1 - \varepsilon_1)} \right), \tag{2.9}$$

$$\bar{I}_1 = \frac{p_1(\alpha_3 - \gamma)}{\beta_2(\alpha_2 + 2\theta - 2\gamma + 3\alpha_3)}, \tag{2.10}$$

$$\bar{I}_2 = \frac{\alpha_1(\alpha_3 - \gamma) - (3 - 4\varepsilon_1)\beta_2^2}{\beta_2^2(1 - \varepsilon_1)} + \frac{3p_1(\alpha_3 - \gamma)\varepsilon_1}{\beta_2(1 - \varepsilon_1)(\alpha_2 + 2\theta - 2\gamma + 3\alpha_3)} \tag{2.11}$$

and

$$p_1 > \frac{(\alpha_2 + 2\theta - 2\gamma + 3\alpha_3)(\alpha_1(\alpha_3 - \gamma) - \beta_2^2(3 - 4\varepsilon_1))}{\beta_2((1 - \varepsilon_1)(\alpha_2 + 2\theta - 2\gamma + 3\alpha_3) - 3(\alpha_3 - \gamma))}. \tag{2.12}$$

The Jacobian matrix for the equilibrium points is (2.13). This gives the linearizing equations of system (2.2),

$$J(\bar{X}) = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \tag{2.13}$$

where

$$\begin{aligned}
 a_{11} &= e^{-r_1 U_1}, & a_{12} &= \frac{3\beta_2(e^{-r_1 U_1} - 1)}{\alpha_1}, & a_{13} &= \frac{\beta_2(e^{-r_1 U_1} - 1)}{\alpha_1}, \\
 a_{21} &= \frac{(3\beta_2(1 - \varepsilon_1) + \beta_2(1 - \varepsilon_2)I_2)(1 - e^{-r_2 U_2})}{\alpha_2}, \\
 a_{22} &= \frac{(\theta + \alpha_2)e^{-r_2 U_2} - \theta}{\alpha_2}, & a_{23} &= \frac{(-\gamma + \beta_2(1 - \varepsilon_2)S)(1 - e^{-r_2 U_2})}{\alpha_2}, \\
 a_{31} &= \frac{(\beta_2\varepsilon_2 + 3\beta_2(1 - \varepsilon_2)I_1)(1 - e^{-r_3 U_3})}{\alpha_3}, \\
 a_{32} &= \frac{(\gamma + \theta + 3\beta_2\varepsilon_1S)(1 - e^{-r_3 U_3})}{\alpha_3}, & a_{33} &= e^{-r_3 U_3}.
 \end{aligned}$$

The characteristic equation is

$$\begin{aligned}
 &\lambda^3 - (a_{11} + a_{22} + a_{33})\lambda^2 - (a_{32}a_{23} + a_{12}a_{21} + a_{31}a_{13} - a_{11}a_{22} - a_{11}a_{33} - a_{22}a_{33})\lambda \\
 &\quad - (a_{11}a_{22}a_{33} + a_{31}a_{12}a_{23} + a_{13}a_{21}a_{32} \\
 &\quad - a_{11}a_{32}a_{23} - a_{12}a_{21}a_{33} - a_{31}a_{13}a_{22}) = 0.
 \end{aligned} \tag{2.14}$$

2.2 Local and global stability of the positive equilibrium

In this part, the local and global stability of system (2.2) will be analyzed. For the proof of Theorem 2.1, Theorem 2.2, and Theorem 2.3, it is assumed that conditions (i)-(viii) hold.

Theorem 2.1 *Let $\bar{X} = (\bar{S}, \bar{I}_1, \bar{I}_2)$ the positive equilibrium point of system (2.2) and assume that*

$$\begin{aligned}
 &\frac{(\alpha_2 + 2\theta - 2\gamma + 3\alpha_3)(\alpha_1(\alpha_3 - \gamma) - \beta_2^2(3 - 4\varepsilon_1))}{\beta_2((1 - \varepsilon_1)(\alpha_2 + 2\theta - 2\gamma + 3\alpha_3) - 3(\alpha_3 - \gamma))} \\
 &< p_1 < \frac{(\alpha_2 + 2\theta - 2\gamma + 3\alpha_3)(\alpha_1(1 - \varepsilon_1)(\alpha_2 - \gamma) + 3\alpha_1\varepsilon_1(\alpha_3 + \gamma) - 3\varepsilon_1\beta_2^2(3 - 4\varepsilon_1))}{3\beta_2\varepsilon_1((1 - \varepsilon_1)(\alpha_2 + 2\theta - 2\gamma + 3\alpha_3) - 3(\alpha_3 - \gamma))}.
 \end{aligned}$$

Furthermore, suppose that

$$\beta_2 < \sqrt{\frac{\alpha_1\alpha_3\theta}{3(1 - \varepsilon_1)(\alpha_2\bar{I}_1 - \alpha_3\bar{I}_2)}}. \tag{2.15}$$

If

$$\bar{U}_1 > \frac{1}{r_1} \ln(K), \tag{2.16}$$

$$\frac{1}{r_2} \ln\left(\frac{3(-\gamma + \beta_2\varepsilon_1\bar{S}) + \alpha_2}{3(-\gamma + \beta_2\varepsilon_1\bar{S}) + \theta}\right) < \bar{U}_2 < \frac{1}{r_2} \ln\left(\frac{\theta + \alpha_2}{\theta}\right), \tag{2.17}$$

$$\bar{U}_3 > \frac{1}{r_3} \ln\left(\frac{\gamma + \theta + 3\beta_2\varepsilon_1\bar{S} + 3\alpha_3}{\gamma + \theta + 3\beta_2\varepsilon_1\bar{S}}\right), \tag{2.18}$$

where

$$K = \frac{\alpha_2 \beta_2^2 (\varepsilon_2 + 3(1 - \varepsilon_2)I_1) + \alpha_1 \alpha_3 \theta}{\alpha_1 \alpha_2 \alpha_3 + \beta_2 \alpha_2 (\varepsilon_2 + 3(1 - \varepsilon_2)I_1) + 3\beta_2^2 \alpha_3 (3(1 - \varepsilon_1) + (1 - \varepsilon_2)I_2) - \alpha_1 (\gamma + \theta + 3\beta_2 \varepsilon_1 S) (-\gamma + \beta_2 (1 - \varepsilon_2) S)},$$

then the positive equilibrium point of system (2.2) is locally asymptotically stable.

Proof To prove Theorem 2.1, we have used the Schur-Cohn criteria (see [15]) to obtain conditions for the local asymptotic stability of the positive equilibrium points of system (2.2). We have

$$(i) \quad p_1 = 1 - (a_{11} + a_{22} + a_{33}) - (a_{32}a_{23} + a_{12}a_{21} + a_{31}a_{13} - a_{11}a_{22} - a_{11}a_{13} - a_{22}a_{33}) \\ - (a_{11}a_{22}a_{33} + a_{31}a_{12}a_{23} + a_{13}a_{21}a_{32} - a_{11}a_{32}a_{23} - a_{12}a_{21}a_{33} - a_{31}a_{13}a_{22}) \\ > 0,$$

$$(ii) \quad (-1)^3 p(-1) = 1 + (a_{11} + a_{22} + a_{33}) \\ - (a_{32}a_{23} + a_{12}a_{21} + a_{31}a_{13} - a_{11}a_{22} - a_{11}a_{33} - a_{22}a_{33}) \\ + (a_{11}a_{22}a_{33} + a_{31}a_{12}a_{23} + a_{13}a_{21}a_{32} \\ - a_{11}a_{32}a_{23} - a_{12}a_{21}a_{33} - a_{31}a_{13}a_{22}) \\ > 0,$$

$$(iii) \quad 1 - \left(- (a_{11}a_{22}a_{33} + a_{31}a_{12}a_{23} + a_{13}a_{21}a_{32} - a_{11}a_{32}a_{23} - a_{12}a_{21}a_{33} - a_{31}a_{13}a_{22}) \right)^2 \\ > \left| - (a_{32}a_{23} + a_{12}a_{21} + a_{31}a_{13} - a_{11}a_{22} - a_{11}a_{13} - a_{22}a_{33}) \right. \\ \left. - \left(- (a_{11}a_{22}a_{33} + a_{31}a_{12}a_{23} + a_{13}a_{21}a_{32} - a_{11}a_{32}a_{23} \right. \right. \\ \left. \left. - a_{12}a_{21}a_{33} - a_{31}a_{13}a_{22}) \right) \left(- (a_{11} + a_{22} + a_{33}) \right) \right|.$$

Considering (i) and (ii) together, we get

$$2 - 2(a_{32}a_{23} + a_{12}a_{21} + a_{31}a_{13} - a_{11}a_{22} - a_{11}a_{33} - a_{22}a_{33}) > 0. \tag{2.19}$$

By simplifying (2.17), we obtain

$$a_{32}a_{23} + a_{12}a_{21} + a_{31}a_{13} < 1 + a_{11}a_{22} + a_{11}a_{33} + a_{22}a_{33}. \tag{2.20}$$

Computations for (2.18) lead to the inequality

$$(\alpha_1 (\gamma + \theta + 3\beta_2 \varepsilon_1 S) (-\gamma + \beta_2 (1 - \varepsilon_2) S) - 3\alpha_3 \beta_2^2 (3(1 - \varepsilon_1) + (1 - \varepsilon_2) I_2) \\ - \alpha_2 \beta_2^2 (\varepsilon_2 + 3(1 - \varepsilon_2) I_1)) + (3\alpha_3 \beta_2^2 (3(1 - \varepsilon_1) + (1 - \varepsilon_2) I_2) \\ + \alpha_2 \beta_2^2 (\varepsilon_2 + 3(1 - \varepsilon_2) I_1)) e^{-r_1 U_1} + (-\alpha_1 (\gamma + \theta + 3\beta_2 \varepsilon_1 S) (-\gamma + \beta_2 (1 - \varepsilon_2) S) \\ + 3\alpha_3 \beta_2^2 (3(1 - \varepsilon_1) + (1 - \varepsilon_2) I_2)) e^{-r_2 U_2} + (-\alpha_1 (\gamma + \theta + 3\beta_2 \varepsilon_1 S) (-\gamma + \beta_2 (1 - \varepsilon_2) S) \\ + \alpha_2 \beta_2^2 (\varepsilon_2 + 3(1 - \varepsilon_2) I_1)) e^{-r_3 U_3} \\ + \alpha_1 (\gamma + \theta + 3\beta_2 \varepsilon_1 S) (-\gamma + \beta_2 (1 - \varepsilon_2) S) e^{-r_2 U_2} e^{-r_3 U_3}$$

$$\begin{aligned}
 & + (-3\alpha_3\beta_2^2(3(1-\varepsilon_1) + (1-\varepsilon_2)I_2))e^{-r_1U_1}e^{-r_2U_2} \\
 & + (-\alpha_2\beta_2^2(\varepsilon_2 + 3(1-\varepsilon_2)I_1))e^{-r_1U_1}e^{-r_3U_3} \\
 & < \alpha_1\alpha_2\alpha_3 - \alpha_1\alpha_3\theta e^{-r_1U_1} - \alpha_1\alpha_3\theta e^{-r_3U_3} + \alpha_1\alpha_3(\theta + \alpha_2)e^{-r_2U_2}e^{-r_3U_3} \\
 & + \alpha_1\alpha_3(\theta + \alpha_2)e^{-r_1U_1}e^{-r_2U_2} + \alpha_1\alpha_2\alpha_3e^{-r_1U_1}e^{-r_3U_3}, \tag{2.21}
 \end{aligned}$$

where we will have

$$\alpha_1\alpha_2\alpha_3 + \alpha_2\beta_2^2(\varepsilon_1 + 3(1-\varepsilon_1)I_1) > 0. \tag{2.22}$$

Since $\varepsilon_1 < 1$, we will also have

$$\alpha_1\alpha_3(\theta + \alpha_2) + 3\alpha_3\beta_2^2(3(1-\varepsilon_1) + (1-\varepsilon_1)I_2) > 0. \tag{2.23}$$

Furthermore, the inequality

$$(\gamma + \theta + 3\beta_2\varepsilon_1S)(-\gamma + \beta_2(1-\varepsilon_1)S) < (\theta + \alpha_2)\alpha_3 \tag{2.24}$$

holds for

$$S < \frac{\alpha_2 - \gamma}{3\beta_2\varepsilon_1} < \frac{\alpha_3 + \gamma}{\beta_2(1-\varepsilon_1)}, \tag{2.25}$$

where $3\alpha_3 > \alpha_2$. By considering both (2.9) and (2.25), we get

$$p_1 < \frac{(\alpha_2 + 2\theta - 2\gamma + 3\alpha_3)(\alpha_1(1-\varepsilon_1)(\alpha_2 - \gamma) + 3\alpha_1\varepsilon_1(\alpha_3 + \gamma) - 3\varepsilon_1\beta_2^2(3-4\varepsilon_1))}{3\beta_2\varepsilon_1((1-\varepsilon_1)(\alpha_2 + 2\theta - 2\gamma + 3\alpha_3) - 3(\alpha_3 - \gamma))}. \tag{2.26}$$

Taking in view (2.12) and (2.26), we obtain

$$\begin{aligned}
 & \frac{(\alpha_2 + 2\theta - 2\gamma + 3\alpha_3)(\alpha_1(\alpha_3 - \gamma) - \beta_2^2(3-4\varepsilon_1))}{\beta_2((1-\varepsilon_1)(\alpha_2 + 2\theta - 2\gamma + 3\alpha_3) - 3(\alpha_3 - \gamma))} \\
 & < p_1 < \frac{(\alpha_2 + 2\theta - 2\gamma + 3\alpha_3)(\alpha_1(1-\varepsilon_1)(\alpha_2 - \gamma) + 3\alpha_1\varepsilon_1(\alpha_3 + \gamma) - 3\varepsilon_1\beta_2^2(3-4\varepsilon_1))}{3\beta_2\varepsilon_1((1-\varepsilon_1)(\alpha_2 + 2\theta - 2\gamma + 3\alpha_3) - 3(\alpha_3 - \gamma))},
 \end{aligned}$$

since $\theta > \gamma$. Additionally, from (2.21), we will have

$$-\alpha_1(\gamma + \theta + 3\beta_2\varepsilon_1S)(-\gamma + \beta_2(1-\varepsilon_2)S) < -3\alpha_3\beta_2^2(3(1-\varepsilon_1) + (1-\varepsilon_2)I_2) \tag{2.27}$$

and

$$-\alpha_1(\gamma + \theta + 3\beta_2\varepsilon_1S)(-\gamma + \beta_2(1-\varepsilon_2)S) < \alpha_2\beta_2(\varepsilon_2 + 3(1-\varepsilon_2)I_1) - \alpha_1\alpha_3\theta. \tag{2.28}$$

Taking in view (2.27) and (2.28), we get

$$3\alpha_3\beta_2^2(3(1-\varepsilon_1) + (1-\varepsilon_2)I_2) > \alpha_2\beta_2^2(\varepsilon_2 + 3(1-\varepsilon_2)I_1) + \theta\alpha_1\alpha_3. \tag{2.29}$$

From (2.29), we get

$$\beta_2^2(9\alpha_3(1 - \varepsilon_1) - \alpha_2\varepsilon_1) + 3\beta_2^2(1 - \varepsilon_1)(\alpha_2I_1 - \alpha_3I_2) - \theta\alpha_1\alpha_3, \tag{2.30}$$

where $3\alpha_3 > \alpha_2$ and

$$\beta_2 > \sqrt{\frac{\theta\alpha_1\alpha_3}{3(1 - \varepsilon_1)(\alpha_2I_1 - \alpha_3I_2)}}. \tag{2.31}$$

Finally, from (2.21) we consider

$$\begin{aligned} &(\alpha_1(\gamma + \theta + 3\beta_2\varepsilon_1S)(-\gamma + \beta_2(1 - \varepsilon_2)S) - 3\alpha_3\beta_2^2(3(1 - \varepsilon_1) + (1 - \varepsilon_2)I_2) \\ &\quad - \alpha_2\beta_2^2(\varepsilon_2 + 3(1 - \varepsilon_2)I_1)) + (3\alpha_3\beta_2^2(3(1 - \varepsilon_1) + (1 - \varepsilon_2)I_2) \\ &\quad + \alpha_2\beta_2^2(\varepsilon_2 + 3(1 - \varepsilon_2)I_1))e^{-r_1U_1} \\ &< \alpha_1\alpha_2\alpha_3 - \alpha_1\alpha_3\theta e^{-r_1U_1}, \end{aligned} \tag{2.32}$$

where we obtain

$$\overline{U}_1 > \frac{1}{r_1} \ln(K), \tag{2.33}$$

where

$$K = \frac{\alpha_2\beta_2^2(\varepsilon_2 + 3(1 - \varepsilon_2)I_1) + \alpha_1\alpha_3\theta}{\alpha_1\alpha_2\alpha_3 + \beta_2\alpha_2(\varepsilon_2 + 3(1 - \varepsilon_2)I_1) + 3\beta_2^2\alpha_3(3(1 - \varepsilon_1) + (1 - \varepsilon_2)I_2) - \alpha_1(\gamma + \theta + 3\beta_2\varepsilon_1S)(-\gamma + \beta_2(1 - \varepsilon_2)S)}.$$

Considering the condition (iii), we investigate

$$1 - c_1^2 > |-c_2 - c_1c_3|, \tag{2.34}$$

where

$$\begin{aligned} c_1 &= a_{11}a_{22}a_{33} + a_{31}a_{12}a_{23} + a_{13}a_{21}a_{32} - a_{11}a_{32}a_{23} - a_{12}a_{21}a_{33} - a_{31}a_{13} + a_{22}, \\ c_2 &= a_{32}a_{23} + a_{12}a_{21} + a_{31}a_{13} - a_{11}a_{22} - a_{11}a_{33} - a_{22}a_{33}, \\ c_3 &= a_{11} + a_{22} + a_{33}; \end{aligned}$$

(2.34) can be written as

$$c_1(c_1 - c_3) - c_2 < c_1(c_1 + c_3) + c_2 < 1. \tag{2.35}$$

In this case, we must only show the condition for the inequality

$$c_1(c_1 + c_3) + c_2 < 0 + 1, \tag{2.36}$$

where $c_1 < 0$. For

$$c_2 < 1 \tag{2.37}$$

we must show

$$a_{32}a_{23} + a_{12}a_{21} + a_{31}a_{13} < 1 + a_{11}a_{22} + a_{11}a_{33} + a_{22}a_{33}. \tag{2.38}$$

So we must only investigate the conditions for the inequality

$$a_{11}a_{22}a_{33} + a_{31}a_{12}a_{23} + a_{13}a_{21}a_{32} < a_{11}a_{32}a_{23} + a_{12}a_{21}a_{33} + a_{31}a_{13}a_{22} \tag{2.39}$$

and

$$\begin{aligned} & -a_{11}a_{22}a_{33} - a_{31}a_{12}a_{23} - a_{13}a_{21}a_{32} \\ & < -a_{11}a_{32}a_{23} - a_{12}a_{21}a_{33} - a_{31}a_{13}a_{22} + a_{11} + a_{22} + a_{33}. \end{aligned} \tag{2.40}$$

By adding (2.39) and (2.40) side by side we get

$$a_{11} + a_{22} + a_{33} > 0, \tag{2.41}$$

which holds for

$$\bar{U}_2 < \frac{1}{r_2} \ln \frac{\theta + \alpha_2}{\theta}. \tag{2.42}$$

Furthermore, if the following conditions hold, then the inequality (2.39) is available.

If

$$a_{12}a_{23} < a_{13}a_{22}, \tag{2.43}$$

then we obtain

$$\bar{U}_2 > \frac{1}{r_2} \ln \left(\frac{3(-\gamma + \beta_2 \varepsilon_1 \bar{S}) + \alpha_2 + \theta}{3(-\gamma + \beta_2 \varepsilon_1 \bar{S}) + \theta} \right), \tag{2.44}$$

where $-\gamma + \beta_2 \varepsilon_1 S > 0$.

If

$$a_{13}a_{32} < a_{12}a_{33} \tag{2.45}$$

we have

$$\bar{U}_3 > \frac{1}{r_3} \ln \left(\frac{\gamma + \theta + 3\beta_2 \varepsilon_1 S + 3\alpha_3}{\gamma + \theta + 3\beta_2 \varepsilon_1 S} \right). \tag{2.46}$$

Considering (2.42), (2.44), and (2.46), we obtain

$$\frac{1}{r_2} \ln \left(\frac{3(-\gamma + \beta_2 \varepsilon_1 S) + \alpha_2}{3(-\gamma + \beta_2 \varepsilon_1 S) + \theta} \right) < \bar{U}_2 < \frac{1}{r_2} \ln \frac{\theta + \alpha_2}{\theta}, \tag{2.47}$$

and we get

$$\bar{U}_3 > \frac{1}{r_3} \ln \left(\frac{\gamma + \theta + 3\beta_2 \varepsilon_1 S + 3\alpha_3}{\gamma + \theta + 3\beta_2 \varepsilon_1 S} \right). \tag{2.48}$$

For the inequality

$$a_{22}a_{33} < a_{32}a_{23} \tag{2.49}$$

let us use (2.47) and (2.48). Also, we can write

$$\frac{\theta}{\theta + \alpha_2} < \exp(-r_2 \bar{U}_2) < \frac{3(-\gamma + \beta_2 \varepsilon_1 S) + \theta}{3(-\gamma + \beta_2 \varepsilon_1 S) + \alpha_2 + \theta} \tag{2.50}$$

and

$$\exp(-r_3 \bar{U}_3) < \frac{\gamma + 3\beta_2 \varepsilon_1 S + \theta}{\gamma + 3\beta_2 \varepsilon_1 S + 3\alpha_3 + \theta}. \tag{2.51}$$

Using (2.50) and (2.51) in the following, we obtain

$$\begin{aligned} a_{22}a_{33} &= \left(\frac{(\theta + \alpha_2) \exp(-r_2 \bar{U}_2) - \theta}{\alpha_2} \right) \exp(-r_3 \bar{U}_3) \\ &< \left(\frac{(\theta + \alpha_2) \frac{3(-\gamma + \beta_2 \varepsilon_1 S) + \theta}{3(-\gamma + \beta_2 \varepsilon_1 S) + \alpha_2 + \theta} - \theta}{\alpha_2} \right) \left(\frac{\gamma + 3\beta_2 \varepsilon_1 S + \theta}{\gamma + 3\beta_2 \varepsilon_1 S + \theta + 3\alpha_3} \right) \\ &= \frac{3(\gamma + 3\beta_2 \varepsilon_1 S + \theta)(-\gamma + \beta_2 \varepsilon_1 S)}{(3(-\gamma + \beta_2 \varepsilon_1 S) + \alpha_2 + \theta)(\gamma + 3\beta_2 \varepsilon_1 S + \theta + 3\alpha_3)} \\ &= \left(\frac{(\gamma + \theta + 3\beta_2 \varepsilon_1 S) \left(1 - \frac{\theta + 3(-\gamma + \beta_2 \varepsilon_1 S)}{3(-\gamma + \beta_2 \varepsilon_1 S) + \alpha_2 + \theta}\right)}{\alpha_3} \right) \left(\frac{(-\gamma + \beta_2 \varepsilon_1 S) \left(1 - \frac{\gamma + \theta + 3\beta_2 \varepsilon_1 S}{\gamma + \theta + 3\beta_2 \varepsilon_1 S + 3\alpha_3}\right)}{\alpha_2} \right) \\ &< \frac{(\gamma + \theta + 3\beta_2 \varepsilon_1 S)(1 - \exp(-r_3 \bar{U}_3))}{\alpha_3} \left(\frac{(-\gamma + \beta_2(1 - \varepsilon_1)S)(1 - \exp(-r_2 \bar{U}_2))}{\alpha_2} \right) \\ &= a_{32}a_{23}. \end{aligned}$$

This completes the proof. □

Theorem 2.2 *Let $\{S(n), I_1(n), I_2(n)\}_{n=0}^\infty$ be a positive solution of system (2.2). The following statements are true.*

(i) *If*

$$\begin{cases} p_1 - \beta_1 I_1(n) - \beta_2 I_2(n) - \alpha_1 S(n) > 0, \\ 1 - I_1(n)(\alpha_2 + \theta) - \gamma I_2(n) \beta_1 (1 - \varepsilon_1) S(n) \beta_2 (1 - \varepsilon_2) S(n) I_2(n) > 0, \\ 1 - \alpha_3 I_2(n) + \beta_2 \varepsilon_2 + (\gamma + \theta) I_1(n) + \beta_1 \varepsilon_1 S(n) I_1(n) > 0, \end{cases} \tag{2.52}$$

then the solution of system (2.2) increases monotonically.

(ii) *If*

$$\begin{cases} p_1 > p_1 - \beta_1 I_1(n) - \beta_2 I_2(n) > \alpha_1 S(n), \\ 1 > 1 + \beta_1 (1 - \varepsilon_1) S(n) - \theta I_1(n) - \gamma I_2(n) + \beta_2 (1 - \varepsilon_2) S(n) I_2(n) \\ > \alpha_2 I_1(n), \\ 1 > 1 + \beta_2 \varepsilon_2 S(n) + (\gamma + \theta) I_1(n) + \beta_1 \varepsilon_1 S(n) I_1(n) > \alpha_3 I_2(n), \end{cases} \tag{2.53}$$

then

$$0 < S(n) < \frac{p_1}{\alpha_1}, \quad 0 < I_1(n) < \frac{1}{\alpha_2} \quad \text{and} \quad 0 < I_2(n) < \frac{1}{\alpha_3}. \tag{2.54}$$

Proof The proof will be left to the readers. □

Theorem 2.3 *Let system (2.2) be written as follows:*

$$F(S(t), I_1(t), I_2(t)) = \begin{cases} S(t + 1) = f(S(t), I_1(t), I_2(t)), \\ I_1(t + 1) = g(S(t), I_1(t), I_2(t)), \\ I_2(t + 1) = h(S(t), I_1(t), I_2(t)), \end{cases} \tag{2.55}$$

where the first order partial derivatives of the functions $f, g,$ and h with regard to $S(t), I_1(t), I_2(t)$ are continuous in $I \subset R^+$ and $f, g, h : V \subset R^{+3} \rightarrow I \subset R^+.$ If

$$p_1 - \beta_1 I_1 - \beta_2 I_2 < \frac{2\alpha_1}{3\beta_2 r_1} \tag{2.56}$$

and

$$1 + \beta_1(1 - \varepsilon_1)S(n) - \gamma I_2(n) - \theta I_1(n) + \beta_2(1 - \varepsilon_2)S(n)I_2(n) < \frac{2\alpha_2}{r_2\theta}, \tag{2.57}$$

then (2.55) has no 2-cycle in $I,$ where $\alpha_1 > 3\beta_2, \alpha_2 > \theta.$

Proof Let the initial values be as follows:

$$\begin{cases} x(0) = (S(0), I_1(0), I_2(0)), x(1) = (S(1), I_1(1), I_2(1)) \in V, \\ y(0) = (S(0), I_1(0), I_2(0)), y(1) = (S(1), I_1(1), I_2(1)) \in V, \\ z(0) = (S(0), I_1(0), I_2(0)), z(1) = (S(1), I_1(1), I_2(1)) \in V, \end{cases} \tag{2.58}$$

$$\begin{cases} f(f(S(0), I_1(0), I_2(0))) = f(x(1)) = x(0), \\ g(g(S(0), I_1(0), I_2(0))) = g(y(1)) = y(0), \\ h(h(S(0), I_1(0), I_2(0))) = h(z(1)) = x(0). \end{cases} \tag{2.59}$$

In this case, we must have

$$\begin{aligned} \int_{x(0)}^{x(1)} \left(1 + \frac{\partial f}{\partial S}\right) dS &\neq 0, & \int_{x(0)}^{x(1)} \left(1 + \frac{\partial f}{\partial I_1}\right) dI_1 &\neq 0, \\ \int_{x(0)}^{x(1)} \left(1 + \frac{\partial f}{\partial I_2}\right) dI_2 &\neq 0, \end{aligned} \tag{2.60}$$

to find that $S(t + 1) = f(S(t), I_1(t), I_2(t))$ has no 2-cycle in $I.$ In view of Theorem 2.2, the following results can be obtained.

(1) $f(S(t), I_1(t), I_2(t))$ has no cycle in $I,$ since for $\alpha_1 > 3\beta_2$ we obtain

$$1 + \frac{\partial f(x)}{\partial S} > 0, \quad 1 + \frac{\partial f(x)}{\partial I_1} > 0, \quad 1 + \frac{\partial f(x)}{\partial I_2} > 0, \tag{2.61}$$

where

$$p_1 - \beta_1 I_1 - \beta_2 I_2 < \frac{2\alpha_1}{3\beta_2 r_1} < \frac{2\alpha_1}{\beta_2 r_1}. \tag{2.62}$$

(2) Similarly, $I_1(t + 1) = g(S(t), I_1(t), I_2(t))$ has no cycle in I , since for $\alpha_2 > \theta$, we get

$$1 + \frac{\partial g(x)}{\partial S} > 0, \quad 1 + \frac{\partial g(x)}{\partial I_1} > 0, \quad 1 + \frac{\partial g(x)}{\partial I_2} > 0, \tag{2.63}$$

where

$$1 + \beta_1(1 - \varepsilon_1)S(n) - \gamma I_2(n) - \theta I_1(n) + \beta_2(1 - \varepsilon_2)S(n)I_2(n) < \frac{2\alpha_2}{r_2 \theta}. \tag{2.64}$$

(3) Finally, $I_2(t + 1) = h(S(t), I_1(t), I_2(t))$ has no cycle in I , since (2.62) and (2.64) hold, and we have

$$1 + \frac{\partial h(x)}{\partial S} > 0, \quad 1 + \frac{\partial h(x)}{\partial I_1} > 0, \quad 1 + \frac{\partial h(x)}{\partial I_2} > 0. \tag{2.65}$$

This completes the proof. □

Some additional assumptions of (2.55) are needed to verify global asymptotic stability for the positive equilibrium point $\bar{x} = (\bar{S}, \bar{I}_1, \bar{I}_2)$.

Let f, g, h be continuous functions and

- i. $f : A \times B \times C \rightarrow A$,
- ii. $g : A \times B \times C \rightarrow B$,
- iii. $h : A \times B \times C \rightarrow C$.

Corollary 2.1 *Let us have a positive equilibrium point of system (2.2). If Theorems 2.1-2.3 hold, then the positive equilibrium point of system (2.2) is globally asymptotically stable.*

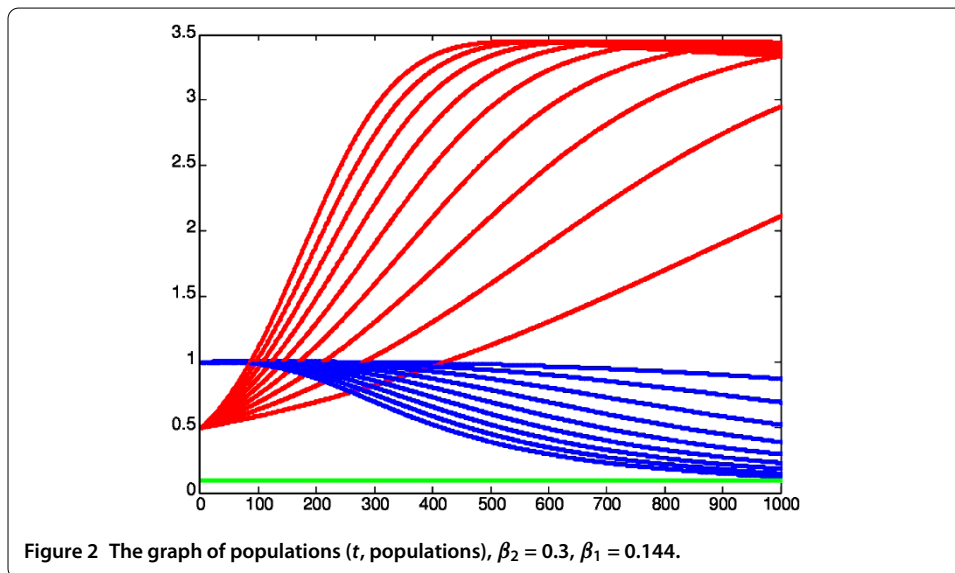
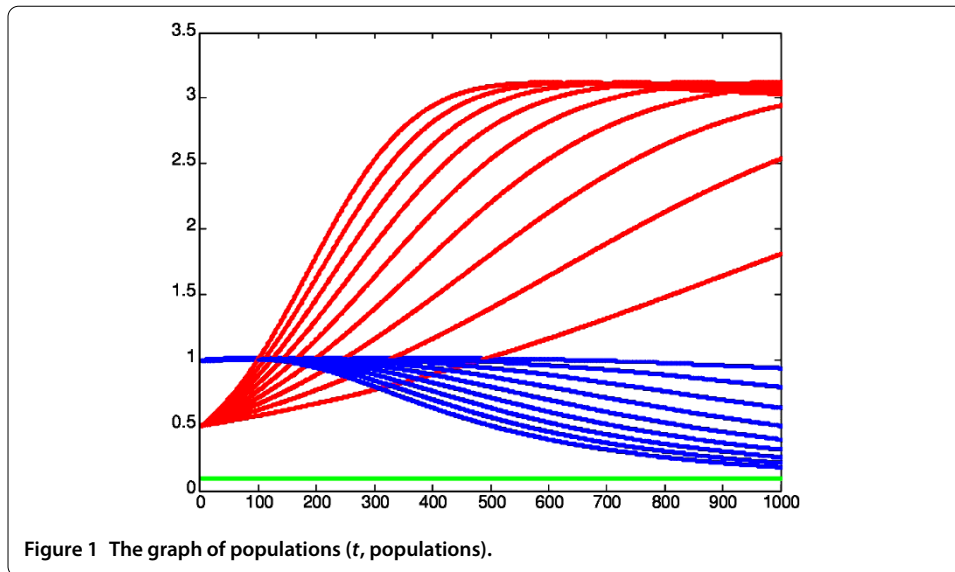
Example

We considered the health system in India in view of the information of [2]. The unit of parameters is in units of per year:

$$\begin{aligned} p_1 &= 2,000, & \gamma &= 0.00001, & \varepsilon_1 &= \varepsilon_2 = 0.01, \\ \theta &= 0.015, & \alpha_1 &= 0.0748, & r_1 &\in [0.001, 0.01], \\ \alpha_2 &= 0.0919, & r_2 &= r_1, & \alpha_3 &= 0.092, & r_3 &= 0.001r_1. \end{aligned}$$

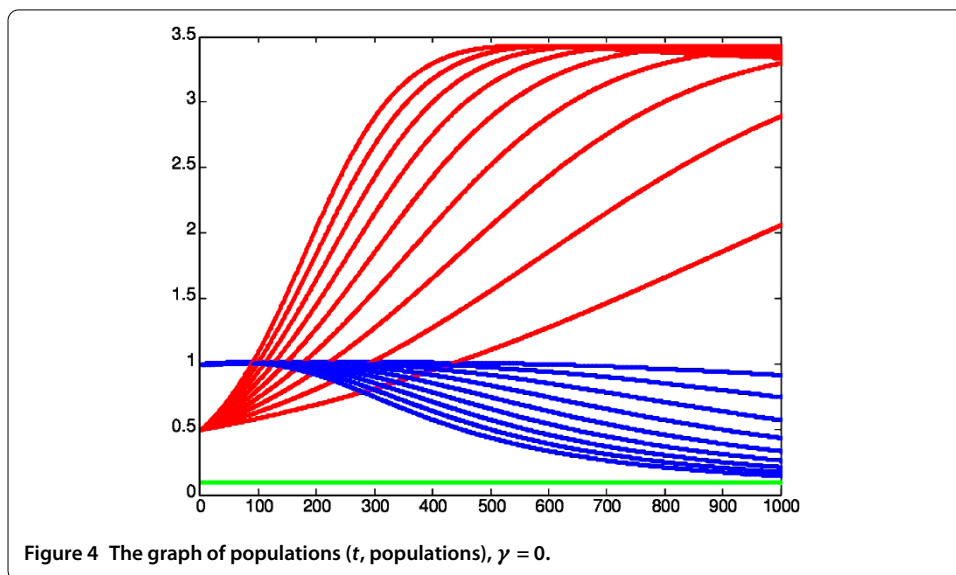
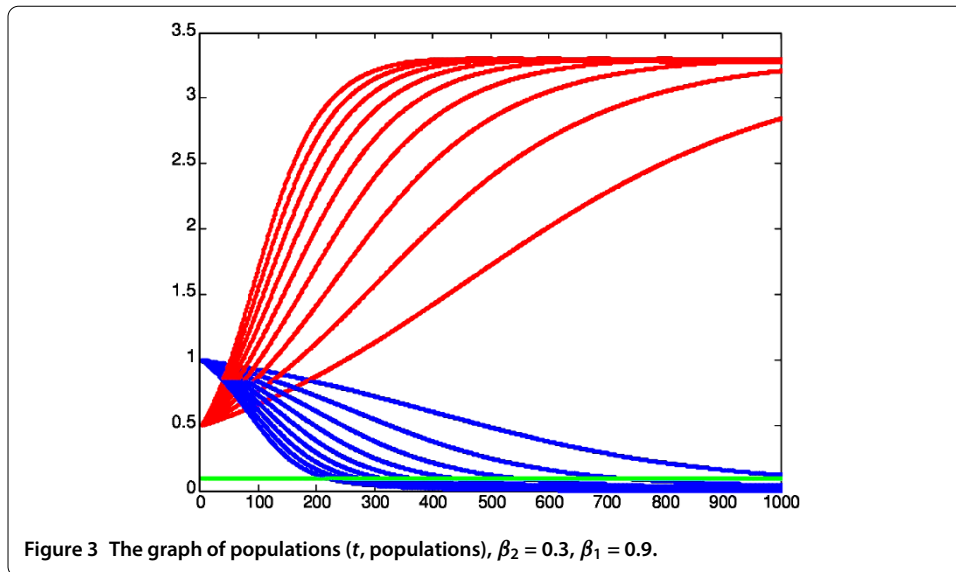
Considering the condition $\beta_2 < \sqrt{\frac{\alpha_1(\alpha_3 - \gamma)}{3 - 4\varepsilon_1}}$, we obtain $\beta_2 = 0.048, \beta_1 = 0.144$. Initial values are selected as $S(0) = 10,000, I_1(0) = 5,000, I_2(0) = 1,000$ in Figure 1.

The health system in India has been considered in view of the information of [2]. Figures 1-8 show the variation of classes for different values of the parameters. In Figures 1-8, blue graph denotes susceptible class ($S(t)$), red graph denotes HIV positives that do not know they are infected ($I_1(t)$), green graph denotes HIV positives that know they are infected. The β_2 value taken in Figure 1 is $\beta_2 = 0.048$. In Figure 2, we have only changed the



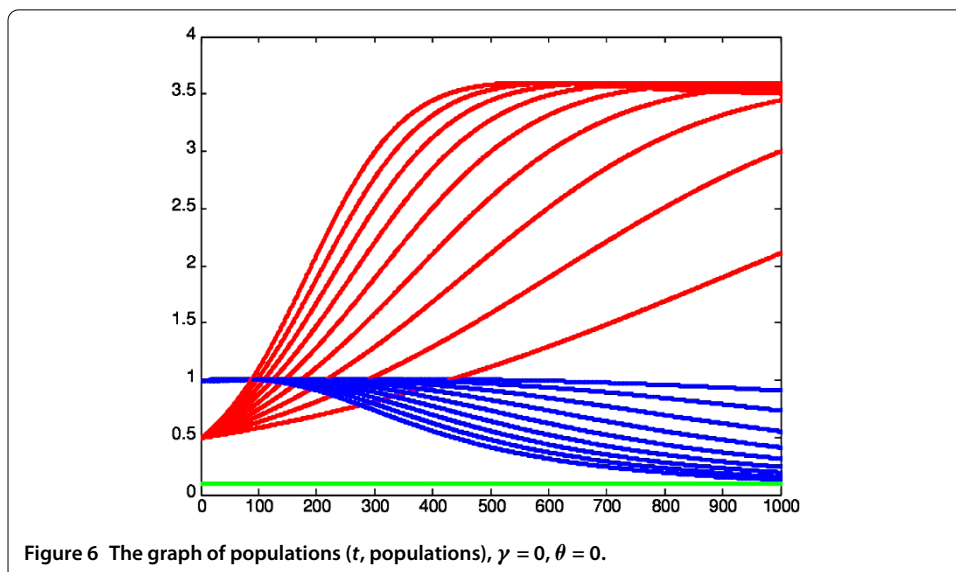
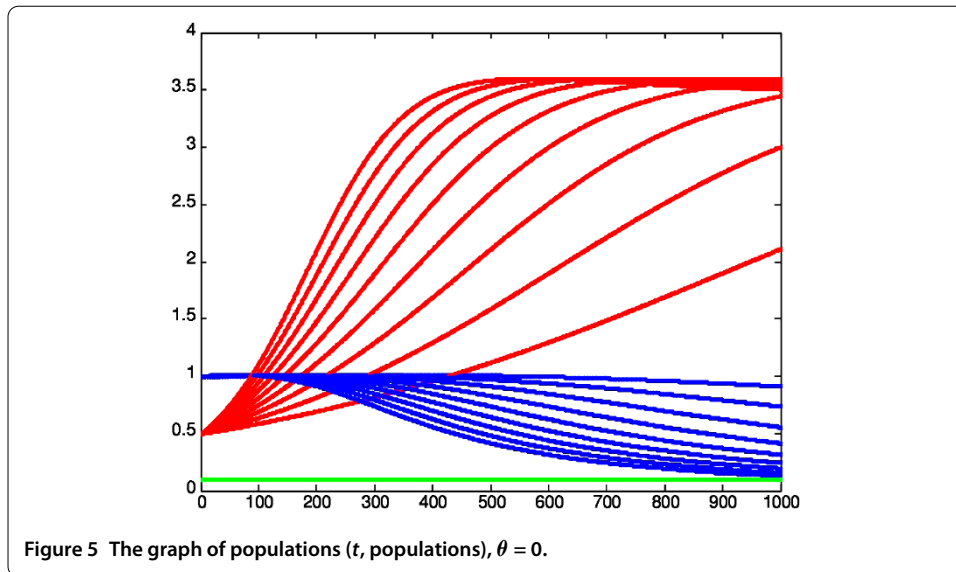
variable $\beta_2 = 0.3$. And also in Figure 3, we have changed $\beta_2 = 0.3$, $\beta_1 = 0.9$. In this case, one can say that β_2 is an important parameter for the spread of the populations over the time t . Considering Figure 1, it is seen that after 75 years an important increase of the I_1 class and decrease of the S class will happen for the selected values in system (2.2). When Figures 1 and 2 are compared, one can see that in the same years the I_1 class increases more in Figure 2 and that the S class decreases more in Figure 2. In Figure 3, for $\beta_2 = 0.3$ and $\beta_1 = 0.9$ after 25 years an important increase of class I_1 and decrease of class S will happen. In that case it is said that as the value of β_2 (per capita contact rate for susceptibles with unaware infectives (I_2)) increases, the I_1 class decreases in earlier years.

In Figures 4-6 the variation of populations for different values of θ and k is shown. It is seen that as infected persons, *i.e.* the unaware HIV infectives become aware about their infection, detect by a rate θ , we may trace out the other infectives which have had the sexual contacts in the past by contact tracing by a rate γ , which results in the decrease of



the number of unaware infectives. It is seen that as the rate of θ and k become zero (see Figure 6) *i.e.* the infectives who do not know that they are infected will continue maintaining sexual relationships in the community which will ultimately increase the infective populations.

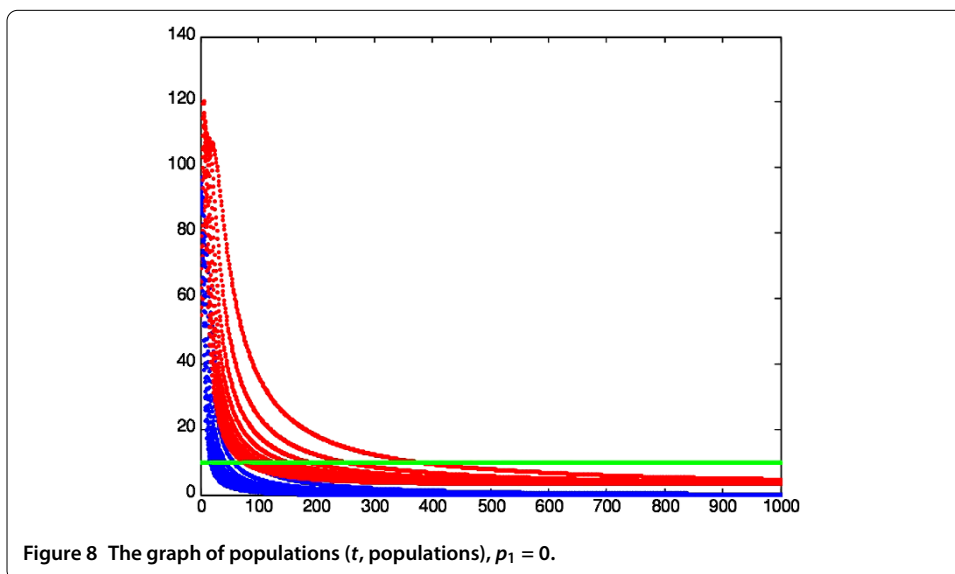
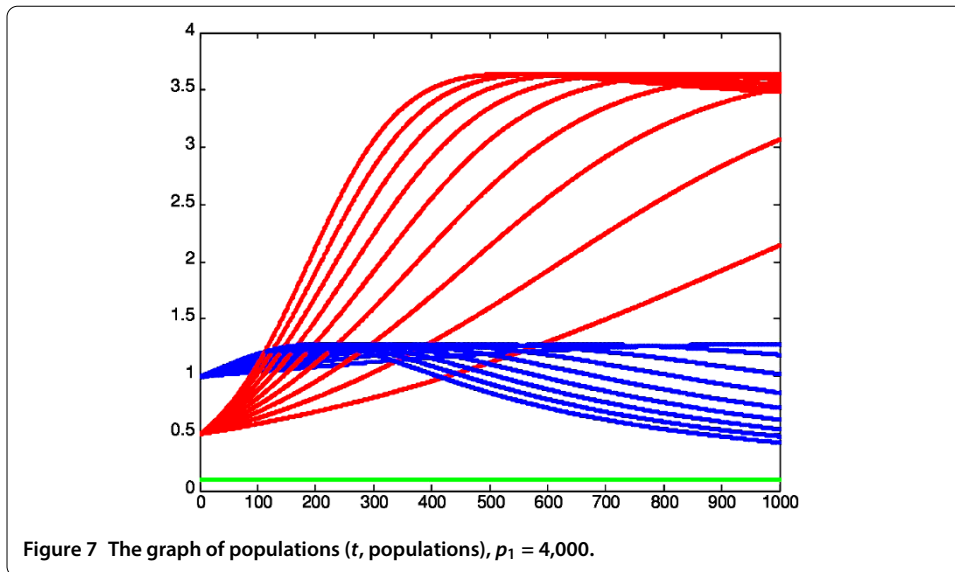
In Figure 1 due to immigration, the susceptible population increases continuously, therefore, infection becomes more endemic and always persists in the population (see also Figure 7). In Figure 8 the distribution of population with time is shown in different classes without immigration. It is seen that in the absence of immigration into the community, the susceptible population decreases continuously as the population is closed, which results in an increase in the infective population first and then it decreases as all infectives will develop AIDS will die out by disease-induced deaths. Thus the total population will be eradicating after some time.



Thus, on changing the behavior and increasing the awareness about the HIV infection in the population the infection can be slowed down and may be kept under control.

Discussion

In this paper, a mathematical model with piecewise constant arguments is proposed to investigate the impact of the parameters (*e.g.* contact tracing, screening) on the spread of HIV in a population with variable size structure. The model is analyzed using Schur-Cohn criteria on difference equations and numerical simulation. The equilibrium point is found to be locally asymptotically stable and globally asymptotically stable under certain conditions. It is observed that if the aware HIV infectives, detected by screening and contact tracing, do not take part in spreading the disease, HIV infection reduces significantly. It is also found that the disease becomes more endemic due to immigration and the endemicity of the disease decreases when the infectives become aware of their infection



after screening and contact tracing and do not take part in sexual interaction, whereas it increases in the absence of contact tracing. In the absence of screening and contact tracing, the infected people continue to spread the disease without taking any precaution due to unawareness of their infection.

Finally from the analysis, it may be speculated that the most effective way to reduce the infection rate and prevalence level is to educate the people about the HIV and make them aware of the consequences of practicing non-safe sex or any other kind of risky behavior. If the population presents a positive attitude toward preventive procedures, the disease may tend to vanish, even for relatively small random screening rates.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

In light of the master's thesis, the authors have had the chance to share this work. All authors have contributed equally. Also they have read and approved the final manuscript.

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